Mathematics For Information Technology

Week 11: Graph theory: Definitions, Types, Paths, Applications

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Outline

- Intended learning outcome
- Graph theory -Definitions
- Types of Graphs
- Paths
- Applications of Graph Theory

Learning outcomes

- Students should have a solid understanding of the fundamental concepts and definitions in graph theory, including graphs, vertices, edges, and adjacency.
- Classify and Analyze Types of Graphs
- Understand the concepts of paths and cycles in graphs.
- Recognize and apply graph theory in various real-world applications

Graph Theory

- A graph is a structure amounting to a set of objects in which some pairs of the objects are in some sense "related".
- The objects of the graph correspond to **vertices** and the relations between them correspond to **edges**.
- A graph is depicted diagrammatically as a set of dots depicting vertices connected by lines or curves depicting edges.
- Formally, "A graph consists of , a non-empty set of **vertices** (or nodes) and, a set of **edges**. Each edge has either one or two vertices associated with it, called its **endpoints**."

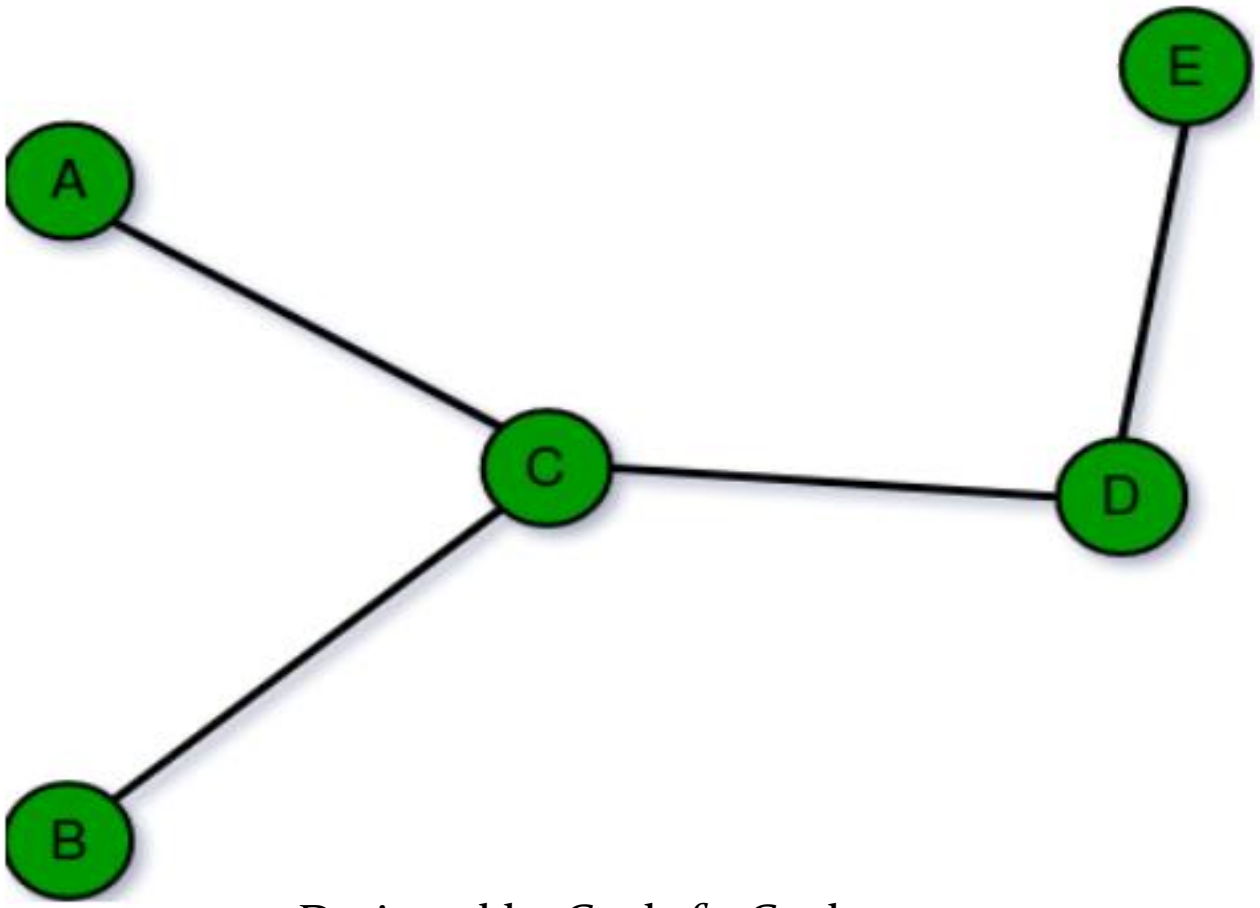
Basic graph Terminology

- **Adjacency** – In a graph G, two vertices u and v are said to be **adjacent** if they are the endpoints of an edge.
- The edge $\{u, v\} - e$ is said to be incident with the vertices.
- In case the edge is directed, u is said to be adjacent to u and v is said to be adjacent from u. Here, u is said to be the **initial vertex** and v is said to be **terminal vertex**.
- **Degree** – The degree of a vertex is the number of edges incident with it, except the self-loop which contributes twice to the degree of the vertex. Degree of a vertex u is denoted as $\deg(u)$.

Types of graphs

Finite Graphs

- A graph is said to be finite if it has a finite number of vertices and a finite number of edges.
- A finite graph is a graph with a finite number of vertices and edges. In other words, both the number of vertices and the number of edges in a finite graph are limited and can be counted.
- Finite graphs are often used to model real-world situations, where there is a limited number of objects and relationships



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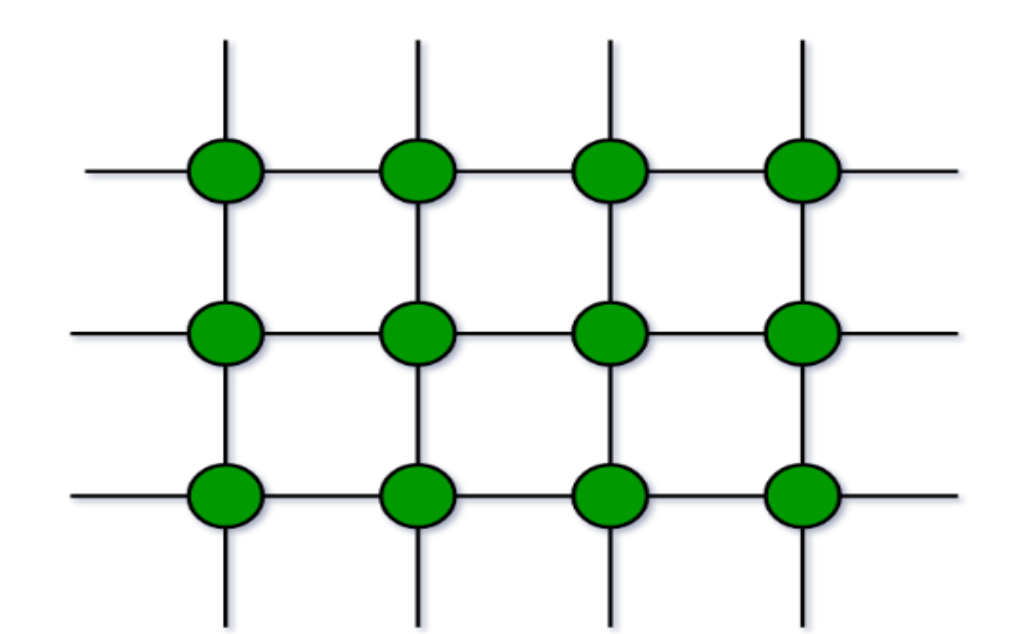
Infinite Graph

