Other Mechanisms of Dispersion

Dispersion is a broad term used to describe all processes, except diffusion, that disperse a patch of tracer and diminish concentration. Typically dispersion coefficients reflect a combination of advection and diffusion processes that are difficult to model separately. For example, the coupled effects of differential longitudinal advection and cross-channel diffusion create shear dispersion. In flow through porous media, the existence of multiple discrete flow paths also creates differential advection that leads to dispersion. The flow paths in a porous medium are constrained to discrete, intertwining pore channels, and multiple flow paths exist between any two points. The time to traverse a given flow path depends on the geometry of the pores. Flow through narrow pores is slower than flow through wider pores. Also, some flow paths are nearly linear, and thus short, and others are very tortuous (bending) and thus long. In addition, each pore channel behaves like a small tube, with higher velocity at the center of the pore than at the grain surface. The pore-scale shear contributes to dispersion in the same manner as channel-scale shear, as described above. Together, the above processes contribute to a longitudinal spreading of tracer particles. That is, tracer particles released together will take different flow paths and get separated (dispersed) both laterally and longitudinally. This is called mechanical dispersion. This form of dispersion can reach a Fickian limit after every tracer particle has sampled a sufficient number of pore channels.

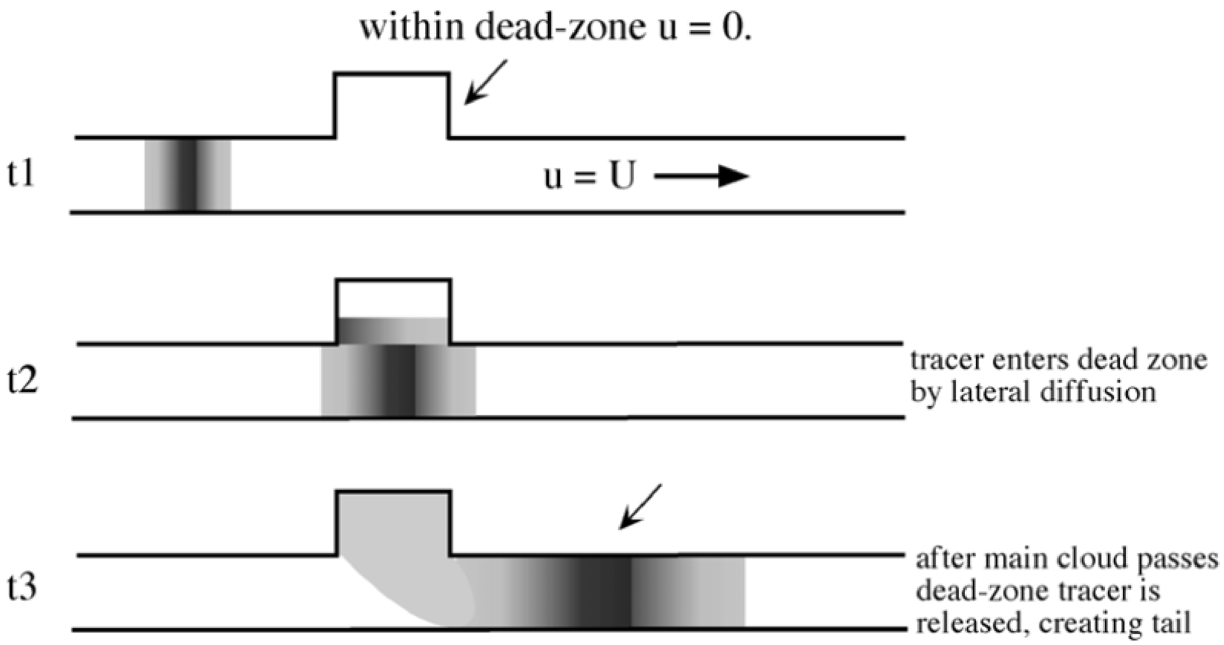


Figure 4. **Dead-zone Dispersion.** The lateral diffusion of particles into a region of zero velocity, called a dead-zone will create longitudinal dispersion. Tracer that enters the dead-zone is held back relative to the tracer that does not. When the tracer is released from the dead-zone, it is separated in space (dispersed) from the rest of the cloud.

