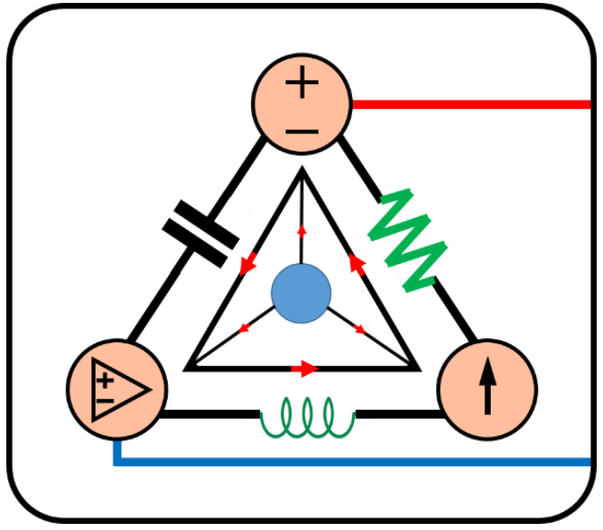
4-Mavzu: Elektr zanjir teoremalari.

(Lecture-4: Circuit Theorems)

4-Mavzuning 2-qismi

(Part 2 of the Lecture-4)



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4-Mavzu: Elektr zanjir teoremalari.

(Lecture-4: Circuit Theorems)

O‘quv rejasi:

4.1. O‘zgarmas elektr tok zanjirlarining chiziqli xossalari.

4.2. Superpozitsiya.

4.3. Manbani o‘zgartirish.

4.4. Tevenin teoremasi.

4.5. Norton teoremasi.

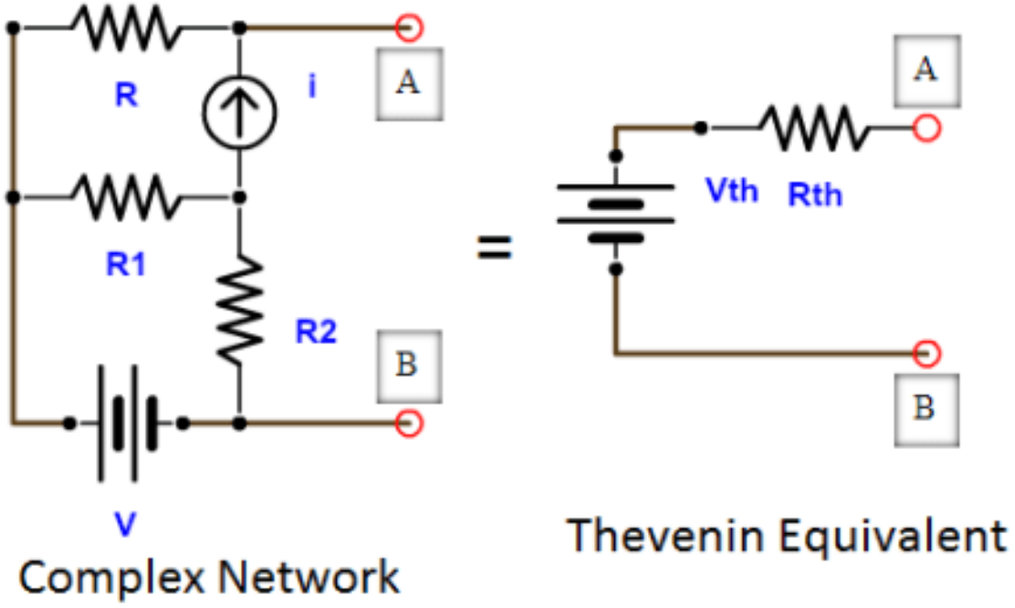
4.6. Maksimal quvvat uzatish.

4.7. Qo‘llanilishi.

4.4. Tevenin teoremasi.

Amaliyotda zanjirdagi ma’lum bir elementlar o‘zgaruvchan (*odatda yuklama deb ataladi*), boshqa elementlar esa o‘zgarmasdir.

Misol uchun, uyimizdagi rozetkasi o‘zgaruvchan yuklamani tashkil etuvchi turli xil qurilmalarga ulanishi mumkin.



Har safar o‘zgaruvchan element o‘zgarganda, butun zanjirni qayta tahlil qilinishi kerak.

Ushbu muammoning oldini olish uchun Tevenin teoremasi (*Thevenin theorem*) zanjir qismini ekvivalent zanjir bilan almashtirish usulini taklif etadi.

Photo source: [11] - https://i0.wp.com/electronicspani.com/wp-content/uploads/2014/02/Conversion_between-_Thevenin_and_Norton_equivalent.png?w=688&ssl=1

Tevenin teoremasiga ko‘ra, 4.9-rasm, *a* dagi chiziqli zanjiri 4.9-rasm, *b* dagi bilan almashtirilishi mumkin (*rasmdagi yuklama bitta rezistor yoki boshqa zanjir bo‘lishi mumkin*).

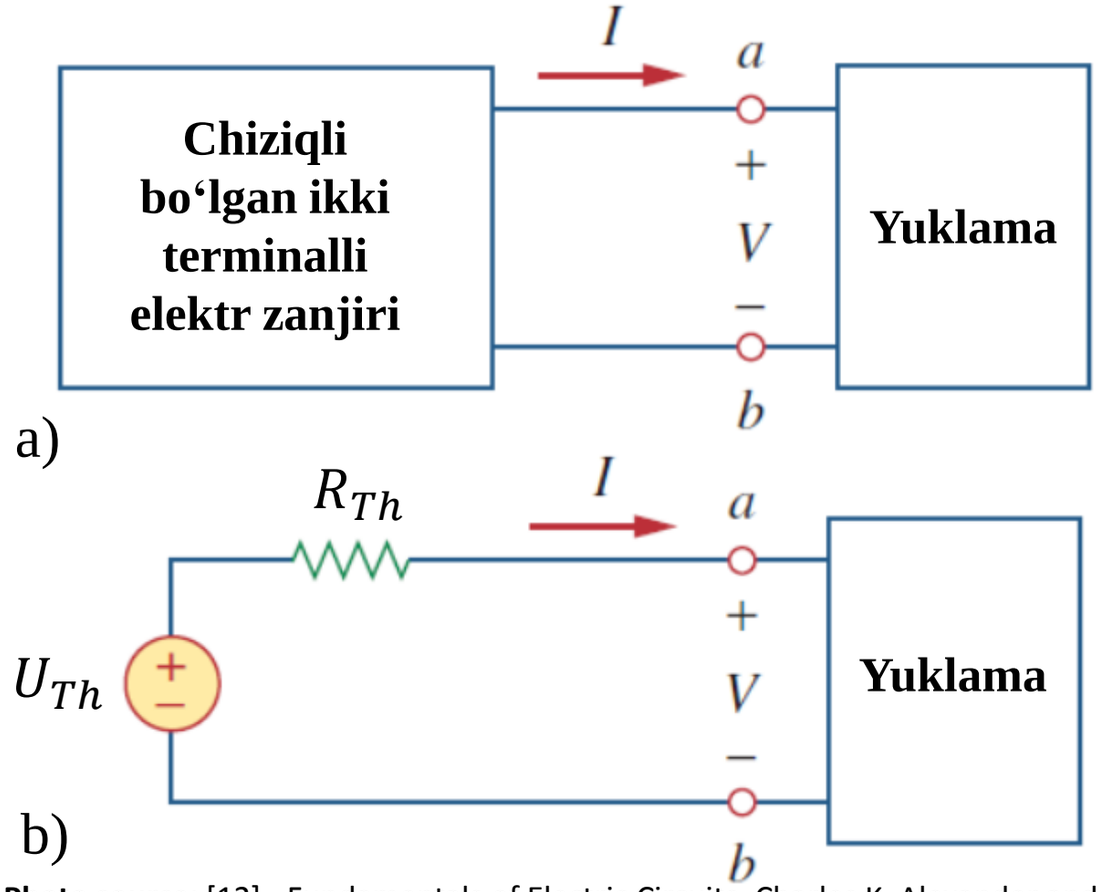


Photo source: [12] - Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 139.

