Course Title

Engineering Economic Analysis

Chapter 4

Comparative Analysis of Alternatives

Lecture 7 (week 7)

Comparing Alternatives having different useful lives using Equivalent worth method & rate of return method and Capitalized worth method.

Associate Prof.   Er. Ishwar Adhikari

Learning Objective

From studying this lecture the students will be able to understand on the topics:

- Comparing the alternatives having different useful life using equivalent worth method.
- Comparing the alternatives having different useful life using rate of return method.
- Capitalized worth method (CW)

In the previous lecture we have discussed about the useful life and the study period of an alternative. Considering these two terms, two cases were developed.

Case 1: Useful lives are the same for all alternatives and equal to the study period.

Case2: Useful lives are different among the alternatives and at least one does not matches the study period.

Among these two cases, only the first case was being discussed in the last lecture. Now, in this lecture we focus on the case 2 where the useful lives among the alternatives are different and unequal to the study period.

4.6 Case2: Useful lives are different among the alternatives and at least one does not matches the study period.

Using Equivalent worth Method

In previous section, we assumed the simplest scenario possible when analysing mutually exclusive projects. The project had useful lives equal to each other and to the required service period. In practice this is seldom the case. Often project lives do not match the required analysis period and/or do not match each other. [1] The equivalent worth of the alternatives must be compared over the same number of years and must end at the same time to satisfy the equal-service requirement. [2] In following sections and examples, we will develop some techniques for dealing with these complications.

The two types of assumptions *repeatability assumption* and *co-terminated assumption* are used for the economic comparison of mutually exclusive alternatives.

4.6.1 Repeatability assumption

- Two alternatives having different useful life are *changed into projects having same useful* life by expanding their life up to least common year.
- The *study period* is equal to the least common multiple (LCM) of the lives of alternatives.
- The economic consequences that are estimated to happen in an alternative's initial life span will also happen in all succeeding life spans

Repeatability assumption is used mostly on public sector facility

Let us consider the two projects having the different useful life of 4 years and 6 years respectively with the cash flow as shown in the figure 4.1(a). We use the repeatability assumption for the analysis. Here the study period for both alternatives will be 12 years (Least common multiple of the useful life of project A and B, i.e. 4 and 6 years). Both the alternatives will be analysed for twelve years as shown in figure 4.1 (b)

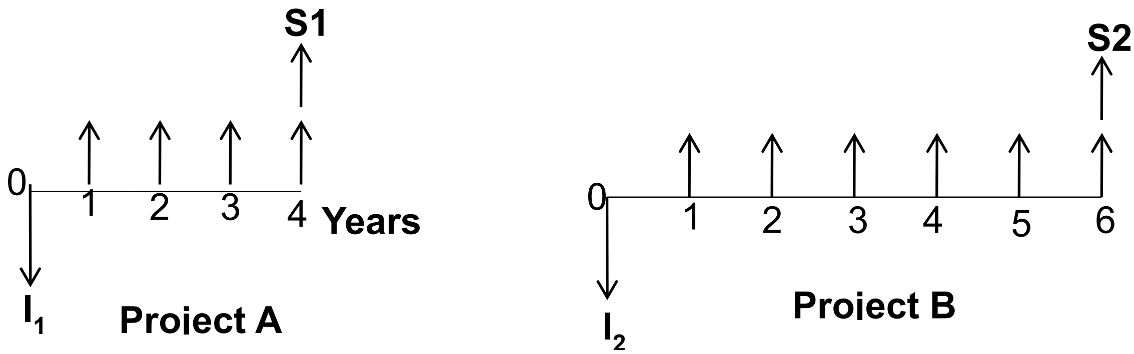
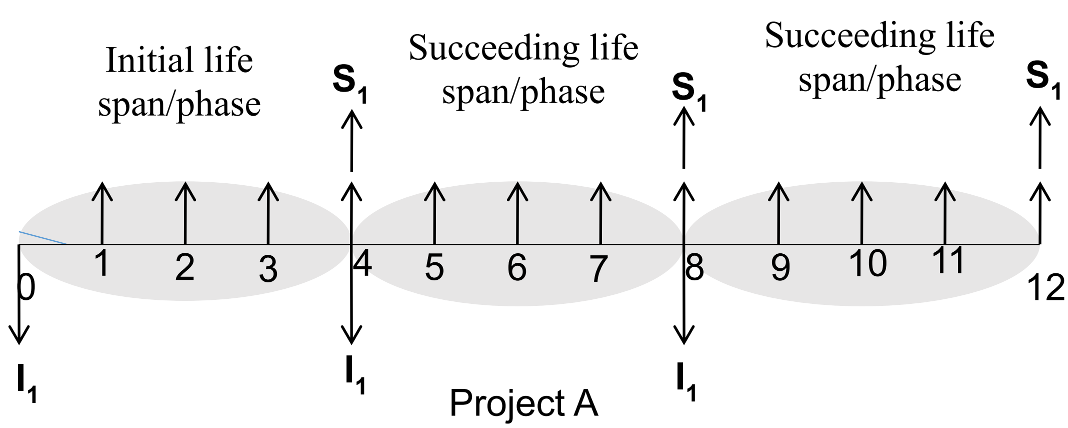


