## Course Title

## Engineering Economic Analysis

## Chapter 3

## Basic Methodologies of Engineering Economics

# Lecture 4 (Week 4) <br> Minimum Attractive Rate of Return (MARR), Equivalent worth Method and Internal Rate of Return 

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## Learning Objective

From studying this lecture the students will be able to understand on the topics:
$>$ The concept of Minimum Attractive rate of return (MARR)
$>$ The concept of Equivalent worth Method (Present worth, Future worth and Annual worth)
$>$ The Concept of Rate of return method and internal rate of return (IRR) method.

### 4.1 Introduction

If an organization have a huge sum of money in the investment pool and there are many alternatives (projects) whose initial investment cost and annual revenues are known then the organization has to select the best alternative among the different projects. The worthiness of the project has to be determined, whether a proposed project can generate revenue or not to recover the invested capital. [1]. There are various methods of analyzing the worthiness of a project. These methods are:

## 1. Equivalent Worth Method

Present worth method (PW)
Future worth method (FW)
Annual worth method (AW)
Equivalent worth method is the absolute measurement and the outcome value from this method is always in absolute term.
2. Rate of Return Method

Internal Rate of Return (IRR)
External rate of return (ERR)

## 3. Benefit Cost Ratio Method

Conventional B/C
Modified B/C
Rate of Return method and the Benefit Cost ratio method is the relative measurement and the outcome value from these methods are always in relative term.

## 4. Pay Back Period Method

Simple Payback
Discounted Payback
Payback period method is not the decision making tool. This method is used only for the screening of the project and the outcome value from this method is always in years.

### 4.2 Minimum Attractive Rate of Return (MARR)

The minimum attractive rate of return (MARR) is the interest rate at which a firm can always earn or borrow money. [2]It is the rate set by an organization to designate the lowest level return that makes an investment acceptable. [3]MARR is determined from the opportunity cost viewpoint, which results from the capital rationing phenomenon.

Capital rationing refers to the situation where the funds available for capital investment are not sufficient to cover potentially acceptable projects.

Opportunity cost refers to the value of benefit scarified in course of selection of alternative.

It is generally dictated by management considering the following points: [2]
> The amount of money available for investment - Source and cost of these funds (equity, borrowed funds etc.)
> The number of good project available for investment
$>$ The amount of perceived risk associated with the investment
$>$ The type of organization involved (government, public, private)

## Example

Consider the firm has
Available amount $=\$ 60$ million
Seven projects available with total amount $=\$ 75$ million


Fig 3.1: MARR Determination [2]

- Last funded project $=$ Project E
- Prospective rate of profit
- Best rejected project $=F$ (by not being able to invest in project $F$, the firm would presumably be forfeiting the chance to realize on $16 \%$ annual return)
- $\operatorname{MARR}=16 \%$ per year (as the amount of investment capital and opportunities available change over time, the firm's MARR will also change)


### 4.3 Equivalent worth Method

Equivalent worth method is used for determining the worthiness of the investment project in an absolute terms. The cash flow occurred over the analysis period of a project is converted into a single equal value in any point in time that is present, future or annual.

## (a) Present worth method (PW) or Net Present Value (NPV)

The net present worth (NPW) or present worth (PW) or net present value (NPV) of a given series of cash flow is the equivalent values of the cash flows at the end of year zero (i.e. beginning of year 1). [4]In other words, how much money we have to set aside to provide for future cash flow. All cash inflows and outflows are discounted to present point in time at MARR.

Net present worth = Equivalent present worth offuture cash flow - Initial investment

## Here we use NPW or NPV as PW

## Present Worth Analysis (PW)

$>$ Determine the interest rate that the firm wishes to earn on their investment, which is referred as either required rate of return or MARR (minimum attractive rate of return). [5]
$>$ Estimate the service life of the project.
$>$ Estimate the cash inflow and out flow for each service period.
$>$ Determine the net cash flows

$$
=\text { Cash inflow }- \text { cash out flow }
$$

Present worth of each net cash flow as shown in figure 3.2, at MARR as


Fig 3.2: Present worth analysis [5]

$$
\begin{aligned}
\mathbf{P W}(\mathbf{i} \%) & =\mathbf{A}_{\mathbf{1}} /(\mathbf{1} \mathbf{+})^{\mathbf{0}}+\mathbf{A}_{2} /(\mathbf{1} \mathbf{+})^{\mathbf{1}}+\mathbf{A}_{\mathbf{3}} /(\mathbf{1} \mathbf{+})^{\mathbf{3}} \ldots \ldots \mathbf{A}_{\mathbf{N}} /(\mathbf{1} \mathbf{+})^{\mathbf{N}} \\
& =\sum_{n=0}^{N} \frac{A_{n}}{(1+i)^{n}} \\
& =\sum_{n=0}^{N} A_{n}(P / F, i \%, N)
\end{aligned}
$$

