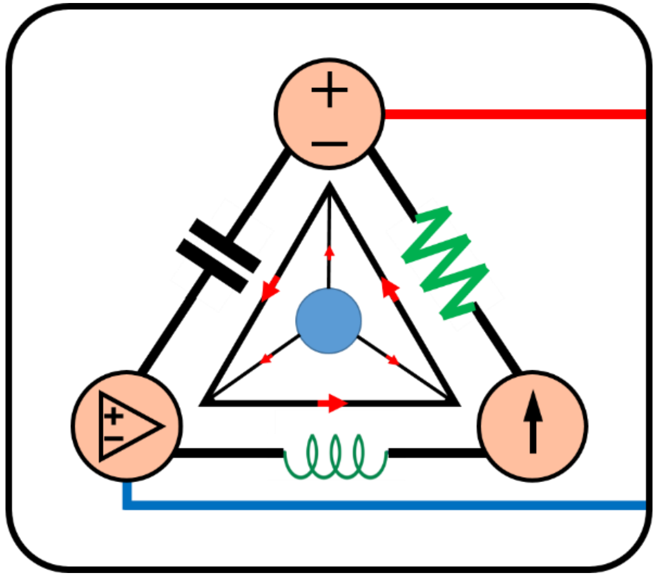
7-Mavzu: Birinchi tartibli elektr zanjiri.

(7th Topic: First-Order Circuit)

7-Mavzuning 1-qismi

(1st part of the 7th Topic)



7-hafta uchun

For the 7th week

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7-Mavzu: Birinchi tartibli elektr zanjiri.

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O‘quv rejasi:

7.1. Umumiy tushunchalar.

7.2. Manbadan holi qarshilik va kondensator (*RC*) zanjiri.

7.3. Manbadan holi qarshilik va induktor (*RL*) zanjiri.

7.4. Yakkalik funksiyalari.

7.1. Umumiy tushunchalar.

Oldingi darslarimizda passiv element (*rezistorlar, kondensatorlar va induktiv g'altaklar)* larni va bitta faol element (*operatsion kuchaytirgich*) ni alohida ko'rib chiqdik.

Endi ikkita yoki uchta passiv elementlarning turli kombinatsiyalarini o'z ichiga olgan elektr zanjirlarni ko'rib chiqamiz.

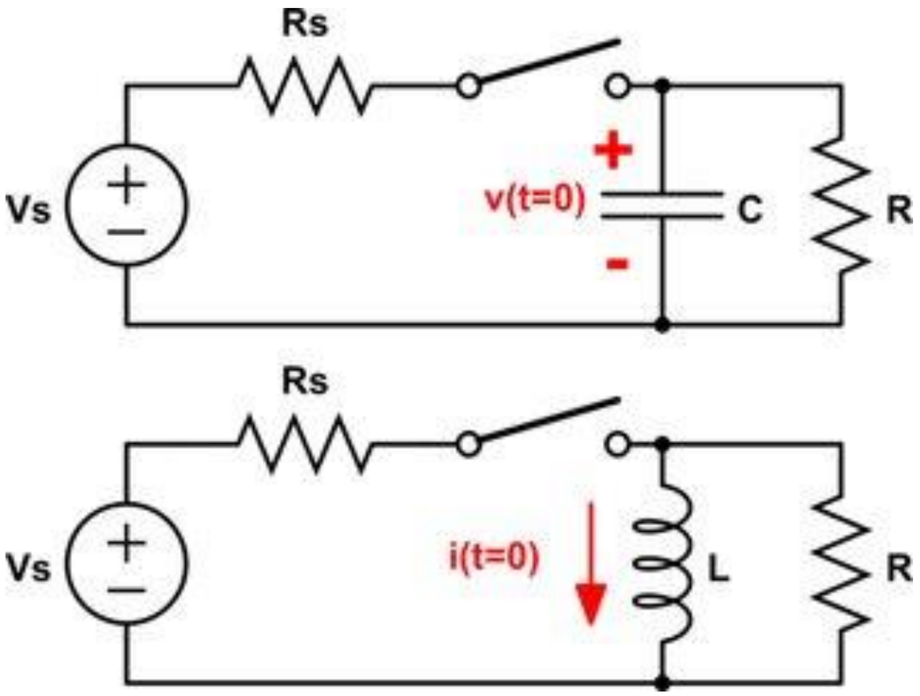


Photo source: [1] - https://i.stack.imgur.com/Zp36Jm.png

Bu mavzuda biz ikki turdagi oddiy zanjirlar bilan tanishamiz:

1) **rezistor** va **kondensatordan** iborat bo'lgan ***RC*** zanjiri;

2) **rezistor** va **induktorni** o'z ichiga olgan ***RL*** zanjiri.

