Course Title

Engineering Economic Analysis

Chapter 2

Interest and Time Value of Money

Lecture 2 (week 2)

Time value of Money, Interest, Single cash flow and Nominal and Effective Interest

Associate Prof. Er. Ishwar Adhikari

Learning Objective

From studying this chapter the students will be able to understand on the topics:

- The concept of time value of money.
- Simple and Compound Interest.
- The formula required to analyse the single cash flow.
- Economic Equivalence
- Nominal and Effective interest rate.

2.1 Time Value of Money

Money has two types of power:

(a) Earning Power and (b) Purchasing Power

Earning power of money is concerned with the ability of a business to earn a profit on invested capital after paying owners and employees, servicing obligations, and fully recognizing its costs while following good accounting practices. [1]

Purchasing Power is concerned with the ability of the money to buy certain commodity. Inflation and Deflation is considered for the analysis.

Time value of money is concerned with earning potential of the money. The simple concept of time value of money is that the value of money received today is more than the value of same money received after a certain period. The change in the amount of money over a given time period is called the time value of money, which is the important concept in engineering economy. [2].This ability of money to change its value over time is due to the interest.

In another words, time value of money can be defined as the relationship between interest and time. The value of money can be analysed either on a given future date or the present date. When the value is calculated at the future date, it is called 'future value' of money. Similarly, when the value is calculated on a present date, it is called the 'present value' of money. The calculation on the future date is called the 'compounding process' and calculation on the present date is called the 'discounting process'.

2.2 Interest

Interest is the manifestation of the time value of money, and is essentially represents 'rent' paid for use of money. In simple terms interest is the fee that is charged for the use of someone else's money. The size of the fee will depend upon the total amount of money borrowed and the length of time over which it is borrowed. [3] Computationally, interest is the difference between the ending amount and beginning amount of money. There is always two perspective to an amount of interest – interest paid and interest earned. [2].

Interest = End amount – Original amount

When the interest over a specific time unit is expressed as a percentage of the original amount (principal), the result is called the interest rate.

$$\textbf{Interest Rate} = \frac{\textbf{(interest accrued per time unit)}}{\textbf{Original amount}} \times 100\%$$

<u>Elements of transaction involving interest</u>

1. An initial amount money including loan or investment is called *principal*.
2. The rate expressed in a percentage per period of time is called *interest rate*
3. A period of time which determines the frequency of interest is called the *interest period*.

4. A specified length of time that marks the duration of the transaction and thereby establishes a certain *number of interest period*. [4]

5. The money that results from the cumulative effects of the interest rate is called *Future amount*.

The elements of interest is shown in figure 2.1.

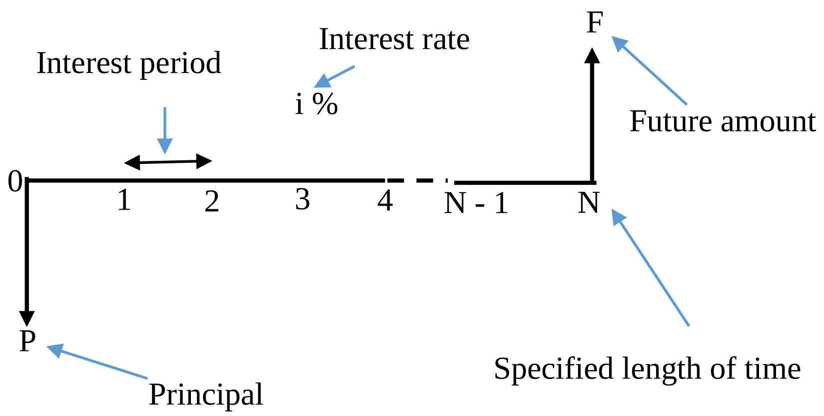


Fig 2.1: Elements of Interest

Calculation of Interest

Money earns the interest in different ways. The interest earned on the principal amount at the end of interest period is calculated according to the interest rate. [4] The two types of techniques are used for the calculation of interest:

1. Simple Interest and
2. Compound Interest

However compound interest is used widely and exclusively.

Simple Interest

When the money is borrowed for certain period of time, the borrower pays interest to the lender, only on the borrowed amount is the simple interest. [5] It uses fixed percentage of the principal (the amount of money borrowed) i.e. if the total amount of interest earned is directly proportional to the initial principal amount, then the interest is said to be simple. It does not earn additional interest on accumulated interest, so it ignores the effect of compounding.

