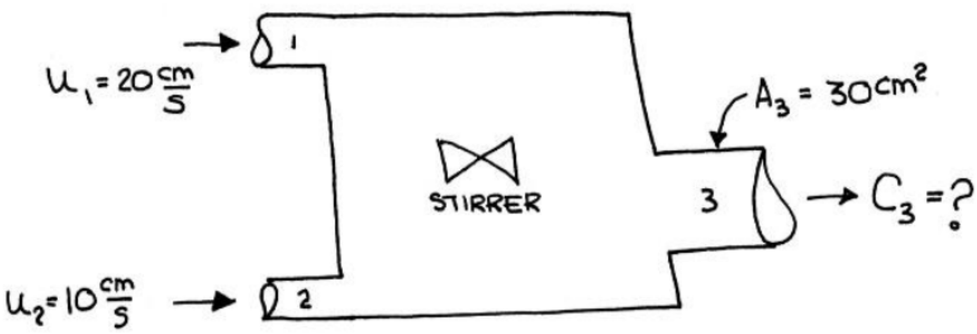
Instructions: Attempt all questions

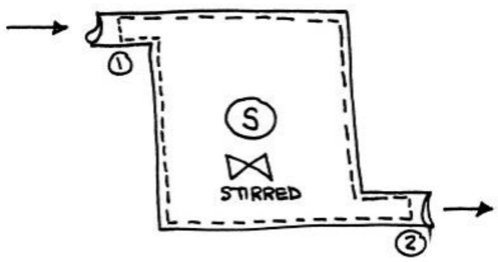
Problem 1



Two pipes, each of $10\,cm^2$ cross-section, carry water into a mixing chamber. The upper pipe carries water saturated in oxygen ($C_1 = 9\,mg/l$), and the lower pipe carries deoxygenated water ($C_2 = 0\,mg/l$). A stirrer within the chamber rapidly mixes the two streams, such that the concentration in the tank is spatially uniform. Assuming the system is at steady state, what is C_3?

Problem 2

A well-stirred tank is fed by an inlet pipe with cross-section, $A_1 = 10\,cm^2$. The inlet velocity is $U_1 = 10\,cm/s$. Inside the tank a plaster ball slowly dissolves supplying a steady source of calcium carbonate to the water, $S = +5\,g/s$. The outlet pipe area is the same as the inlet. There is no calcium carbonate in the inflow. At steady state, what is the outlet concentration?



Problem 3

Two water pipes of equal cross-section, $A = 20\,cm^2$, join to form a single pipe of cross-section, $A_3 = 40\,cm^2$. The two incoming pipes carry water of different temperature, $T_1 = 10\,°C$ and $T_2 = 20\,°C$, respectively. If the velocity in the two upstream pipes is the same, what is the temperature in the pipe downstream of the junction? Assume that all pipes are perfectly insulated.

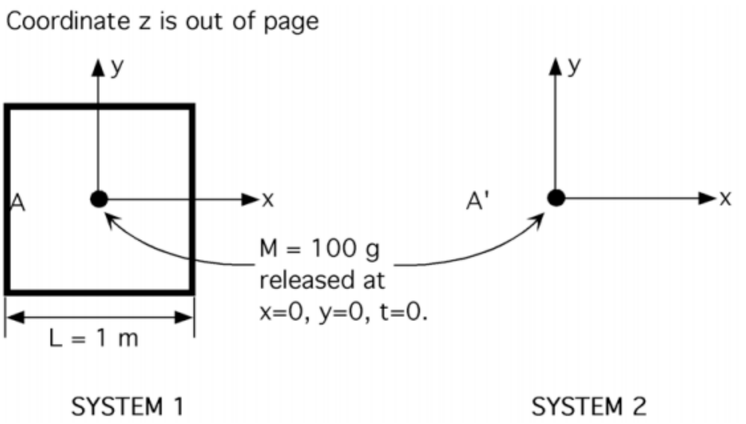
Problem 4

A shallow river flows out of a shaded, wooded region into an open plain at $x = 0$. Once in the open region ($x > 0$) the river begins to receive solar radiation at $H_3 = 800\,watts\,m^{-2}$. If the river emerges from the forest at a constant temperature, T_0, find the gradient of temperature along the river, $\partial T/\partial x$, for $x > 0$. The river is $h = 1\,m$ deep, $b = 10\,m$ wide and flows at $U = 1\,ms^{-1}$. The diffusion coeffient is homogeneous and isotropic, $D = 0.1\,m^2s^{-1}$.