- A graph is said to be infinite if it has an infinite number of vertices as well as an infinite number of edges.



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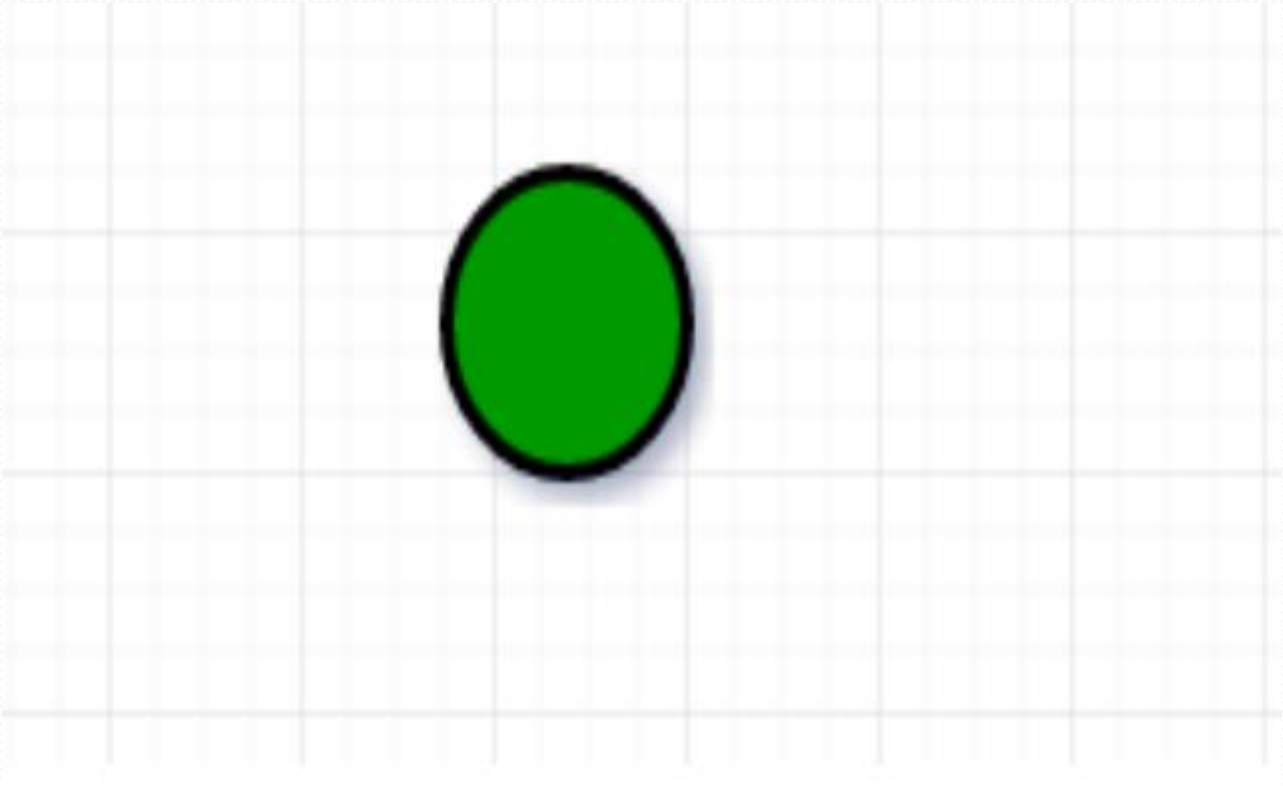
Trivial Graph

- A graph is said to be trivial if a finite graph contains only one vertex and no edge.
- A trivial graph is a graph with only one vertex and no edges. It is also known as a singleton graph or a single vertex graph.



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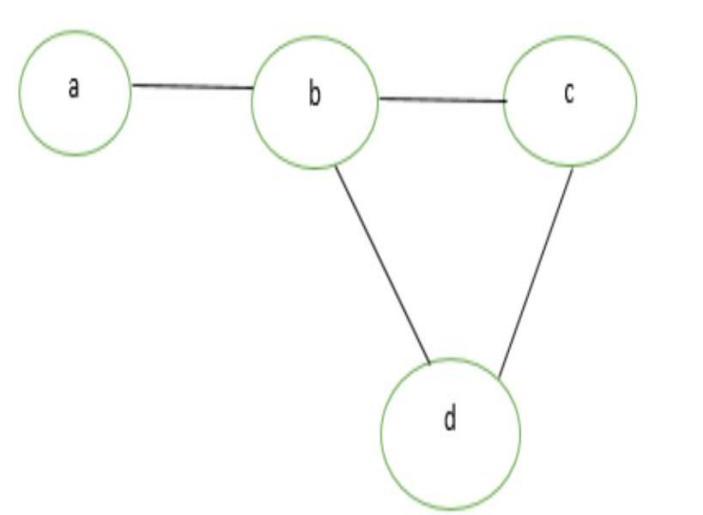
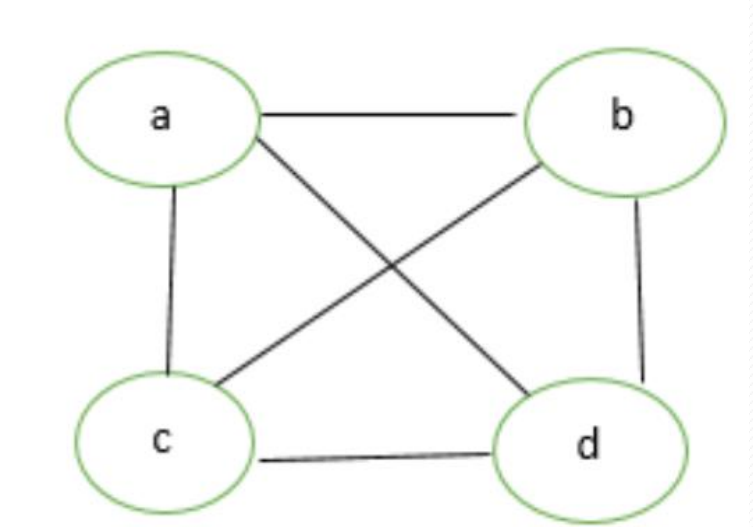
Simple Graph:

- A simple graph is a graph that does not allow loop and parallel edges.





