Mathematics For Information Technology

Week 3: Systems of linear equation: Solution by elimination, substitution

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outline

- Intended learning outcome
- Introduction
- Linear equation in one variable
- Elimination method: two and three variables
- Substitution method
- Application of systems of linear equations

Learning outcomes

- Understanding the concept of elimination and how it is used to solve systems of linear equations.
- Understanding the difference between elimination and substitution methods and when to use each method.
- Being able to use the elimination method to solve systems of linear equations with two or more variables.
- Understanding the substitution method and how it is used to solve systems of linear equations.

Introduction

- A linear equation in the variables $x_1, x_2 \dots \dots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b \dots \dots \dots (1)$$

Where b and the coefficients $a_1, a_2 \dots., a_n$ are real numbers usually known.

The subscript n may be any positive integer.

- The equations $4x_1 - 5x_2 + 2 = x_1$and $x_2 = 2\left(\sqrt{6} - x_1\right) + x_3$ are both linear because they may be rearranged algebraically as in $eqn(1)$

- A system of linear equations is a collection of one or more linear equations involving the same set of variables i.e. $x_1, x_2 \ldots \ldots, x_n$

- The set of all possible solutions is called the solution set of the linear systems

- Finding the solution set of a system of two linear equations in two variables is easy because it amounts to finding the intersection of two lines.

- A system of linear equations has either
  - No solution
  - Exactly one solution
  - Infinitely many solutions

$$x_1 - 2x_2 = -1 \ldots.(i)$$

$$-x_1 + 2x_2 = 3 \ldots.(ii)$$

$$when\ we\ add\ the\ two\ equations\ we\ obtain$$

$$0 = 2$$

Implying that these equation have no solution

$$x_1 - 2x_2 = -1 \ldots.(i)$$

$$-x_1 + 2x_2 = 1 \ldots..(ii)$$

$$when\ we\ add\ the\ two\ equations\ we\ obtain$$

$$0 = 0$$

These equations have infinitely many solution

Linear equations in one variable

- Examples

$$a + 5 = 7$$
$$a = 7 - 5$$
$$a = 2$$

$$a^2 - 25 = 0$$
$$a^2 = 25$$
$$a = \pm\mathbf{5}$$

- $x + 2x = 6$

$$3x = 6$$
$$x = 2$$

Elimination method

- It involves eliminating one variable from the equations by adding or subtracting them together, in order to solve for the other variable.
- The steps to use the elimination method are:
- Write the two equations in standard form, with the variables on the left side and the constants on the right side.

- Choose one of the variables to eliminate, and multiply one or both of the equations by a constant so that the coefficients of that variable are the same in both equations.
- Add or subtract the two equations to eliminate the chosen variable. This will leave an equation in one variable.
- Solve for the remaining variable by substituting its value into either of the original equations.
- Check the solution by substituting the values into both equations and verifying that they are true.

Elimination: Linear simultaneous equations in two variables

Examples

- Solve the following simultaneous equations

$$3x + 5y = 25 \ldots\ldots(i)$$

$$2x + 4y = 18 \ldots\ldots(ii)$$

- $multiply\ eqn(i) by\ 2\ and\ multiply\ eqn\ (ii) by\ 3$

$$6x + 10y = 50 \ldots\ldots\ldots(iii)$$

$$6x + 12y = 54 \ldots\ldots(iv)$$

$$subtract\ eqn\ iv\ from\ eqn\ (iii)\ to\ eliminate\ x$$

$$-2y = -4$$

$$y = 2$$

$$substitute\ \ y = 2\ into\ eqn\ (i)$$

$$3x + 5 \times 2 = 25$$

$$3x = 15,$$

$$x = 5$$

$$5a + 2b = 20 \ldots\ldots(i)$$

$$6a + 3b = 15 \ldots\ldots(ii)$$

- $we\ shall\ eliminate\ b\ by\ multiplying\ 3\ by\ eqn\ (i) and\ 2\ by\ eqn\ (ii)$

$$15a + 6b = 60 \ldots\ldots(iii)$$

$$12a + 6b = 30 \ldots\ldots(iv)$$

- $subtracting\ eqn\ (iii) and\ eqn\ (iv) we\ obtain$

$$3a = 30$$

$$a = 10$$

- $substitute\ a = 10\ into\ eqn\ (i)$

$$5(10) + 2b = 20$$

$$2b = -30$$

$$\mathrm{b} = -15$$

Linear simultaneous equations in three variables

Example

$$3x - y - 2z = 0 \ldots\ldots.(i)$$
$$x + 3y - z = 5 \ldots\ldots(ii)$$
$$2x - y + 4z = 26 \ldots.(iii)$$