Fig 4.1 (a): Repeatability assumption example

LCM of 4 and 6 is 12 years, so we assume study period as 12 years

Project A is repeated 3 times



Project B is repeated 2 times

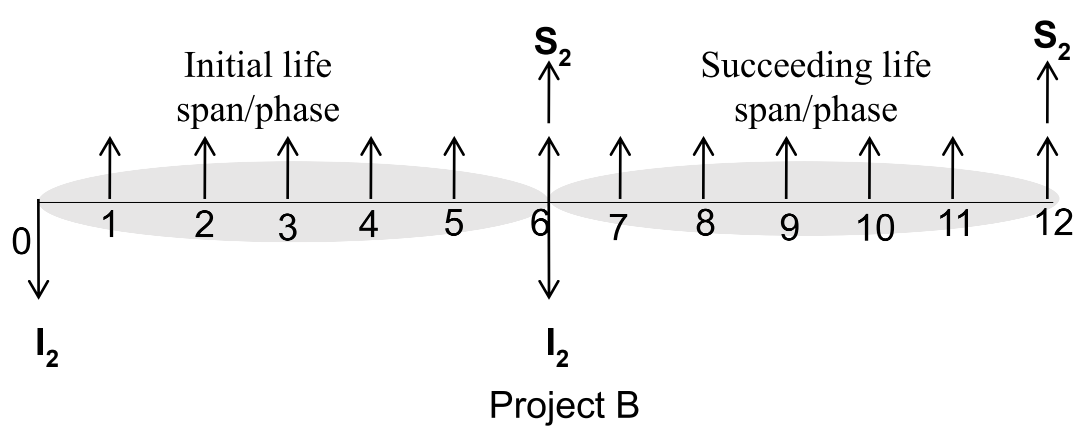


Fig 4.1 (b): Repeatability assumption example

Example 4.2

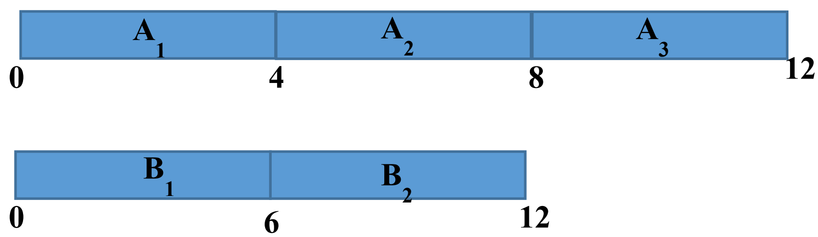
Select the best project using equivalent worth methods. MARR = 10%. Salvage value at the end of useful life of each project is 0. Use repeatability assumption. [3]

	Project A	Project B
Initial Investment	- $3,500	- $5,000
Annual revenue	$1,900	$2,500
Annual cost	- $645	- 1,020
Useful life	4 years	8 years

Study Period:

Least common multiple (LCM) of the useful lives of project A & project B = 12 years.

Identical replacement of project A at the end of year 4 and year 8 occurs (i.e. project A is repeated 3 times and project B is repeated 2 times.



