

Course Title

Engineering Economic Analysis

Chapter 6

Risk Analysis

Lecture 11 (Week 11)

Sensitivity Analysis, Decision Tree Analysis, Scenario Analysis and Risk Adjusted MARR

Associate Prof. Er. Ishwar Adhikari

Learning Objective

From studying this chapter the students will be able to understand on the topics:

- Sensitivity Analysis with example
- Concept of Decision Tree Analysis with example
- Concept of Scenario Analysis and Risk adjusted MARR

6.3 Methods of describing project risk

6.3.2 Sensitivity Analysis

For the economic analysis of any engineering project we need different parameters or variables like initial investment, annual revenues, annual expenses, salvage value, useful life and interest rate. All the estimate made of these parameters are ‘best estimates’ made by the experienced personnel i.e. the same value will occur during the implementation phase of the project. But in the real practice, the budgeting or estimate of these variables are uncertain i.e. there are variables which may not occur as expected. Sensitivity analysis tries to seek the solution what the outcome of the project will be if one of these variables changes from the best estimates.

Sensitivity analysis tries to answer the question of

‘If the economic variables changes from expectation, what will the effect be on business, model system, or whatever being analyzed, and which variable are causing the largest deviations?’[1]

Sensitivity analysis determines the effect on the NPW of variations in the input variables (such as revenues, expenses, investment, salvage value etc.) used to estimate after tax cash flow. [2]. “In Engineering economic studies, sensitivity analysis is a general non probabilistic methodology, to provide information about the potential impact of uncertainty in selected factor estimates”. [3] A Financial Sensitivity Analysis, also known as a What-If analysis or a What-If simulation exercise, is most commonly used by financial analysts to predict the outcome of a specific action when performed under certain conditions. [4] For example: What if the investment increases by 20% of the initial investment, what will be the PW/IRR? What if the net annual revenue decreases by 10%, what will be the changes in the PW/IRR? What if the useful life increases by 15%, what will be the changes in PW/IRR? Sensitivity analysis tries to answer these questions and is considered as what if analysis.

Steps for sensitivity analysis

- It begins with the base case situation, which is developed using the most likely values for each input. Develop the prime equation.
- Justify the project economically.
- Change the specific variable of interest by several specified percentages above and below the most – likely value, while holding other variables constant.
- Calculate a new PW/FW/AW/IRR/BCR for each of these values.
- Present the results of a sensitivity analysis in the sensitivity graph.
- The slope of the line shows how sensitive the NPW is to changes in each of the inputs.
- The steeper the slope, the more the sensitive the NPW is to change in a particular variable

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Interpretation of Sensitivity Graph

On a plot, there are two directions to measure uncertainty:

- On X-axis, the uncertainty in the input variable is measured.
- On Y-axis, the impact of that uncertainty on PW is measured.
- The slope of the line shows how sensitive the PW is to change in each of the inputs. The steeper the slope, the more sensitive the PW is to a change in a particular variable.
- The graph allows us to identify the crucial variables that most affect the final outcome
- A properly drawn spider plot shows the following:
 - Limits of uncertainty for each cash flow element.
 - Impact of each cash flow element on the PW or IRR (each can be used as Y-axis)
 - Identification of each cash flow element that might change the recommendation.

Example 6.3 [3]

Perform the Sensitivity Analysis using the PW formulation over a range of $\pm 40\%$ changes in the estimates for – 1. Capital Investment (I) 2. Annual net Cash flow (A) 3. Market value (MV) 4. Useful life (N). Based on these best estimates, plot a diagram that summarizes the sensitivity of present worth to percent deviation changes in each separate factor estimate when MARR =10% per year

Capital Investment (I)	- \$ 11,500
Revenues / yr	\$ 5,000
Expenses / yr	- \$ 2,000
Market Value (MV)	\$1,000
Useful Life (N)	6 years

Step 1

Developing base case situation i.e. The PW of the project on the best estimate of the given factors is

