6. Continuous point sources

In contrast to previous sections, this chapter describes the continuous release of chemicals into a fluid (e.g. smoke released from stacks, groundwater flux into a river, industrial waste streams etc.). A chemical released continuously into a moving fluid will form a plume that grows in the lateral dimension through diffusion and extends downstream via advection. This chapter describes the concentration field of the plume in both bounded and unbounded domains. In a bounded domain (e.g. a narrow, shallow channel), the steady-state solution is obtained by a simple mass balance; the transient period before steady-state is far more interesting, and the analytical expression describing the concentration front is derived here. The animation highlights how the observed concentration front varies with distance from the source.

The sample problems test the user's ability to simplify problems based on the geometry of the domain, and also to obtain information on the diffusion coefficient based on the observed concentration front.

6. Continuous Release - Point Source

A scalar released continuously into a moving fluid and from a discrete point will form a plume that grows in the lateral dimension through diffusion and extends downstream via advection. Because the concentration profile perpendicular to the flow is established by Fickian diffusion from the centerline, it has a Gaussian distribution. For this reason the plume is called a Gaussian Plume. Some common examples that approximate a Gaussian plume include smoke released from a stack, a waste stream released into river, and the contaminant plume emerging from a leaking drum buried underground.

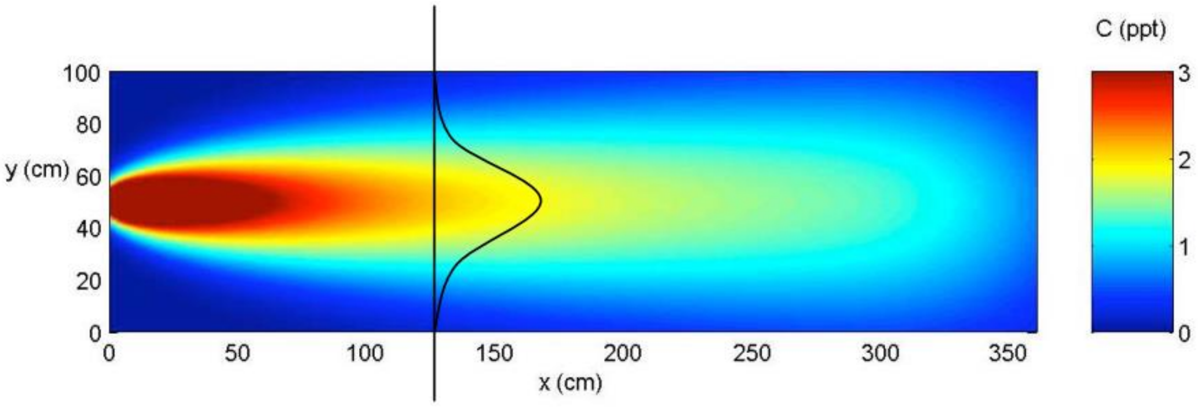


Figure 1. Concentration contours for a Gaussian plume originating at x=0, y=50 cm, z=0. The concentration profiles C(y) and C(z) are Gaussian. For example, see the profile C(x=125, y, z=0) shown above in black.

A release may be considered continuous if the advection time scale is short compared to the duration of the release. And a release approximates a point source if the scale of the source is small compared to the distance from the source and the width of the plume at that distance. Whether a source approximates a continuous or instantaneous release, and whether it approximates a point or distributed source depends not only on the configuration of the source, but also on the spatial position of interest. Thus, the choice of approximations does not depend on the absolute spatial and temporal scales of the source, but rather on the relative scales. For example, consider a river with mean flow U. The concentration observed during and after a ten-minute release ($T_{release}$) of chemical into this river will approach the predictions for a continuous release for downstream locations $x << UT_{release}$, but will approach an instantaneous release at $x >> UT_{release}$.

Steady-State, One-Dimensional Solution for a Continuous Release

Consider a long, narrow channel of width L_y, depth L_z, unidirectional flow U, and isotropic diffusion, D. At mid-width and mid-depth in this channel we release a continuous stream of tracer at a rate, $\dot{m}$ [kg s^{-1}]. Neglecting reaction, the Conservation of Mass equation (transport equation) is,

$$(1) \qquad \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right].$$

Typically a river is advection dominated such that Pe >> 1 for any distance of interest downstream (Pe = xU/D >> 1 for any x of interest). For this condition, the longitudinal diffusion term will be negligible when compared to the longitudinal advection term, *i.e.* $D\partial^2C/\partial x^2 << u\partial C/\partial x$, and the former term will be dropped. We assume that the release has been occurring sufficiently long that the conditions in the river have reached steady-state ($\partial C/\partial t = 0$). Specifically, in an advected dominated system and at a distance x downstream of the release the steady-state will be reached at time $t >> x/U$. Furthermore, we assume that the time-scale to mix the tracer uniformly over the width and depth, T_{mix}, is sufficiently short that we may assume the tracer is instantly uniform over the cross-section. Then $\partial C/\partial y = \partial C/\partial z = 0$. This is satisfied at distances $x >> UT_{mix}$. With these additional assumptions, (1) reduces to

$$(2) \qquad u\frac{\partial C}{\partial x} = 0,$$

or simply, $C \neq f(x)$ in the far field of a continuous, bounded release. To find the far-field concentration value we enforce mass balance within a control volume around the release.