<u>Using PW and FW Method</u>

PW of Project A (10%) = - 3500 - 3500 [(P/F, 10%, 4) + (P/F, 10%, 8)] + (1900 - 645) (P/A, 10%, 12)

= $ 1028

FW of Project A (10%) = - 3500 (F/P, 10%, 12) - 3500 (F/P, 10%, 8) - 3500 (F/P, 10%, 4) + (1900 - 645) (F/A, 10%, 12)

= - 23611 + 26837

= $ 3226

PW of Project B (10%) = - 5000 - 5000 (P/F, 10%, 6) + (2500 - 1020) (P/A, 10%, 12)

= $ 2262

FW of Project B (10%) = - 5000 (F/P, 10%, 12) - 5000 (F/P, 10%, 6) + (2500 - 1020) (F/A, 10%, 12) = $ 7099.6

FW of Project B (10%) > FW of Project A (10%) Therefore, Select the Project B

FW of Project B (10%) > FW of Project A (10%) Therefore, Select the Project B

Using AW Method

In case of AW method, the AW value of all the cash flow in one cycle is equal in all the cycle. Therefore we calculate the AW value of one cycle.

AW of Project A (10%) = - 3500 (A/P, 10%, 4) + (1900 - 645)

= $151

AW of Project B (10%) = - 5000 (A/P, 10%, 6) + (2500 - 1020)

= $332

From AW method, value of B > value of A, Select B.

4.6.2 Co – terminated assumption

The repeatability assumption has limited use in engineering practice, because actual situation seldom meet both condition assumed in repeatability method. [4]If the repeatability assumption is not applicable to a decision situation, then an appropriate study period needs to be selected i.e. co terminated assumption. [3]This is the most frequently used approach most frequently used in engineering practice as product life cycle are becoming shorter.

The co terminated assumption uses a finite and identical study period for all alternatives. The planning horizon combined with appropriate adjustments to the estimated cash flows plus the alternatives on a common and comparable basis. The planning horizon chosen could be

- Life of shorter lived alternative.
- Life of longer lived alternative.
- Less than shorter lived alternative
- Greater than longer lived alternative
- In between the shortest and longest lived alternative.

Two cases are involved in the co-terminated assumption

Case 1: Study period longer than the Useful life.

Case 2: Study period shorter than the Useful life.

Case 1: Study period longer than the Useful life.

A common instance of project lives that are longer than study period occurs in construction industry, where a building project may have a relatively short completion time but the equipment purchased has a much longer useful life. [1]

Two assumptions are made in this case:

- Cash flow accumulated at the end of useful life will be reinvested for the extended period.
- Replacement / reinvestment is necessary for remaining period and economic consequences will also happen in succeeding life as on initial life span.

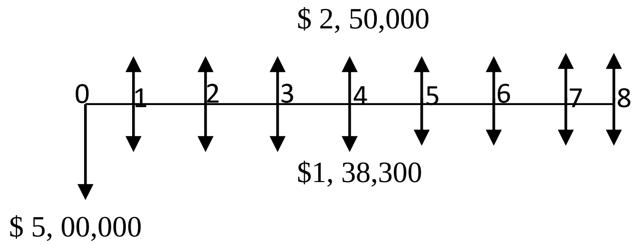
Example 4.3

Consider the following mutually exclusive projects. MARR = 10%

	A	**B**
Investment	\$ 350000	\$500000
Annual revenue	\$ 190000	\$ 250000
Annual cost	\$ 64500	\$ 138300
Useful life	4 years	8 years
Salvage value	0	0

Which alternative is more desirable based on the co-terminated assumption.

Taking analysis period as 8 years (the value should be taken in such a way that the study period is either equal to or greater than useful lives of all the alternatives). If lesser useful life is taken than we have to curtail down the cash flow to the end of study period and suitable market value should be assigned to the alternatives.



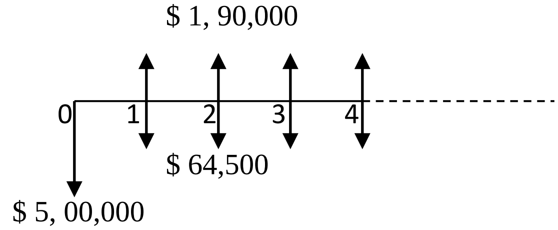
Cash flow diagram of Project B

There is no adjustment required for alternative B. The adjustment is required in case of A, which study period is 4 years greater than its useful life.