Another form of dispersion arises when the flow field contains zones with zero velocity, called dead-zones. Tracer particles can enter these zones through turbulent or molecular diffusion. After some mean residence time, τ, within the dead-zone, the tracer is again released to the advecting zone. Any tracer that enters the dead-zone will be held back relative to the tracer that does not, so that, when the tracer is released from the dead-zone, it is separated (dispersed) from the rest of the cloud. For a dead-zone to contribute to dispersion, its residence time must be long compared to the time scale of passage of the original tracer cloud. Otherwise there is no delay associated with entering the dead-zone, and thus no dispersion. Possible dead-zones include pore spaces with no outlet, a side-pool in a river, and a wake behind an obstruction. If the dead-zones are distributed through out the flow field, then dead-zone dispersion can reach a Fickian limit after a sufficient number of dead-zones have been sampled by the tracer cloud. This can be the case for dead-end pores in a porous medium and for the wakes behind individual stems within a wetland. If only a few dead-zones can be experienced, *e.g.* a small number of pools and riffles along a river reach, then the Fickian Limit cannot be reached. The trapping and delay associated with a single or small number of dead-zones (as in Figure 4) produce non-Gaussian (non-Fickian) dispersion, and the longitudinal distribution of concentration, $\overline{C}(x)$, will be skewed, with a long tail on the upstream end associated with the tracer temporarily trapped in the dead-zone. An example of this skewed distribution is shown at t1 in Figure 3, which also represents a condition for which the Fickian limit has not been reached by the dispersion process.

A third process of dispersion can arise from chemical reaction with the stationary substrate, *e.g.* grain walls in porous media or channel boundaries. Consider the tracer depicted in Figure 5, which can adsorb to the solid boundary. When the dissolved chemical (white circles) is released into the channel, it begins to adsorb to the boundary (black circles) with reaction rate k. The chemical associated with the boundary is stationary, and thus will be delayed relative to the dissolved phase, which continues to move downstream at speed U. Once the local dissolved concentration drops to zero, the adsorbed chemical is released back to the moving fluid (gray circles, t1 and t2, Figure 5), also at rate k. The cloud is dispersed longitudinally because some fraction of the mass is delayed by adsorption to the boundary. As with dead-zone dispersion, adsorption to the solid boundary can only result in dispersion if the time scale for the adsorption processes (1/k) is long compared to transport time-scale of the cloud, such that each adsorption event results in some delay. In Chapter 9 we will discuss the effect of adsorption under conditions for which the time-scale of the adsorption/desorption reaction is very short compared to the transport time-scales, such that the concentration in the dissolved and adsorbed phases always remain at equilibrium.

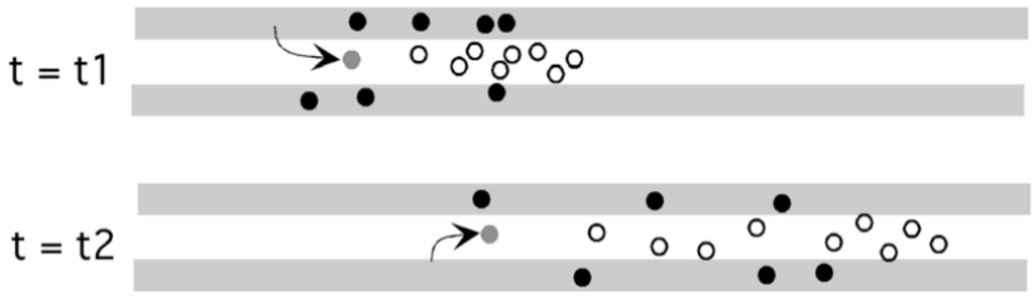


Figure 5. Some fraction of the dissolved chemical (white circles) may adsorb to the solid, stationary boundary (black circles). The adsorbed chemical is released from the boundary (gray circles) after the dissolved cloud has passed. The adsorption and desorption of chemical results in a transport delay that lengthens (disperses) the tracer cloud.