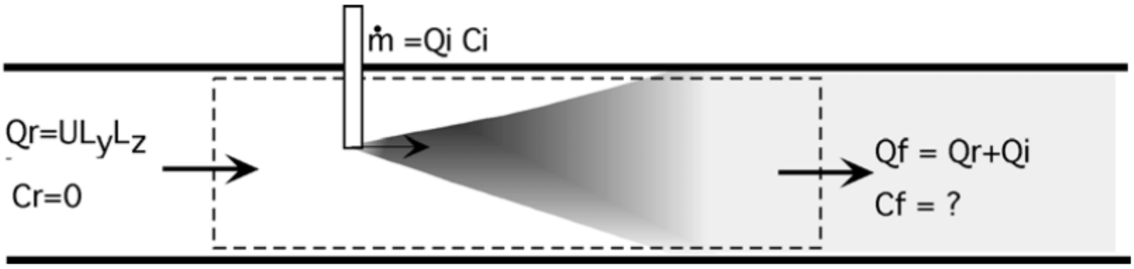


Figure 2. Mass Balance within a Control Volume around a Steady, Continuous Source. Top view of the channel with flow Qr and continuous injection of concentration Ci at rate Qi.

In steady-state, the total mass in the control volume is not a function of time, and the sum of influxes and outfluxes of mass must balance, *i.e.*,

$$\underbrace{Qr\,Cr + Qi\,Ci}_{\text{influxes}} = \underbrace{Qf\,Cf}_{\text{outflux}} = (Qr + Qi)\,Cf$$

With Cr = 0, we find the far-field, steady concentration.

$$(3) \qquad Cf = (Qi\,Ci)/(Qi + Qr).$$

Dilution Gauging:

The steady, far-field solution is sometimes used to estimate the volumetric flow in rivers with shallow complex bathymetry, for which the deployment of velocity meters is impractical. The tracer release is designed so that the injection flow rate, Qi, is far less than the stream flow, Qr, so that ($Qr+Qi \approx Qr$). The injection concentration and injection rate are known, and the far-field concentration, Cf, is measured. Then from (3),

$$(4) \qquad Qr = Qi\,Ci / Cf.$$

Recall that (4) requires the tracer to be well-mixed over the channel cross-section at the point that Cf is measured. The concentration of a conservative tracer (no sources or sinks) will cease to be a function of downstream distance once it is well-mixed over the cross-section. So, to establish the position at which (4) can be applied, measure the concentration progressively downstream until it asymptotes to a constant value.

Steady, Continuous, Point-Source in an Unbounded Domain

Next consider a steady, continuous, point source in an unbounded domain. Without boundaries a well-mixed condition can never be reached, so all dimensions must be retained. We assume that v = w = 0, neglect reaction; and use Pe >> 1 to neglect longitudinal diffusion relative to advection. For generality we allow anisotropy in diffusion [$D_x \neq D_y \neq D_z$]. The equation governing steady transport in this system is,

$$(5) \qquad u\frac{\partial C}{\partial x} = D_y\frac{\partial^2 C}{\partial y^2} + D_z\frac{\partial^2 C}{\partial z^2}.$$

The above equation indicates that advection and diffusion are decoupled by coordinate. That is, transport in the x-direction is by advection only and transport in the y- and z-directions is by diffusion only. To solve (5) we recast it in a frame of reference moving with the flow. Let $\tau = x/u$, then $u\partial C/\partial x = \partial C/d(x/u) = \partial C/\partial\tau$, and (5) becomes

$$(6) \qquad \frac{\partial C}{\partial \tau} = D_y\frac{\partial^2 C}{\partial y^2} + D_z\frac{\partial^2 C}{\partial z^2}.$$

Equation (6) describes the evolution of tracer within a thin slab of fluid moving downstream at speed u. The slab has length dx. As the slab passes the injection point, it receives a slug of mass $M = \dot{m}\,(dx/u)$, where dx/u represents the duration of time the slab is exposed to the injection. If the injection is located at x = 0, the slab receives the slug of mass at $\tau = 0$. Thus, the initial condition within the slab is $C(\tau = 0) = M\,\delta(y)\,\delta(z)$, for an injection located at y = z = 0. The solution to (6) with this initial condition was given in Chapter 3, equation (23). Modifying that solution to the coordinates used here,

$$(7) \qquad C(y,z,\tau) = \frac{M}{dx\, 4\pi\tau\sqrt{D_y D_z}}\exp\left(-\frac{y^2}{4D_y\tau} - \frac{z^2}{4D_z\tau}\right).$$

We return (7) to the stationary frame by substituting $\tau = x/u$. Additionally, $M = \dot{m}\,(dx/u)$. This gives us the concentration field for a steady, unbounded, 3-D, continuous release ($\dot{m}$) from point source at (x, y, z) = 0

$$(8) \qquad C(x,y,z) = \frac{\dot{m}}{4\pi\sqrt{D_y D_z}\,x}\exp\left(-\frac{uy^2}{4D_y x} - \frac{uz^2}{4D_z x}\right)$$

Downstream Evolution of Continuous Plume in a Channel

Now we consider the downstream evolution of a continuous source (Fig. 3). The release is at mid-depth (z = 0), mid-width (y = 0) and x = 0. To account for the boundary conditions at the channel walls and the water surface we must add image sources to the unbounded solution given in (8). Recall from Chapter 4 that a no-flux boundary can be satisfied with a positive image source and a totally absorbing boundary can be satisfied with a negative image source. If the chemical in the plume is not volatile, the water surface acts as a no-flux boundary. The four no-flux boundaries are satisfied with a sum of positive images located at ($x=0$, $y=nL_y$, $z=0$) and ($x=0$, $y=0$, $z=nL_z$) with $n = \pm 1, \pm 2, \pm 3$ upward to ± infinity.

