Unit Title: Calculus I

Instructor: Kahenya NP

Trimester: 1st Semester

Time: 2 hours

Instructions:

1) Attempt all questions.
2) Show all your working.
3) Total marks – 50 marks

Section A

1) Determine the gradient of any tangent to curve $f(x) = x^2 + 4x - \sin x$ (1 mk)
2) Determine the point where the following function does not exist; f(x)= $\frac{1-\cos x}{\sin x}$ (1 mk)
3) Is the following state true of false: The limit of the function $f(x) = \frac{5x^3+3x-7}{10x^3-7}$ as x approaches infinity does not exist? Support your answer. (1 mk)
4) Explain when it is essential to use the L'Hopital's rule. (2 mks)
5) Evaluate $\frac{d}{dx}\left(e^{2x^2+x}\right)$ (2 mks)
6) At what point(s) will the function $f(x) = \frac{2x-3}{2x^2+11x-21}$ approaches as x tends 1.5 from both sides. (3 mks)
7) Assume that a bow is shaped as the curve of $f(x) = 3x^2 - 7x - 5$ and that the arrow is to be shot at $\text{at point } x = 1$. If the arrow is perpendicular to the tangent to the bow at this point, determine the gradient of the arrow. (2 mks)
8) Determine whether Rolles' theorem applies to the following function over the given interval. (4 mks)
   i. $f(x) = 5 + 2x - 3x^2, [-\infty, \infty]$
9) The radius of a circle is increased from 5 cm to 5.01 cm. Use differentiation to find the approximate increase in the area. (3 mks)
10) Use the first principle to show that given $f(x) = 3x^2$ $\text{then}\frac{df}{dx} = 6x$. (3 mks)
11) Determine $\frac{dy}{dx}$ given $f(x, y) = 3x^2 - 4x^3y^2 + x\sin y - 2y\cos x$ (3 mks)

Section B

a) Use the definition $\varepsilon - \delta$ definition of a limit l of a function $f(x)$ as x approaches a point a to show that the $\lim_{x\to 2}(3x - 1) = 5$. (3 marks)

b) (i) Determine if the function $f(x) = \begin{cases} 7x - 3 \text{ if } x < 3 \\ 25 - x \text{ if } x \geq 3 \end{cases}$ is continuous at $x = 3$ (3 marks)

(ii) Illustrate the above functions on a graph (You can also draw manually and embed the graph on the main answer sheet to submit one file). (2 marks)

c) Investigate the turning points for $f(x) = 5x^3 - \frac{1}{2}x^2 - 2x + 3$ (3 marks)

d) If $y = \sqrt{1 + x^2}$ find the approximate change in y if x decreases from 3 to 2.999 (answer in 4 significant figure). (3 marks)

e) Determine at what point(s) the function below fail to exist. (3 marks)

$$f(x) = \frac{19x^3 + 7}{3x^2 - 50x - 437}$$

f) A juakali artisan intends to construct an open box from a rectangular sheet of metal of sides 28 metres by 20 metres. He has four equal square portions removed at the corners and the sides are then turned up to form an open box. Determine its maximum volume. (4 marks)

g) Given the equation $x^3y^2 + 3x^2y^2 + 2x^4 = 3y + 3$ determine the gradient $\frac{dy}{dx}$ of the normal at point $x = y = 1$. (4 marks)

'So will I sing praise unto thy name for ever, that I may daily perform my vows'

Psalms 61: 8 (KJV)