EXERCISES WITH SOLUTIONS

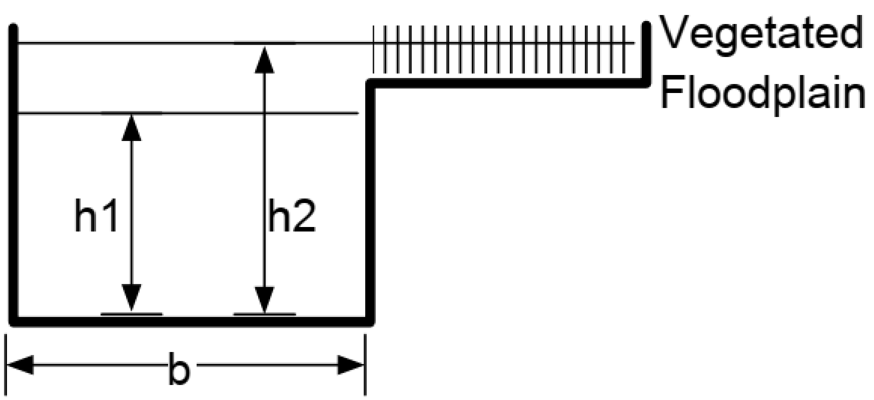
Problem 9.1

The bed slope of a wide river changes from S1 to S2, where S2 < S1. Assume the channel substrate is the same in both sections. For steady, uniform flow in both reaches, determine which section has the greater longitudinal dispersion? Consider two cases,

a) the channel depth is constant
b) the channel width is constant

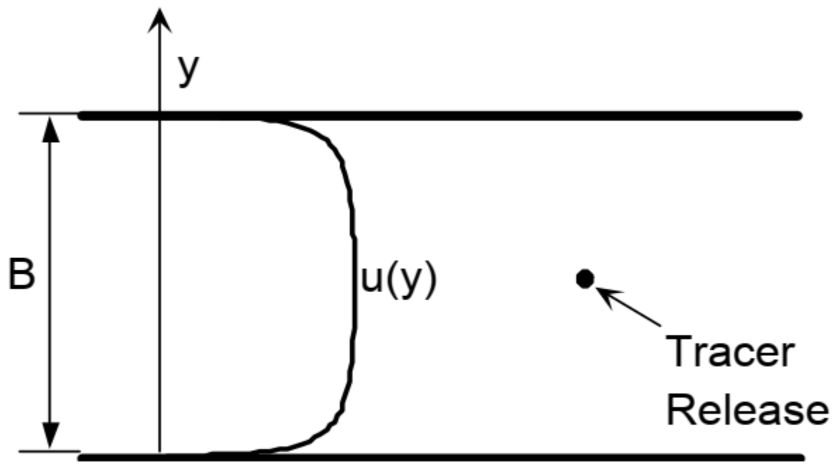
Problem 9.2

The cross-section below corresponds to a long, straight river, and is identical along the entire river. When the flow depth is h1, the channel cross-section is rectangular. When the flow depth is h2, the flood plain becomes part of the channel. Assume that the same flow rate, Q, is observed under both conditions. Will the longitudinal dispersion change as the flow depth changes from h1 to h2? Provide an explanation with sketches.



Problem 9.3

A small slug of tracer is released at mid-channel, shown below. The channel is very deep, such that vertical shear and vertical domain limits can be neglected. The channel width is B, and isotropic diffusivity D. Describe the evolution of this cloud at t1 << $B^2/4D$ and t2 >> 0.4 B^2/D. For each time period, describe the shape of the cloud and the rate at which its length increases.



EXERCISES - SOLUTIONS

Answer 9.1

The expression for longitudinal dispersion in a wide channel is $K_x = 5.93u_* h$, where u_* is the shear velocity and h the water depth. We wish to make the comparison,

$$\frac{K_{X1}}{K_{X2}} = \frac{u_*1\,h1}{u_*2\;h2} \qquad (1)$$

First consider how u_* will change between section 1 and 2. For steady uniform flow driven by bed slope, the momentum balance requires that

$$u_* = \sqrt{ghS} \qquad \text{Equation 23, Chapter 7.}$$

Such that $u_*1/u_*2 = \sqrt{(h_1S_1)/(h_2S_2)}$ (2)

If the channel depth is constant, then from (1)

$$\frac{K_{X1}}{K_{X2}} = \frac{u_*1}{u_*2} = \sqrt{\frac{S_1}{S_2}} > 1.$$

The dispersion is greater in the steeper channel.