(9)

$$C(x,y,z) = \frac{\dot{m}}{4\pi\sqrt{D_y D_z}\,x}\left[\sum_{n=-\infty}^{\infty}\exp\left(-\frac{u(y+nL_y)^2}{4D_y x}-\frac{uz^2}{4D_z x}\right)+\sum_{\substack{n=-\infty\\ \text{but } n\neq 0}}^{\infty}\exp\left(-\frac{uy^2}{4D_y x}-\frac{u(z+nL_z)^2}{4D_z x}\right)\right]$$

In the second sum n = 0 is excluded as it represents the real source which is already accounted for in the first sum.

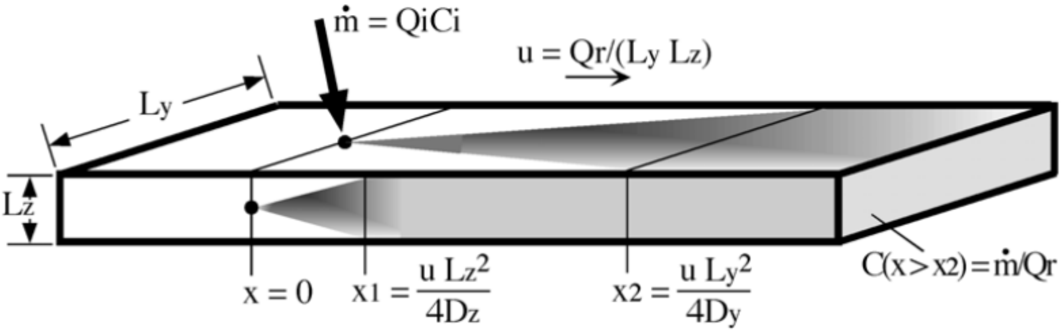


Figure 3. Continuous Release at mid-width (y=0), mid-depth (z = 0) and x = 0. The points at which the plume becomes uniformly mixed over depth (x1) and width (x2) are indicated.

The plume concentration will be well-mixed over depth at $x > u\,L_z^2/4D_z$, and $\partial C/\partial z = 0$ beyond this distance (Figure 3). This removes the vertical diffusion term from (5). Following the same mathematical development following (5) for a three-dimensional plume, an unbounded two-dimensional plume released from (x,y)=0 has the concentration field.

$$(10)\qquad C(x,y) = \frac{\dot{m}/u}{L_z\sqrt{4\pi D_y (x/u)}}\exp\left(-\frac{uy^2}{4D_y x}\right)$$

With $\partial C/\partial z = 0$, the no-flux boundaries at the surface and bed are automatically satisfied. We only need to account for the side boundaries with image sources at ($x=0$, $y=nL_y$, $z=0$) for $n = \pm 1, \pm 2$, upward to ± infinity. The concentration in the plume between x1 and x2, as shown in Figure 3, is then approximately

$$(11) \qquad C(x,y) = \frac{\dot{m}/u}{L_z\sqrt{4\pi D_y(x/u)}} \sum_{n=-\infty}^{\infty} \exp\left(-\frac{u\left(y+nL_y\right)^2}{4D_y x}\right),$$

where again, $n = 0$ represents the real source.

Transient, One-Dimensional Solution for Continuous, Point Source in a Channel

Figure 3 represents the spatial evolution downstream of a continuous source after steady-state has been reached. Now we consider the transient period between when the injection starts ($t = 0$) and the concentration field reaches a steady state ($t \gg x/U$). Specifically, we allow $\partial C/\partial t \neq 0$. For simplicity we assume a well-mixed condition over width and depth, such that $\partial C/\partial y = \partial C/\partial z = 0$, and we assume no reaction.

$$(12) \qquad \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D_x \frac{\partial^2 C}{\partial x^2}$$

We retain the longitudinal diffusion term, even though we expect $Pe \gg 1$. In Figure 4, below, we see that the diffusion term creates spatial detail around the front, or leading edge of the plume. Once the injection is initiated, each slab of fluid, of width dx, passing the release point receives mass $M = \dot{m}\,(dx/u)$. We assume that the mass instantly mixes within the slab, *i.e.* over the channel depth and width and over the length dx, such that the initial concentration in the slab is $C(x=0) = C_0 = M/(L_y L_z\, dx) = \dot{m}/(u L_y L_z)$. The boundary conditions then become,

$$(13) \qquad \begin{array}{ll} C(x) = 0 & \forall x,\ t < 0 \\ C_0 = \dfrac{\dot{m}}{u\, L_y\, L_z} & x = 0,\ t \geq 0 \end{array}$$

The solution to (12) with boundary conditions in (13) is

One-Dimensional, Transient Solution to Continuous Release

$$(14) \qquad C(x,t) = \frac{\dot{m}}{2\, u\, L_y\, L_z}\,\mathrm{erfc}\left(\frac{x - ut}{\sqrt{4D_x t}}\right) = \frac{C_0}{2}\,\mathrm{erfc}\left(\frac{x - ut}{\sqrt{4D_x t}}\right).$$

The function, *erfc*, is called the complementary error function, and is defined in Table 1 below. The distribution in (14) is called a *Break-Through Curve*, because it represents the temporal evolution of concentration when tracer first *breaks through* at a specific distance downstream of an injection. Figure 4 provides an example for a tracer injected at x = 0 beginning at t = 0 into a channel with current speed u. If the tracer mixes quickly over depth, L_z, and width, L_y, then (14) predicts the concentration measured at x = L_x.