<u>Considering assumption 1</u>

FW_B (10%) = -5, 00, 000 (F/P, 10%, 8) + (25,000- 1, 38,300) (F/A, 10%, 8)
= -5, 00,000 $\{(1.1)^8\}$ + 1, 11,700 $\{(1.1)^8$ -1/0.1$\}$
= -10, 71,794.405 + 12, 77, 388.701

FW_B (10%) = \$ 2, 05,594.2958



Cash flow diagram of Project A

FW $_A$ (10%) = {-3, 50,000(F/P, 10%, 4) + (1, 90, 000- 64500) (F/A, 10%, 4)} (F/P, 10%, (8-4))

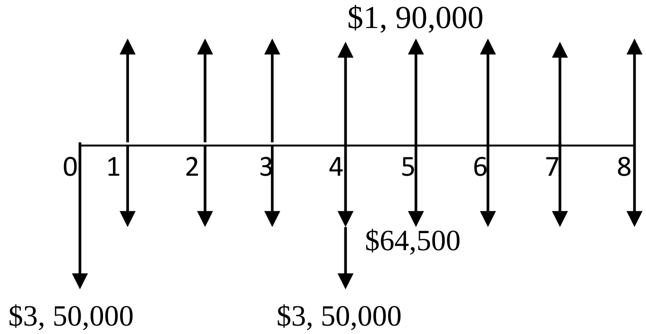
= {-3, 50,000 $(1.1)^4$ + 1, 25,500($(1.1)^4$ -1)/ 0.1} $(1.1)^4$

= (-5, 12,435 + 5, 82,445.5) $(1.1)^4$

FW $_A$ (10%) = $ 1, 02,502.37

FW $_B$ (10%) > FW $_A$ (10%), select alternative B

<u>Considering assumption 2</u>



Cash flow diagram of Project A

FW $_A$(10%) = -3,50,000 (F/P, 10%, 8) -3,50,000 (F/P, 10%,4) + (1,90,000 -64,500) (F/A,10%,8)

FW $_A$ (10%) = $ 1, 72,510

FW $_B$ (10%) > FW $_A$ (10%), select alternative B

Case 2: Study period shorter than the Useful life.

When study period is less than useful life than we have to calculate the imputed market value (IMV) at the end of study period occurring at the end of useful life. IMV of an assets is the estimated market value of the asset based on the economic equivalence of the investment amount 'I' and the salvage value 'S' of the asset. [5]If we simply convert the salvage value occurring at the end of useful life to end of study period, the value obtained will be incorrect.

The imputed market value technique which is sometimes called the implied market value can be used for for comparison with market place values when current data are available. If an imputed market value is needed for a piece of equipment, say at the end of T<useful life, the estimate is calculated based on the sum of two parts as follows.

MV_T = [PW at end of year T of remaining capital recovery amounts] + [PW at end of year T of original market value at end of useful life]

Where,

MV = market value

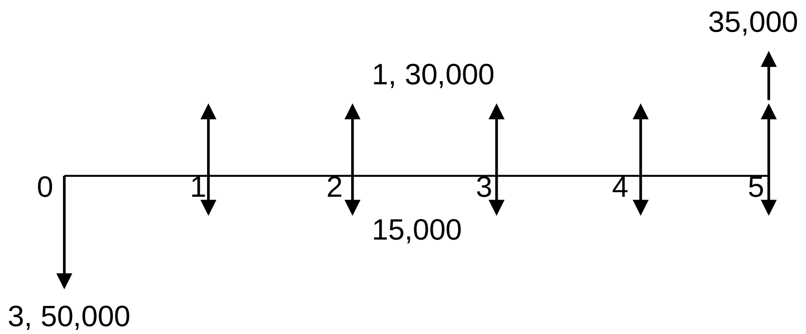
T = study period

Imputed Market Value $(IMV)_N = (PW)_{CR} + (PW)_{MV}$

Example 4.4

Using co-terminated assumption recommend the best project taking study period as 5 years

Project	A	B
Initial Investment ($)	3,50,000	5,00,000
Annual Revenues ($)	1,30,000	1,75,000
Annual cost ($)	15,000	25,000
Salvage value ($)	35,000	50,000
Useful life	5 years	8 years
MARR	10%	