Problem 5

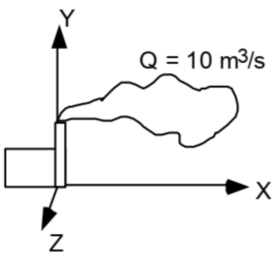
Consider the two systems shown below. System 1 is enclosed by no-flux walls which define a domain of dimensions 1m x 1m x 0.1m. System 2 is defined by parallel, horizontal (x-y plane), no-flux boundaries at z = ± 0.1 m, but is otherwise unconstrained. Both systems have an isotropic diffusivity of $D = 2\ cm^2 s^{-1}$. At t = 0 a mass, M = 100g, is released into both systems at x=0, y=0, z=0. A concentration probe (A and A') is located in each system at the position (x = -0.5 m, y = 0, z = 0). The detection limit of these probes is 10 ppm (gm^{-3}).

a) Estimate the time at which the concentration measured at A and A' begin to diverge?
b) What is the final concentration measured in each system, and when is this concentration achieved?
c) Describe the evolution of the concentration field in each system, i.e. C(x,y,z,t).



Problem 6

A chemical plant releases exhaust gas at a flow rate of $10\ m^3/s$ through a smokestack of height H = 20 m. A failure in the filter system allows a pulse of sulfur dioxide, SO_2, to be released with the exhaust. The concentration of SO_2 in the exhaust is normally zero, but during the failure increases to $10\ mg/m^3$. This release occurs for $T_R = 5$ minutes. Assume an isotropic diffusivity, $D = 1\ m^2/s$, and uniform wind speed, U = 2 m/s.



a) Using the coordinate system shown in the sketch above, and assuming the release occurs as an instantaneous slug of mass, M, write an expression for concentration of chemical XX downstream. Assume that the ground is a no-flux boundary.

b) Calculate the Pe# for x = 500 m and explain what the value indicates about transport.

c) At what distance downstream will the release appear as an instantaneous release?

d) Your home is 500 m downwind of the plant. What is the maximum concentration you would expect to observe at ground level at your house?

Problem 7

Consider a long, narrow channel connected to a harbor that receives drainage water from the city at its landward end (x = 0) at an average rate, $Q = 0.005 m^3/s$. The channel is 75-m long, 2-m deep, and 10-m wide. One morning a boat spills a tank of soluble degreasing fluid at x = 25 m. Use the following questions to assess the impact of this spill on the harbor. Assume an isotropic diffusivity, $D = 0.1\ m^2/s$.

a. Estimate the Peclet No., $Pe = \frac{UL}{D}$, for the system and explain its meaning.

b. What time do I have to execute preventive measures to keep the contaminant out of the harbor?

c. Estimate the time scale for the contaminant to become well-mixed in the lateral and vertical.

d. To describe the concentration of contaminant observed at the harbor (x = 75 m), should I use the 1-D, 2-D, or 3-D evolution equation? Justify your answer.

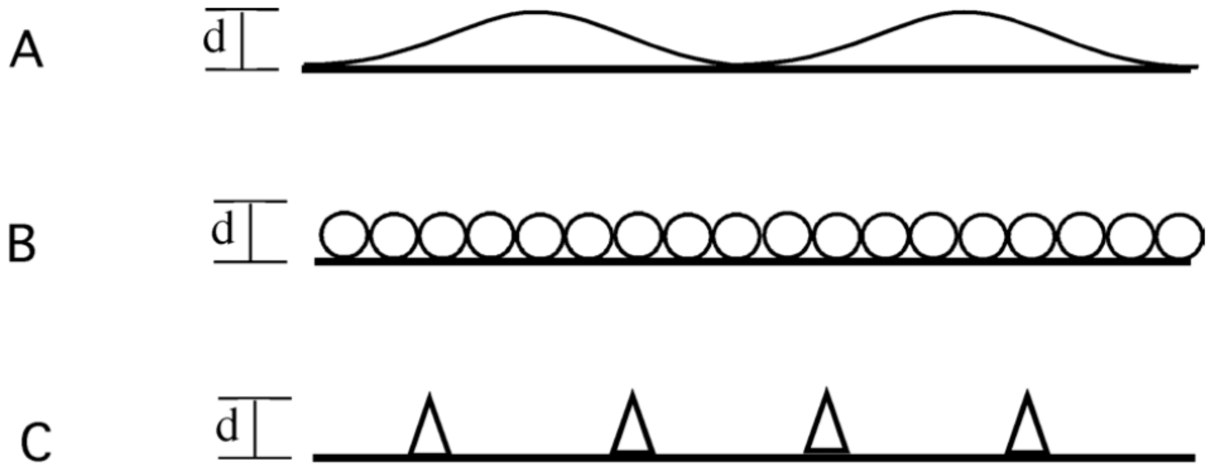
e. Using symbolic language [U, D, x, t, M, etc] write a mathematical expression that describes the evolution of the spill in time and space, after the concentration has become well-mixed across the width and depth of the channel. Assume that the contaminant cannot pass through the back wall of the inlet at x = 0.