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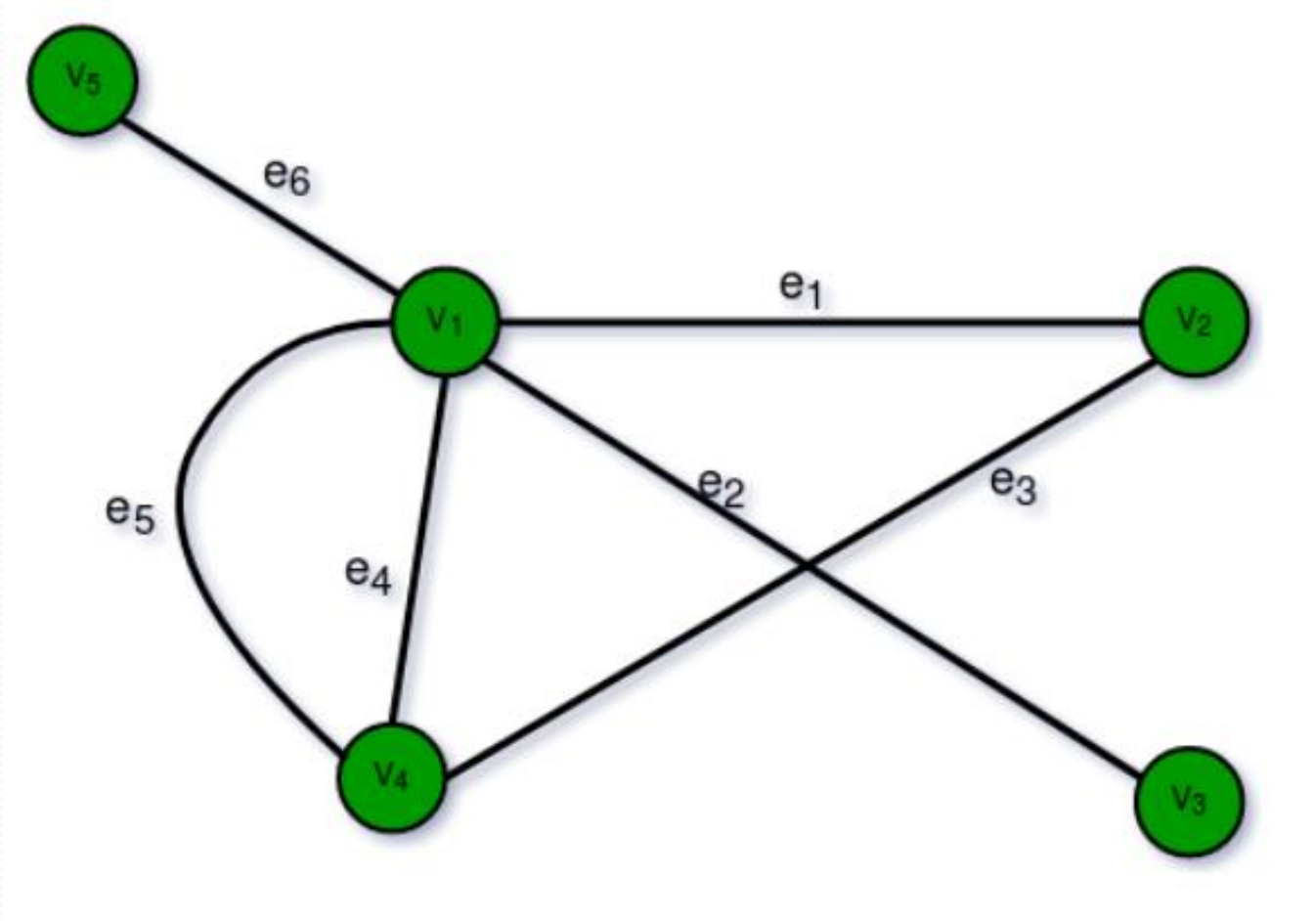
Multi Graph

- Any graph which contains some parallel edges but doesn't contain any self-loop is called a multigraph. For example a Road Map.
- **Parallel Edges:** If two vertices are connected with more than one edge then such edges are called parallel edges that are many routes but one destination.
- **Loop:** An edge of a graph that starts from a vertex and ends at the same vertex is called a loop or a self-loop.



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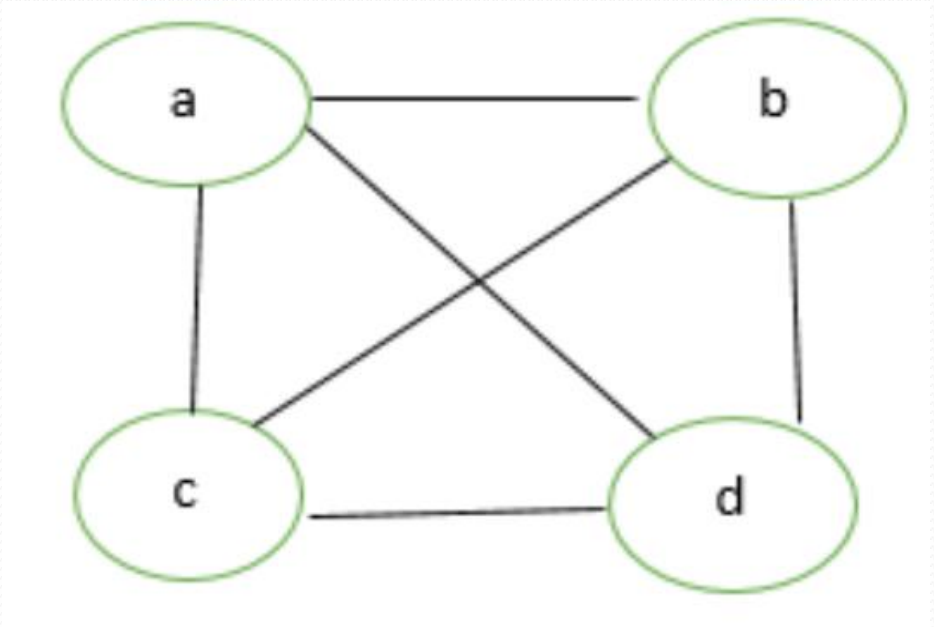
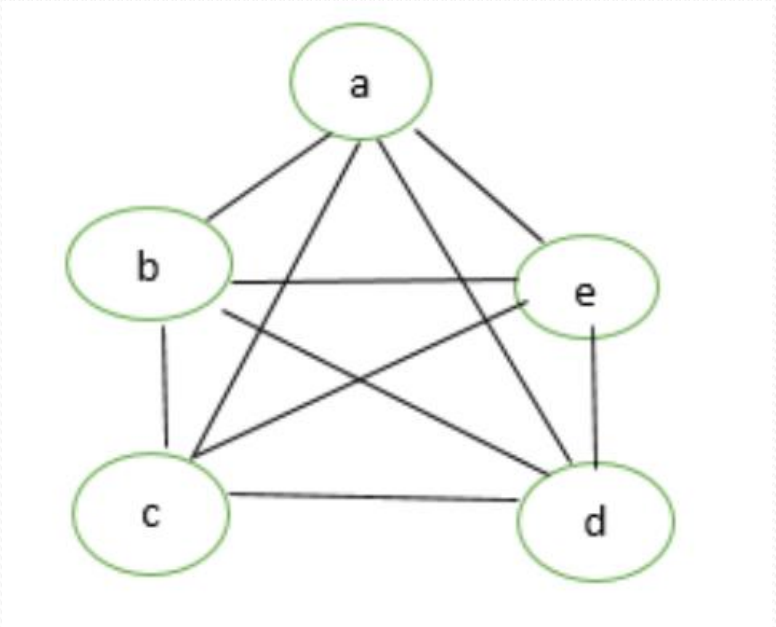
Complete Graph

