# Mathematics For Information Technology

Week 1: Algebra : Indices, Logarithm

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# Outline

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6. Application of indices and logarithms in IT

# Course description

- Mathematics for I T is a course designed to provide students with the mathematical foundation necessary to understand and apply concepts in information technology.

- students will have the mathematical skills necessary to succeed in more advanced courses in computer science and related fields.

# Course goals

- Develop a strong foundation in mathematical concepts and methods that are essential for understanding and applying information technology.
- Understand how mathematical principles are applied in various areas of information technology, such as computer science, data science.
- Develop critical thinking, problem-solving, and analytical skills that are essential for success in information technology.

# Intended Learning Outcomes

- Understanding the basic concepts of indices and logarithms, including the laws of indices and logarithms.
- Solving problems that involve the use of indices and logarithms, such as simplifying expressions and solving equations.
- Applying indices and logarithms in practical IT applications, such as working with computer memory, data storage, and encryption algorithms.

# INDICES

- Powers as indices.
- Indices can be expressed in the form below
  - $a \times a \times a = a^3$
  - $a \times a = a^2$
  - $a \times a \times a \times a = a^4$
- And such powers are what we call the indices i.e. 3 is the index and $a$ is the base

**Example:** Simplify the following

$a^3 \times a^2$

$$= a \times a \times a \times a \times a$$

$$= a^5$$

$a^4 \div a^3$

$$= \frac{a \times a \times a \times a}{a \times a \times a}$$

$$= a$$

$(a^3)^2$

$$= (a \times a \times a) \times (a \times a \times a)$$

$$= a^6$$

# Laws of indices

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{-m} = \frac{1}{a^m}$

- $a^0 = 1$

- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

- $\sqrt{a} = a^{\frac{1}{2}}$

- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

# Simplification of expressions

Find the value of

*1.* $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

$$= \left(\frac{8}{27}\right)^{\frac{2}{3}}$$

$$= 8^{\frac{2}{3}}/27^{\frac{2}{3}}$$

$$= 2^{3^{\frac{2}{3}}}/3^{3^{\frac{2}{3}}}$$

$$=\frac{4}{9}$$

*2.* $(8^{\frac{1}{6}} \times 4^{\frac{1}{3}}) / (32^{\frac{1}{6}} \times 16^{\frac{1}{12}})$

$$=2^{\frac{1}{2}} \times 2^{\frac{2}{3}} / 2^{\frac{5}{6}} \times 2^{\frac{1}{3}}$$

$$= 2^{\frac{1}{2}+\frac{2}{3}-\frac{5}{6}-\frac{1}{3}}$$

$$= 2^{0}$$

$$= 1$$

Simplify the following

$8^n \times 2^{2n} \div 4^{3n}$

$$= 2^{3n} \times 2^{2n} \div 2^{6n}$$

$$= 2^{5n} \div 2^{6n}$$

$$= 2^{-n}$$

$$3^{n+1} \times 9^{n} \div 27^{\left(\frac{2}{3}\right)^{n}}$$

$$= 3^{n+1} \times 3^{2n} \div 3^{3^{\left(\frac{2}{3}\right)^{n}}}$$

$$= 3^{n+1+2n-2n}$$

$$= 3^{n+1}$$

- $6^{\left(\frac{1}{2}\right)^n} \times 12^{n+1} \times 27^{\left(-\frac{1}{2}\right)^n} \div 32^{\left(\frac{1}{2}\right)^n}$

$$= (2 \times 3)^{\frac{n}{2}} \times (4 \times 3)^{n+1} \times 3^{-\frac{3n}{2}} \div 2^{\frac{5n}{2}}$$

$$= 2^{\frac{n}{2}} \times 3^{\frac{n}{2}} \times 2^{2n+2} \times 3^{n+1} \times 3^{-\frac{3n}{2}} \div 2^{\frac{5n}{2}}$$

$$= 2^{\frac{n}{2}+2n+2-\frac{5n}{2}} \times 3^{\frac{n}{2}+n+1-\frac{3n}{2}}$$

$$= 2^{\frac{4}{2}} \times 3^{\frac{2}{2}}$$

$$= 4 \times 3$$

$$= 12$$

# LOGARITHMS

- Having just considered indices, we can use the same idea to logarithms because a logarithm is an index.