U 1883 yilda fransuz telegraf muhandisi M.Leon Tevenin (1857–1926) tomonidan ishlab chiqilgan.

4.9-rasm, *b* dagi *a-b* terminallarining chap tomonidagi zanjir Tevenin ekvivalent zanjirini hosil qiladi.

4.9-rasm. Chiziqli ikki terminalli elektr zanjirni Tevenin ekvivalenti bilan almashtirish:
a) asl zanjir; b) Tevenin ekvivalent zanjiri.

Tevenin teoremasiga ko‘ra, chiziqli ikki qutbli zanjirni R_{Th} rezistor bilan ketma-ket ulangan U_{Th} kuchlanish manbaidan tashkil topgan ekvivalent zanjir bilan almashtirish mumkin.

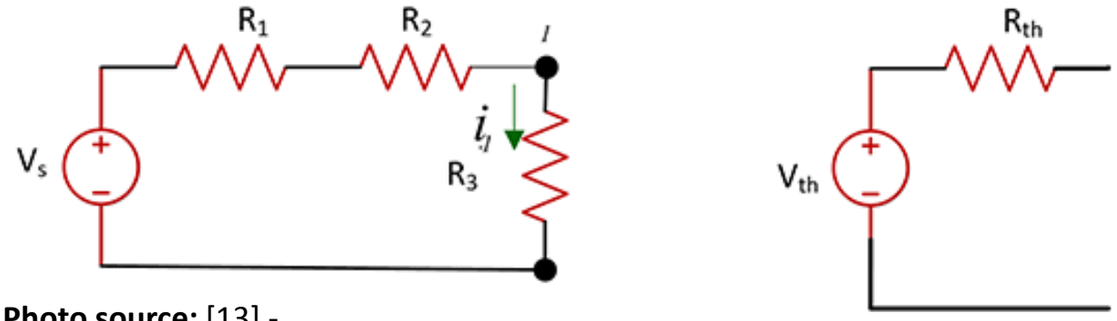


Photo source: [13] - https://www.cpp.edu/~elab/projects/project_08/images/Thevenin-1.gif

U_{Th} - terminallardagi ochiq zanjir kuchlanishi;

R_{Th} - mustaqil manbalar o‘chirilganda terminallardagi kirish yoki ekvivalent qarshiligidir.

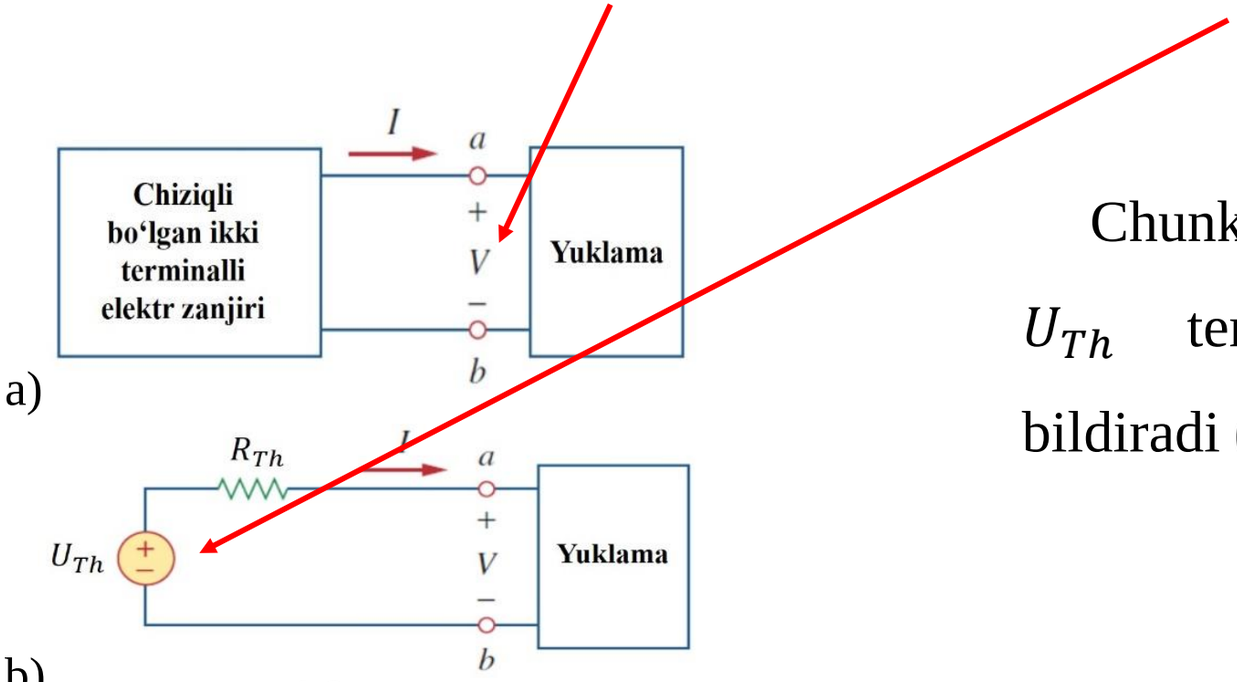
Maqsad: Tevenin ekvivalent kuchlanish - U_{Th} va qarshilik - R_{Th} ni qanday topishni o‘ranishdir.

Agar ularning terminallarida kuchlanish va tok kuchlarining nisbati bir xil bo‘lsa, ikkala zanjir ekvivalent deb ataladi.

Buning uchun 4.9-rasmdagi ikkita zanjirni ekvivalent deb faraz qilib, ikkita zanjirni nima ekvivalent qilishini bilib olamiz.

Agar *a*-*b* terminallari ochiq zanjir bo‘lsa (*yuklamani olib tashlash orqali*), tok kuchi o‘tmaydi.

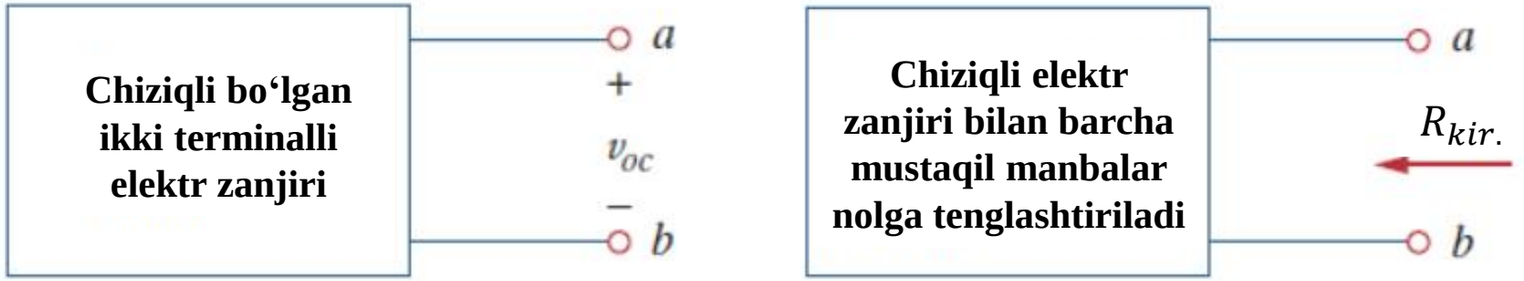
Demak, *a*-*b* terminali ochiq zanjir kuchlanishi, U_{Th} kuchlanish manbaiga teng bo‘lishi kerak.



4.9-rasm.

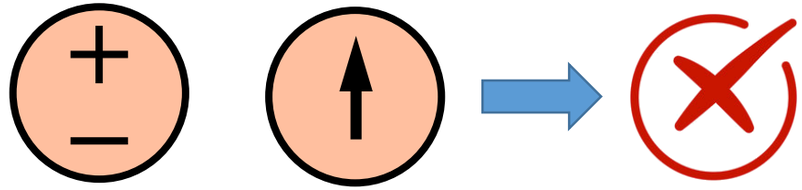
Chunki, ikkala zanjirlar ekvivalentdir. Shunday qilib, U_{Th} terminallar bo‘ylab ochiq zanjir kuchlanishini bildiradi (4.10-rasm, *a*). Ya’ni,

$$\boldsymbol{U_{Th} = u_{oc}} \qquad (4.6)$$



4.10-rasm. U_{Th} va R_{Th} ni aniqlash.

