Solutions to Calculus 1_Exam

1) Determine the gradient function of the tangent to curve $f(x) = x^2 + 4x - \sin x$ (1 mk)

$$\frac{dy}{dx} = x + 4 - \cos x$$

2) Determine the point where the following function does not exist; f(x)= $\frac{1-\cos x}{\sin x}$ (1 mk)

At x = 0

3) Find the limit of $\frac{5x^3+3x-7}{10x^3-7}$ as x approaches infinity. (1 mk)

$$\lim_{x\to\infty}\left(\frac{5+\frac{3}{x^2}-\frac{7}{x^3}}{10-\frac{7}{x^3}}\right) = \frac{1}{2}$$

4) Explain when it is essential to use the L'Hopital's rule. (2 mks)

Given $f(x) = \frac{g(x)}{h(x)}$ and that $\lim_{x\to a} f(x) = \frac{\pm\infty}{\pm\infty}$ or $\frac{0}{0}$ i.e. indeterminate form

5) Evaluate $\frac{d}{dx}\left(e^{2x^2+x}\right)$ (2 mks)

Let $2x^2 + x = u \Rightarrow du = (4x+1)dx$

Thus $f(x) = e^u \Rightarrow \frac{df}{dx} = e^u$

Hence;

$$\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx} = e^u \cdot (4x+1) = (4x+1)e^{2x^2+x}$$

6) Work out the limit of $f(x) = \frac{2x-3}{2x^2+11x-21}$ as x approaches 1.5 from both sides. (3 mks)

$$\lim_{x\to\frac{3}{2}}\left(\frac{2x-3}{2x^2+11x-21}\right) = \lim_{x\to\frac{3}{2}}\left(\frac{(2x-3)}{(2x-3)(x+7)}\right) = \lim_{x\to\frac{3}{2}}\left(\frac{1}{x+7}\right) = \frac{2}{17}$$

7) Assume that a bow is shaped as the curve of $f(x) = 3x^2 - 7x - 5$ and that the arrow is to be shot at at point $x = 1$. If the arrow is perpendicular to the tangent to the bow at this point, determine the gradient of the arrow. (2 mks)

$$\frac{dy}{dx} = 6x - 7 \Rightarrow \frac{dy}{dx} = -1$$

$\Rightarrow$ gradient of the normal is 1

8) Determine whether Rolles' theorem applies to the following function over the given interval. (4 mks)

$f(x) = 5 + 2x - 3x^2, [-\infty, \infty]$

$$\frac{dy}{dx} = 2 - 6x$$

$$2 - 6c = 0 \therefore c = \frac{1}{3}$$

9) The radius of a circle is increased from 5 cm to 5.01 cm. Use differentiation to find the approximate increase in the area. (3 mks)

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\delta r = 0.01 \text{ when } r = 5.0$$

$$\frac{\delta A}{\delta r} \approx \frac{dA}{dr}$$

$$\delta A \approx \frac{dA}{dr} \cdot \delta r = 2\pi r \cdot \delta r = 2\pi \cdot 5 \cdot 0.1 = \pi$$

% Increase in area is

$$\frac{\pi}{\pi r^2} \times 100 = \frac{100}{25} = 4\%$$

10) Use the first principle to show that given $f(x) = 3x^2$ then $\frac{df}{dx} = 6x$. (3 mks)

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left(\frac{3(x+h)^2 - 3x^2}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{6xh + 3h^2}{h} \right) = \lim_{h \to 0} (6x + 3h) = 6x \ \square$$

11) Determine $\frac{dy}{dx}$ given $f(x, y) = 3x^2 - 4x^3y^2 + x\sin y - 2y\cos x$ (3 mks)

$$f_x = 6x - 12x^2y^2 + \sin y + 2y\sin x$$

$$f_y = -8x^3y + x\cos y - 2\cos x$$

$$\frac{dy}{dx} = \frac{12x^2y^2 - 6x - \sin y - 2y\sin x}{(x\cos y - 2\cos x - 8x^3y)}$$