Prime Equation

$$\begin{aligned} \text{PW (10\%)} &= - \$ 11,500 + \$ 3,000 (P/A, 10\%, 6) + \$ 1,000 (P/F, 10\%, 6) \\ &= \$ 2,130 > 0 \text{ (economically accepted)} \end{aligned}$$

Step 2

When the capital Investment (I) varies with the increment of 10% up to $\pm 40\%$)

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$$PW(10\%) = - \$ 11,500 (1 \pm p \%) + \$ 3,000 (P/A, 10\%, 6) + \$ 1,000 (P/F, 10\%, 6)$$

$$\text{For } + 10\%, \quad PW(10\%) = +\$980 \qquad \text{For } - 10\%, \quad PW(10\%) = +\$3280$$

$$\text{For } + 20\%, \quad PW(10\%) = - \$170 \qquad \text{For } - 20\%, \quad PW(10\%) = +\$4430$$

$$\text{For } + 30\%, \quad PW(10\%) = -\$1320 \qquad \text{For } - 30\%, \quad PW(10\%) = +\$5580$$

$$\text{For } + 40\%, \quad PW(10\%) = -\$2470 \qquad \text{For } - 40\%, \quad PW(10\%) = +\$6730$$

Step 3

When the Annual Cash Flow (A) varies with the increment of 10% up to $\pm 40\%$

$$PW(10\%) = - \$ 11,500 + \$ 3,000 (1 \pm p \%) (P/A, 10\%, 6) + \$ 1,000 (P/F, 10\%, 6)$$

$$\text{For } + 10\%, \quad PW(10\%) = +\$3,436 \qquad \text{For } - 10\%, \quad PW(10\%) = +\$823.42$$

$$\text{For } + 20\%, \quad PW(10\%) = +\$4743 \qquad \text{For } - 20\%, \quad PW(10\%) = -\$483.16$$

$$\text{For } + 30\%, \quad PW(10\%) = +\$6049 \qquad \text{For } - 30\%, \quad PW(10\%) = -\$1789.74$$

$$\text{For } + 40\%, \quad PW(10\%) = +\$7356 \qquad \text{For } - 40\%, \quad PW(10\%) = -\$3096.32$$

Step 4

When the market value (SV) varies with the increment of 10% up to $\pm 40\%$

$$PW(10\%) = - \$ 11,500 + \$ 3,000 (P/A, 10\%, 6) + \$ 1,000 (1 \pm p \%) (P/F, 10\%, 6)$$

$$\text{For } + 10\%, \quad PW(10\%) = +\$2,186.44 \qquad \text{For } - 10\%, \quad PW(10\%) = +\$2073.56$$

$$\text{For } + 20\%, \quad PW(10\%) = +\$2243 \qquad \text{For } - 20\%, \quad PW(10\%) = +\$2017.12$$

$$\text{For } + 30\%, \quad PW(10\%) = +\$2299 \qquad \text{For } - 30\%, \quad PW(10\%) = +\$1960.68$$

$$\text{For } + 40\%, \quad PW(10\%) = +\$2356.04 \qquad \text{For } - 40\%, \quad PW(10\%) = +\$1904.24$$

Step 5

When the useful life (N) varies with the increment of 10% up to $\pm 40\%$

$$PW(10\%) = - \$ 11,500 + \$ 3,000 \{P/A, 10\%, 6 (1 \pm p \%)\} + \$ 1,000 \{P/F, 10\%, 6(1 \pm p \%)\}$$

$$\text{For } + 10\%, \quad N=6.6 \quad PW(10\%) = +\$3040 \qquad \text{For } - 10\%, \quad N=5.4 \quad PW(10\%) = +\$1167$$

$$\text{For } + 20\%, \quad N=7.2 \quad PW(10\%) = +\$3900 \qquad \text{For } - 20\%, \quad N=4.8 \quad PW(10\%) = +\$147$$

$$\text{For } + 30\%, \quad N=7.8 \quad PW(10\%) = +\$4710 \qquad \text{For } - 30\%, \quad N=4.2 \quad PW(10\%) = -\$933$$