- A simple graph with n vertices is called a complete graph if the degree of each vertex is $n - 1$, i.e.
- One vertex is attached with $n - 1$ edges or the rest of the vertices in the graph.
- A complete graph is also called Full Graph.





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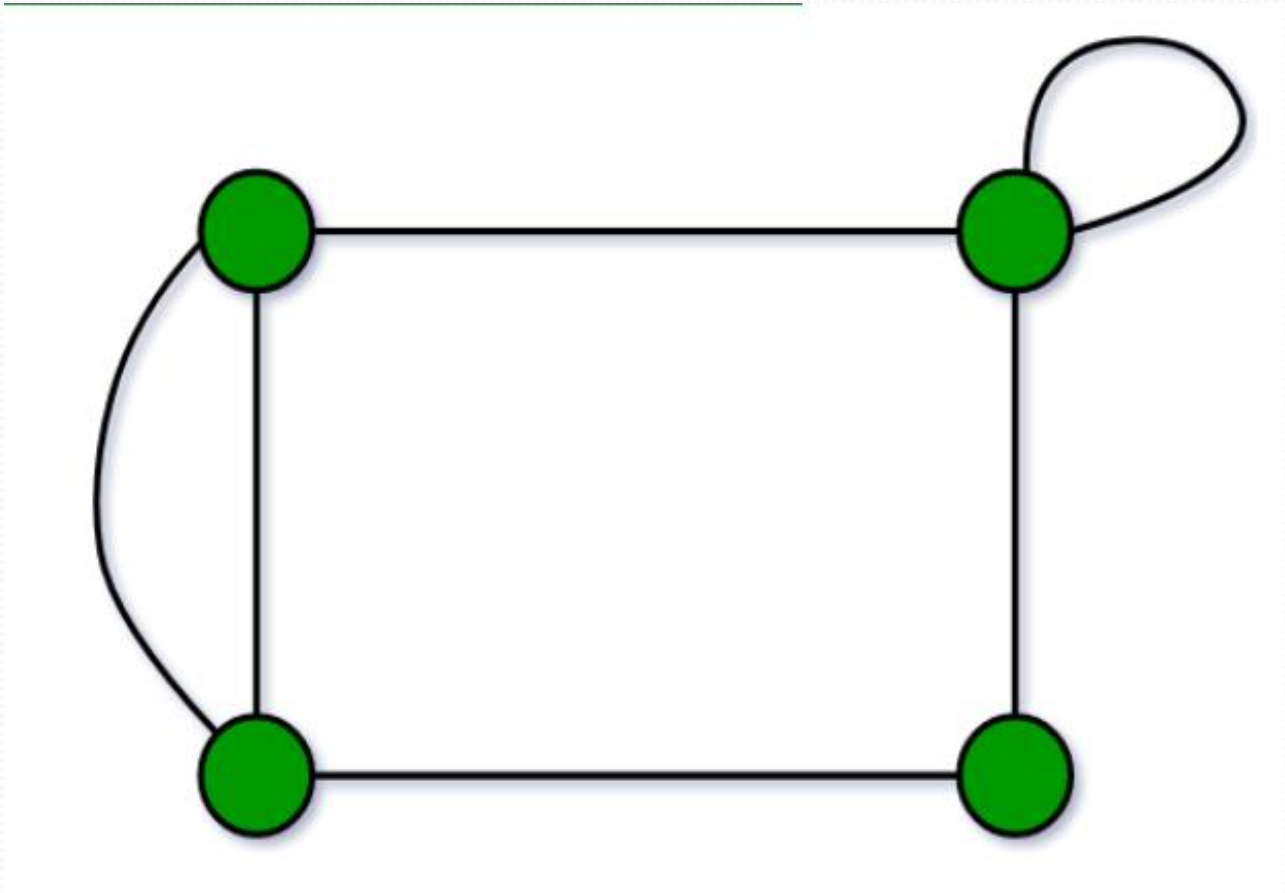
Proof for complete graph

- Consider a complete graph with n nodes. Each node is connected to other $n-1$ nodes.
- Thus it becomes $n(n-1)$ edges.
- But this counts each edge twice because this is a undirected graph so divide it by 2.
- Thus it becomes $\frac{n(n-1)}{2}$

Pseudo Graph

- A graph G with a self-loop and some multiple edges is called a pseudo graph.
- A pseudo graph is a type of graph that allows for the existence of loops (edges that connect a vertex to itself) and multiple edges (more than one edge connecting two vertices).
- In contrast, a simple graph is a graph that does not allow for loops or multiple edges.