$\mathrm{A}_{\mathrm{n}}$ will be +ve if the corresponding period has a net cash inflow and -ve if there is net cash outflow. Positive 'NPW' means the equivalent worth of cash inflows is greater than equivalent worth of cash out flows and vice versa.

## Decision Rule for PW

$$
\begin{array}{ll}
\text { If } P W(i)>0, & \text { accept the investment } \\
\text { If } P W(i)=0, & \text { remain indifferent } \\
\text { If } P W \text { (i) }<0, & \text { reject the investment }
\end{array}
$$

## (b) Future worth method (FW) or Net Future Value (NFV)

Net present worth measures the surplus in an investment project at time zero whereas net future worth measures this surplus at time period other than zero. [5]Net future worth analysis is particularly useful on an investment solution where we need to compute the equivalent worth of a project at the end of investment period rather than its beginning.

$$
\begin{aligned}
\mathrm{FW}(\mathrm{i} \%) & =\mathrm{A}_{\mathrm{o}}(1+\mathrm{i})^{N}+\mathrm{A}_{1}(1+\mathrm{i})^{N-1}+\mathrm{A}_{2}(1+\mathrm{i})^{N-2}+\cdots-\cdots---+\mathrm{A}^{\mathrm{N}} \\
& =\sum_{n=o}^{N} \mathrm{~A}_{\mathrm{n}}(1+\mathrm{i})^{N-n} \\
& =\sum_{n=o}^{N} \mathrm{~A}_{\mathrm{n}}(\mathrm{~F} / \mathrm{P}, \mathrm{i} \%,+\mathrm{N}-\mathrm{n})
\end{aligned}
$$

## Decision Rule for FW

The decision rule is same as PW
If FW (i) $>0$, accept the investment
If $F W$ (i) $=0, \quad$ remain indifferent
If FW (i) $<0$, reject the investment

## (c) Annual worth Method (AW) or Net Annual worth Method (NAW)

Annual worth method provides the basis for measuring investment worth by determining equal payments on an annual basis. [5]The AW of a project is its annual equivalent receipts $(\mathrm{R})$ minus annual equivalent expenses ( E ) minus annual equivalent capital Recovery (CR). R, E, and CR are calculated at MARR (Sullivan)

$$
\mathbf{A W}(\mathbf{i} \%)=\mathbf{R}-\mathbf{E}-\mathbf{C R}
$$

Where,
$\mathrm{R}=$ annual revenues,
$\mathrm{E}=$ annual expenses,
CR = capital recovery
Knowing that any lump sum cash amount can be converted into a series of equal annual payments we may first find the present worth of the original series and then multiply this amount by the capital recovery factor:

$$
\mathrm{AW}(\mathbf{i})=\mathbf{P W}(\mathbf{i})(\mathbf{A} / \mathbf{P}, \mathbf{i}, \mathbf{N})
$$

## Decision rule

The decision rule is same as PW and FW

$$
\begin{array}{ll}
\text { If } A W(i)>0, & \text { accept the investment } \\
\text { If } A W(i)=0, & \text { remain indifferent } \\
\text { If } A W(i)<0, & \text { reject the investment }
\end{array}
$$