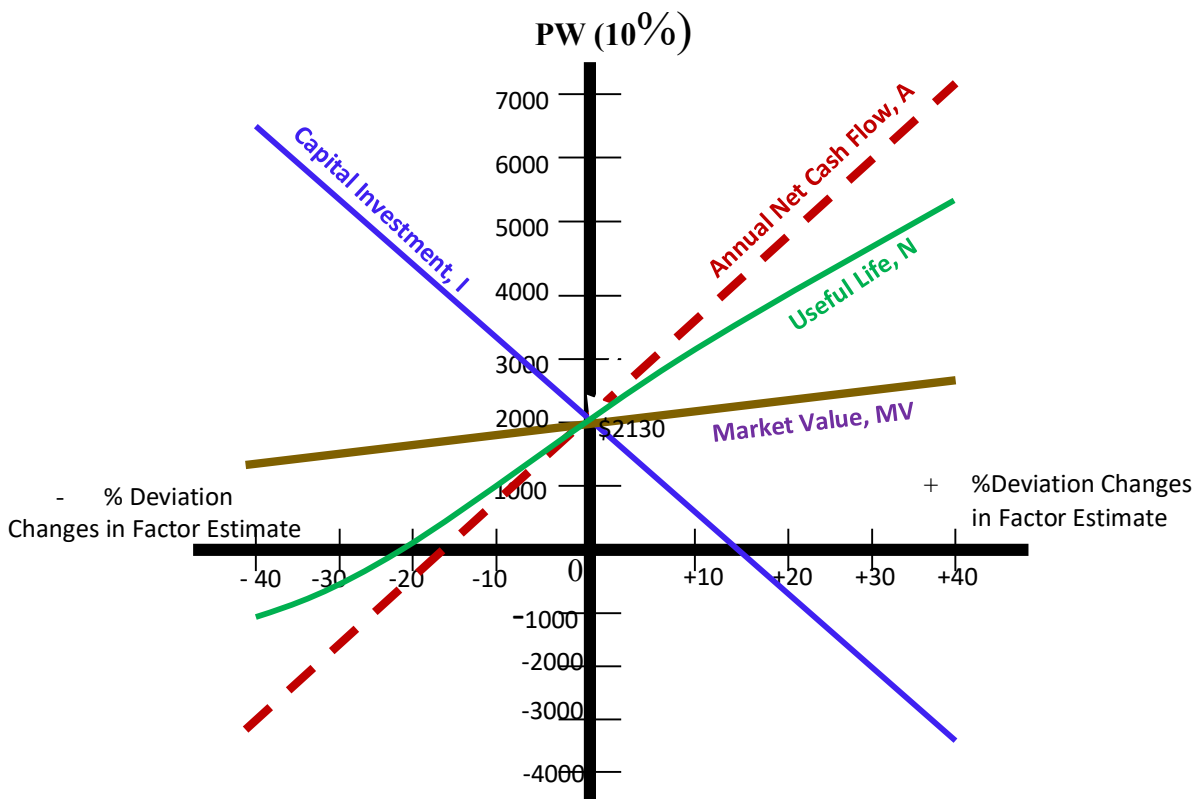
$$\text{For } + 40\%, \quad N=8.4 \quad PW(10\%) = +\$5477 \qquad \text{For } - 40\%, \quad N=3.6 \quad PW(10\%) = -\$2077$$

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Present Worth (PW)									
	-40%	-30%	-20%	-10 %	0 %	+10%	+ 20%	+30%	+ 40%
(I)(\$)	6730	5,580	4430	3,280	2,130	980	- 170	-1320	- 2,470
(A)(\$)	-3096	-1,789	- 483	823	2,130	3,436	4,743	6,049	7356
(MV)(\$)	1,904	1,960.	2,017	2,073	2,130	2,186	2,243	2,299	2,356
(N)(\$)	- 2786	-933	147	1167	2,130	3040	3,900	4,710	5,477

Step 6

Sensitivity Graph (Spider plot) Of Four Factors



Step 7

Revelations of Spider plot

- Shows the sensitivity of the present worth to percent deviation changes in each factor's best estimate
- Other factors are assumed to remain at their best estimate values
- The relative degree of sensitivity of the present worth to each factor is indicated by the slope of the curves (the "steeper" the slope of a curve the more sensitive the present worth is to the factor)
- In this example: Present worth is insensitive to MV and N, Present worth is sensitive to I, A,