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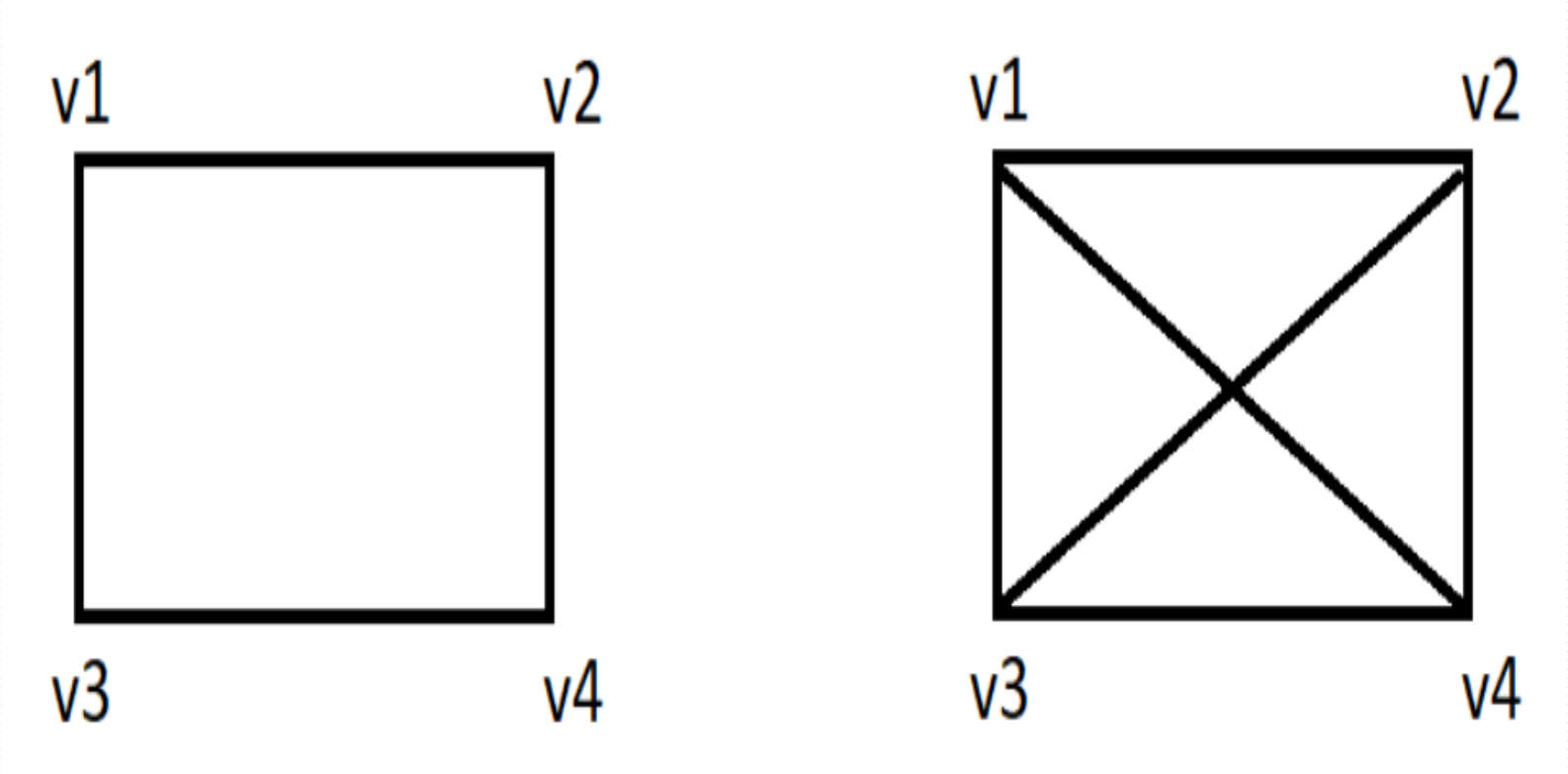
Regular Graph

- A simple graph is said to be regular if all vertices of graph G are of equal degree.
- All complete graphs are regular but vice versa is not possible.
- A regular graph is a type of undirected graph where every vertex has the same number of edges or neighbors.
- In other words, if a graph is regular, then every vertex has the same degree.



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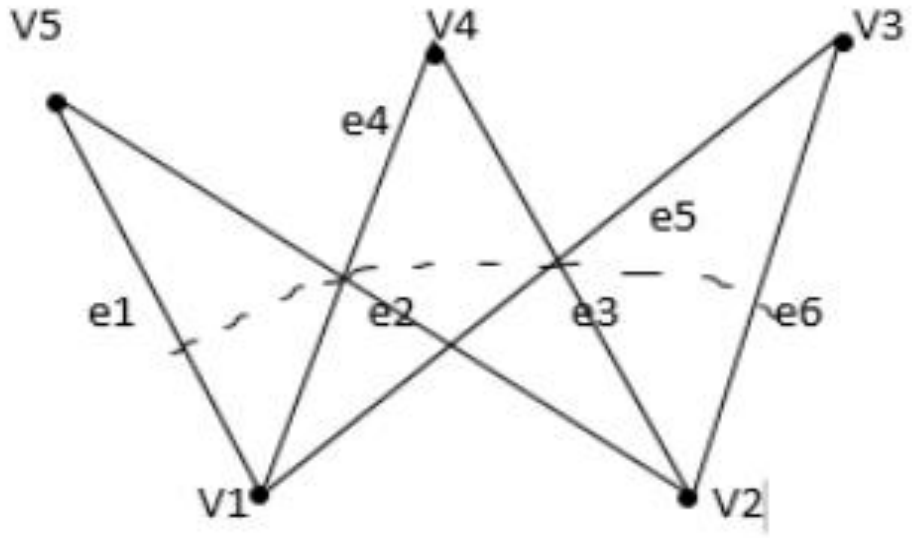
Bipartite Graph

- A graph $G = (V, E)$ is said to be a bipartite graph if its vertex set $V(G)$ can be partitioned into two non-empty disjoint subsets.
- $V_1(G)$ and $V_2(G)$ in such a way that each edge E of E(G) has one end in $V_1(G)$ and another end in $V_2(G)$.
- The partition $V_1 \cup V_2 = V$ is called Bipartite of G.
- Here in the figure: $V_1(G) = \{V5, V4, V3\}$ and $V_2(G) = \{V1, V2\}$



