

Course Title

Engineering Economic Analysis

Chapter 4

Comparative Analysis of Alternatives

Lecture 6 (week 6)

Comparing mutually exclusive project, Useful Life, Study Period, Comparing Alternatives having same useful lives using Equivalent worth method, rate of return and B/C method

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Learning Objective

From studying this chapter the students will be able to understand on the topics:

- Basic concept of comparing alternatives
- The useful life and study period
- Comparing the alternatives having same useful life using equivalent worth method.
- Comparing the alternatives having same useful life using rate of return and B/C method.

4.1 Introduction

Before this chapter we have used the different methods for analyzing the single project only. Most engineering projects can be accomplished by more than one feasible design alternative. These projects are funded from the capital budget. [1]When the selection of one of these alternatives excludes the choice of any of the others, the alternatives are called mutually exclusive. Typically, the alternatives being considered require the investment of different amounts of capital, and their annual revenues and costs may vary. In this chapter we discuss on the analysis of the two or more than two project. How the decision is made while comparing the alternatives? The method like equivalent worth (PW/FW/AW), Rate of return (IRR/ERR) and B/C method is used for the comparing the alternatives.

4.2 Comparison of Alternatives.

As discussed in principle of engineering economics, the first principle is ‘develop the alternative’. For most of the engineering projects, equipment etc., there are more than one feasible alternative. It is the duty of the project management team (comprising of engineers, designers, project managers etc.) of the client organization to select the best alternative that involves less cost and results more revenue. For this purpose, the economic comparison of the alternatives is made.

The cost and the revenues associated with each alternative is different from each other. So comparing the alternatives with each other becomes inevitable to determine the differential cost. Also, by applying the different methods like equivalent worth, rate of return and B/C method gives the decision to choose among the alternatives.

The problem of deciding which mutually exclusive alternatives should be selected is made easier if we adopt a rule in which:

“The alternative that requires the minimum investment of capital and produces satisfactory functional results will be chosen unless the incremental (additional) capital associated with an alternative having a larger investment can be justified with respect to its incremental benefits”.

[1]

“If the extra benefits obtained by investing additional capital are better than those that could be obtained from investment of the same capital elsewhere in the company at the MARR, the investment should be made”. [1]

4.3 Useful Life and Study and Period

Useful Life

It is the length of time an asset might reasonably be expected to be useful in the production of income [2]. The useful life of an asset is the time period during which it is kept in productive use in a trade or business. The time denoted by the manufacturer or producers of an assets or products.

Study Period

The analysis period is a time span over which the economic effects of an investment will be evaluated. Also called planning horizon or analysis period. [3] The length of the analysis period is determined by company policy, the service period, the useful life of the shorter lived alternative, the useful life of longer lived alternative etc.

The useful lives of alternatives being compared, relative to the selected study period, can involve two situations.

Case 1: Useful lives are the same for all alternatives and equal to the study period.

Case2: Useful lives are different among the alternatives and at least one does not matches the study period.

4.4 Case 1: Useful lives are the same for all alternatives and equal to the study period.

4.4.1 Equivalent worth method

The economic comparison of mutually exclusive alternatives can be carried out by different equivalent worth methods namely present worth method, future worth method and annual worth method. In these methods all the cash flows i.e. cash outflows and cash inflows are converted into equivalent present worth, future worth or annual worth considering the time value of money at a given interest rate per interest period. [4]

When equivalent worth methods are used, consistency of alternative selection results from the equivalency relationship. [1] Also the economic ranking of mutually exclusive alternatives will be the same using the three methods. Consider the general case of two alternatives, A and B.

If

$$PW_A (i \%) < PW_B (i \%)$$

$$PW_A (i \%) \{A/P, i\%, N\} < PW_B (i \%) \{A/P, i\%, N\}$$

$$AW_A (i \%) < AW_B (i \%)$$

Similarly,

$$PW_A (i \%) \{F/P, i\%, N\} < PW_B (i \%) \{F/P, i\%, N\}$$

$$FW_A (i \%) < FW_B (i \%)$$

Ranking is always consistent in the equivalent method.