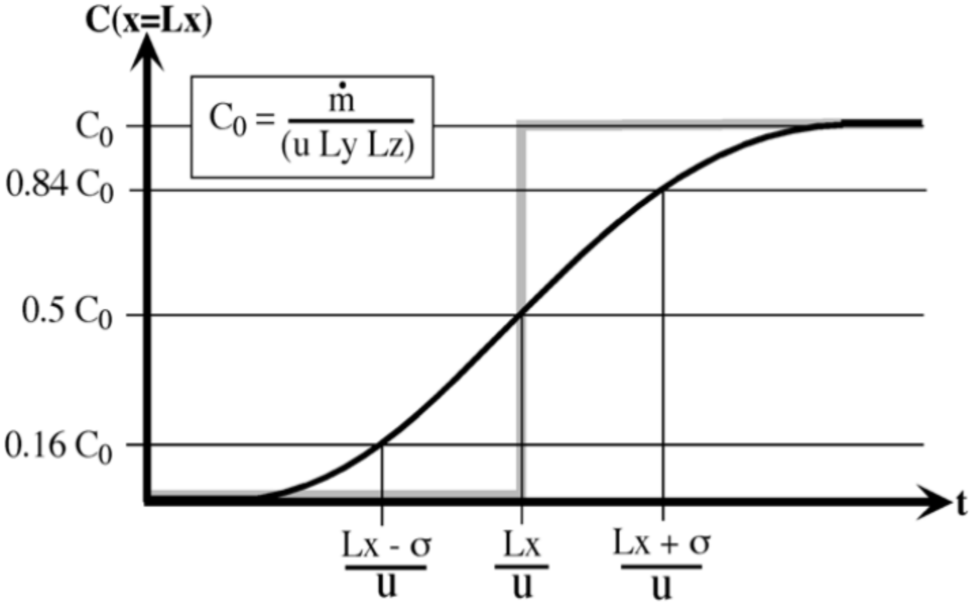


Figure 4. **Break-Through Curve**. Injection of tracer at rate $\dot{m}$ is initiated at t = 0. The concentration observed at x = L_x, shown as a black curve, follows (14). The break-through curve shown in gray results if diffusion is neglected, $D_x = 0$.

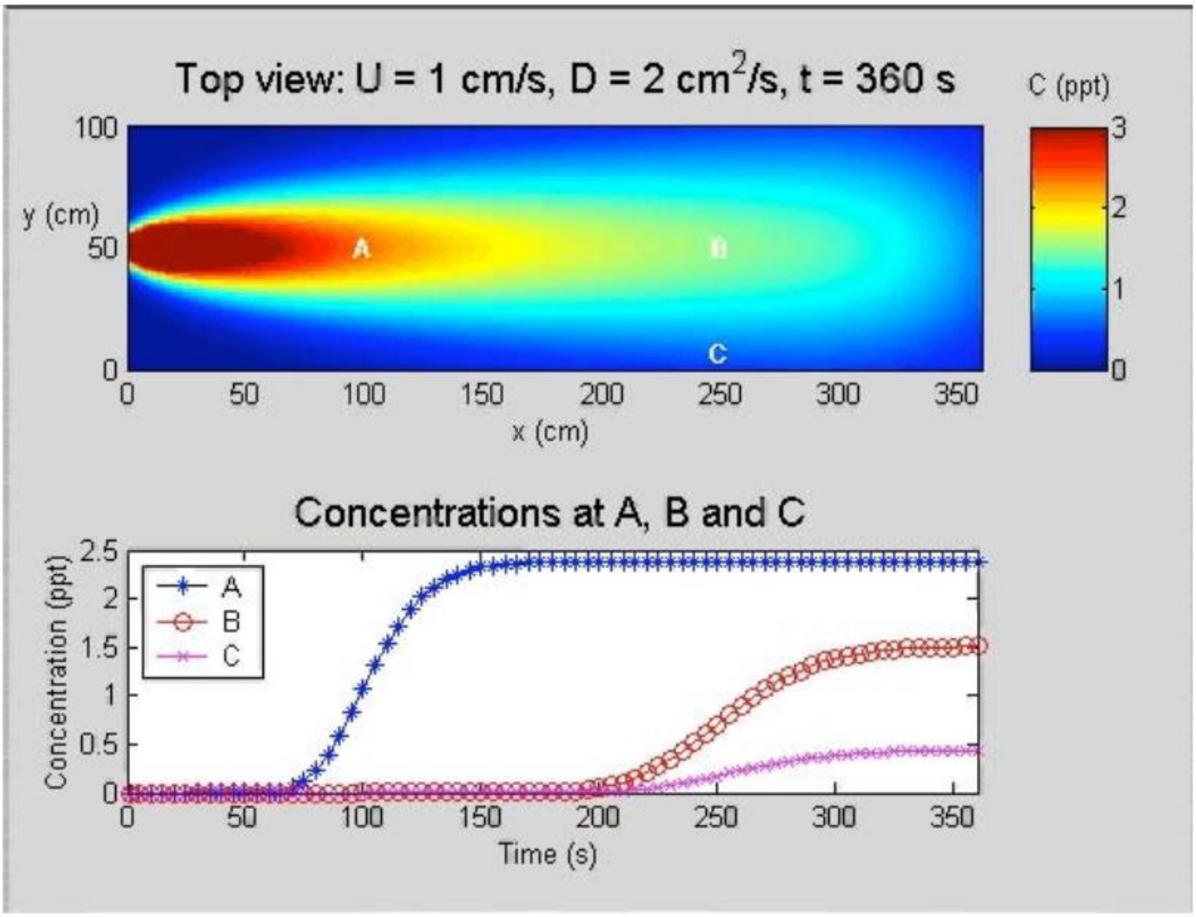
Table 1. The error function, *erf*(a), is the integral of the Gaussian distribution from zero to a. The complementary error function is the integral of the Gaussian distribution from a to infinity, or simply, *erfc*(a) = 1 – *erf* (a). In addition, *erfc*(-a) = 2-*erfc*(a).

