Mathematics For Information Technology

Week 12: Set theory : Sub-sets, Operations (Union, Intersection, complements)

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outline

- Intended Learning Outcomes
- Sets (Definition, types)
- Set Operations i.e. Union of sets, Intersection of sets, Complement of a set

Intended learning outcome

- Understand the concept of subsets and be able to determine if one set is a subset of another. Use the notation $A \subseteq B$ to denote that set A is a subset of set B.

- Perform Set Operations i.e. Union ($A \cup B$), Intersection ($A \cap B$), Complement (A')

Set theory

Definition:

- A set is a collection of objects or members, which are related in some way.
- Sets are denoted by capital letters, e.g. set **A**, **B**, **M** etc.

Representation of Sets

- Sets can be represented in two ways:
- **Roster Form**: In roster form, all the elements of the set are listed, separated by commas and enclosed between curly braces { }.
- **Set Builder Form**: If set **S** has all the elements which are even prime numbers, it is represented as:

$$S = \{\, x\colon\ x\ is\ an\ even\ prime\ number \}$$

- where 'x' is a symbolic representation that is used to describe the element.

': ' means 'such that', '{}' means 'the set of all'

- So, $S\ = \{\, x\colon x\ is\ an\ even\ prime\ number\ \}$ is read as 'the set of all x such that x is an even prime number'.

Terms used:

Member (element) of a set

- The objects in a set are called members or elements of the set enclosed in curly brackets {}.
- The symbol Є is a short form of saying "is a member of" and " $\notin$ " is for "not a member of".
- Consider set $\boldsymbol{A} = \{1, 3, 4, 6\}$. Then $1 \in \boldsymbol{A}, 3 \in \boldsymbol{A}, 4 \in \boldsymbol{A}$, and $6 \in \boldsymbol{A}$. Whereas 7, 8, 9 etc. are not members of set **A**, i.e. $7 \notin \boldsymbol{A}, 8 \notin \boldsymbol{A}, 9 \notin \boldsymbol{A}$, etc.

Subsets

- Set **A** is said to be a subset of set **B** if every element of set **A** is also in set **B**. E.g. given that $\boldsymbol{B} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $\boldsymbol{A} = \{1, 3, 5, 7\}$.
- Here every element of set **A** is also in set **B**.
- Therefore, set **A** is a subset of **B**.
- The symbol ⊂ is for ‘’subset of’’ and ⊄ is for ‘’not a subset of’’.
- Therefore $A \subset \boldsymbol{B}$ **or** $\boldsymbol{B} \supset \boldsymbol{A}$ but $B \not\subset A$ because not all elements of B are in A.

Empty set (null set)

- An empty set is a set with no element. It is at time called null set.
- The null set is denoted by the symbol { } or ϕ.

Note:

- The empty set { } is not the same as {0}.
- This is because the set {0} has one element which is o whereas the set { } has no element.

Finite sets

- The set is called finite if the elements of the set can be counted.

Example

- Consider the following sets:

$$D = \{days\ of\ the\ week\},$$

$$F = \{Factors\ of\ 12\}$$

$$G = \{whole\ numbers\ greater\ than\ 5\ but\ less\ than\ 11\}$$

- We can list all the members of these sets.

$$D = \{Monday, Tuesday, Wednesday,$$

$$Thursday, Friday, Saturday, and\ Sunday\}$$

$$F = \{1, 2, 3, 4, 6, and\ 12\}$$

$$G = \{6, 7, 8, 9, and\ 10\}$$

Infinite sets

- These are sets with unlimited number of elements.

Example

- Given the following sets:

$$W = \{whole\ numbers\}$$

$$R = \{real\ numbers\}$$

$$M = \{multiple\ of\ 3\}$$

- Here, we cannot list all the members of these sets.

$$W = \{0,1,2,3,4,5,6, \ldots ..\}$$

$$R = \{\ldots \ldots -2, -1, 0, 1, 2\ \ \ldots\}$$

$$M = \{3, 6, 9, 12, \ldots .\}$$

- All members of these sets cannot be exhausted so they are infinite sets.