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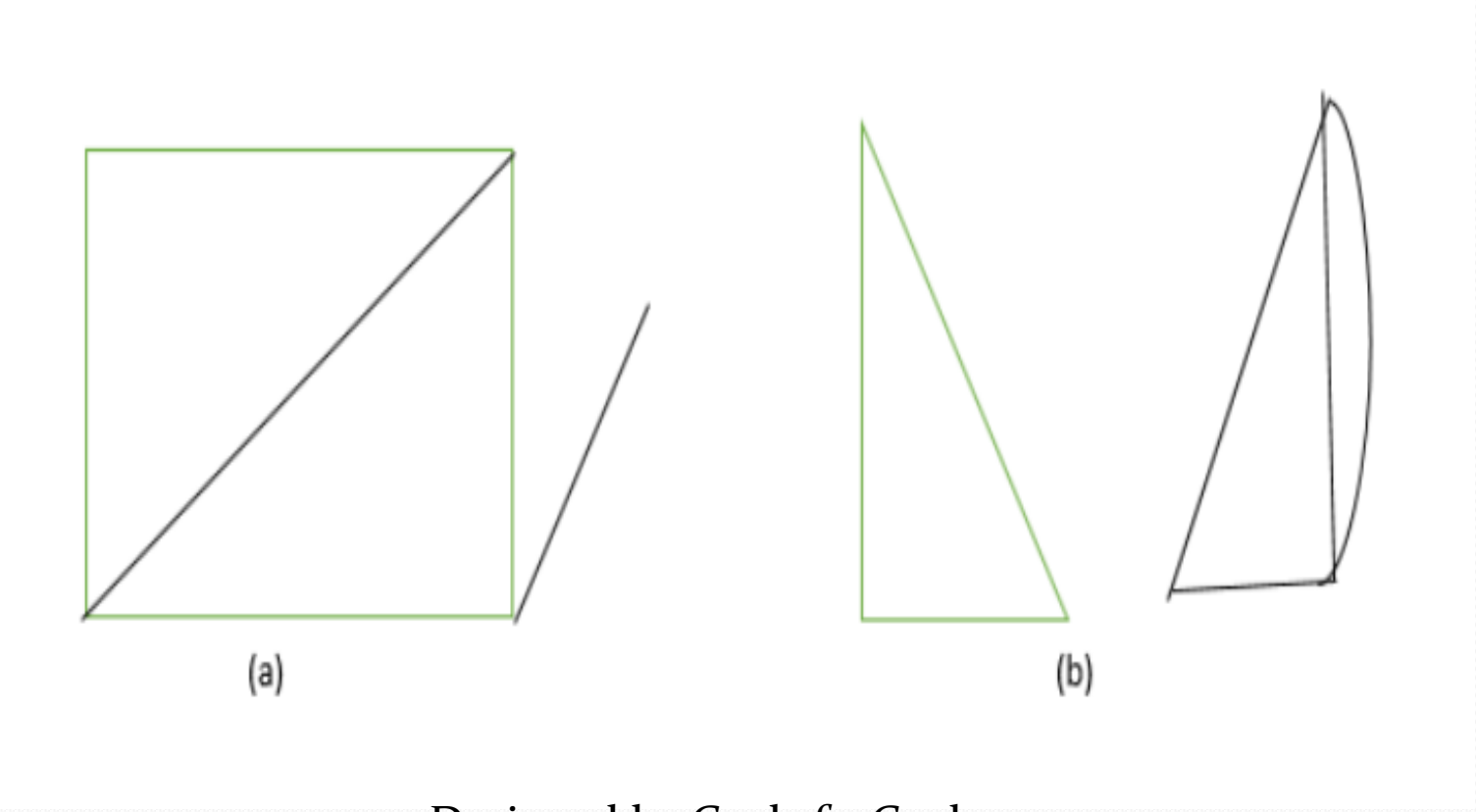
Connected or Disconnected Graph

- Graph G is said to be connected if any pair of vertices (V_i, V_j) of a graph G is reachable from one another.

Or

- A graph is said to be connected if there exists at least one path between each and every pair of vertices in graph G, otherwise, it is disconnected.
- A null graph with n vertices is a disconnected graph consisting of n components.
- Each component consists of one vertex and no edge.



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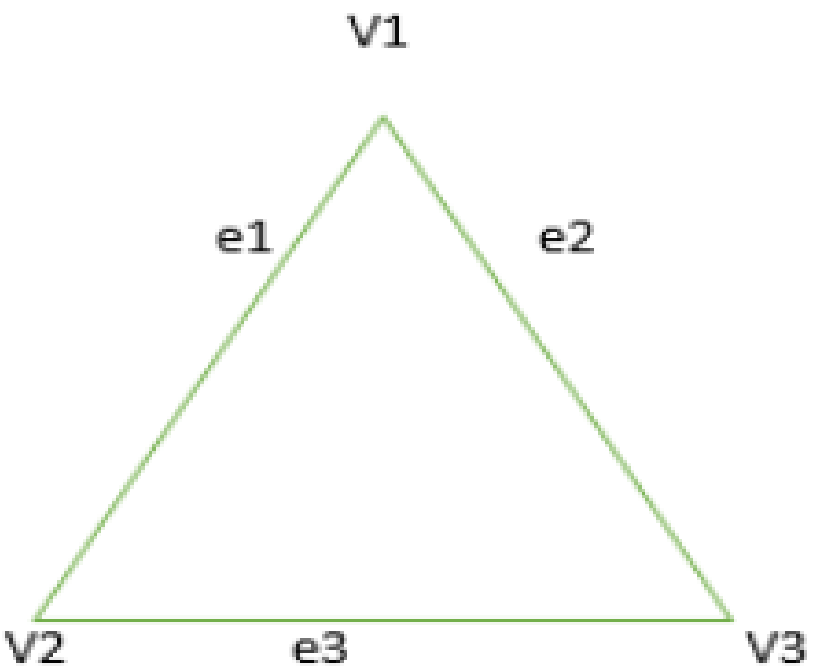
Cyclic Graph

- A graph G consisting of n vertices and $n >= 3$ that is $V_1, V_2, V_3 - - - - V_n$ and edges
- $(V_1, V_2), (V_2, V_3), (V_3, V_4) - - - - (V_n, V_1)$ are called cyclic graph otherwise its acyclic graph.



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Advantages of graphs

- Graphs can be used to model and analyze complex systems and relationships.
- They are useful for visualizing and understanding data.
- Graph algorithms are widely used in computer science and other fields, such as social network analysis, logistics, and transportation.
- Graphs can be used to represent a wide range of data types, including social networks, road networks, and the internet.