If the channel width is constant, then from continuity the channel depth must change inversely with the depth-averaged velocity, U. Specifically, U1h1=U2 h2, or

$$h1/h2 = U2/U1. \qquad (3)$$

If the substrate is the same, we can infer that U/u_*, is constant. More formally, using the drag coefficient for the bed, C_b, we can write

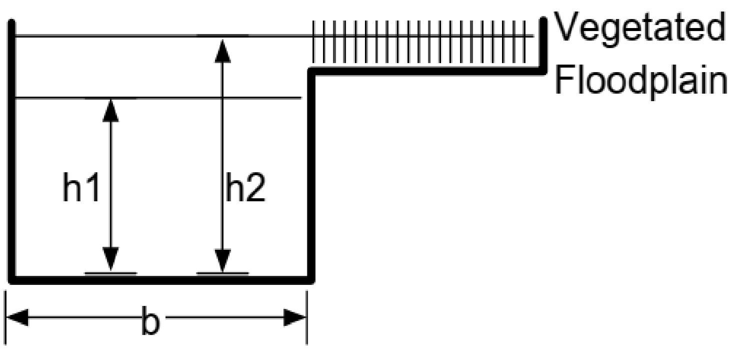
$$\tau_{bed}=\rho C_b U^2=\rho u_*^2,$$

which also indicates that for constant C_b the ratio U/u_* is constant, *i.e.*

$$U1/u_*1 = U2/u_*2. \qquad (4)$$

Combining (3) and (4), $h1/h2 = u_*2/u_*1$. Using this ratio in (1), $K_{X1}/K_{X2} = 1$. That is, for a constant channel width, the longitudinal dispersion is the same in both sections.

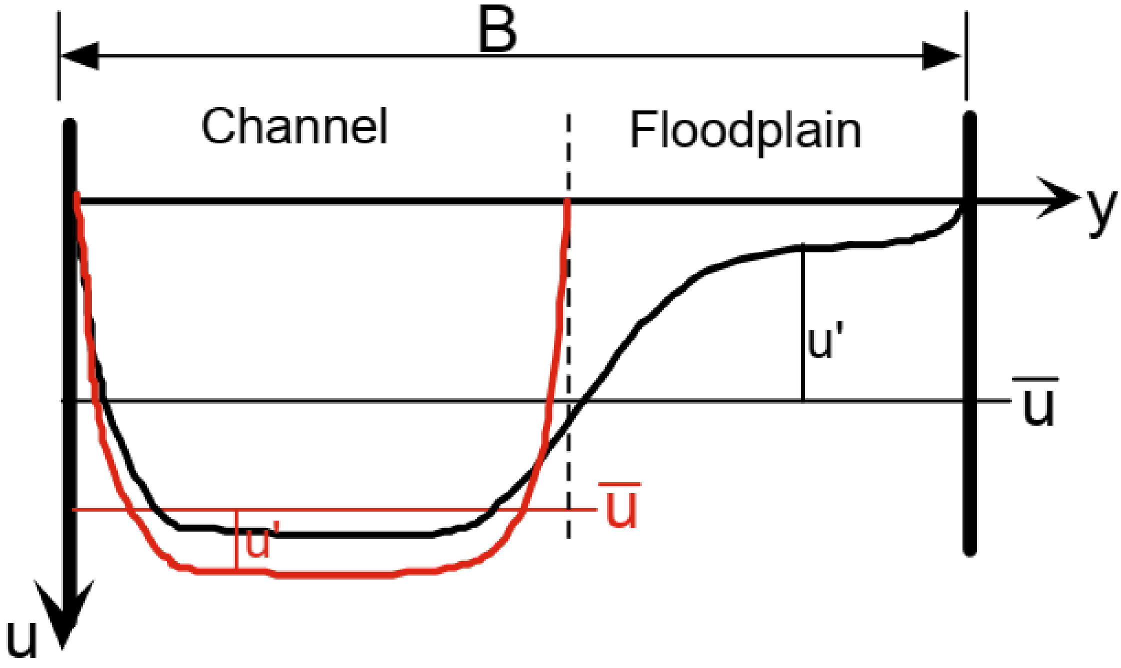
Answer 9.2



When the water depth increases from h1 to h2, flow enters the floodplain. Vegetative drag and the shallow depth combine to retard flow on the floodplain relative to the channel. This produces strong lateral shear that augments dispersion. The lateral profile of depth-averaged velocity is shown below for flow depth h1 (red) and h2 (black). The velocity fluctuations, u', are deviations from the channel mean velocity, $\bar{u}=Q/A_{total}$. Adapting the expression for shear dispersion (eq. 16, chap. 8) to the lateral shear,