Number of elements in a set

- The number of element in a finite set can be counted.
- The number of elements of set **A** is denoted by $n(A)$ *and it's the total number of elements in set A*

Example

- Find the number of elements in the following sets.

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 12\}$$

$$B = \{2, 4, 6, 8, 9\}$$

Solution

- $n(A) = 9$
- $n(B) = 5$

Example

Given that set $B = \{factors\ of\ 24\}$, Write out set B in full and

Find $n(B)$

Solution

$$B = \{1, 2, 3, 4, 6, 8, 12, 24\}$$

$$n(B) = 8$$

Example

Given that set

$$N = \{natural\ numbers\ from\ 2\ to\ 11\}$$

Write out set N in full and find $n(N)$

Solution

a) $N = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

b) $n(N) = 10$

Equal sets

- Two or more sets are equal if they contain the same elements. e.g.

- Given set $\boldsymbol{A} = \{1, 3, 5, 7\}$ and $\boldsymbol{B} = \{1, 3, 5, 7\}$ are equal sets.

- Here $A \subset B \;\; and \; B \subset A$

Equivalent sets

- Two or more sets are said to be equivalent if they contain the same number of elements e.g. set $\boldsymbol{A} = \{a, e, i, o, u\}$ and $\boldsymbol{B} = \{2, 4, 6, 8, 10\}$.
- Sets **A** and **B** contain the same number of elements which is 5.
- We therefore say that they are equivalent sets.

Union of sets

- The union of two or more sets is the set of containing all elements of the given sets.
- The symbol for union is ∪.
- Suppose the union of two sets X and Y can be represented as $X \cup Y$

Union of Sets Definition

- The union of two sets X and Y is equal to the set of elements that are present in set X, in set Y, or in both the sets X and Y.
- This operation can be represented as;

$$\boldsymbol{X \cup Y = \{a : a \in X \ or \ a \in Y\}}$$

- Let us consider an example, say; set $A = \{1, 3, 5\}$ and set $B = \{1, 2, 4\}$ then;

$$A \cup B = \{1, 2, 3, 4, 5\}$$

Example

- Given that: $M = \{1, 2, 3, 4\}$
  $N = \{3, 4, 6, 7\}$

a) List $M \cup N$

b) Find $n(M \cup N)$

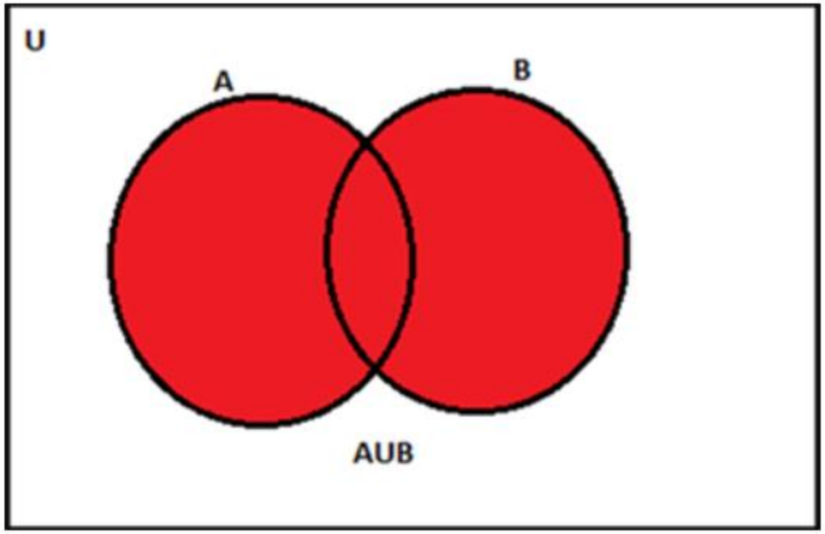
Solution

1) $M \cup N = \{1,2,3,4,6,7\}$

2) $n(M \cup N) = 6$

Venn Diagram of Union of Sets

- Let us consider a universal set U such that A and B are the subsets of this universal set.



Designed by BYJUS

- Thus, the union of two sets A and B is given by a set C, which is also a subset of the universal set U such that C consists of all those elements or members which are either in set A or set B or in both A and B i.e.,

$$C = A \cup B = \{x : x \in A \, or \, x \in B\}$$

- The union of set A and set B is equal to the set containing all the elements in A and B. This is represented as $A\ U\ B$ and can be read as "A union B" or "A or B".