f. State two limitations of the model you have chosen.

Problem 8

Nikuradse used uniform sand grains of diameter ε to characterize the impact of surface texture on velocity profiles. Based on his experiments the length scale ε, now called the equivalent sand grain diameter, has become a standard for describing roughness of any shape. The ‘equivalency’ is interpreted as providing the same drag. The equivalent roughness is estimated by measuring the velocity profile and fitting it to a logarithmic profile to determine y_o. If the flow is Rough Turbulent, then $y_o = \varepsilon/30$. This tells us that the roughness provides the same drag to the flow as uniform sand grains of diameter 30 y_o.

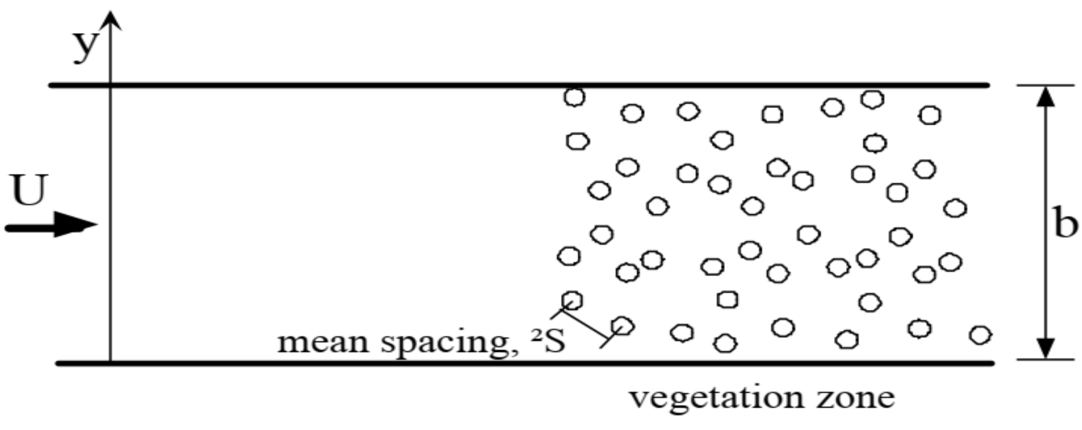
Consider the roughness elements depicted below. Although the shapes have the same physical scale, d, they will not necessarily offer the same roughness to the flow, that is they will not necessarily produce the same y_o. Consider two cases, $d \ll \delta_s$, and $d \gg \delta_s$, and order the three cases from highest to lowest y_o.



Problem 9

Below is a top view of a channel of width b. The cross section of the channel is constant, so that the velocity, U, is also constant along the channel. Part of the channel is filled with vegetation whose morphology is uniform over depth, and emerges through the water surface. The mean stem diameter is d, and the mean spacing between stems is ΔS. Consider the model, $D_{t,y} \sim v' \; l_y$, to describe the lateral diffusivity.

a) How will the lateral turbulent diffusivity change as the flow enters the vegetation?
b) Compare the diffusivity in the vegetated zone for $Ud/\nu = 1$ versus $Ud/\nu = 1000$?
c) Suppose the flow is unconfined, *i.e.* no side-walls, but the lateral extent of the vegetated zone is unchanged, how will the turbulence scales, turbulence intensity, and diffusivity differ in the vegetated and unvegetated zones?



Problem 10

The velocity profile shown below was measured in a stream above a gravel bed for which the mean gravel size was 1 cm.

a) Find the friction velocity, u_*.
b) What is the thickness of the laminar sub-layer?
c) Does the gravel's roughness contribute to the resistance at the bed?
d) What diameter of spherical sand grain would produce the same flow resistance?