- To use elimination we are going to consider eliminating any variable by considering any two equations

$$considering\ equation\ (i) and\ (ii) to\ eliminate\ x$$

- $we\ shall\ multuiply\ 1\ by\ eqn\ (i) and\ 3\ by\ eqn\ (ii) to\ obtain$

$$3x - y - 2z = 0 \ldots\ldots (i)$$

$$3x + 9y - 3z = 15 \ldots\ldots (ii)$$

$$subtracting\ eqn\ (i) and\ eqn\ (ii)$$

$$-10y + z = -15 \ldots\ldots\ldots. (iv)$$

- $also\ considering\ another\ set\ of\ equations\ (i) and\ (iii)$
- $to\ eliminate\ x\ by\ multiplying\ 2\ by\ eqn\ (i) and\ 3\ by\ eqn\ (iii)$

$$6x - 2y - 4z = 0$$

$$6x - 3y + 12z = 78$$

$$subtracting\ the\ new\ eqns\ (i) and\ (iii)$$

we obtain the following

$$y - 16z = -78 \ldots \ldots \ldots \ldots . . (v)$$

- $combining\ eqn\ (iv) and\ (v)\ by\ eliminating\ z\ to\ obtain\ y$

$$-10y + z = -15 \ldots \ldots \ldots .(iv)$$

$$y - 16z = -78 \ldots \ldots \ldots \ldots ..(v)$$

- $multipy\ eqn(iv) by\ 16\ \ then\ add\ to\ equation\ (v)$

$$-159y = -318$$

$$y = 2$$

- Substituting y into equation (iv)

$$z = 5$$

- Substituting y and z into equation (ii)

$$x + 6 - 5 = 5$$

$$x = 4$$

Therefore the values of $x = 4, y = 2\ and\ z = 5$

$$x + 2y - 3z = 0 \ldots\ldots (i)$$
$$3x + 3y - z = 5 \ldots\ldots. (ii)$$
$$x - 2y + 2z = 1 \ldots\ldots (iii)$$

Here we can also consider equation $(i)$and equation (iii) and we realize that we have the same coefficient of x which makes x to be eliminated easily

Therefore we are going to eliminate y by adding eqn $(i)$and eqn (iii)

$$2x - z = 1 \ldots\ldots\ldots. (iv)$$

We also consider eqn (i) and eqn (ii) and eliminate y

$$multiply\ eqn\ (i) by\ 3\ and\ eqn\ (ii) by\ 2$$

$$then\ we\ subtract\ to\ eliminate\ y$$

$$-3x - 7z = -10 \ldots\ldots\ldots\ldots (v)$$

$$we\ shall\ eliminate\ z\ by\ multiplying\ 7\ by\ eqn\ (iv)\ and$$

$$then\ subtract\ with\ eqn\ (v)$$

$$17x = 17$$

$$x = 1$$

$we\ substitute\ x = 1$ into equation eqn $(iv)$to obtain the value of z

$$z = 1$$

We then substitute the value of x and z into any of the above equations

i.e. substituting into eqn (i)

$$y = 1$$

- Therefore the values of $x = 1, y = 1\ and\ z = 1$

Substitution method

- The substitution method is a way to solve a system of linear equations by solving one of the equations for one variable in terms of the other variables, and then substituting that expression into the other equation(s) to eliminate that variable.

- The resulting equation(s) will be in terms of the remaining variable(s), which can then be solved using basic algebraic techniques.