$$erf(a) = \frac{2}{\sqrt{\pi}}\int_0^a e^{-r^2}\,dr \qquad erfc(a) = \frac{2}{\sqrt{\pi}}\int_a^\infty e^{-r^2}\,dr$$

a	***erf*(a)**	***erfc*(a)**	**a**	***erf*(a)**	***erfc*(a)**
0	0	1.0	1.4	0.952285	0.047715
0.7	0.677801	0.322199	2.0	0.995322	0.004678
1.0	0.842701	0.157299	3.0	0.999978	0.000022

Example: Break-Through Curves with Gaussian Plume.
You can open the animation on the Chapter 6 homepage. The animation depicts the evolution of the concentration field downstream of a point source located at (x = 0, y = 50 cm, z = 0) and initiated at t = 0 in a channel with steady flow u=1 cm/s and longitudinal diffusion coefficient D = 2 cm^2/s. The concentration is measured at three points all at elevation z = 0. At each point the center of the front, defined by C = 0.5 C_{final}, arrives at the advection time scale, x/u. The duration of the front, which is the time required for the concentration to rise from C=0 to C_{final}, is $4\sigma_i/u$, where σ_i is the length-scale of the front at t = x/u, *i.e.* $\sigma_i = \sqrt{2D_x x/u}$. Use the table below to compare the transport time-scales with the concentration record at each Probe.

Probe	x[cm]	y[cm]	x/u [s]	σ_i[cm]	$4\sigma_i/u$ [s]
A	100	50	100	20	80
B	250	50	250	32	128
C	250	0	250	32	128



EXERCISES WITH SOLUTIONS

Problem 6.1

A small stream was found to be contaminated with Lindane, a pesticide known to cause convulsions and liver damage. Groundwater wells in the same region have also been found to contain Lindane, and so you suspect that river contamination is due to groundwater inflow. To test your theory you conduct a dye study. Based on the information given below, estimate the ground water volume flux, Q_{GW}, and the concentration of Lindane, C_{LGW}, in the groundwater.

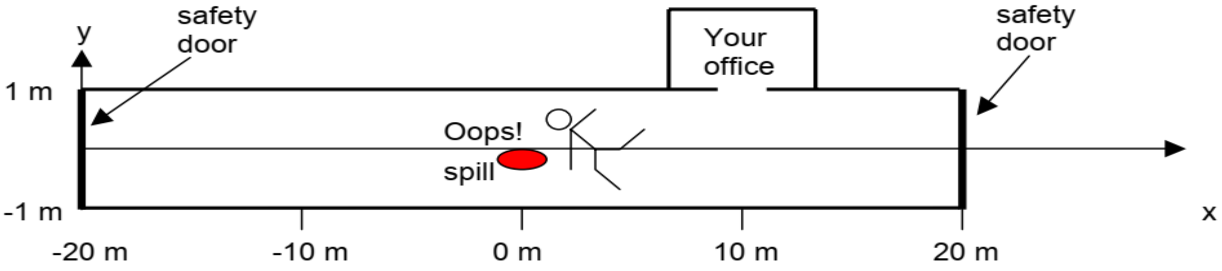
Station 1:	A 50-mg/l solution of tracer is injected at the rate of $Q_i = 100 cm^3/s$.
Station 2:	Located 100-m downstream of Station 1. Dye concentration, $C_{dye} = 10$ µg/l Lindane concentration, $C_{L2} = 0.5$ µg/l
Station 3:	Located 200-m downstream of Station 1. Dye concentration, $C_{dye} = 8$ µg/l Lindane concentration, $C_{L3} = 0.9$ µg/l

Problem 6.2

A clumsy professor trips and spills 1 g of chemical 10 m down the hall from your door. The hallway is 2 m wide and 2 m high. The chemical evaporates in 25 seconds. Assume isotropic turbulent diffusion $D = 0.05 m^2/s$.

Case 1. The safety doors are closed so that there is no air current in the hall. Write an expression for the concentration at your door, justifying all assumptions. Plot the exposure concentration at your door versus time, indicating the maximum and final concentration.

Case 2. The safety doors have been propped open (bad, bad, bad), so that there is a 4 m/s breeze blowing down the hall from the spill towards your door. Calculate the magnitude and duration of the maximum concentration at your door. Sketch the concentration, c(t), clearly indicating the maximum and final concentration.



Problem 6.3

Indeck Corporation has proposed to construct a new 350 MW gas-fired power plant in Smithfield, Rhode Island. Environmental reports submitted for the plant indicate it would discharge 150 metric tons per year of carbon monoxide from a 57.9-m high stack. Your house is located 1-km downwind of the proposed stack, so you are very interested in the potential carbon monoxide exposure. Assume a steady wind of $U = 2$ m/s, a vertical turbulent diffusion coefficient of $D_z = 1\ m^2/s$, and a horizontal turbulent diffusion coefficient of $D_x = D_y = 2\ m^2/s$.

a. Write the equation for the carbon monoxide concentration downwind of the stack.

b. Plot the vertical profile of concentration, C(z), at your house. If your house is 10 m high, what is the maximum concentration you might experience?