Number of Elements in A union B

- Consider two sets, A and B, such that the number of elements in the union of A and B can be calculated as follows.

$$n(A\ U\ B) = n(A) + n(B) - n(A \cap B)$$

Here,

- $n(A\ U\ B)$ = Total number of elements in A U B; is called the cardinality of a set A U B

- $n(A)$ = Number of elements in A; is called the cardinality of set A

- $n(B)$ = Number of elements in B; is called the cardinality of set B

- $n(A \cap B)$ = The number of elements that are common to both A and B; is called the cardinality of set $A \cap B$

Properties of Union of Sets

Commutative Law: The union of two or more sets follows the commutative law i.e., if we have two sets A and B then,

$$A \cup B = B \cup A$$

Example: A = {a, b} and B = {b, c, d}

So, $A \cup B = \{a, b, c, d\}$

$$B \cup A = \{b, c, d, a\}$$

- Since, in both the union, the group of elements is same.
- Therefore, it satisfies commutative law.

$$A \cup B = B \cup A$$

Associative Law: The union operation follows the associative law i.e., if we have three sets A, B and C then

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Example:

$A = \{a, b\}$ and $B = \{b, c, d\}$ and $C = \{a, c, e\}$

$$(A \cup B) \cup C = \{a, b, c, d\} \cup \{a, c, e\} = \{a, b, c, d, e\}$$

$$A \cup (B \cup C) = \{a, b\} \cup \{b, c, d, e\} = \{a, b, c, d, e\}$$

Hence, associative law proved.

Identity Law: The union of an empty set with any set A gives the set itself i.e.,

$$A \cup \emptyset = A$$

- Suppose, $A = \{a, b, c\}$ and $\emptyset = \{\}$
- Then, $A \cup \emptyset = \{a, b, c\} \cup \{\} = \{a, b, c\}$

Idempotent Law: The union of any set A with itself gives the set A i.e.,

$$A \cup A = A$$

- Suppose, $A = \{1, 2, 3, 4, 5\}$
- Then $A \cup A = \{1, 2, 3, 4, 5\} \cup \{1, 2, 3, 4, 5\}$

$$= \{1, 2, 3, 4, 5\} = A$$

Domination Law: The union of a universal set U with its subset A gives the universal set itself.

$$A \cup U = U$$

- Suppose, $A = \{1, 2, 4, 7\}$ and $U = \{1, 2, 3, 4, 5, 6, 7\}$
- Then $A \cup U = \{1, 2, 4, 7\} \cup \{1, 2, 3, 4, 5, 6, 7\}$

$$= \{1, 2, 3, 4, 5, 6, 7\} = U$$

- Hence, proved.

Example :

Let U be a universal set consisting of all the natural numbers until 20 and set A and B be a subset of U defined as

$$A = \{2, 5, 9, 15, 19\} \text{ and } B = \{8, 9, 10, 13, 15, 17\}.$$

Find A ∪ B.