- Is an exponent that incase the power or index to which a number (base) is raised to produce a given number i.e.

if $10^a = b$, then

- 10 is the base
- $a$ is a logarithm
- $b$ is the number

- Another form of writing $10^a = b$ in logarithm form

$$10^a = b$$

$$\log_{10} 10^a = \log_{10} b$$

$$alog_{10}10 = \log_{10} b$$

$$a = \log_{10} b$$

# LAWS OF LOGARITHMS

- $\log_{10} a + \log_{10} b = \log_{10}(ab)$

- $\log_{10} a - \log_{10} b = \log_{10}\left(\frac{a}{b}\right)$

- $\log_{10} a^n = nlog_{10}a$

# LAWS OF LOGARITHMS…..

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a -\frac{1}{a} = \log_a a^{-1}$
- $\log_a x = 1/\log_x a$

# Change of base

- Given that $\log_3 x = p$ and $\log_{18} x = q$. Show that $\log_6 3 = \frac{q}{p-q}$

$$from\ \log_3 x = p$$

$$x = 3^p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (i)$$

$$\log_{18} x = q$$

$$\boldsymbol{x} = 18^q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Equating eqn(i) and (ii)

$$3^p = 18^q$$

Introduce log base 6 on both sides

$$\log_6 3^p = \log_6 18^q$$

$$p \log_6 3 = q(1 + \log_6 3)$$

$$\log_6 3 = \frac{q}{p - q}$$

1. Express $\log_{10}\frac{a^2b^2}{100\sqrt{c}}$ in terms $of \log_{10} a, \log_{10} b, \log_{10} c$

$$= \log_{10}(a^2b^2) - \log_{10} 100\sqrt{c}$$

$$= \log_{10} a^2 + \log_{10} b^2 - (\log_{10} 10^2 + \log_{10} \sqrt{c})$$

$$= 2\log_{10} a + 2\log_{10} b - 2 - \frac{1}{2}\log_{10} c$$

Given $5^x . 25^{2y} = 1$ and $3^{5x} . 9^y = \frac{1}{9}$. Calculate the values of $x \ and \ y$

$$from \ 5^x 25^{2y} = 1$$

Introduce log base 5 on both sides

$$\log_5 5^{x+4y} = \log_5 1$$

$$5^{x+4y} = 5^0$$

$$x + 4y = 0 \ldots \ldots . (1)$$

From $3^{5x}.\, 9^{y} = \frac{1}{9}$

$$3^{5x}.\, 3^{2y} = 3^{-2}$$

$$5x + 2y = -2 \ldots \ldots \ldots (2)$$

Solving eqn (1) and (2) simultaneously gives

$$x = -\frac{4}{9}, \qquad y = 1/9$$

- Given that $\log_2 x + 2\log_4 y = 4$, show that $xy = 16$.

$$\log_2 x + \log_4 y^2 = 4$$

$$let\ \log_4 y^2\ be\ t$$

$$y^2 = 2^{2t}$$

Introducing log base 2 on both sides

$$2\log_2 y = 2t$$

$$t = \log_2 y$$

$$\log_2 x + \log_2 y = 4$$

$$\log_2(xy) = 4$$

$$xy = 2^4$$

$$xy = 16$$

Solve for the values of $x$ of the following

1. $9^x - 3^{x+1} = 10$

$$3^{2x} - 3^{x+1} = 10$$

$$2x \log_{10} 3 - (x+1) \log_{10} 3 = \log_{10} 10$$

$$(x-1) \log_{10} 3 = \log_{10} 10$$

$$x = \frac{\log_{10} 10}{\log_{10} 3} + 1$$

$$x = 2.0959 + 1$$

$$x = 3.0959$$

- $\log_4 x^2 - 6\log_x 4 - 1 = 0.$

Change base x to base 4

$$\log_4 x^2 - \frac{6}{\log_4 x} - 1 = 0$$

Let $\log_4 x \; be \; m$

$$2m - \frac{6}{m} - 1 = 0$$
$$2m^2 - 6 - m = 0$$

This is a quadratic equation with roots (-4,3)

$$(m-2)(2m+3) = 0$$
$$m = 2, m = -\frac{3}{2}$$

- But remember $\log_4 x = m$
- $\log_4 x = 2$

$$x = 4^2, x = 16$$

Also $\log_4 x = -\frac{3}{2}$

$$x = 4^{-\frac{3}{2}}$$

$$x = 0.125$$

# Application of Indices and logarithms in IT

- A computer has 16GB of memory. How many bytes of memory does it have?

To convert 16GB to bytes, we need to multiply 16 by the number of bytes in one GB.

$$1\ GB\ = 10^9 = 1{,}000{,}000{,}000\ bytes$$

So, $16\ GB\ =\ 16\ x\ 1{,}000{,}000{,}000\ bytes$

$$=\ 16{,}000{,}000{,}000\ bytes$$

- Therefore, the computer has 16,000,000,000 bytes of memory.

- The number of data packets transmitted over a network doubles every 5 minutes. If there were 100 packets transmitted at time t=0, how many packets will be transmitted at t=20 minutes?

$P(0) = 100$ (the initial number of packets at t=0)

$$P(t) = P(0)\ x\ 2^{\frac{t}{5}}$$

(the number of packets at time t, doubling every 5 minutes)

$$P(20) = 100 \times 2^{\frac{20}{5}}$$

$$P(20) = 100 \times 2^{4}$$

$$P(20) = 100 \times 16$$

$$P(20) = 1600$$

# References


- Stewart, J. (2015). Calculus: Early transcendental (8th ed.). Cengage Learning.
- Strang, G. (2016). Linear algebra and its applications (5th ed.). Cengage Learning.

End of lecture 1

Thank you