EXERCISES - SOLUTIONS

Answer 6.1

Station 1:	A 50-mg/l solution of tracer is injected at the rate of $Q_i = 100\ cm^3/s$.
Station 2:	Located 100-m downstream of Station 1. Dye concentration, $C_{dye} = 10\ \mu g/l$ Lindane concentration, $C_{L2} = 0.5\ \mu g/l$
Station 3:	Located 200-m downstream of Station 1. Dye concentration, $C_{dye} = 8\ \mu g/l$ Lindane concentration, $C_{L3} = 0.9\ \mu g/l$

If dye is well-mixed and concentration is steady, then the flow at stations 2 and 3 is

$$Q_2 = Q_iC_i/C_{dye} = (100\ cm^3s^{-1})(50\ mgl^{-1})/(0.01\ mgl^{-1}) = 0.50\ m^3s^{-1}$$

$$Q_3 = Q_iC_i/C_{dye} = (100\ cm^3s^{-1})(50\ mgl^{-1})/(0.008\ mgl^{-1}) = 0.63\ m^3s^{-1}$$

If no tributaries or sewer outfalls exist between stations 2 and 3, then we expect the groundwater inflow between these stations to be

$$Q_{GW} = Q_3 - Q_2 = 0.13\ m^3s^{-1}.$$

To find the Lindane concentration in the groundwater we evaluate conservation of mass within the river between station 2 and 3. Influxes of Lindane to this control volume are at station 2 and from groundwater. The outflow is at station 3. Such that

$$Q_2C_{L2} + Q_{GW}C_{LGW} = Q_3C_{L3}$$

Solving for the groundwater concentration

$$C_{LGW} = \frac{Q_3C_{L3} - Q_2C_{L2}}{Q_{GW}} = 2.4\ \mu g/l$$

Answer 6.2

Case 1. The safety doors are closed so that there is no air current in the hall.

Use time scales to determine if, for the exposure at your office, the release is instantaneous or continuous. The distance to your door is dx=10-m. With no advection (zero current), the transport time scale is $T_{\text{diffusion}} = dx^2/8D$, based on the arrival of the 2σ contour. Thus, $T_{\text{diffusion}} = (10\text{-m})^2/(8 \times 0.05\text{m}^2\text{s}^{-1}) = 250$ s. This is much longer than the release time scale (25 s), so the concentration observed at your office will appear as if from an instantaneous release.

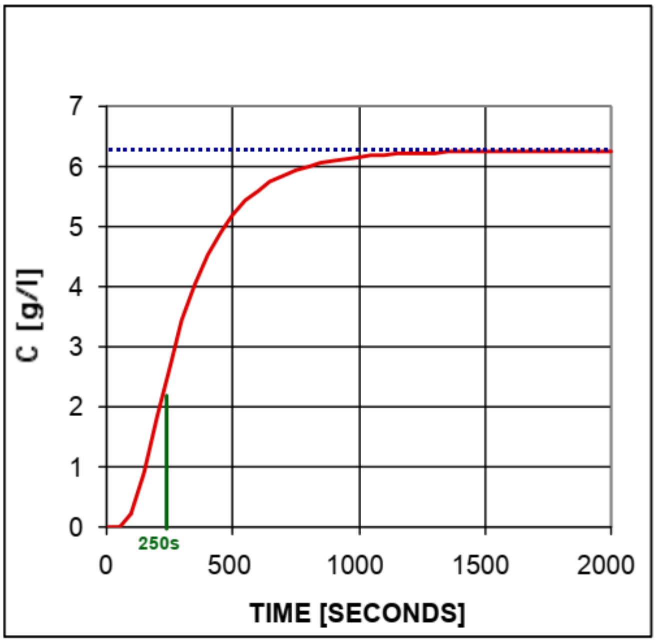
Similarly, use a comparison of time-scales to determine if the concentration at your door appears as if from a 1-D, 2-D, or 3-D source. If we assume the release occurs at mid-width, the time-scale to mix across the hallway is, $t_{\text{mix},y} = (2\text{m})^2/(4 \times 0.05\ \text{m}^2\text{s}^{-1}) = 20$ s. For the vertical mixing time-scale, note that the spill is at the floor boundary. A release at the boundary must diffuse across twice the distance, as compared to a release at mid-height, so the length-scale must be doubled. The time-scale to mix over the hall height is thus, $t_{\text{mix},z} = (2\text{x}2\text{m})^2/(4\text{x}0.05\text{m}^2\text{s}^{-1}) =$ 80s. Since $t_{\text{diffusion}} >> t_{\text{mix}}$, the concentration at your door appears as if released as a one-dimensional source, *i.e.* distributed evenly over y and z.

Finally, with the closed doors at x = ±20 are no-flux boundaries. To account for these boundaries we put in an infinite set of images located at x = 40n, where n = ±1,±2, etc. The real source is located at n=0. The concentration observed at your door then follows,

$$C(x,t) = \frac{M}{A\sqrt{4\pi\,Dt}} \sum_{n=-\infty}^{\infty} \left(\exp\left(-(x+40n)^2/4Dt\right)\right),$$

with x = 10 m. The cross-sectional area of the hallway is A = 4 m^2.