Solution

Given,

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

$$A = \{2, 5, 9, 15, 19\}$$

$$B = \{8, 9, 10, 13, 15, 17\}$$

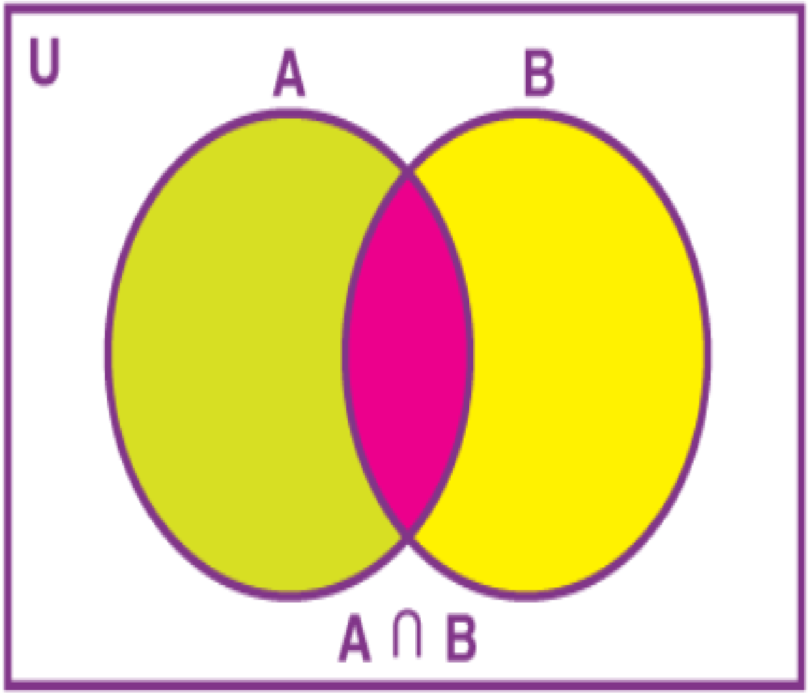
$$A \cup B = \{2, 5, 8, 9, 10, 13, 15, 17, 19\}$$

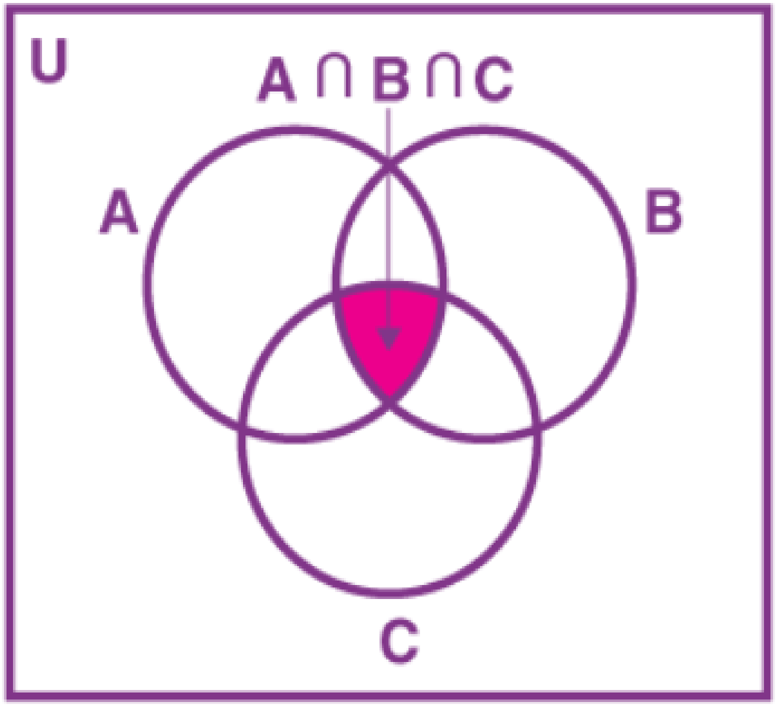
Intersection of sets

- The intersection of two sets or more sets is the set of elements that are in both sets.
- It is denoted by the " ∩ " symbol. All those elements which belong to both A and B represent the intersection of A and B. Thus we can say that,

$$A \cap B = \{x : x \in A \, and \, x \in B\}$$

- For n sets $A_1, A_2, A_3, \ldots, An_,$ where all these sets are the subset of universal set U, the intersection is the set of all the elements which are common to all these n sets.





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Intersection of Two sets

- If A and B are two sets, then the intersection of sets is given by:

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

where

- n(A) is the cardinal number of set A,
- $n(B)$ is the cardinal number of set B,
- $n(A \cup B)$ is the cardinal number of the union of sets A and B.