Let,
P = Principal amount (borrowed amount)
i = Interest rate per period of time
N = Specified length of time
The simple interest (I) is calculated as:

$$\mathbf{I = P * N * i}$$

The Future sum of value at the end of interest period (N) will be

$$F = P + I$$
$$F = P + P * N * i$$
$$\mathbf{F = P\ (1 + Ni)}$$

Example 1

Suppose that $ 1000 is being deposited into the saving account which earns 7% simple interest. What is the accumulated value at the end of 2^{nd} year?

Here, principal amount (P) = $ 1000

Interest rate (i%) = 7%

Time (N) = 2 year

Simple Interest = P * N * i = 1000*0.07 *2

= $140

Future Value (F) = 1000 +140 = $1140.

Compound Interest

When the total time period is subdivided into several interest periods (one year, half yearly, quarterly, monthly etc.), interest is credited at the end of each interest period. The credited interest is allowed to accumulate from one interest period to next, then the interest is said to be compound interest. [4]

Suppose that if you invest $ 1000 for 2 years at an interest rate at 10% for 2 years then: The interest earned in first year is $1000*0.1 = $ 100 and the future value after one year is $1000+100 = $ 1100. In the second year interest is charged for $ 1100 and the future value will be $1100 +$1100*0.1 = $1210.

2.3 Single Cash Flow formula

Single cash flow involves the equivalence of a single present amount and its future worth. The single cash flow formulas deal with the only two amounts; a single present amount P and its future worth F as shown in figure 2.2

To find the single future value (F) of Initial payment (P)

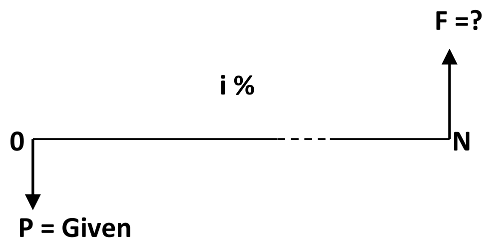


Fig 2.2: Single Cash Flow

For the 1st interest period,

Interest I_1= i*P

Total accumulated amount at the end of 1st year

F_1= P + I_1 = P + i*P = P (1+i)

For the 2nd interest period

Interest I_2= i* F_1 = i* (1+i)

Total accumulated amount at the end of 2nd year

$F_2 = F_1 + I_2 = P (1+i) + i * (1+i) P = P (1+i)^2$

For the 3rd interest period

$I_3 = F_2 * i = P(1+i)^2 * i$
Total accumulated amount at the end of 2nd year
$F_3 = F_2 + I_3 = P(1+i)^2 + P(1+i)^2 * i = P(1+i)^3$
If we continue compounding until Nth interest period we get:

$$\mathbf{F = P\{(1+i)^N\}}$$

The factor in the bracket is called the ***Single Payment compound amount factor***

Functionally, $\boldsymbol{F = P(F/P, i\%, N)}$

<u>To find the initial payment (P) of the single future sum (F)</u>

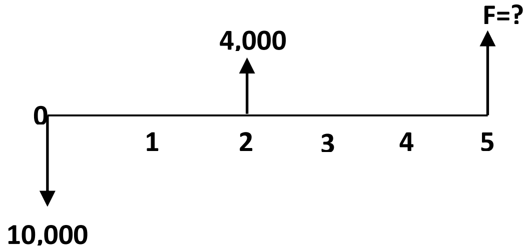
$$\mathbf{F = P\{(1+i)^{-N}\}}$$

The factor in the bracket is called the ***Single Payment Present/Discount amount factor***

Functionally, $\boldsymbol{P = F\,(P/F, i\%, N)}$

<u>Example 2</u>

Mr. X deposits \$ 10,000 now in a bank which gives 8% interest per year. He draws \$ 4,000 at the end of 2^{nd} year. What will be the remaining amount at the end of 5^{th} year?



At the end of the 2nd year, the accumulated amount will be

10,000 (F/P, 8%, 2) = 10,000 $(1+0.08)^2$ = \$ 11,664

After drawing 4000, the remaining deposit amount at the end of 2nd year will be,

11,664 – 4,000 = \$ 7,664

At the end of the fifth year the total accumulated amount will be

7,664 (F/P, 8%, 3) = 7,664 $(1+0.08)^3$ = \$ 9,654.5 (Ans)

<u>RULE OF 72 (Doubling the Money)</u>

Rule of 72 can determine approximately how long it will take for a sum of money to 'double'. The rule states that "to find the time it takes for the present sum of money to grow by a factor of 2, we divide 72 by the interest rate".

2.4 Economic Equivalence

Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence. The time value of money and the interest rate helps to develop the concept of economic equivalence which means that different sums of money at different times are equal in economic value [2].