With the doors closed, the hallway is a contained system, and the final concentration is simply that established when the mass released in fully mixed over the volume of the hallway. We assume that the doors to individual rooms are closed, so that no fumes enter the rooms. The final concentration is then, $C_{\text{final}} = 1\text{g} / (2 \times 2 \times 40\ \text{m}^3) = 6.25$ g/l.



The concentration observed at your door ($x = 10$ m) is plotted above. Note the concentration at your door at the diffusion time scale, 250 s, is not negligible. This time scale is based on the arrival of the 2σ contour, not on a specific value of concentration. The time-scale based on cloud size (2σ) is not necessarily a good indicator of exposure risk. Also, if the absolute concentration at 2σ is below the detection limit, you would not detect it, even though you predict it has arrived. The final concentration is reached at around 1600 s, which is shorter than $t_{mix,x} = L_X^2/4D = 8000$ s, with $L_X = 40$m (defined in chap. 4). This again emphasizes the conservative nature of the standard mixing-time estimator, $L^2/4D$.

Case 2. Safety doors are open, u= 4 m/s from the spill towards your door.
The advection time scale is, $T_{advection} = dx/u = 10\text{-m}/4\text{ms}^{-1} = 2.5 \text{ s} << T_{diffusion}$ and $<< T_{release}$. So, the transport is dominated by advection, and the chemical arrives at your door before the release ends. So, during most of the release the concentration at your door appears as if released from a continuous source. In addition, the mixing time scales derived above still hold, so we can write, $T_{mix,y}$ and $T_{mix,z} > T_{advection}$, such that the chemical cloud is not mixed over y and z at the door. The cloud is thus fully three-dimensional at your door. For simplicity we assign the origin of the release to be ($x = 0$, $y = 0$, $z = 0$). Image sources are need to account for the no-flux boundaries at $y = \pm$ 1-m and $z = 0$ and 2m. Consider the four closest images, (y,z)=[(0,0); (-2,0); (2,0); (0,4)]. During most of the release, the concentration at your door will be,

$$C(x, y, z) =$$

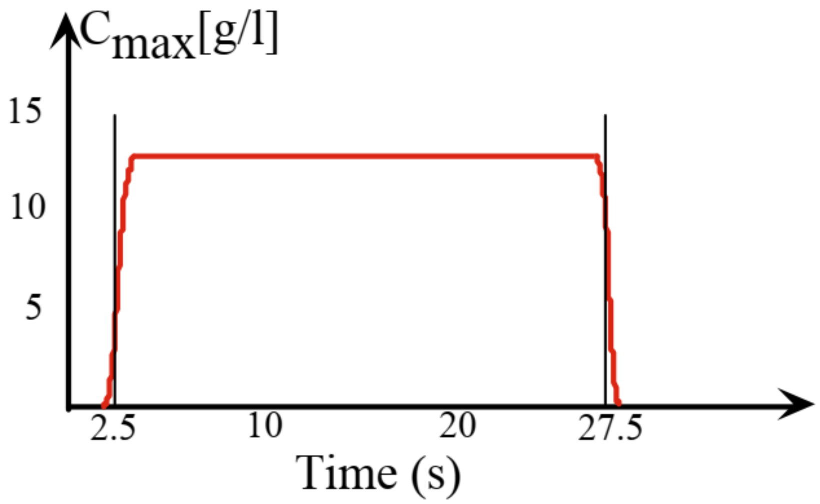
$$\frac{\dot{m}}{4\pi D x}\left[\underbrace{2\exp\left(-\frac{u(y^2+z^2)}{4Dx}\right)}_{\text{real + image at (y=0,z=0)}}+\underbrace{\exp\left(-\frac{u((y-2)^2+z^2)}{4Dx}\right)}_{\text{image at (y = 2,z = 0)}}+\underbrace{\exp\left(-\frac{u((y+2)^2+z^2)}{4Dx}\right)}_{\text{image at (y = -2, z = 0)}}+\underbrace{\exp\left(-\frac{u(y^2+(z-4)^2}{4Dx}\right.}_{\text{image at (y = 0,z = 4)}}\right.$$

The maximum concentration at the door will be at the floor - in line with the source. So, we evaluate the above equation for ($x = 10$m, $y = 0$, $z = 0$).

$$C_{max} = \frac{\dot{m}}{4\pi D x}\left[2 + 2\exp\left(-\frac{u(2m)^2}{4Dx}\right) + \exp\left(-\frac{u(4m)^2)}{4Dx}\right)\right] =$$

$$= \frac{(1g/25s)}{4\pi(0.05m^2s^{-1})(10m)}\left[2 + 2(0.00034) + 1.2x10^{-14}\right] = 12.7g/l$$

One can see from the values in the last expression that only the image coincident with the real source contributes significantly to the solution. The above concentration is observed at the door only during the duration of the release. More specifically, the concentration is observed starting approximately at $t = 2.5$ s (the advection time scale for the front) and ending at $(25 + 2.5) = 27.5$ seconds, the duration of the release plus the advection time scale. The maximum exposure concentration at your door is depicted below. Because of longitudinal diffusion, the front and tail of the passing cloud are not sharp, but will have a transition region of length $\approx 4\sigma$, where σ is estimated at the advection time-scale, 2.5s. Thus, the transition region at the front and tail of the exposure cloud will $4\sqrt{2(0.05m^2s^{-1})(2.5s)} = 2$ m long, which translates into an transient period of (2-m)/(4m/s) = 0.5 s. Note that the final concentration is zero, everything eventually flushes out of the hall by the air current.