Tevening qarshiligi R_{Th} ni aniqlash uchun zanjirdagi yuklama uzib qo‘yiladi va *a*-*b* terminallar ochiq zanjir holatida bo‘lib,



R_{Th} mustaqil manbalar o‘chirilganda terminallardagi kirish qarshiligidir (4.10-rasm, *b*). Ya’ni,

$$\boldsymbol{R_{Th} = R_{kir.}(R_{in})} \qquad (4.7)$$

Tevenin qarshiligi R_{Th} ni topishda quyidagi holatlarni ko‘rib chiqishimiz kerak. Ya’ni:

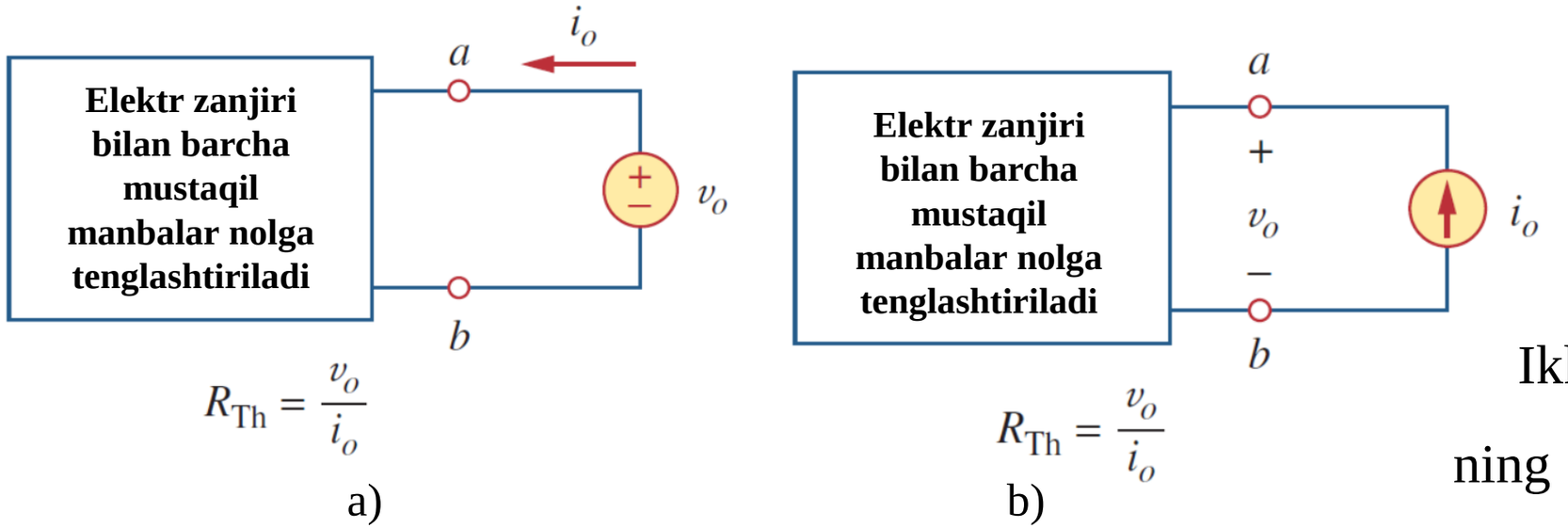
1-holat: Agar tarmoqda bog‘liq manbalar bo‘lmasa, biz barcha mustaqil manbalarni o‘chirib qo‘yamiz.

2-holat: Agar tarmoqda bog‘liq manbalar bo‘lsa, biz barcha mustaqil manbalarni o‘chirib qo‘yamiz.

Superpozitsiya usulida bo‘lgani kabi, bog‘liq manbalar o‘chirilmaydi, chunki ular zanjir o‘zgaruvchilari tomonidan boshqariladi.

Biz a va b terminallarida u_0 kuchlanish manbasini qo‘llaymiz va natijada i_0 tok kuchini aniqlaymiz.

U holda, $R_{Th} = \frac{u_0}{i_0}$ (4.11-rasm, *a*). Shu bilan bir qatorda, 4.11-rasm, *b* da ko‘rsatilganidek, *a*-*b* terminallariga i_0 tok kuchi manbasini kiritishimiz va terminal kuchlanish u_0 ni topishimiz mumkin.



4.11-rasm. Zanjirda bog‘liq manbalar bo‘lganda R_{Th} ni aniqlanishi.

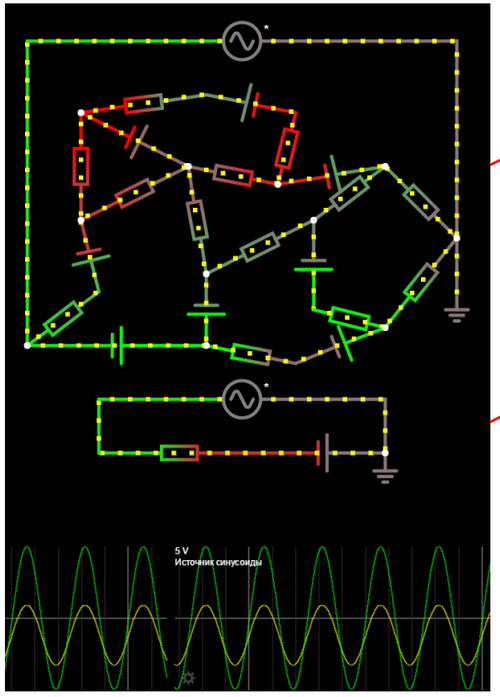
Ikkala yondashuvning har biri bir xil natijani beradi.

Ikkala yondashuvda biz u_0 va i_0 ning istalgan qiymatini qabul qilishimiz mumkin.

Masalan, biz $u_0 = 1\,V$ yoki $i_0 = 1A$ dan foydalanishimiz hattoki u_0 yoki i_0 ning aniqlanmagan qiymatlaridan foydalanishimiz ham mumkin.

Ko‘pincha R_{Th} salbiy qiymat oladi. Bunday holda, salbiy qarshilik ($u = -iR$) zanjirning quvvat bilan ta’minlanishini bildiradi. Bu **nomustaqil manbalarga** ega bo‘lgan zanjirida bo‘lisi mumkin.

Tevenin teoremasi zanjirlarni tahlil qilishda **juda muhimdir**. **Bu zanjirni soddalashtirishga yordam beradi.**



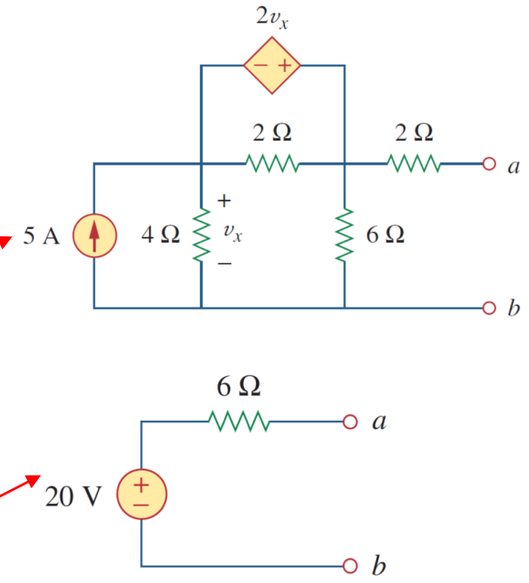
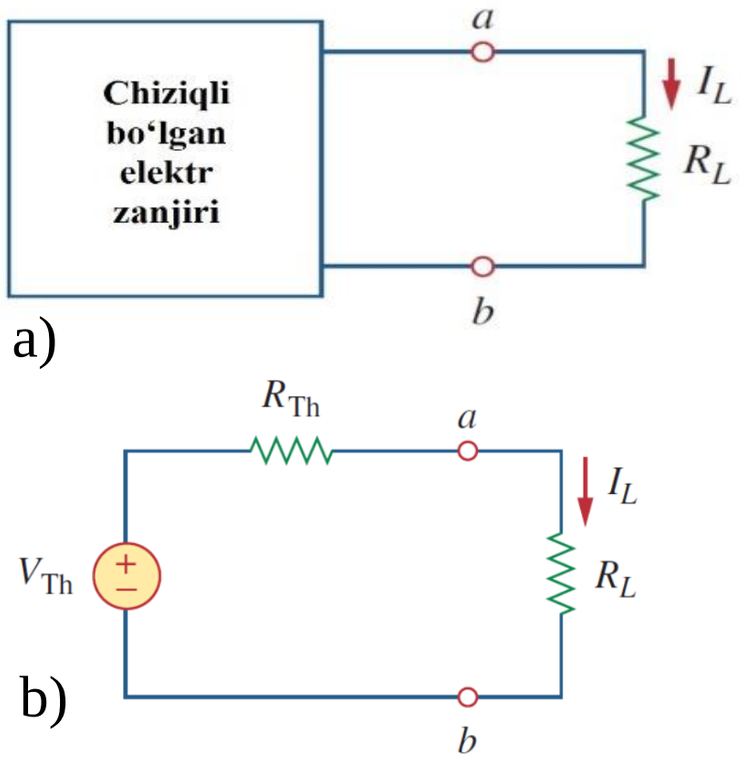