Steps for equivalent worth method

- Calculate PW/FW/AW at the MARR for each alternative.
- If PW/FW/AW is greater than zero for all alternatives, then the alternative having the highest PW/FW/AW or least negative PW/FW/AW is selected.

For example

PW_A	PW_B	Selected Alternatives
-\$1500	-\$500	B
-\$500	+\$1000	B
+\$2500	-\$1000	A
+2500	+\$1000	A

4.4.2 Rate of return method & Benefit Cost ratio method

Under Equivalent worth Method, the mutually exclusive project with the highest worth figure was preferred. Unfortunately, the same principle cannot be applied to IRR, ERR and BCR analysis. The project with highest IRR, ERR and BCR may not be the preferred alternative.

Let us consider the two mutually exclusive alternatives with 1 year of service life. (Table 4.1)

N	A₁	A₂
0	-\$1,000	-\$5,000
1	\$2,000	\$7,000
IRR	100%	40%
BCR (10%)	1.82	1.27
PW (10%)	\$ 818	\$ 1364

Table 4.1 Example of ROR and B/C method for comparing alternatives: [3]

We can see that,

A₂ is preferred over A₁ by PW method and

A₁ is preferred over A₂ by IRR and BCR methods.

This inconsistency is because equivalent worth method (AW, PW, FW) are *absolute method* whereas IRR/ERR and B/C is relative (percentage) measure and cannot be applied in the same way. For this purpose *Incremental Analysis* should be done.

Incremental Analysis

Incremental analysis, also called cost approach, marginal analysis and differential analysis, is a comparative decision-making process. [5] Incremental Analysis evaluates difference, or the ‘increment’ between two or more mutually exclusive alternatives. When independent projects are evaluated, no incremental analysis is necessary between projects. Each project is evaluated separately from others, and more than one can be selected.

Steps for Incremental Analysis

1. **Identify** all the alternatives
2. **Compute the IRR/ERR/BCR** of each alternative. Any alternative with $IRR < MARR/ERR < MARR/BCR < 1$ should be rejected.
3. **Order alternatives in increasing order** of investment cost to ensure that the increments have cash flow corresponding to investments.
4. **Establish a base alternative:** Alternative having least capital investment is established as the base alternative and should have been pre-qualified i.e. $IRR > MARR/ERR > MARR/BCR > 1$
5. **Perform an incremental analysis** between the base alternative and the alternative with the next higher initial cost. If the incremental $IRR \geq MARR/ERR \geq MARR/BCR \geq 1$ “reject” the base alternative and “accept” the higher cost alternative and “retain” it as base alternative.
6. **Select the next higher cost alternative** and perform the incremental analysis until all the alternatives have been evaluated.

“Evaluation should always be done based on same study period for all alternatives”

Example (Incremental Analysis)

The cash flows for the two mutually exclusive alternatives are given as follows (Table 4.2)

EOY	A	B
0	- \$ 3,000	- \$ 12,000
1	\$ 1,350	\$ 4,200
2	\$ 1,800	\$ 6,225
3	\$ 1,500	\$ 6,330

Table 4.2 Example of Incremental analysis: [3]

Which project would be selected based on IRR criterion, at $MARR = 10\%$

Calculating the IRR of each alternative.

$IRR_A = 25\% > 10\%$ (MARR) (Justified)

$IRR_B = 17.43\% > 10\%$ (MARR) **(Justified)**

EOY	A	B	B – A
0	- \$ 3,000	- \$12,000	- \$9,000
1	\$ 1,350	\$ 4,200	\$ 2,850
2	\$ 1,800	\$ 6,225	\$ 4,425
3	\$ 1,500	\$ 6,330	\$ 4,830
IRR	25%	17.43%	?

Performing the incremental analysis.

We choose the base alternative in this case as alternative A because the Alternative A has the least investment among the two alternatives.

Why did we choose increment B – A instead of A – B??

The first flow of the incremental cash flow series should be investment flow. If we have chosen the increment A – B instead of B – A, the first cash flow would have been positive. It means we are borrowing the money at the very beginning of project instead of investing. If we calculate the rate of return of (A – B), then this would be Borrowing Rate of Return (BRR) instead of internal rate of return (IRR).