Example

Given two sets:

$$A \ = \ \{-1, 0, 4, 5, 6, 7\} \text{ and } B \ = \ \{-1, 6, 8, 10\}$$

Find: $n(AnB)$

Solution

$$A \cap B = \{ -1,6\}$$

$$\mathrm{n}(A \cap B) = 2$$

Properties of Intersection of a Set

- **Commutative Law:** The intersection of two sets A and B follow the commutative law, i.e., $A \cap B = B \cap A$
- **Associative Law:** The intersection operation follows the associative law, i.e., If we have three sets A , B and C then $(A \cap B) \cap C = A \cap (B \cap C)$
- **Identity Law:** The intersection of an empty set with any set A gives the empty set itself i.e., $A \cap \emptyset = \emptyset$

- **Idempotent Law:** The intersection of any set A with itself gives the set A i.e., $A \cap A = A$

- **Law of U:** The intersection of a universal set U with its subset A gives the set A itself. $A \cap U = A$

- **Distributive Law:** According to this law:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Disjoint set

- When the intersection of the two sets is empty, the two sets are called disjoint sets e.g. given that $\boldsymbol{P} = \{1, 3, 5, 7\}$ and $\boldsymbol{Q} = \{2, 4, 6, 8\}$.
- Here $(P \cap G) = \{\}$

Complement of set

- If U is a universal set and A be any subset of U then the complement of A is the set of all members of the universal set U which are not the elements of A.

$$A' = x: x \in U \; and \; x \notin A$$

- Alternatively it can be said that the difference of the universal set U and the subset A gives us the complement of set A. i.e.
- Consider two sets: $\boldsymbol{A} = \{a, b, c, d\}$ and $\boldsymbol{B} = \{a, b, c, d, e, f\}$.
- From the two sets above:

$$A' = \{e, f\}$$

$$A'nB = \{e, f\}$$

Example:

Let U be the universal set which consists of all the integers greater than 5 but less than or equal to 25. Let A and B be the subsets of U defined as:

$$A = x: x \in U \text{ and } x \text{ is a perfect square}$$

$$B = \{7, 9, 16, 18, 24\}$$

Find the complement of sets A and B and the intersection of both the complemented sets.

Solution:

- The universal set is defined as:

U

$= \{6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25\}$

- Also, $A = \{9,16,25\}$ and $B = \{7,9,16,18,24\}$
- The complement of set A is defined as:
- Therefore,

$$A' = \{6,7,8,10,11,12,13,14,15,17,18,19,20,21,22,23,24\}$$

- Similarly the complement of set B can be given by:

$$B' = \{6,8,10,11,12,13,14,15,17,19,20,21,22,23,25\}$$

- The intersection of both the complemented sets is given by $A' \cap B'$.

- Right arrow

$$A' \cap B' = \{6, 8, 10, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23\}$$

- We can see from the above discussions that if a set A is a subset of the universal set U then the complement of set A, i.e. A' is also a subset of U.