Photo source: [14] - Circuit_Simulator_1.2.0_x64_setup program. (Thevenin Theorem)

Katta zanjir bitta mustaqil kuchlanish manbai va bitta qarshilik bilan almashtirilishi mumkin.

Ushbu almashtirish usuli zanjirni loyihalashda muhim instrument bo‘lib xizmat qiladi.



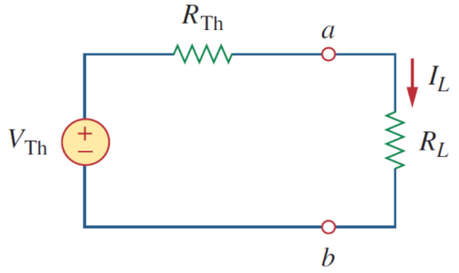
4.12-rasm. Yuklangan zanjir:

a) asl zanjir; b) Tevenin ekvivalent zanjiri.

O‘zgaruvchan yuklanishga ega chiziqli elektr zanjiri yuklamani hisobga olmaganda Tevenin ekvivalenti bilan almashtirilishi mumkin.

4.12-rasm, *a* da ko‘rsatilganidek, R_L yuklanish bilan tugatilgan chiziqli elektr zanjirini tahlil qilamiz.

4.12-rasm, *b* da yuklama terminallarida zanjir uchun Tevenin ekvivalenti olingandan so‘ng, yuklama bo‘ylab tok kuchi I_L va yuklama bo‘ylab kuchlanish U_L ni aniqlash oson.



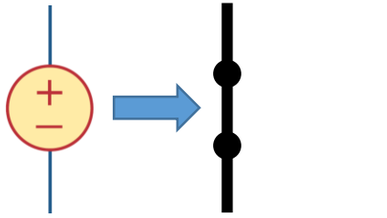
$$I_L = \frac{U_{Th}}{R_{Th}+R_L} \qquad (4.8\ a)$$

$$U_L = R_L I_L = \frac{R_L}{R_{Th}+R_L} U_{Th} \qquad (4.8\ b)$$

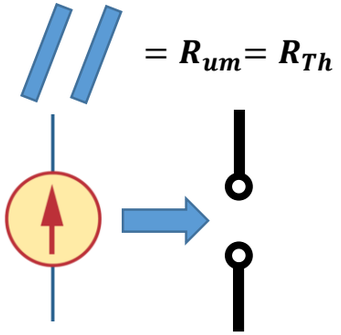
4.12-rasm, *b* ga e'tibor bersak, Tevenin ekvivalenti oddiy kuchlanish bo'luvchisi bo'lib, $U_L$ni shunchaki tekshirish orqali beradi.

Zanjirlarni hisoblashda Tevenin teoremasidan foydalanish tartibi:

1. R_{Th} =?

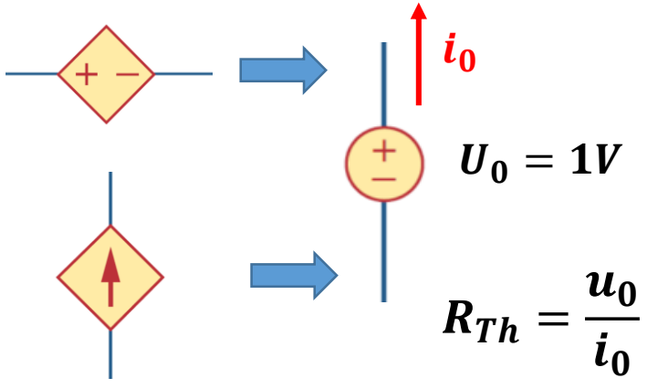


Qisqa tutashuv zanjir (*short circuit*)



Ochiq zanjir (*open circuit*)

2. R_{Th} =?

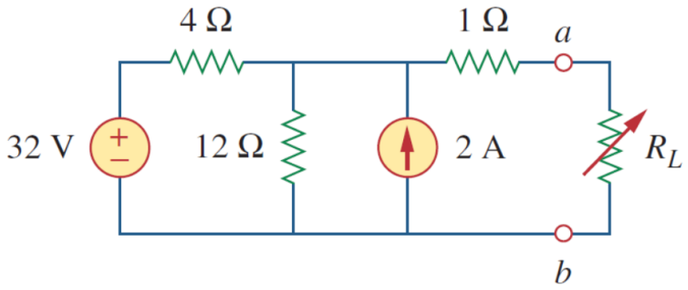


3. U_{Th} =?

- KVL;
- KCL;
- Mesh tahlili;
- Tugun tahlili;
- va boshqalar.

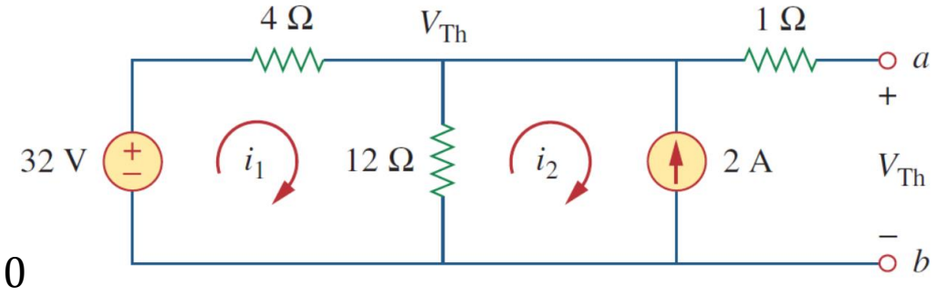
4.4.1-masala: *a-b* terminallarining chap tomonidagi Tevenin ekvivalent zanjirini toping

(4.13-rasm). So‘ng $R_L = 6, 16$ va 36 Om bo‘lgan qarshiliklar orqali tok kuchi I ni toping.



4.13-rasm.