Bu zanjirlar qanchalik sodda bo‘lsa ham ular elektronika, aloqa va boshqaruv tizimlarida keng qo‘llaniladi.

RC va ***RL*** zanjirlari qarshilik zanjirlari kabi Kirxgof qonunlari orqali tahlil qilinadi.

Yagona farq shundaki, Kirxgof qonunlarini sof qarshilikli zanjirlarga qo‘llash natijasida algebraik tenglamalar hosil qilinsa, ***RC*** va ***RL*** zanjirlarida algebraik tenglamalarga qaraganda **yechish qiyinroq bo‘lgan differensial tenglamalar hosil qilinadi.**

RC va ***RL*** zanjirlarini tahlil qilish natijasida **birinchi tartibli differensial tenglamalar** hosil bo‘ladi.

$$a\frac{du(t)}{dt} + bu(t) = d$$

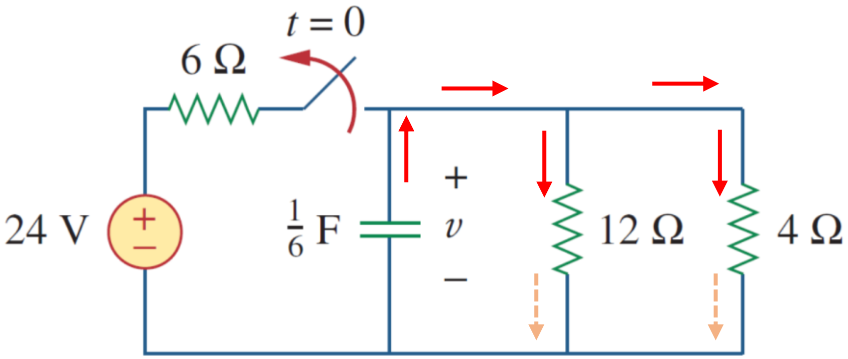
Demak, zanjirlar birgalikda birinchi tartibli elektr zanjirlari deb ataladi.

Birinchi tartibli elektr zanjir birinchi tartibli differensial tenglama bilan tavsiflanadi.

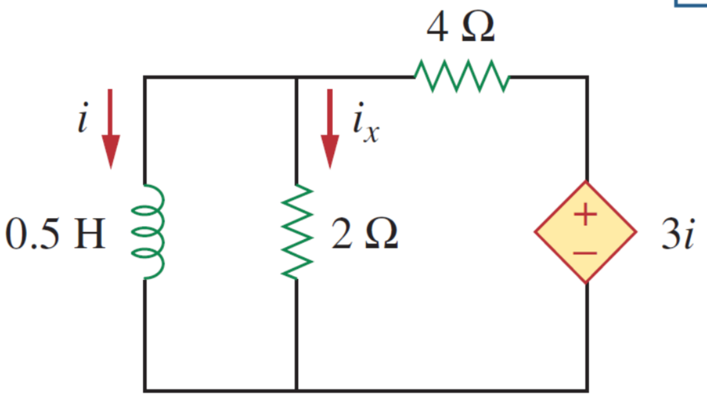
Ikki turdagi birinchi tartibli elektr zanjirlarini (***RC*** va ***RL***) oʻrganishning ikkita usuli mavjud.

Birinchi usul – zanjirlardagi elementlarni saqlashning dastlabki shartlari.

Manbasiz deb nomlanadigan bu zanjirlarda biz energiyani dastlab sigʻimli yoki induktivlik elementlarda saqlanadi deb taxmin qilamiz.



Energiya tok kuchini zanjirda oqishiga olib keladi va rezistorlarda asta-sekin tarqaladi.