## Capital Recovery (CR)

When only cost are involved, the AW method is sometimes called the annual equivalent cost method. In this case, two types of costs are involved i.e. operating and capital cost. Operating cost (labors and raw materials etc.) is recurred over the life of project and they are estimated on annual basis whereas capital costs (purchasing assets or establishing company etc.) tend to be one time cost.
So for the purpose of annual equivalent cost analysis this onetime cost (capital cost) must be translated into its annual equivalent over the life of the project. This annual equivalent of capital cost is given a special name: Capital Recovery cost designated as CR (i). It covers depreciation and interest on invested capital. Two general monetary transactions are associated with the purchase and retirement of capital asset: Initial cost (I) and its Salvage value (S). The Capital Recovery cost can be calculated as follows:


$$
\text { CR }(\mathrm{i} \%)=I(\mathrm{~A} / \mathrm{P}, \mathrm{I} \%, \mathrm{~N})-\mathrm{S}(\mathrm{~A} / \mathrm{F}, \mathrm{i} \%, \mathrm{~N})
$$

### 4.4 Rate of Return Method

A rate of return (RoR) is the net gain or loss of an investment over a specified time period, expressed as a percentage of the investment's initial cost. [6] Rate of Return is the interest rate earned on the unpaid balance of an amortized loan. [5]Equivalent worth method is absolute measurement whereas the rate of return is the relative measurement used for comparing mutually exclusive alternatives.

Suppose that a bank lends $\$ 10,000$ and it is repaid $\$ 4021$ at the end of each year for 3 years.
How can be the interest rate determined that the bank charges on this transaction?

$$
\begin{aligned}
\mathrm{P} \quad & =\mathrm{A}(\mathrm{P} / \mathrm{A}, \mathrm{i} \%, 3) \\
\$ 10,000 & =\$ 4021(\mathrm{P} / \mathrm{A}, \mathrm{i} \%, 3) \\
\mathrm{i} \quad & =10 \%
\end{aligned}
$$

The bank will earn $10 \%$ return annually on its investment of $\$ 10,000$. [5]

## .4.1 Internal Rate of Return (IRR)

$>\operatorname{IRR}$ is that interest rate (return rate) which equates the equivalent worth of an alternative's cash inflows to the equivalent worth of cash outflows. [1]
> IRR is the interest rate that is charged on the un-recovered project balance of the investment such that, when the project terminates, the un-recovered balance will be zero. [5]
$>$ At this particular rate, the equivalent worth of revenue generated by project is enough to bear the equivalent worth of expenses absorbed by project without any financial burden.

## Mathematical relation of IRR

A project's return is referred as internal rate of return (IRR) promised by an investment project over its useful life. (Figure: 3.3)

Based on PW formulation
PW ( $\mathrm{i} * \%$ ) $=0$
PW inflow -PW outflow $=0$
Based on FW formulation
FW ( $\mathrm{i}^{*} \%$ ) $=0$
$\mathrm{FW}_{\text {inflow }}-\mathrm{FW}_{\text {outflow }}=0$
Based on AW formulation


Fig 3.3: Mathematical relation of IRR [2]

AW (i* \%) $=0$
$\mathrm{AW}_{\text {inflow }}-\mathrm{AW}{ }_{\text {oufflow }}=0$

## Method of Calculating IRR

## $>$ Direct Solution Method

For the very special case of a project with only a two-flow transaction (an investment followed by a single future payment) or service life of 2 years of return, we can apply direct mathematical solution for determining the rate of return. [5]

## $>$ Trial and error method

The following steps are followed to calculate the IRR of any project's cash flow:
$>$ Develop an equation for equivalent worth of any point of time indicating rate of interest by $\mathrm{i} * \%$ whose value is to be found.
$>$ Equate the developed equation as to zero.
$>$ Solve it to get the value of $\mathrm{i} * \%$ which would be IRR

## 3. Computer Solution Method

$>$ Click Formula
$>$ Click Auto Sum
$>$ Click More Function
$>$ Type IRR in Search for a Function, Click Ok
$>$ Insert the value of Cash Flow
$>$ Click Enter,
$>$ The result gives the value of IRR.

## Decision Rule of IRR



## Graphical representation of IRR



Fig 3.4: Investment Balance Diagram [2]

## Drawbacks of IRR

> The recovered funds are re-invested at $\mathrm{i} * \%$ rather than MARR, which leads to the concept of External rate of return (ERR).
$>$ It needs trial and error approach for the calculation.
$>$ If the algebraic sum of the cash flow changes in the middle of the project more than two times, we might obtain multiple IRR values.
$>$ When choosing between the mutually exclusive alternatives, IRR method can be misleading and does not compare the scale of investment.

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[6] https://www.investopedia.com/terms/r/rateofreturn.asp (viewed on September 2022)