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6.3.3 Decision Tree Analysis

Alternative evaluation may require a series of decisions where the outcome from one stage is important to the next stage of decision making. When each alternative is clearly defined and probability estimates can be made to account for risk, it is helpful to perform the evaluation using decision tree. [5]A decision tree is a flowchart that starts with one main idea and then branches out based on the consequences of your decisions. Decision tree is a powerful means of facilitating the analysis of important problems, especially those that involve sequential decisions and variable outcomes over time. [3]It's called a "decision tree" because the model typically looks like a tree with branches. [6]In short, decision tree is the graphic representation of various alternative solutions that are available to solve a given problem, in order to determine the most effective courses of action. [7]

A decision tree includes: [5]

- More than one stage of alternative selection.
- Selection of an alternative at one stage that leads to another stage.
- Expected results from a decision at each stage.
- Probability estimates for each outcome.
- Estimate of economic value (cost or revenue) for each outcome.
- Measure of worth as the selection criterion such as E (PW).

Components of Decision Tree

The decision tree is constructed left to right and includes each possible decision and outcome (Figure 6.3)

- **Decision node:** A square represents a decision node for making decision by a decision maker.
- **Branch:** It is a line connecting nodes from the left to the right of the diagram.
- **Probability node (chance node):** A circle represents probability node with the possible outcomes and estimated probabilities on the branches.

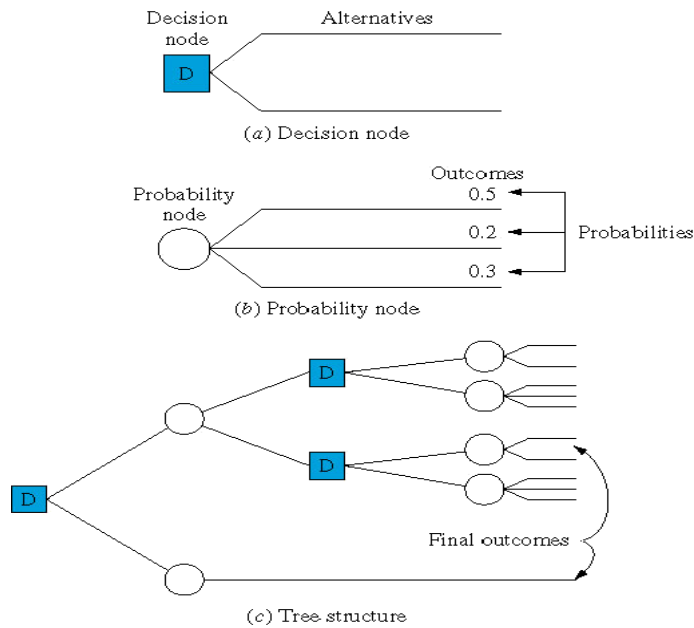


Fig 6.3: Decision tree diagram showing decision node, probability node and tree structure [5]

For the Evaluation and Selection of the alternative, the following information is necessary. [5]

- The probability that is estimated must sum to 1.0 for each set of outcomes (branches) that results from decision.
- Economic information for each decision alternative and possible outcome, such as initial investment and estimated cash flows.

Procedure for Solving Decision Tree Using PW analysis

- Start at the top right of the tree. Determine the PW value for each outcome branch.
- Calculate the Expected value for each decision alternative
- $E(\text{decision}) = \Sigma (\text{outcome estimate}) P(\text{outcome})$
- At each decision node, select the best E (decision) value – minimum cost or maximum value (if both cost and revenues are estimated).
- Continue moving to the left of the tree to the root decision in order to select the best alternative.