Disadvantages of graphs

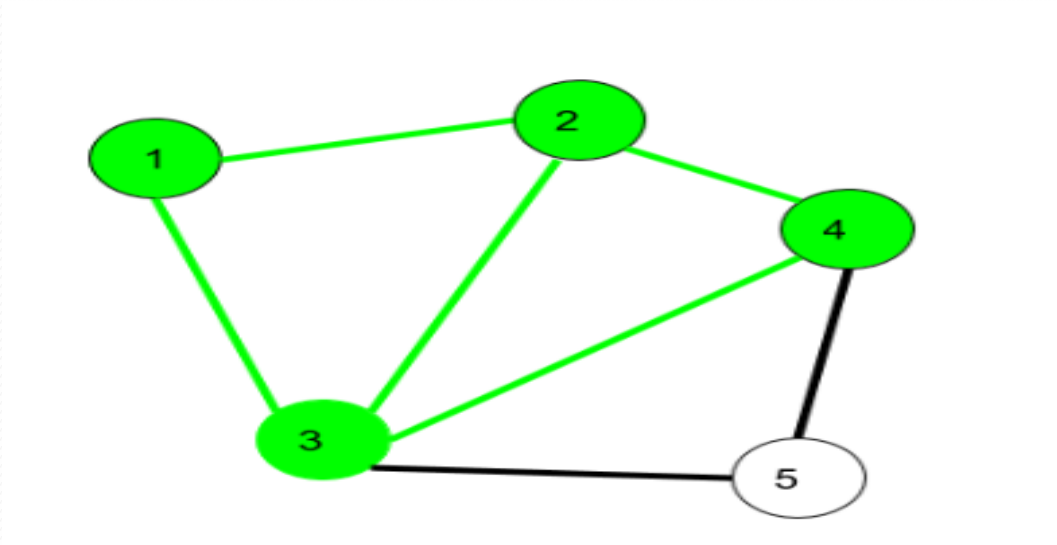
- Large graphs can be difficult to visualize and analyze.
- Graph algorithms can be computationally expensive, especially for large graphs.
- The interpretation of graph results can be subjective and may require domain-specific knowledge.
- Graphs can be susceptible to noise and outliers, which can impact the accuracy of analysis results.

More Terms used in graph theory

Walk –

- A walk is a sequence of vertices and edges of a graph i.e. if we traverse a graph then we get a walk.

- Note: No edge appears more than once while as a vertex, however may appear more than once.



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Here, 1 –> 2 –> 3 –> 4 –> 2 –> 1 –> 3 is a walk.

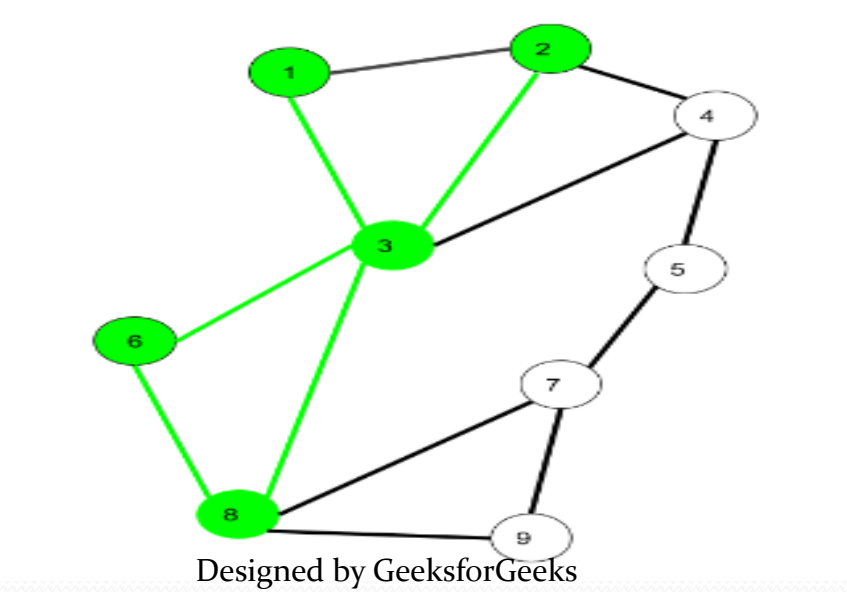
- **Open walk-** A walk is said to be an open walk if the starting and ending vertices are different i.e. the origin vertex and terminal vertex are different.
- **Closed walk-** A walk is said to be a closed walk if the starting and ending vertices are identical i.e. if a walk starts and ends at the same vertex, then it is said to be a closed walk.
- In the above diagram:

1 –> 2 –> 3 –> 4 –> 5 –> 3 is an open walk.

1 –> 2 –> 3 –> 4 –> 5 –> 3 –> 1 is a closed walk.

Trail –

- Trail is an open walk in which no edge is repeated.
- Vertex can be repeated.



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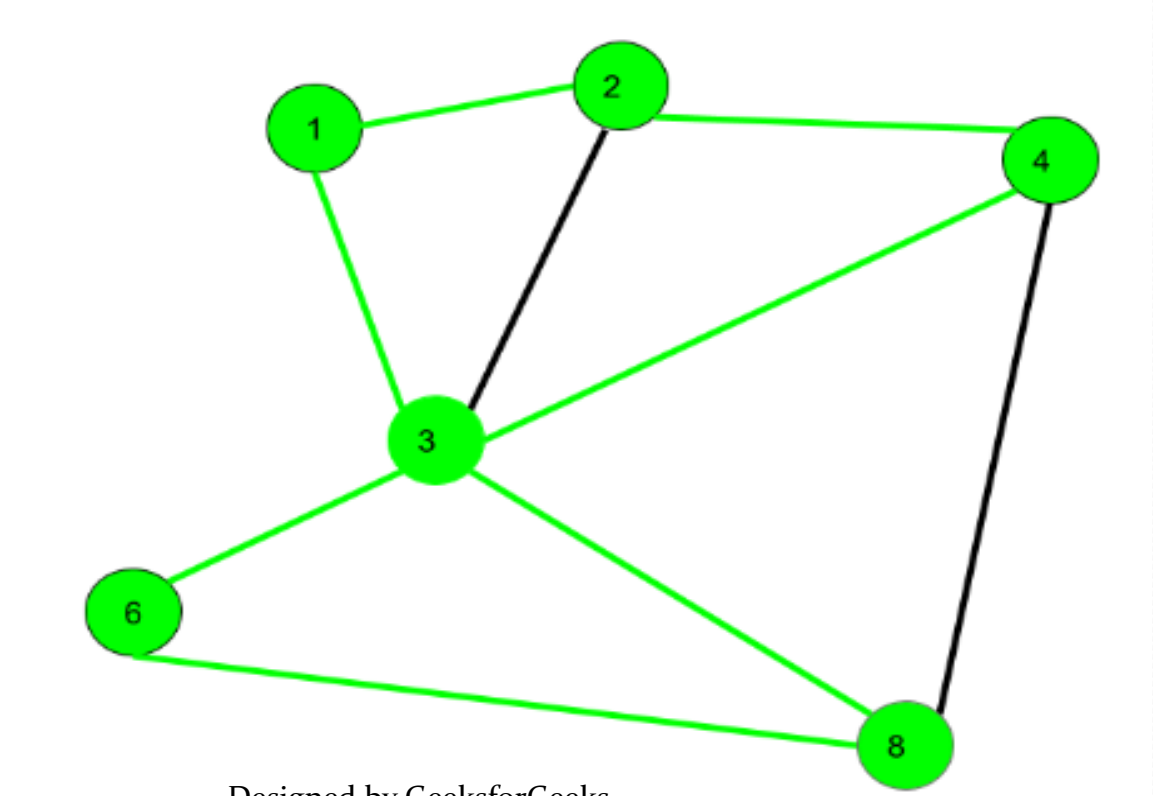
Here 1->3->8->6->3->2 is trail

Also 1->3->8->6->3->2->1 will be a closed trail

Circuit

- Traversing a graph such that no edge is repeated but vertex can be repeated and it is closed also i.e. it is a closed trail.
- Vertex can be repeated.
- Edge can not be repeated.