$$K_X = -\frac{1}{BD_y}\int_0^B u' \int_0^y \int_0^y u' \, dy dy dy,$$

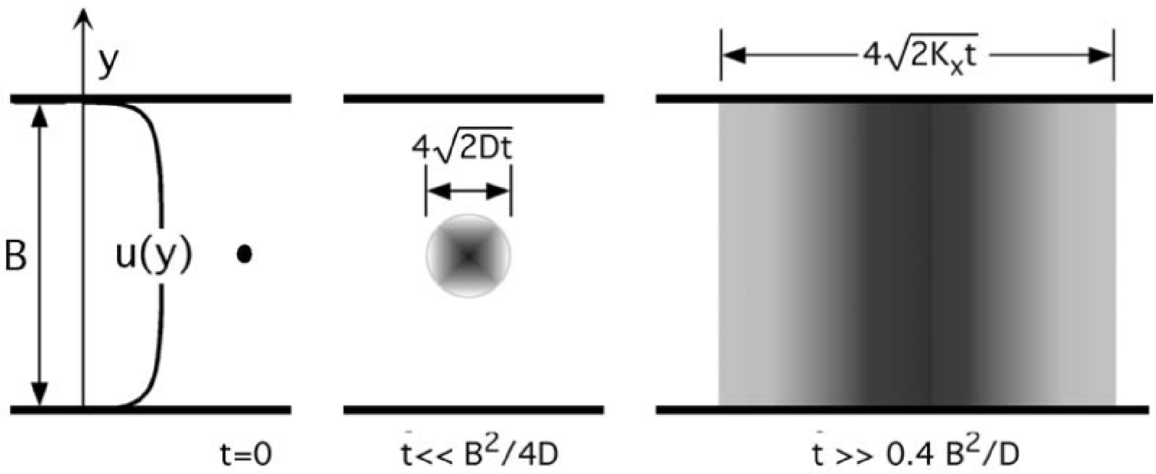
where B is the total channel width with the flood plain. From the sketch below, it is clear that the sum of spatial fluctuations, u', is greater when the water depth permits flow on the floodplain, and so we expect K_X to increase at the greater water depth. This is a greater effect than the increase lateral diffusivity that might occur with increasing width, which according to the above equation would decrease K_X. In particular the lateral diffusivity cannot increase significantly through the floodplain because of obstruction by vegetation.



Answer 9.3

Time t1<< B^2/4Dy: When the slug is initially released, it is very small compared to the width of the channel. Released in the center of the channel, the variation in velocity (shear) across the patch is negligible, and the entire patch advects at the same speed. Because the patch is not experiencing differential advection, the spreading of the cloud in the longitudinal direction is due to longitudinal diffusion only. The cloud's longitudinal length scale is $4\sqrt{2Dt}$. More specifically, letting the release point be (x, y, z) = (0, 0, 0), the concentration field evolves as,

$$C(x,y,z,t) = \frac{M}{(4\pi Dt)^{3/2}} \exp\left(-\frac{(x-ut)^2}{4Dt} - \frac{y^2}{4Dt} - \frac{z^2}{4Dt} \right)$$



Time t2>> 0.4 B^2/Dy. By this time the patch has grown to uniformly fill the lateral dimension of the channel. In addition, sufficient time has passed for the longitudinal dispersion due to the lateral shear to reach Fickian behavior. The longitudinal length-scale of the patch is now $4\sqrt{2K_x t}$, i.e. the patch growth rate is dictated by the dispersion coefficient, K_X. In the vertical direction the cloud continues to grow via vertical diffusion, such that the concentration field evolves as,

$$C(x,z,t) = \frac{M}{B\, 4\pi \sqrt{DK_x}\, t} \exp\left(-\frac{(x-ut)^2}{4K_x t} - \frac{z^2}{4Dt} \right).$$