Example 6.4

A company is planning for its plants by investing \$ 3, 60,000. Estimates for efficiency of design goals, their probabilities and corresponding annual expenses saving are as follows:

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Design goal met (%)	Probability	Annual expenses saving (\$)
85	0.35	40,000
60	0.50	33,000
50	0.15	26,000

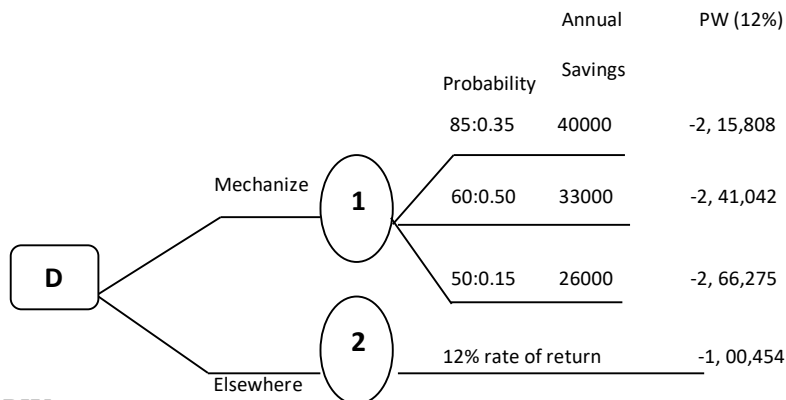
The Company has another option to invest this money that earns 20% per year elsewhere. Based on the Expected PW as the decision criterion, determine whether the mechanizing is preferable or not? MARR = 12% and analysis period is 5 years.

The PW corresponding to each efficiency goals are:

- $PW(12\%)_{85} = -3,60,000 + 40,000(P/A, 12\%, 5) = -\$2,15,808$
- $PW(12\%)_{60} = -3,60,000 + 33,000(P/A, 12\%, 5) = -\$2,41,042$
- $PW(12\%)_{50} = -3,60,000 + 26,000(P/A, 12\%, 5) = -\$2,66,275$

For Option 2 (Invest elsewhere that earns 20% per year)

- Annual earning = $(0.2 * 3,60,000) = \$72,000$
- $PW(12\%) = -3,60,000 + 72,000(P/A, 12\%, 5) = -\$1,00,454$



Expected PW

For option 1

$$E(PW) = -215808(0.35) + (-241042)(0.50) + (-266275)(0.15)$$

$$= -\$2,35,995$$

For Option 2

$$E(PW) = -\$100454$$

The analysis indicates that none of the options are preferable and even if we make the decision, the option 2 should be selected.

6.3.4 Scenario Analysis

It is also called the Optimistic- pessimistic – most likely estimation. Sensitivity and break even analysis is useful to show the effect of one common variable and NPW. In reality there may be difficult to specify precisely the relationship between a particular variable and the NPW because there may be interdependence between the variable. [8]Hence the relationship is further complicated by inter dependencies among the variables at a time. Scenario analysis is the technique that considers the sensitivity of NPW due to change in key variables at a time and the range of likely value of those variables. For example, the decision maker may examine three extreme cases i.e. best case scenario, worst case scenario and the most likely scenario.

6.3.5 Risk Adjust MARR

Uncertainty causes factors inherent to engineering economy studies, such as cash flows and project life, to become random variables in the analysis. A widely used industrial practice for including some consideration of uncertainty is to increase the MARR when a project is thought to relatively uncertain. Hence a procedure has emerged that employs risk adjust interest rates. If two or more investments delivered the same return over a given time period, the one that has the lowest risk will have a better risk-adjusted return. [9]

References:

- [1] <https://www.slideshare.net/lashinialahendra/sensitivity-analysis> (Viewed November 2022)
- [2] *Contemporary Engineering Economics*, Chan S. Park Second Edition, Addison-Wesley Publishing Company, 1997.
- [3] *Engineering Economy*: William G. Sullivan, James A. Bontadelli & Elin M. Wicks, Eleventh Edition, Pearson Educations, Inc. 2000
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- [8] *A Textbook of Engineering Economics*: Damodar Adhikari, First Edition, Dreamland Publication Pvt. Ltd. Kathmandu, Nepal, 2019.
- [9] <https://www.investopedia.com/terms/r/riskadjustedreturn.asp> (Viewed November 2022)