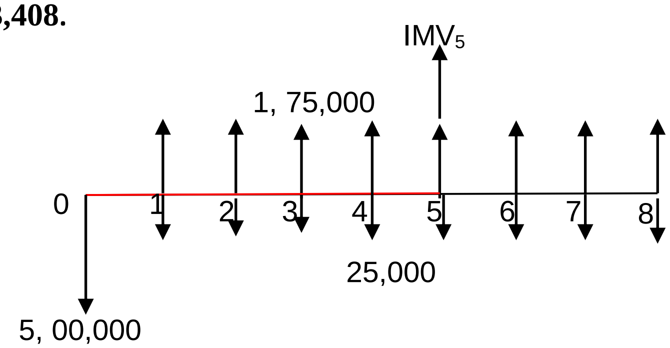
Cash Flow Diagram of Project A

<u>Using FW formulation,</u>

FW_A (10%) = -3, 50,000(F/P, 10%, 5) + (1, 30,000 -15,000) (F/A, 10%, 5) + 35,000

= - 5, 63,678.5 + 702086.5 + 35,000

= **$ 1, 73,408.**



Cash Flow Diagram of Project B

Applying imputed market value calculation,

CR (10%) = 5, 00,000 (A/P, 10%, 8) – 50,000 (A/F, 10%, 8)

= 93,722 – 4,372

= $. 89,350

Present worth (at year 5) of CR for remaining 3 years

PW_{CR} (10%) = 89,350 (P/A, 10%, 3)

= $ 2, 22,200

Present worth (at year 5) of Market value (MV) for remaining 3 years

PW_{MV} (10%) = 50,000(P/F, 10%, 3)

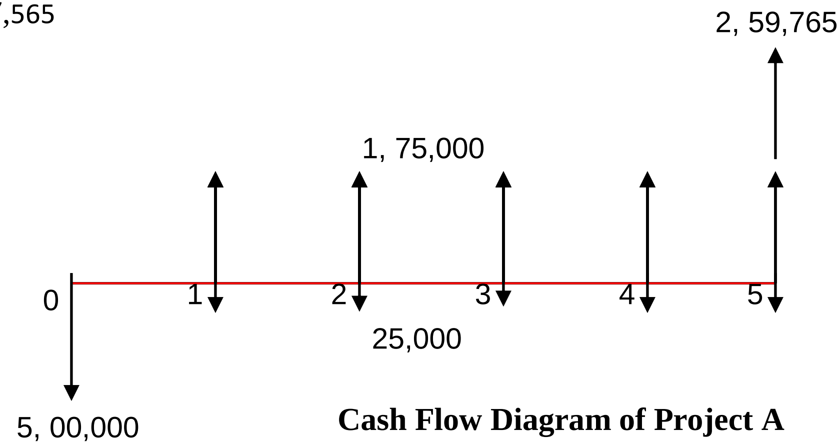
= $ 37,565

Market value at the study period. i.e. year 5

$MV_5 = PW_{CR}(10\%) + PW_{MV}(10\%)$

= 2, 22,200 + 37,565

= **$ 2, 59,765**



Cash Flow Diagram of Project A

<u>Using FW formulation,</u>

FW_B (10%) = -5,00,000(F/P,10%,5) + (1,75,000 -25,000) (F/A,10%,5) + 2,59,765

= - 8, 05,255+ 9, 15,765 + 2, 59,765

= **$. 3, 70,275**.

FW_B (10%) > FW_A (10%), Recommend Project B

4.7 IRR method for the Unequal project lives

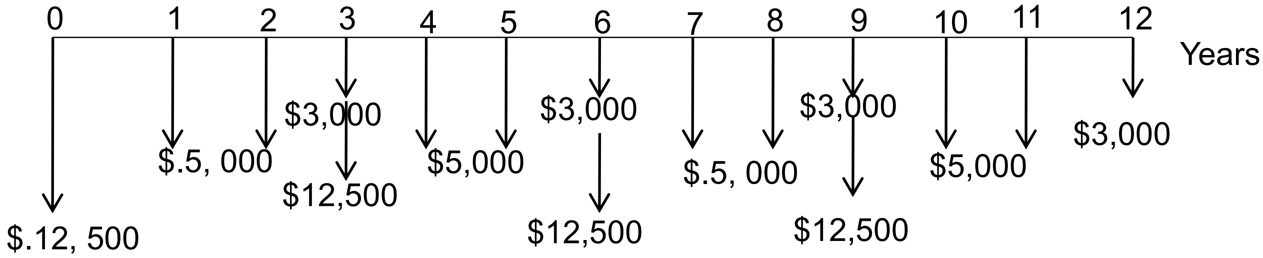
The IRR method can also be used to compare projects with unequal lives, it is necessary to find overall time length that is multiple of the lives of the alternatives. [6] This can be performed by using the Repeatability as well as co-terminated assumptions.