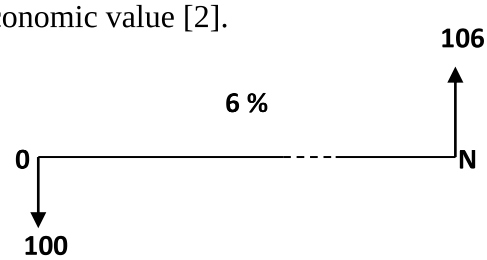


Fig 2.3: Economic Equivalence

For example, if the interest rate 6% per year, $ 100 today is equivalent to $106 one year from today as shown in figure: 2.3. if you receive $ 100 today or $ 106 one year from today, it makes no difference. Both the values are same. If the interest rate is varied, $ 100 is not equivalent to $106 one year from today. Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence. [4]. Economic equivalence exists between cash flows that have same economic effect.

<u>Economic Equivalence Principles</u>

<u>Principle 1:</u> Equivalence calculations made to compare alternatives requires a common time basis.

If we had been given magnitude of each cash flow and had been asked to determine their equivalency, we should choose the reference point and find the value of each cash flow at that point. For selecting the reference point, commonly present time (present worth) or some point in future (future worth) is used. The choice of point is chosen as per convenience.

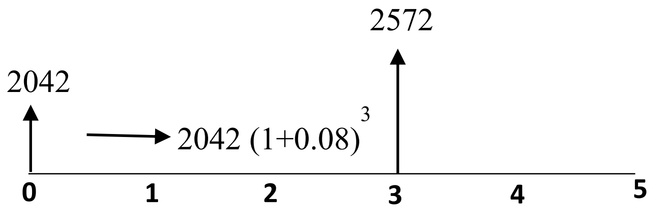


Fig: 2.4 (a)

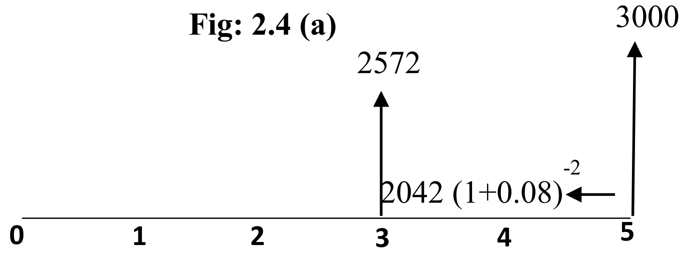


Fig: 2.4 (b)

Figure 2.4: Equivalence in common time basis [4]

At a given interest rate of 8% per year, receiving $ 2042 today is equivalent to receiving $. 3000 in 5 years. These values are also equal at the end of year 3 as shown in figure 2.4 (a) and (b) respectively.

<u>Principle 2:</u> Equivalence depends on interest rate

The equivalence between two cash flows is a function of both the cash flow pattern and the interest rate that operates on those cash flows. Change in the interest rate will destroy the equivalence between these two sums. In previous figure 4, the two cash flows were equal at 8%. If the interest is changed to 10%, the Future sum of $2042 will be $3829 which is greater than $3000

<u>Principle 3:</u> Equivalence Calculations may require the conversion of multiple payment cash flows to a single cash flow.

Let us consider the plan as shown in figure 2.5(a) and (b) as option 1 and option 2 respectively.

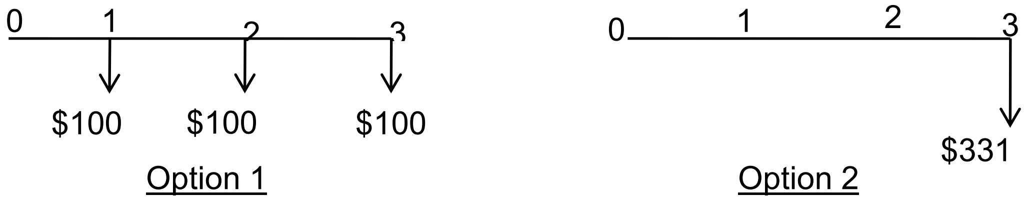


Fig 2.5 (a) **Fig 2.5 (b)**

Figure 2.5: Conversion of multiple payment to single cash flow [4]

Since option 2 is already a single payment at n=3 years, it is simplest to convert option 1 cash flow pattern to a single value at n=3.

We must convert the three disbursement of option 1 to their respective values at n=3.

F_3 for \$100 at n=1: \$100 $(1+0.10)^{3-1}$ = \$121

F_2 for \$100 at n=2: \$100 $(1+0.10)^{3-2}$ = \$110

F_1 for \$100 at n=3: \$100 $(1+0.10)^{3-3}$ = \$100

Total = $ 331

2.5 Nominal and Effective interest rate

If a financial institution uses a unit of time other than a year – a month or a quarter or semiannual etc. (e.g. when calculating interest payments), the institution usually quotes the interest rate on an annual basis.