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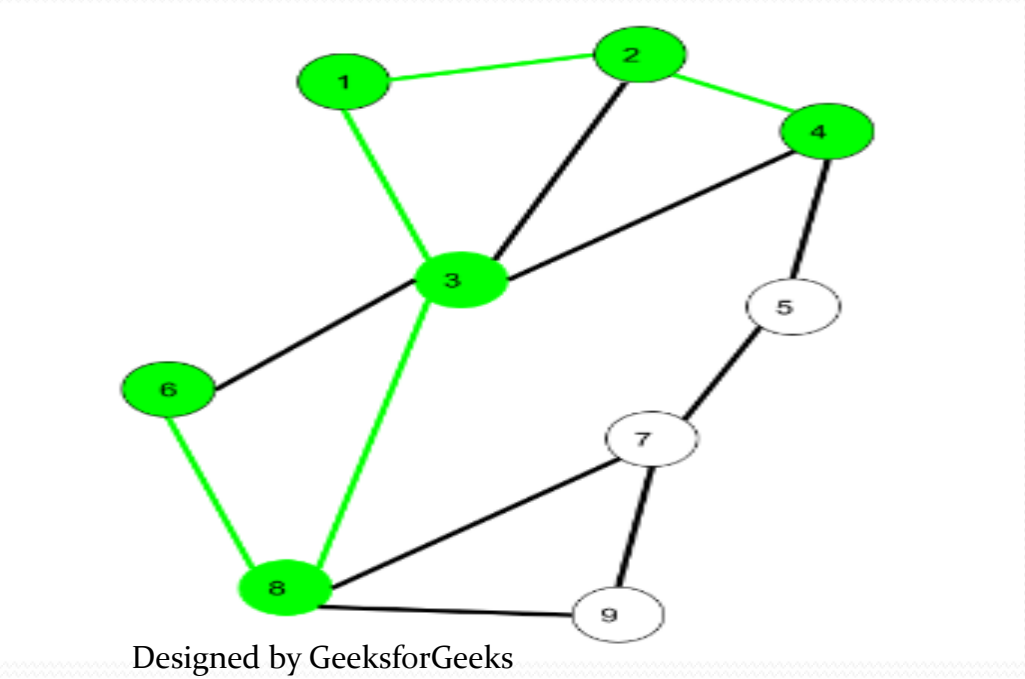
Here 1->2->4->3->6->8->3->1 is a circuit

Path

- It is a trail in which neither vertices nor edges are repeated i.e. if we traverse a graph such that we do not repeat a vertex and nor we repeat an edge.
- As path is also a trail, thus it is also an open walk.
- Another definition for path is a walk with no repeated vertex.
- Vertex not repeated
- Edge not repeated



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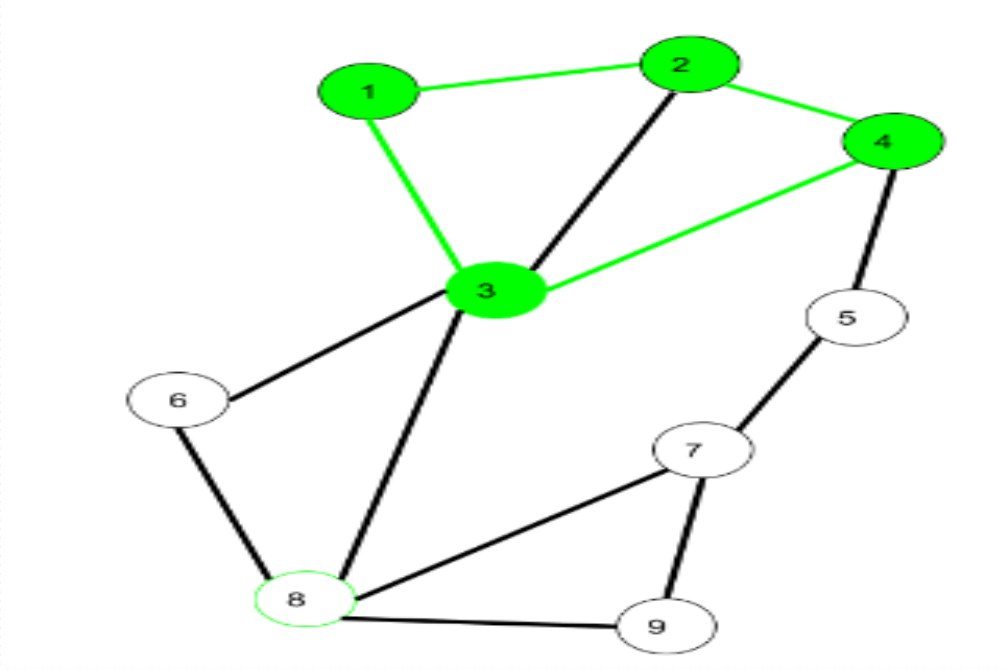
Here 6->8->3->1->2->4 is a Path

Cycle

- Traversing a graph such that we do not repeat a vertex nor we repeat an edge but the starting and ending vertex must be same i.e. we can repeat starting and ending vertex only then we get a cycle.
- Vertex not repeated
- Edge not repeated



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Here 1->2->4->3->1 is a cycle.

Application of graph theory

Graph is a data structure which is used extensively in our real-life.

- Social Network: Each user is represented as a node and all their activities, suggestion and friend list are represented as an edge between the nodes.
- Google Maps: Various locations are represented as vertices or nodes and the roads are represented as edges and graph theory is used to find shortest path between two nodes.

- Recommendations on e-commerce websites: The “Recommendations for you” section on various e-commerce websites uses graph theory to recommend items of similar type to user’s choice.

- Graph theory is also used to study molecules in chemistry and physics.

- Bioinformatics: Graph theory is used in bioinformatics to represent and analyze biological data. For example, it is employed to model protein-protein interactions, gene regulatory networks, metabolic pathways, and phylogenetic trees.

- Electrical and Telecommunications Networks: Graph theory is applied to analyze electrical circuits and telecommunications networks. It helps model power grids, circuit boards, telecommunication networks, and signal processing.
- Game Theory: Graph theory is used in game theory to model and analyze strategic interactions among players. It helps study game structures, equilibrium points, and optimal strategies.

Reference

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- West, D. B. (2001). Introduction to graph theory (2nd ed.). Prentice Hall.
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End of lecture 11

Next topic: Set theory

Thank you