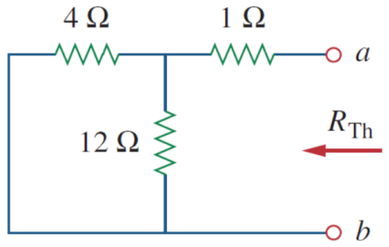
Yechish:



$$-32 + 4i_1 + 12(i_1 - i_2) = 0$$
$$-32 + 4i_1 + 12i_1 - 12i_2 = 0$$
$$-32 + 16i_1 - 12i_2 = 0 \quad (4.4.1)$$

i_1 - KVL: $u = u_1 + u_2 \rightarrow 32 = 4i_1 + 12i_2 \quad (4.4.2)$

KCL: $i_1 + i_3 = i_2 \rightarrow i_3 = 2 \rightarrow i_2 = i_1 + 2 \quad (4.4.3)$



$$R_{Th} = 4||12 + 1 = \frac{4 \cdot 12}{16} + 1 = 4\ \Omega$$

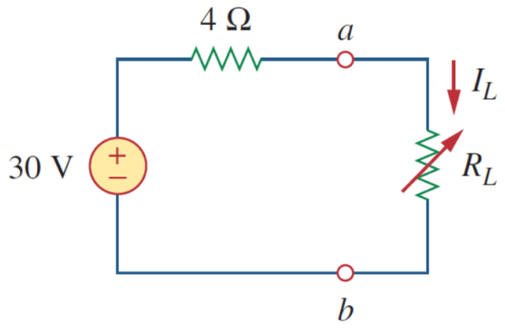
(4.4.3) tenglamani (4.4.2) tenglamaga almashtiramiz.

$$32 = 4i_1 + 12(i_1 + 2) \rightarrow i_1 = 0{,}5\ A \quad (4.4.4)$$

(4.4.4) tenglamani (4.4.1) tenglamaga almashtiramiz.

$$-32 + 16i_1 - 12i_2 = 0 \rightarrow i_2 = -2\ A \quad (4.4.5)$$

$$U_{Th} = 12(i_1 - i_2) = 12\big(0{,}5 - (-2)\big) = 30\ V$$



$$\boldsymbol{I_L = \frac{U_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}}$$

$R_L = 6$

$$I_L = \frac{30}{4 + 6} = 3\ A$$

$R_L = 16$

$$I_L = \frac{30}{4 + 16} = 1{,}5\ A$$

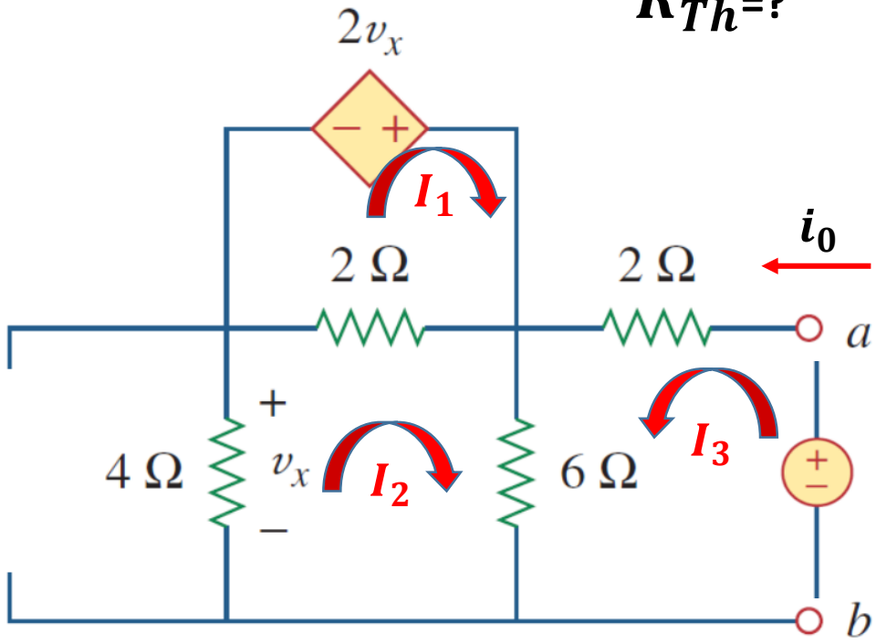
$R_L = 36$

$$I_L = \frac{30}{4 + 36} = 0{,}75\ A$$

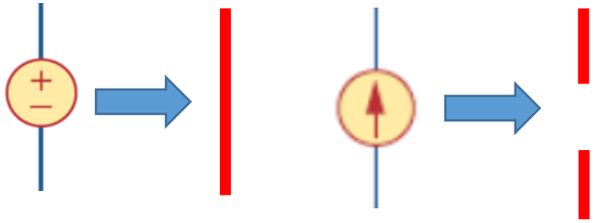
4.4.2-masala: Quyida keltirilgan elektr zanjiridagi *a-b* terminallarida zanjirning **Tevenin** ekvivalentini hosil qiling. So‘ng kuchlanish U_{Th} va qarshilik R_{Th}ni toping

Yechish:

R_{Th}=?



$U_0 = 1V$



$$R_{Th} = \frac{U}{I} = \frac{U_0}{i_0}$$

KVL: $\Sigma U=0$

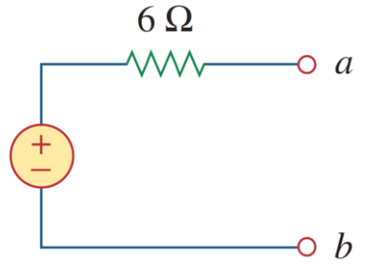
Mesh-1: $2(I_3 - I_2) + (-2U_x) = 0$

$U_x = 4(-I_2)$

Mesh-2: $4I_2 + 2(I_2 - I_1) + 6\,(I_2 + I_3) = 0$

Mesh-3: $1 + 2I_3 + 6\,(I_3 + I_2) = 0$

Zanjirning Tevenin ekvivalenti chiziladi



$$R_{Th} = \frac{1}{\frac{1}{6}} = 6\ Om$$

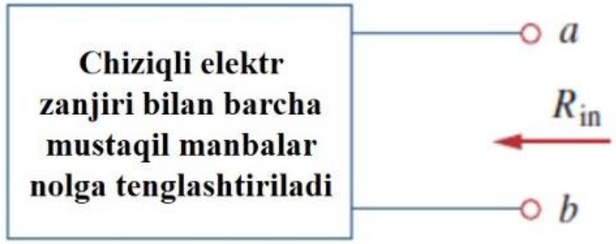
$U_{Th} = 19,98\ V$

TEVENIN TEOREMASI

Chiziqli ikki terminalli elektr zanjirni Tevenin ekvivalenti bilan almashtirish.