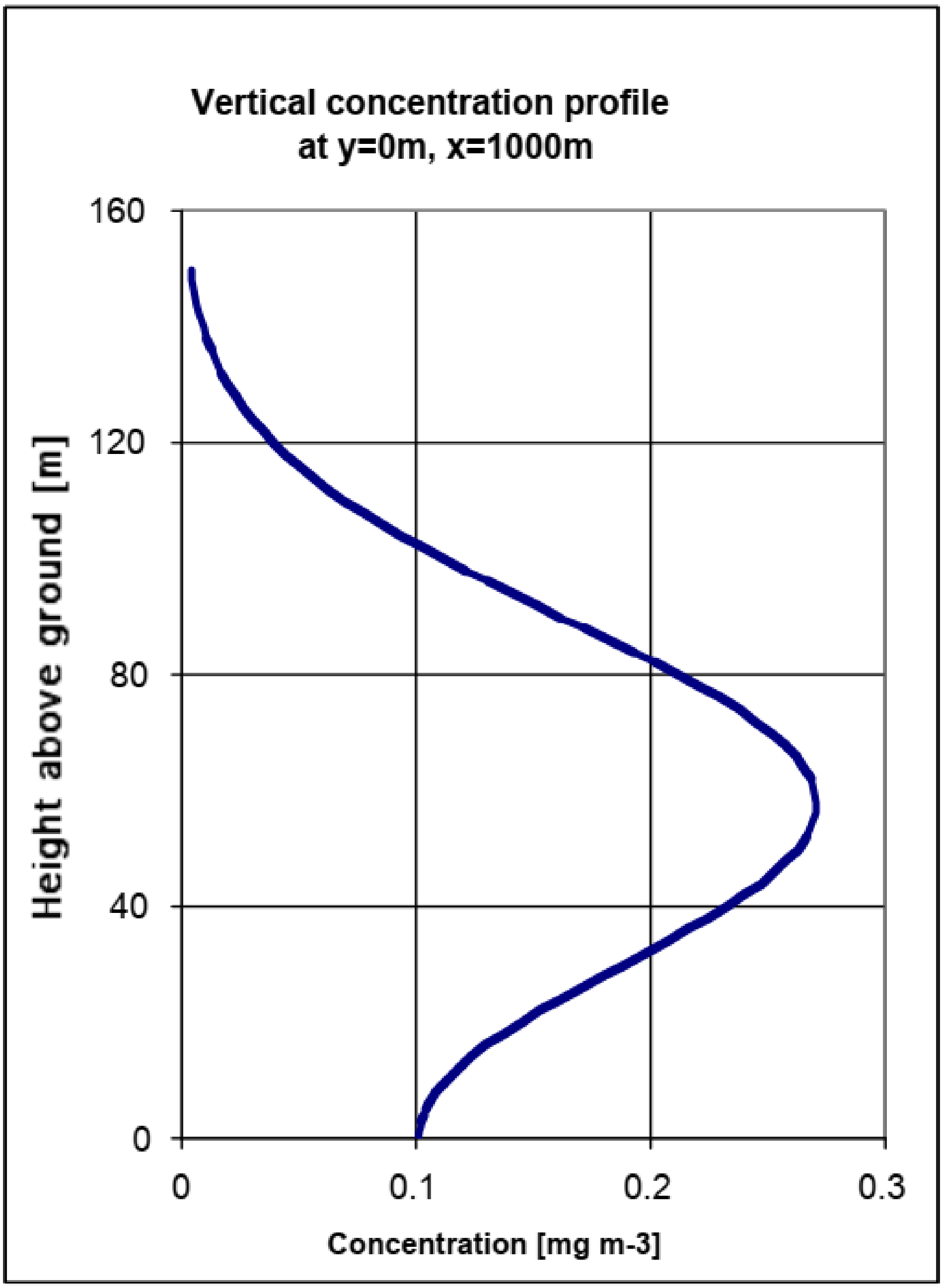


Answer 6.3.
The stack will run continuously, so use a continuous release solution. Let H = 57.9-m be the stack height. With z = 0 at the ground, the source is at (x=0, y=0, z = H). One image source is needed at (x = 0 , y = 0, z = -H) to account for the no-flux boundary at the ground.

$$C(x,y,z) = \frac{\dot{m}}{4\pi x\sqrt{D_y D_z}}\left[\underbrace{\exp\left(-\frac{uy^2}{4D_y x}-\frac{u(z-H)^2}{4D_z x}\right)}_{\text{real source}}+\underbrace{\exp\left(-\frac{uy^2}{4D_y x}-\frac{u(z+H)^2}{4D_z x}\right)}_{\text{image source}}\right]$$

This equation is valid when Pe=Ux/D_X >>1. This is true for x >> D_X/U = 1-m. Since the house is directly downwind, we let y = 0, x = 1000-m in the above equation. The rate of release is, $\dot{m}$ = 150,000kg/yr = 4.8 gs^{-1}. See plot of C(z) below. The maximum concentration at your home will be at the roof, C(z = 10-m) = 0.11 mg/m^3 = 0.11 µg/l. Finally, note the no-flux boundary is expressed in the profile as $\partial C/\partial z = 0$ at z = 0.

Refer to the image in the next page


Vertical concentration profile
at y=0m, x=1000m
Height above ground [m]
160
120
80
40
0
0
0.1
0.2
0.3
Concentration [mg m-3]