The steps to solve a system of linear equations using the substitution method are:

1. Choose one equation and solve for one variable in terms of the other variables.
2. Substitute the expression found in step 1 into the other equation(s) to eliminate that variable.
3. Solve the resulting equation(s) for the remaining variable(s).
4. Substitute the values found in step 3 back into any of the original equations to solve for the other variable(s).

$$3x - y - 2z = 0 \ldots\ldots (i)$$

$$x + 3y - z = 5 \ldots\ldots. (ii)$$

$$2x - y + 4z = 26 \ldots.. (iii)$$

Basing on the above equations we see that equation (ii)

Can be considered by making x the subject

$$x = 5 - 3y + z \ldots\ldots\ldots (iv)$$

Substitute $eqn(iv)$ into the remaining equations

From equation (i)

$$3(5 - 3y + z) - y - 2z = 0$$

$$15 - 9y + 3z - y - 2z = 0$$

$$-10y + z = -15 \ldots\ldots\ldots.. (v)$$

From equation (iii)

$$2(5-3y+z)-y+4z=26$$

$$10-6y+2z-y+4z=26$$

$$-7y+6z=16 \dots\dots\dots.(vi)$$

Using elimination method on equation $(v) and\ (vi)$

Eliminating y and find the value of z

$$-70y+7z=-105$$

$$-70y+60z=160$$

Subtracting to get the value of z

$$z=5$$

Substitute z into eqn (v)

$$-10y + 5 = -15$$

$$-10y = -20$$

$$y = 2$$

Substituting y and z into equation (iv)

$$x = 5 - 6 + 5$$

$$x = 4$$

Therefore the values of $x = 4, y = 2\ and\ z = 5$

Application of systems of linear equations

A company is designing a network topology to connect its five offices (A, B, C, D, and E) located in different cities. Each office has different requirements for the bandwidth capacity to the server. The company wants to minimize the cost of data transfer between the offices while meeting the minimum bandwidth requirements of each office. How can systems of linear equations be used to optimize the network topology and routing protocols?

To use systems of linear equations to optimize the network topology and routing protocols for the five offices, the following steps can be taken:

1. Define the variables: The variables can represent the amount of data traffic between each pair of offices, which can be measured in megabits per second (Mbps).

- For example, xAB can represent the data traffic between offices A and B, xBC can represent the data traffic between offices B and C, and so on.

2.Formulate the constraints: The minimum bandwidth requirements for each office can be translated into constraints.

For example, the constraint for office A can be written as

$xAB + xAC + xAD + xAE \geq 100\, Mbps$, since office A requires at least 100 Mbps to the server.

Similarly, the constraints for offices B, C, D, and E can be written as

$$xBA + xBC + xBD + xBE \geq 50\ Mbps,$$

$$xCA + xCB + xCD + xCE \geq 75\ Mbps,$$

$$xDA + xDB + xDC + xDE \geq 60\ Mbps,$$ and

$$xEA + xEB + xEC + xED \geq 40\ Mbps,$$ respectively

3. Formulate the objective function: The objective function can represent the total cost of data transfer between the offices, which can be a function of the data traffic between each pair of offices and the cost per unit of data transfer.

- For example, if the cost per unit of data transfer is $0.05 per Mbps, the objective function can be written as

$$0.05(xAB + xBA + xAC + xCA + xAD + xDA + xAE + xEA + xBC + xCB + xBD + xDB + xBE + xEB + xCD + xDC + xCE + xEC + xDE + xED).$$

4. Solve the system of linear equations: The system of linear equations can be solved using a matrix equation or other linear algebra techniques to find the optimal data traffic between each pair of offices that meets the minimum bandwidth requirements and minimizes the cost of data transfer.

5. **Implement the optimal network topology and routing protocols:** Based on the solutions obtained in step 4, the optimal network topology and routing protocols can be implemented to minimize the cost of data transfer while meeting the minimum bandwidth requirements of each office.

This can involve configuring the network hardware, software, and protocols to route the data traffic between the offices in the most efficient and cost-effective way.

References

- Strang, G. (2016). Linear algebra and its applications (5th ed.). Cengage Learning

Next week we shall look at

Systems of linear equations: Crammer's rule

Thanks for Listening to me