Example 4.5

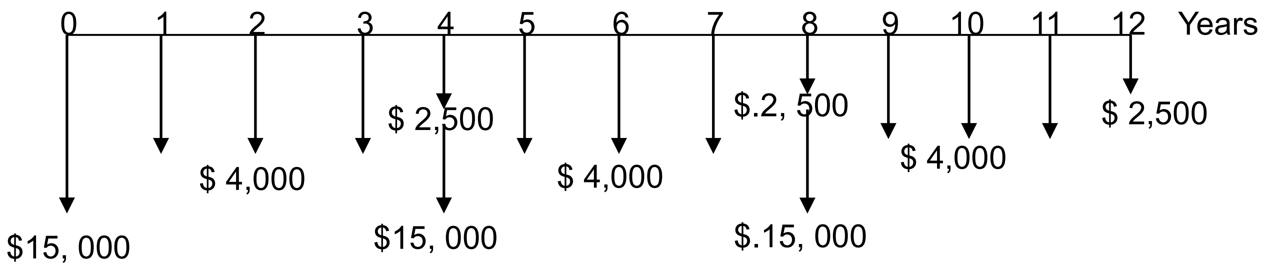
Consider the Two mutually exclusive projects. Recommend the best project using the IRR method. MARR =15%.

EOY	A	B
0	- $ 12,500	- $ 15,000
1	- $ 5,000	- $ 4,000
2	- $ 5,000	- $ 4,000
3	-$ 5,000+ $ 2000	- $ 4,000
4		- $ 4000+ $ 1,500

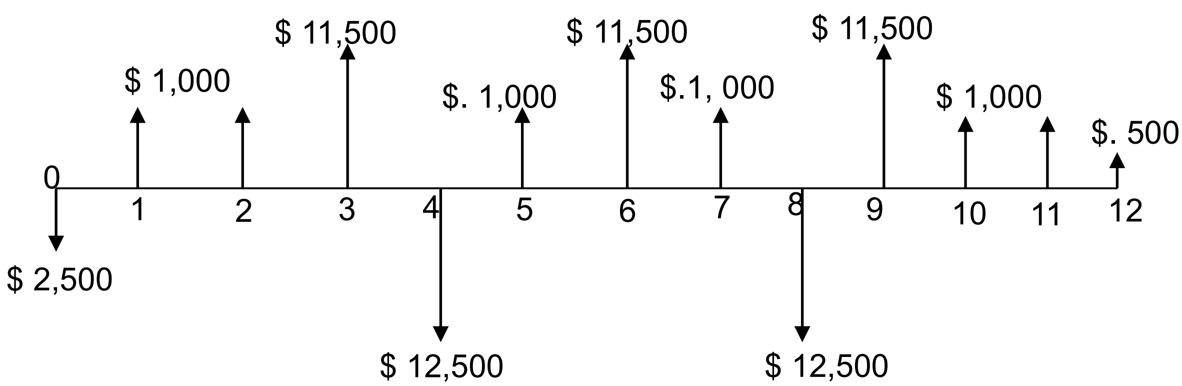
Since the Study period is LCM of 3 and 4 i.e. 12 years, we may compute the incremental cash flow over this 12-year period. As shown in figure (c), we subtract cash flows from Alternative A from Alternative B to form the increment of investment. (We want the first cash flow difference to be negative). Alternative A is repeated 4 times and alternative B is repeated 3 times.



Cash flow diagram of Alternative A



Cash flow diagram of Alternative B



Incremental Cash flow (Alternative B – Alternative A)

Here, five sign changes in the incremental cash flow, indicating non-simple incremental investment. This results in the multiple rate of return. Therefore we abandon the rate of return analysis and use the PW criterion.