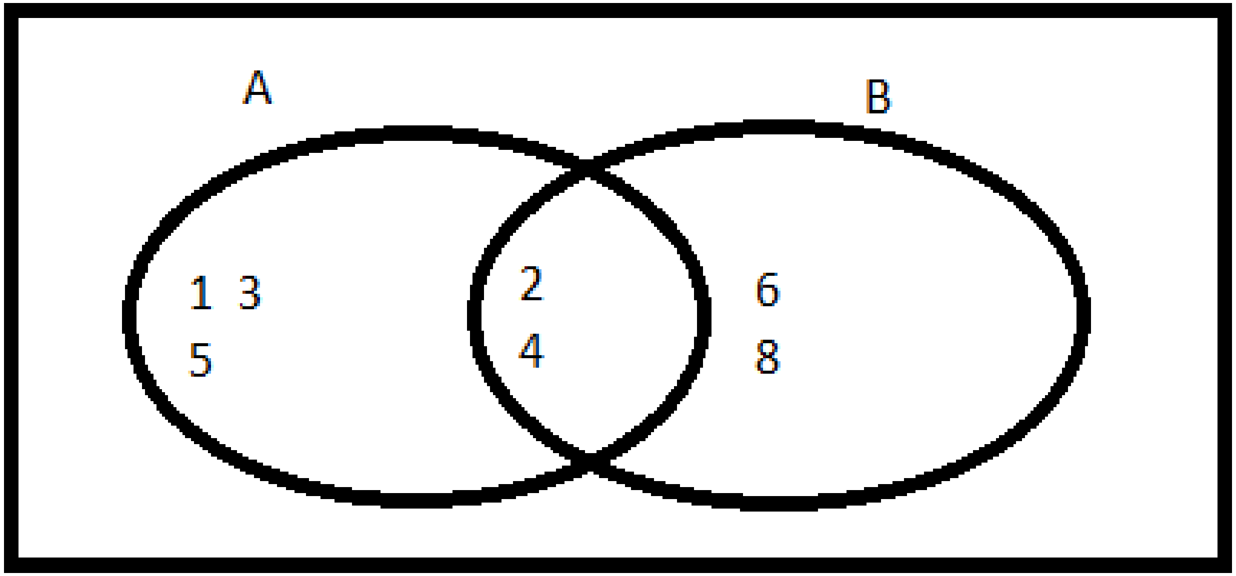
The Venn diagram

- The Venn diagram is used to simplify solving problems in sets. It can also be used to illustrate sets.

Example

Given that set $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8\}$. Use a Venn diagram to find:

1) $A \cap B$
2) $n(A \cap B)$
3) $A \cup B$
4) $n(A \cup B)$



a) $A \cap B = \{2,4\}$

b) $n(A \cap B) = 2$

c) $A \cup B = \{1,2,3,4,5,6,8\}$

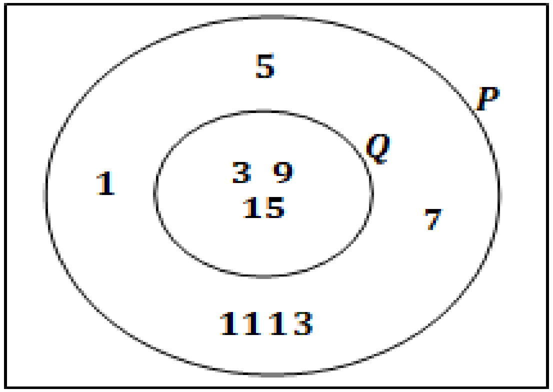
d) $n(A \cup B) = 7$

Example

Given that $P = \{1, 3, 5, 7, 9, 11, 13, 15\}$ and $Q = \{3, 9, 15\}$. Illustrate this information in a Venn diagram.

Solution

Since $Q \subset P$, Q is drawn inside P



$$P \cap Q = \{3, 9, 15\} = Q$$

$$P \cup Q = \{1, 3, 5, 7, 9, 11, 13, 15\} = P$$

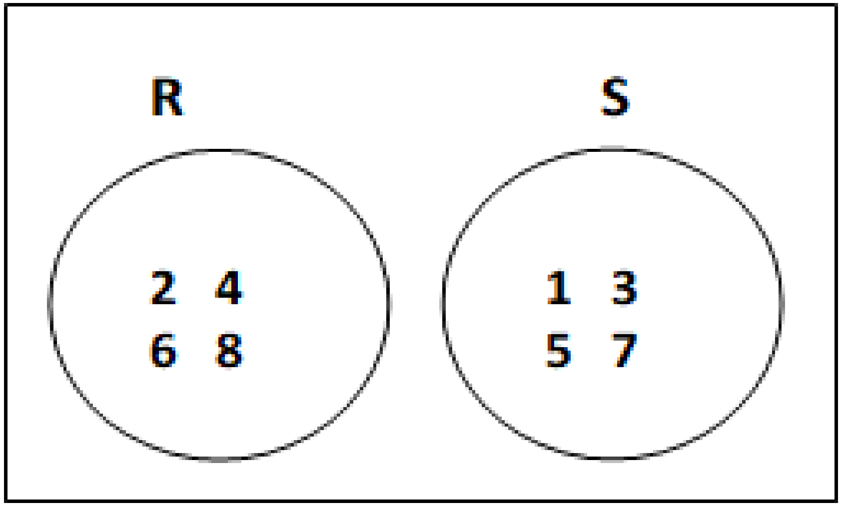
Example

Given that, $R = \{2, 4, 6, 8\}$ and $S = \{1, 3, 5, 7\}$. Show this information in a Venn diagram.

Solution

Here R and S are disjoint sets i.e., $R \cap S = \{\}$

So R and S are drawn separately as shown below.



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End of lecture 12

Next topic: Set theory : Applications (Mutually exclusive, independent events)

Thank you