Calculating IRR of the incremental Cash flow.

$$PW (i \%^*)_{B-A} = 0$$

$$- 9000 + 2850 (P/F, i\%^*, 1) + 4425 (P/F, i\%^*, 2) + 4830 (P/F, i\%^*, 3) = 0$$

$$(i \%^*)_{B-A} = 15\% > 10\% \text{ (MARR)}$$

Hence, Option B is selected.

Decision Rule

IF, $IRR_{B-A} > MARR / ERR_{B-A} > MARR / BCR_{B-A} > 1$, Select B (higher first cost alternative)

$IRR_{B-A} = MARR / ERR_{B-A} = MARR / BCR_{B-A} = 1$, Select either one

$IRR_{B-A} < MARR / ERR_{B-A} < MARR / BCR_{B-A} < 1$, Select A (lower first cost alternative)

Example 4.1

Consider the following three set of mutually exclusive alternatives:

Alternatives			
EOY	D ₁	D ₂	D ₃
0	- \$ 2000	- \$ 1000	- \$ 3000
1	\$ 1500	\$ 800	\$ 1500
2	\$ 1000	\$ 500	\$ 2000
3	\$ 800	\$ 500	\$ 1000

Which project would you select based on IRR, ERR and BCR methods on incremental investment assuming that $MARR = \epsilon = 15\%$?

Solution

Calculate the IRR of the each project.

IRR of D₁

$$PW = 0$$

$$-2,000 + 1,500 (P/F, i^*, 1) + 1,000 (P/F, i^*, 2) + 800 (P/F, i^*, 3) = 0$$

$$i^* = 34.37\% > 15\% \text{ (MARR) (Justified)}$$

IRR of D₂

$$PW = 0$$

$$-1,000 + 800 (P/F, i^*, 1) + 500 (P/F, i^*, 2) + 500 (P/F, i^*, 3) = 0$$

$$i^* = 40.76\% > 15\% \text{ (MARR) (Justified)}$$

IRR of D₃

$$PW = 0$$

$$-3,000 + 1,500 (P/F, i^*, 1) + 2,000 (P/F, i^*, 2) + 1,000 (P/F, i^*, 3) = 0$$

$$i^* = 24.81\% > 15\% \text{ (MARR) (Justified)}$$

Calculate the BCR of the each project using PW formulation

$$\text{B/C of D}_1 = PW (\text{Benefits}) / PW (\text{Costs})$$

$$\begin{aligned} PW (\text{Benefits}) &= 1,500 (P/F, 15\%, 1) + 1,000 (P/F, 15\%, 2) + 800 (P/F, 15\%, 3) \\ &= 1,500 * 0.8696 + 1,000 * 0.7561 + 800 * 0.6575 \\ &= \$ 2,586.5 \end{aligned}$$

$$PW (\text{Costs}) = \$ 2,000$$

$$\text{B/C of D}_1 = 2,586.5 / 2,000 = 1.293 > 1 \text{ (Justified)}$$

$$\underline{\text{B/C of D}_2} = \text{PW (Benefits)} / \text{PW (Costs)}$$

$$\begin{aligned}\text{PW (Benefits)} &= 800 (\text{P/F}, 15\%, 1) + 500 (\text{P/F}, 15\%, 2) + 500 (\text{P/F}, 15\%, 3) \\ &= 800 * 0.8696 + 500 * 0.7561 + 500 * 0.6575 \\ &= \$ 1,402.48\end{aligned}$$

$$\text{PW (Costs)} = \$ 1,000$$

$$\underline{\text{B/C of D}_2} = 1,402.48 / 1,000 = \mathbf{1.402 > 1 \text{ (Justified)}}$$

$$\underline{\text{B/C of D}_3} = \text{PW (Benefits)} / \text{PW (Costs)}$$

$$\begin{aligned}\text{PW (Benefits)} &= 1,500 (\text{P/F}, 15\%, 1) + 2,000 (\text{P/F}, 15\%, 2) + 1,000 (\text{P/F}, 15\%, 3) \\ &= 1,500 * 0.8696 + 2000 * 0.7561 + 1000 * 0.6575 = \text{Rs. } 3,474.1\end{aligned}$$

$$\text{PW (Costs)} = \$ 3,000$$

$$\underline{\text{B/C of D}_3} = 3,474.1 / 3,000 = \mathbf{1.15 > 1 \text{ (Justified)}}$$