PW (15%) $_{B-A}$ = -$ 2, 500 + $1,000(P/F, 15%, 1) +…………… + $500 (P/F, 15%, 12)
= **$ 5, 123 > 0**

This indicates that PW (15%) $_{B}$ > PW (15%) $_{A}$, **select project B.**

4.8 Capitalized worth method (CW)

The sum of the first cost and the present worth of disbursements assumed to be last forever is called a capitalized cost. [6]Special case of PW criterion is useful when the life of a proposed project is *perpetual* or study period is extremely long (30, 40 years or more) or forever. Example for capitalized worth are Public projects such as bridges, waterway constructions, irrigation systems, hydroelectric dams, roads etc. falls into this category. This criterion for evaluating and comparing the alternatives is useful in places where the repeatability assumption is applicable. In this section capitalized equivalent [CW (i| %)] method for evaluating such project is examined.

Let us consider the cash flow as shown in the figure 4.2 below

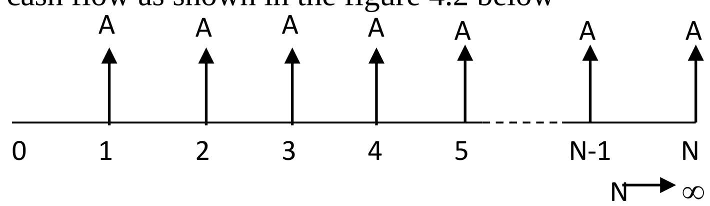


Fig 4.2: Equivalent present worth of an infinite cash flow series

The capitalized cost represents the amount of money that must be invested today to yield a certain return A at the end of each and every period forever, assuming interest rate i.

The formula to calculate CW is derived from the relation

P = A (P/A, i, n), where $N = \infty$.

The equation of P using the P/A factor formula is

$$PW (i \%) = A [(1+i)^{N} - 1] / [i * (1+i)^{N}]$$

Divide the numerator and denominator by $(1+i)^{N}$

$$PW (i \%) = A \{1/i - 1/ (1+i)^{N}\}$$

As N approaches ∞, the bracketed term becomes 1/i, and the symbol CW replaces PW or P.

$$CW (i \%) = A \{1/i - 1/\infty\} = A/i$$

$$\mathbf{CW (i\%) = (AW/i) (i\%) \ldots\ldots\ldots\ldots (1)}$$

Calculation Procedure of Capitalized Worth Method

The following procedure assists in calculating the Capitalized cost (CC) for an infinite sequence of cash flow: [7]

- Draw a cash flow diagram showing all the non-recurring and at least two cycles of all recurring cash flows.

- Find the present worth of all non-recurring amounts. This is their CC value.
- Find the equivalent uniform annual worth (A value) through one life cycle of all recurring cost. This is the same value in all succeeding life cycles.
- Add this A value to all other uniform amounts occurring in years 1 through infinity and result is the total equivalent uniform annual worth (AW)
- Divide this value of AW obtained by the interest rate (i%) to obtain a CC value. This is an application of equation 1.
- Add the CC value obtained in the step second and second last step.

References:

[1] *Contemporary Engineering Economics,* Chan S. Park Second Edition, Addison-Wesley Publishing Company, 1997.

[2] https://engmohannadb.github.io/etccourse21/inner-page/U6-L3.html (Viewed on October 2022)

[3] *Engineering Economy:* William G. Sullivan, James A. Bontadelli & Elin M. Wicks, Eleventh Edition, Pearson Educations, Inc. 2000.

[4] (http://civil.pcampus.edu.np/wp-content / uploads / 2019 / 07 / 06082019123 - MANUAL-OF-ENGINEERING-ECONOMY.pdf (Viewed on October 2022).

[5] (https://www.slideshare.net/abidnadeem/imputed-market-value-of-an-asset (Viewed on October 2022)

[6] *Engineering Economics:* James L. Riggs, David D. Bedworth and Sabah U. Randhawa, Fourth Edition, Tata McGraw Hill Education Private Limited, New Delhi, India, 2004.

[7] *Basics of Engineering Economy:* Leland Blank and Anthony Tarquin, Indian Edition, Tata McGraw Hill Education Private Limited, New Delhi, India, 2013.