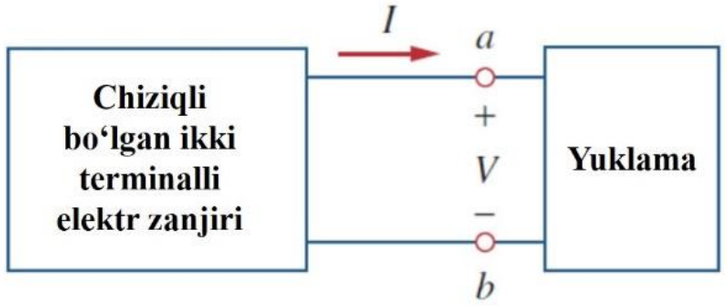
$$U_{Th} = U_{OC}$$

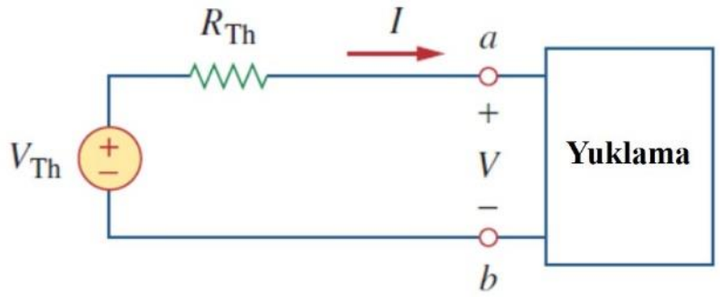
$$V_{\mathrm{Th}} = v_{oc}$$

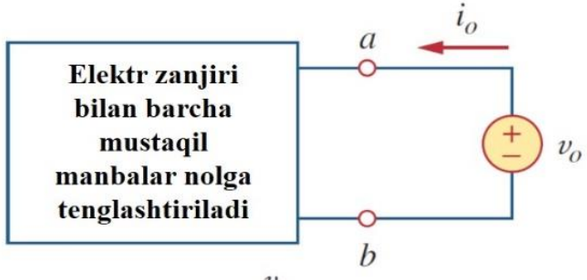


$$R_{Th} = R_{in}$$

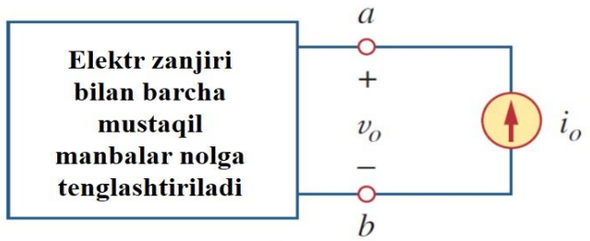
$$R_{\mathrm{Th}} = R_{\mathrm{in}}$$





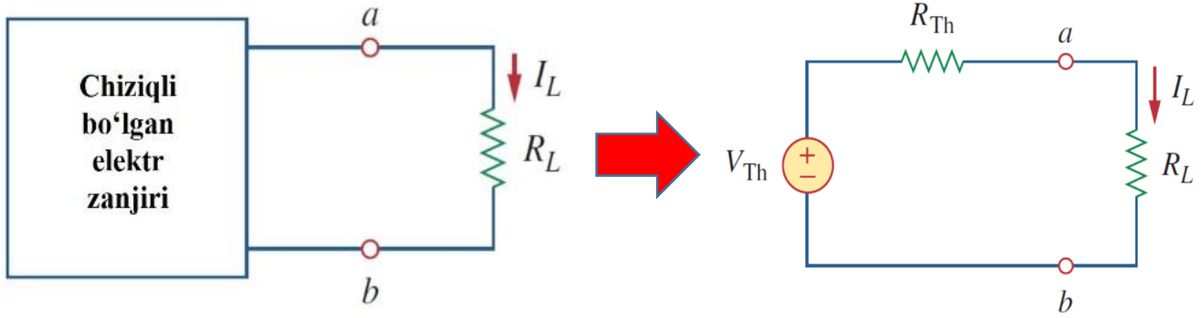


$$R_{\mathrm{Th}} = \frac{v_o}{i_o}$$



$$R_{\mathrm{Th}} = \frac{v_o}{i_o}$$

$$R_{Th} = \frac{U_0}{i_0}$$

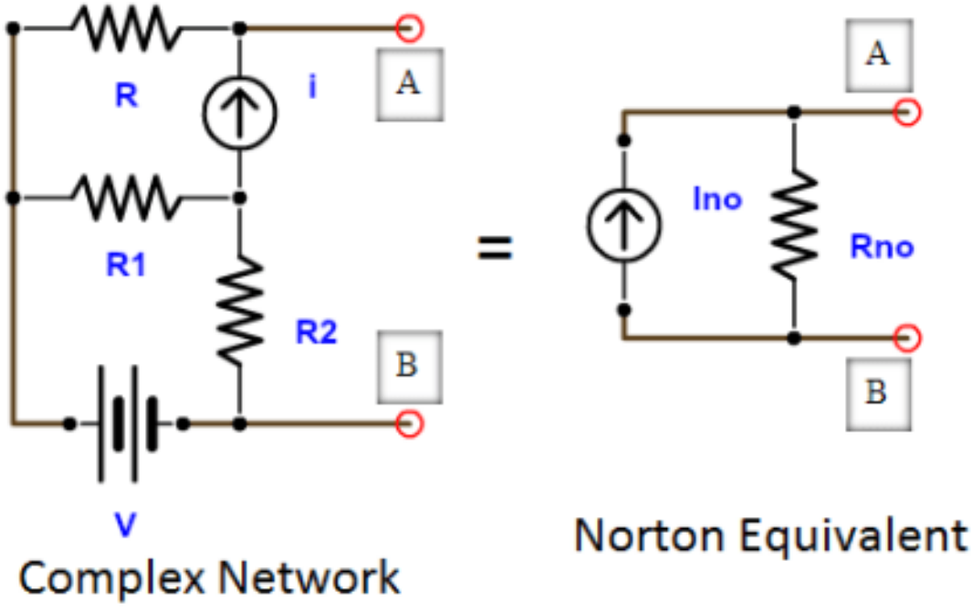


$$I_L = \frac{U_{Th}}{R_{Th} + R_L} \qquad U_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} U_{Th}$$

4.5. Norton teoremasi.

1926 yilda, Tevenin o‘z teoremasini e’lon qilganidan taxminan 43 yil o‘tgach, Bell Telefon Laboratoriyasida amerikalik muhandis E.L. Norton shunga o‘xshash teoremani taklif qildi.

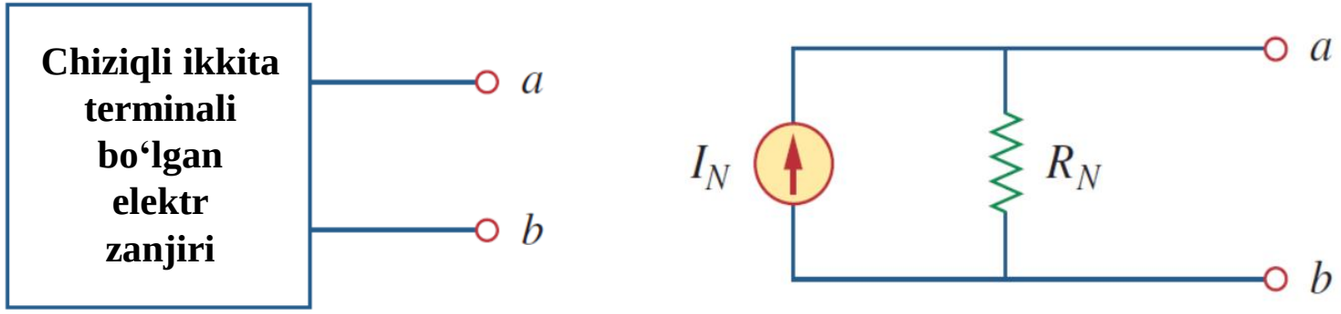
Norton teoremasiga asosan chiziqli ikki terminalli zanjirni qarshilik R_N bilan parallel ravishda tok kuchi manbaidan I_N tashkil topgan ekvivalent zanjir bilan almashtirish mumkin bo‘ldi.



Bu yerda: I_N – terminallardagi qisqa tutashuv zanjiridagi tok kuchi va R_N - mustaqil manbalar o‘chirilganda terminallardagi kiruvchi yoki ekvivalent qarshiligidir.

Photo source: [11] - https://i0.wp.com/electronicspani.com/wp-content/uploads/2014/02/Conversion_between-_Thevenin_and_Norton_equivalent.png?w=688&ssl=1

Shunday qilib, 4.16-rasm, *a* dagi zanjirni 4.16-rasm, *b* dagi zanjir bilan almashtirilishi mumkin.



a) b)

4.16-rasm.

a) asl zanjir; b) Norton ekvivalent zanjiri.

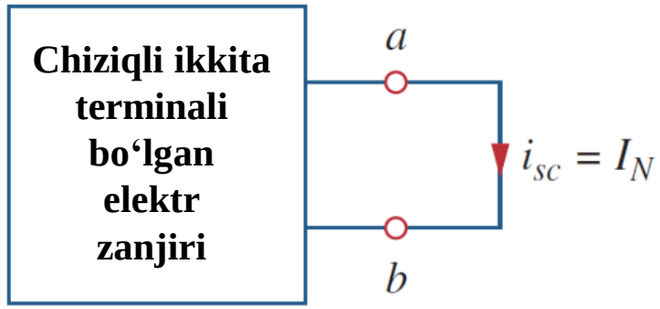
Ma’lumki, murakkab zanjirlarda manba transformatsiyasini qo‘llaganda qarshiliklar bir xil bo‘lgani kabi, Tevenin va Norton usullarida ham qarshiliklar teng bo‘ladi. Ya’ni,

$$\boldsymbol{R_N = R_{Th}} \qquad (4.9)$$

Photo source: [15] - Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 145.

Norton tok kuchi I_N ni topish uchun biz 4.16-rasmdagi ikkala zanjirda *a* terminalidan *b* ga oqib keluvchi qisqa tutashuv zanjir tokini aniqlaymiz.

4.17-rasmdagi qisqa tutashuv toki I_N dir.



$$\boldsymbol{I_N = i_{sc}} \qquad (4.10)$$

Nomustaqil va mustaqil manbalar Tevenin teoremasidagi kabi bir xil usuldan foydalaniladi.

4.17-rasm. Norton tok kuchi I_N ni aniqlanishi.