Calculate the ERR of the each project

ERR of D₁

- Compound all cash inflow to the future value
 $3000 (\text{F/P}, 15\%, 2) + 1500 (\text{F/P}, 15\%, 1) + 800 = \$ 3933.75$
- Discount all cash outflow to the present value = 2000
- Making the equivalence of two equation
 $2000 (1+i)^3 = 3933.75$
 $(1+i)^3 = 1.966$
i = 25.29% >MARR (acceptable)

ERR of D₂

- Compound all cash inflow to the future value
 $800 (\text{F/P}, 15\%, 2) + 500 (\text{F/P}, 15\%, 1) + 500 = \$ 2133$
- Discount all cash outflow to the present value = 1000
- Making the equivalence of two equation
 $1000 (1+i)^3 = 2133$
 $(1+i)^3 = 2.133$
i = 28.72% >MARR (acceptable)

ERR of D₃

- Compound all cash inflow to the future value
 $1500 (F/P, 15\%, 2) + 2000 (F/P, 15\%, 1) + 1000 = \$ 5283.75$
- Discount all cash outflow to the present value = 3000
- Making the equivalence of two equation
 $3000 (1+i)^3 = 5283.75$
 $(1+i)^3 = 1.76$
i = 20.76% >MARR (acceptable)

Performing Incremental Analysis

- Select D₂ as the base alternative because it has the lower initial cost (\$1000).
- Compare D₁ with base alternative D₂ which is the next higher initial cost (\$2000).
- Calculate the incremental cost and incremental benefits (D₁-D₂)
- Calculate the IRR of the incremental cash flow.
- $-1,000 + 700 (P/F, i^*, 1) + 500 (P/F, i^*, 2) + 300 (P/F, i^*, 3) = 0$
- $i^* = 27.61 > 15\%$ (MARR)
- Eliminate D₂ from the consideration because alternative D₁ gives the higher return.
- Select D₁ as the base alternative which initial cost is lower than the next remaining alternative D₃.
- Compare alternative D₁ with D₃ and compute incremental cash flow. (D₃-D₁)
- Calculate the IRR of the D₃-D₁.

EOY	D ₂	D ₁ -D ₂	D ₃ -D ₁
0	-1000	-1000	-1000
1	800	700	0
2	500	500	1000
3	500	300	200
Incremental IRR	40.76%	27.61%	8.8%
Is increment justified		Yes	No
		Select D₁ Reject D₂	Reject D₃ Select D₁

Similar process is applied for B/C method and ERR and method.

EOY	D ₂	D ₁ -D ₂	D ₃ -D ₁
0	-1000	-1000	-1000
1	800	700	0
2	500	500	1000
3	500	300	200
Incremental BCR	1.402	1.18	0.88
Is increment justified		Yes	No
		Select D₁ Reject D₂	Reject D₃ Select D₁

EOY	D ₂	D ₁ -D ₂	D ₃ -D ₁
0	-1000	-1000	-1000
1	800	700	0
2	500	500	1000
3	500	300	200
Incremental BCR	1.402	1.18	0.88
Is increment justified		Yes	No
		Select D₁ Reject D₂	Reject D₃ Select D₁

References:

[1] *Engineering Economy*: William G. Sullivan, James A. Bontadelli & Elin M. Wicks, Eleventh Edition, Pearson Educations, Inc. 2000.

[2] *Engineering Economics*: James L. Riggs, David D. Bedworth and Sabah U. Randhawa, Fourth Edition, Tata McGraw Hill Education Private Limited, New Delhi, India, 2004.

[3] *Contemporary Engineering Economics*, Chan S. Park Second Edition, Addison-Wesley Publishing Company, 1997.

[4][https://nitsri.ac.in/Department/Civil%20Engineering/ Lecture_3,4.pdf](https://nitsri.ac.in/Department/Civil%20Engineering/Lecture_3,4.pdf) (Viewed September 2022)

[5]<https://www.indeed.com/career-advice/career-development/incremental-analysis> (Viewed September 2022)