Commonly this rate is stated as

r% Compounded M-ly

Where, r = the nominal interest rate per year

M = the compounding frequency or the number of interest periods per year

r/M = the interest rate per compounding period.

Suppose that if a bank express the interest rate as “18% compounded monthly”, we say that 18% is the nominal interest rate or annual percentage rate (APR) and the compounding frequency is monthly i.e. number of interest period per year is 12.

A nominal interest rate r may be stated for any time period – 1 year, 6 months, quarter, month, week day etc. Let us see the following examples for expressing the nominal interest rate.

r = 12% compounded semi-annually, M=2,
i.e.12%/2 (6% per 6 months)
r = 12% compounded quarterly, M=4,
i.e. 12%/4 (3% per 3 months)
r = 18% compounded monthly, M=12,
i.e. 18%/12 (1.5% per month)
r = 15%compunded weekly, M= 52
i.e.15%/52 (0.29% per week)

Suppose that $ 1,000 to be invested at a nominal rate of 12% compounded semi-annually.

The interest earned during first six months is 1,000* 0.12/2 = $ 60

Total principal at the end of the first six months = $ (1,000+60) = $ 1,060

Interest earned during the second six months is $ 1,060*0.12/2 = $ 63.60

Total interest at the end of 1 year = $ 60+ $ 63.60 = $ 123.60

The effective annual interest rate for the entire year = 123.60/1,000 * 100 = 12.36%

The *exact or the actual rate* of interest earned on the principal during one year is known as the ***effective interest (i).*** The effective interest rates are always expressed on an annual basis unless specifically stated otherwise.

Difference between Nominal and Effective interest rate

- Nominal interest rate is expressed on an annual basis but the compounding period is less than annual where as effective interest rate is expressed on an annual basis and compound is also on annual basis.
- Nominal interest rate is always less than effective interest rate on an annual basis where as Effective interest rate is always greater than the nominal interest rate.

Relation between effective (i) and nominal (r) interest rate

$\mathbf{i} = (\mathbf{1} + \mathbf{r}/\mathbf{M})^{\mathbf{M}}$-1, M is the compounding period per year.
As from the above example, the effective interest rate for 12% compounded semi-annually,
$i = (1 + r/M)^{M}$-1 = $(1+0.12/2)^{2}$-1 = 12.36%

<u>Example 3</u>

What is the effective interest rate of the nominal interest rate 9% per year if the compounding is a) yearly b) quarterly c) monthly (d) daily?

Solution
For compounding yearly,
$i = (1+0.09/1)^{1} - 1 = 0.09 = 9\%$
For compounding quarterly,
$i = (1+0.09/4)^{4} - 1 = 0.09308 = 9.308\%$
For compounding monthly,
$i = (1+0.09/12)^{12} - 1 = 0.09380 = 9.380\%$
For compounding daily,
$i = (1+0.09/365)^{365} - 1 = 0.0941 = 9.41\%$

Here from the above example, it can be noted that the when the compounding period is lesser, the final interest earned is greater.

Example 4.

A person deposits a sum of $ 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity of the deposit after 10 years.

Given Parameters,
Initial Principal (P) = $ 5,000,
Length of Time (N) = 10 years,
Interest rate (i%) =12%compounded quarterly

As Interest is compounded quarterly, Interest period in a year = 4

Total interest period for 10 years = 40
Interest per period (i%) = 12%/4 = 3%
Using single payment compound amount factor
$F = P\,(F/P, 3\%, 40) = 5{,}000\,(1+0.03)^{40}$
= **$ 16,310.**
Alternatively

$i_{effective} = (1+0.12/4)^{4} - 1 = 12.5508\%$ per year , N =10 years

$F = 5{,}000\,(F/P, 12.55\%, 10) = 5{,}000\,(1+0.125508)^{10}$

= **$ 16,310**.

References:

[1] https://marketbusinessnews.com/financial-glossary/earning-power/ (Viewed September 2022).

[2] *Basics of Engineering Economy:* Leland Blank and Anthony Tarquin, Indian Edition, Tata McGraw Hill Education Private Limited, New Delhi, India, 2013.

[3]*Engineering Economics:* Jose A. Sepulveda, William E. Souder and Byron S. Gottfried, Tata McGraw – Hill Publishing Company Limited, New Delhi, India, 2005.

[4] *Contemporary Engineering Economics*, Chan S. Park, Second Edition, Addison-wesley Publishing Company, 1997.

[5] *Engineering Economics and Costing:* Dr. K.K. Patra & Dhiraj Bhattacharjee, First Edition, S. Chand and Company Ltd, 2013.