Norton va Tevenin teoremalari orasidagi yaqin bog‘liqlikka e’tibor bersak, (4.9) tenglamadagi kabi $R_N = R_{Th}$. Shunday qilib,

$$\boldsymbol{I_N = \frac{U_{Th}}{R_{Th}}} \qquad (4.11)$$

Bunday munosabatni aslida manba o‘zgarishi deyiladi. Shu sababli, manba transformatsiyasi ko‘pincha Tevenin-Norton transformatsiyasi deb ataladi.

Chunki U_{Th}, I_N va R_{Th} (4.11) tenglamaga muvofiq bog‘langan. Tevenin yoki Norton ekvivalent zanjirini aniqlash uchun biz quyidagilarni topishimiz kerak:

- *a* va *b* terminallaridagi ochiq zanjir kuchlanishi (u_{oc});
- *a* va *b* terminallaridagi qisqa tutashuvdagi (*berk*) zanjir tok kuchi (i_{sc});
- Barcha mustaqil manbalar o‘chirilganda *a*-*b* terminallaridagi ekvivalent yoki kiruvchi $R_{kir.}$ qarshiligi.

Biz eng kam kuch sarflaydigan usul yordamida uchtadan ikkitasini hisoblashimiz va Om qonuni yordamida uchinchisini olish uchun foydalanishimiz mumkin. Demak,

$$U_{Th} = u_{oc} \qquad (4.12\ a)$$

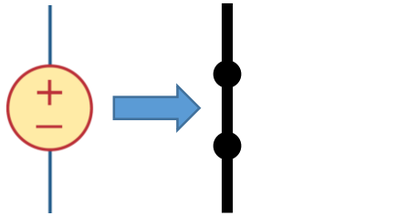
$$I_N = i_{sc} \qquad (4.12\ b)$$

$$R_{Th} = \frac{u_{oc}}{i_{sc}} = R_N \qquad (4.12\ c)$$

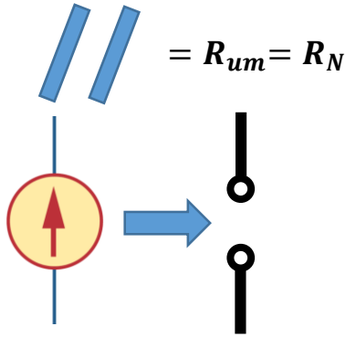
Bundan tashqari, ochiq zanjir va yopiq zanjir sinovlari kamida bitta mustaqil manbani o‘z ichiga olgan har qanday Tevenin yoki Norton ekvivalentini topish uchun yetarlidir.

Zanjirlarni hisoblashda Norton teoremasidan foydalanish tartibi:

1. $\boldsymbol{R_{Th} = R_N}$

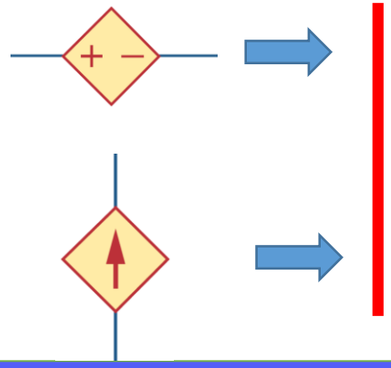


Qisqa tutashuv zanjir
(*short circuit*)



Ochiq zanjir
(*open circuit*)

2. $\boldsymbol{R_N}$ =?



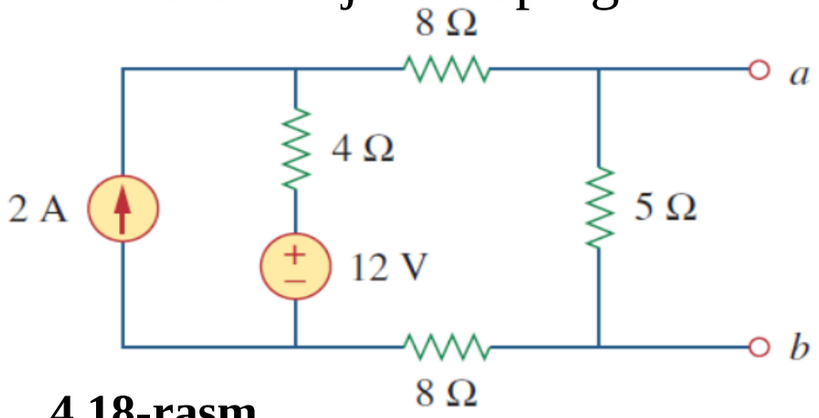
I_{sc}

$$R_N = \frac{U_{oc}}{I_{sc}}$$

3. $\boldsymbol{I_N}$ =?

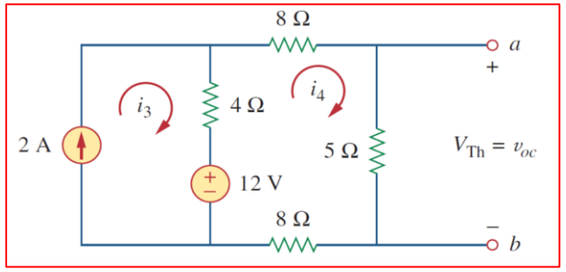
- KVL;
- KCL;
- Mesh tahlili;
- Tugun tahlili;
- va boshqalar.

4.5.1-masala: 4.18-rasmdagi elektr zanjirining *a-b* terminallarida Norton ekvivalent elektr zanjirini toping.



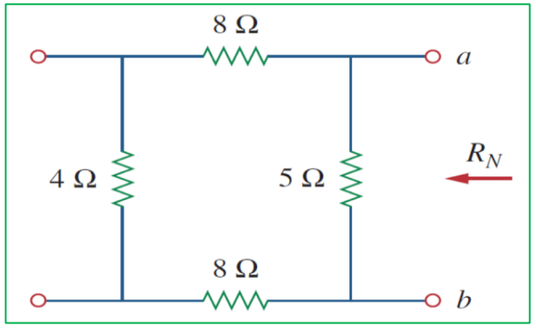
4.18-rasm.

2-usul:



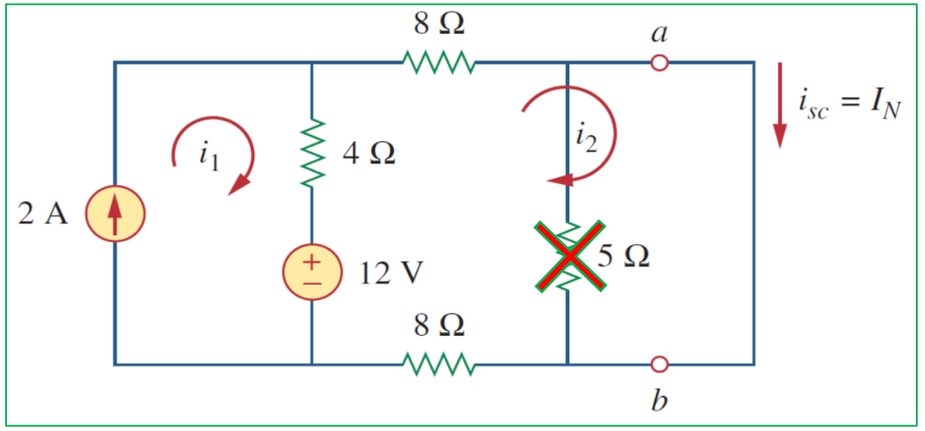
$$I_3 = 2\ A$$
$$-12 + 4(I_4 - I_3) + 21I_4 = 0$$
$$-12 + 4I_4 - 4 * 2 + 21I_4 = 0$$
$$25I_4 = 20 \quad \rightarrow \quad \boldsymbol{I_4 = 0,8\ A}$$

Yechish: **1-usul:**



$$\boldsymbol{R_N = 5||(8 + 4 + 8) == \frac{20 \cdot 5}{20 + 5}}$$
$$\boldsymbol{= 4\ Om}$$

$$\boldsymbol{R_{Th} = \frac{U_{OC}}{I_{SC}} = \frac{4}{1} = 4\ Om}$$

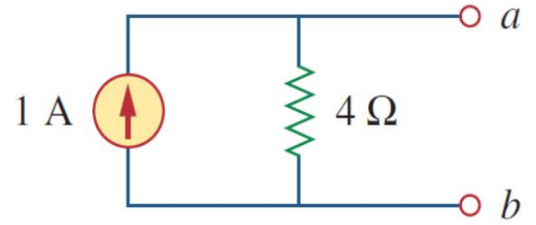


$$I_1 = 2\ A$$
$$-12 + 4(I_2 - I_1) + 16I_2 = 0$$
$$-12 + 4I_2 - 4 * 2 + 16I_2 = 0$$
$$20I_2 = 20 \quad \rightarrow \quad I_2 = 1\ \text{A}$$
$$\boldsymbol{I_N = I_2 = 1\ \text{A}}$$

$$\boldsymbol{U_{OC} = V_{Th} = 5I_4 = 5 * 0,8 = 4\ V}$$

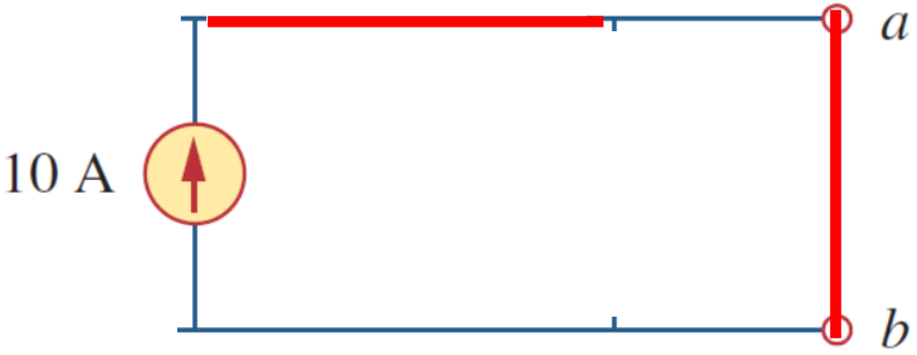
$$\boldsymbol{I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1\ A}$$

Zanjirning Norton ekvivalenti

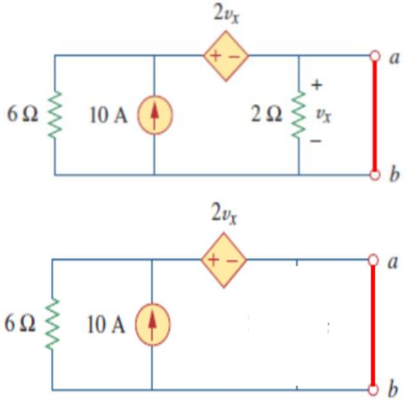


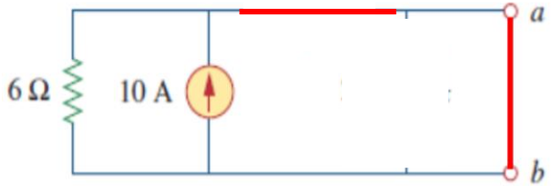
4.5.2-masala: Elektr zanjirining a-b terminallarida **Norton** ekvivalent elektr zanjirini hosil qiling.

So‘ng qarshilik R_N va tok kuchi I_N ni toping.



$I_N = ?$





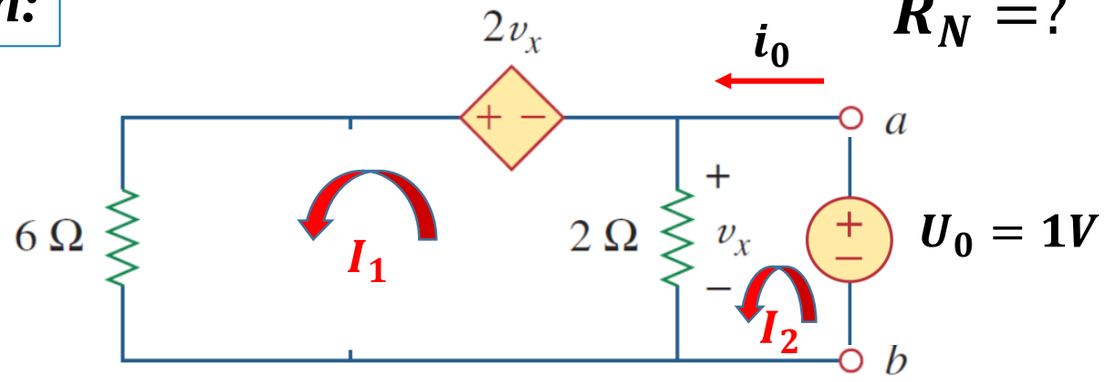
$U_x = 0$

$2U_x = 0$

$I_N = 10\ A$

Yechish:

$R_N = ?$



KVL: Loop-1

$$6I_1 + 2(I_1 - I_2) - 2u_x = 0$$

$$2u_x = 2(I_1 - I_2)$$

$$6I_1 + 2(I_1 - I_2) - 2(2(I_1 - I_2)) = 0$$

$$I_1 + (I_1 - I_2) = 0$$

$$2I_1 - I_2 = 0$$

KVL: Loop-2

$$-1 + 2(I_2 - I_1) = 0$$

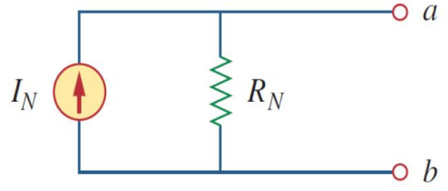
$$-2I_1 + 2I_2 = 1$$

\+

$$-2I_1 + 2I_2 = 1$$

$$I_2 = I = 1\ A$$

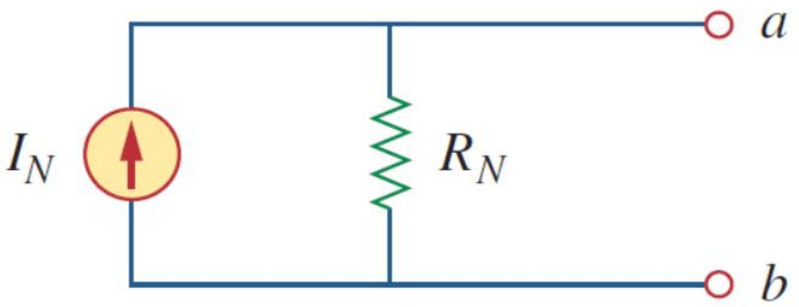
$$R_N = \frac{u_0}{i_0} = \frac{1}{1} = 1\ \Omega$$

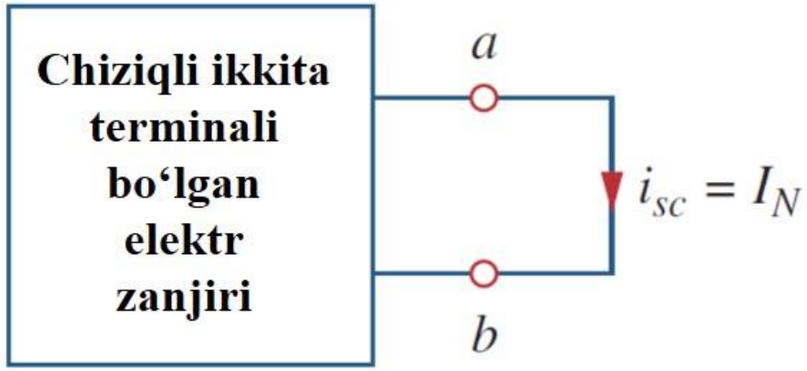


NORTON TEOREMASI

$$I_N = \frac{U_{Th}}{R_{Th}}$$

$$R_N = R_{Th}$$





$$I_N = I_{SC}$$

$$U_{Th} = U_{OC}$$

$$R_{Th} = \frac{U_{OC}}{I_{SC}} = R_N$$

U_{OC} - open circuit

I_{SC} - short circuit

FOYDALANILGAN MANBALAR:

11. https://i0.wp.com/electronicspani.com/wp-content/uploads/2014/02/Conversion_between-_Thevenin_and_Norton_equivalent.png?w=688&ssl=1
12. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 139.
13. https://www.cpp.edu/~elab/projects/project_08/images/Thevenin-1.gif
14. Circuit_Simulator_1.2.0_x64_setup program. (Thevenin Theorem)
15. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 145.

E'TIBORINGIZ UCHUN RAHMAT!!!