Garchi manbasiz zanjirlar ta’rifiga koʻra mustaqil manbalardan holi boʻlsa-da, ular bogʻliq manbalarga ega boʻlishi mumkin.

Ikkinchi usul - mustaqil manbalar.

Biz ko‘rib chiqadigan mustaqil manbalar o‘zgarmas tok (*DC*) manbalaridir.

Birinchi tartibli zanjirlarning ikki turi va ularni qamrovining ikkita usulini bu mavzuda o‘rganadigan to‘rtta mumkin bo‘lgan vaziyatni birlashtiradi.

Shunday qilib, ***RC*** va ***RL*** zanjirlarining to‘rtta tipik qo‘llanilishini ko‘rib chiqamiz:

- **Tortish va Rele zanjirlari;**
- **Fotoflesh bloki;**
- **Avtomobilning o‘t oldirish tizimi zanjiri.**

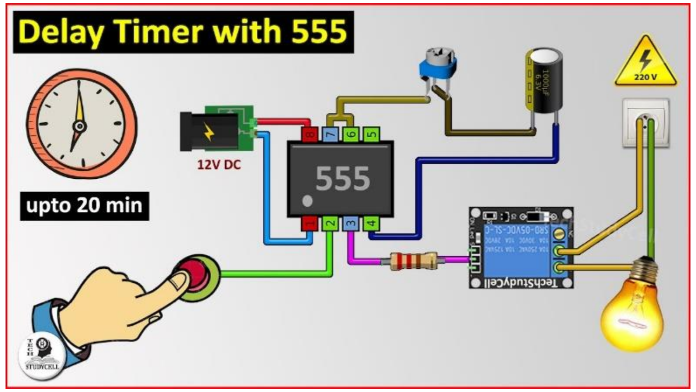


Photo source: [2] - https://i.ytimg.com/vi/wgvo4aTATsU/maxresdefault.jpg

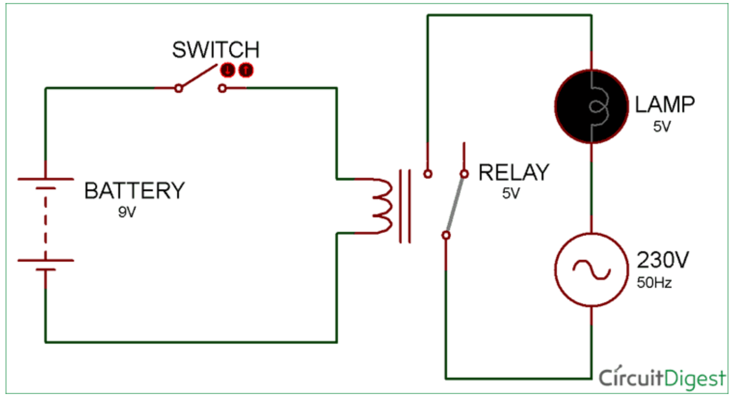


Photo source: [3] - https://circuitdigest.com/sites/default/files/circuitdiagram/Simple-relay-switch-circuit-diagram_0.png

Photo source: [4] - https://cdn11.bigcommerce.com/s-6eafq5t88w/images/stencil/500x659/products/33412/54192/743181__06345.1644440897.jpg?c=1

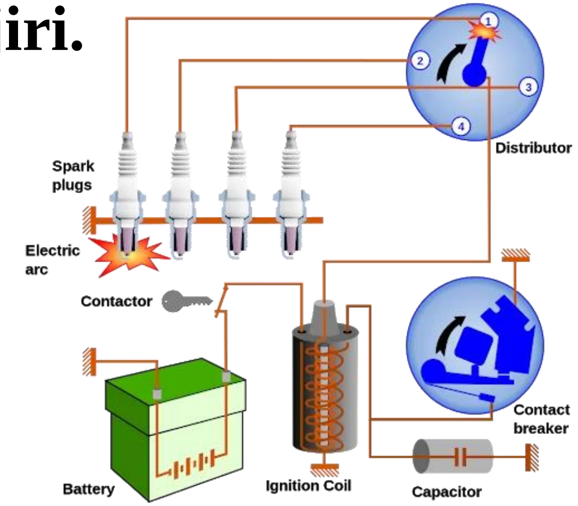
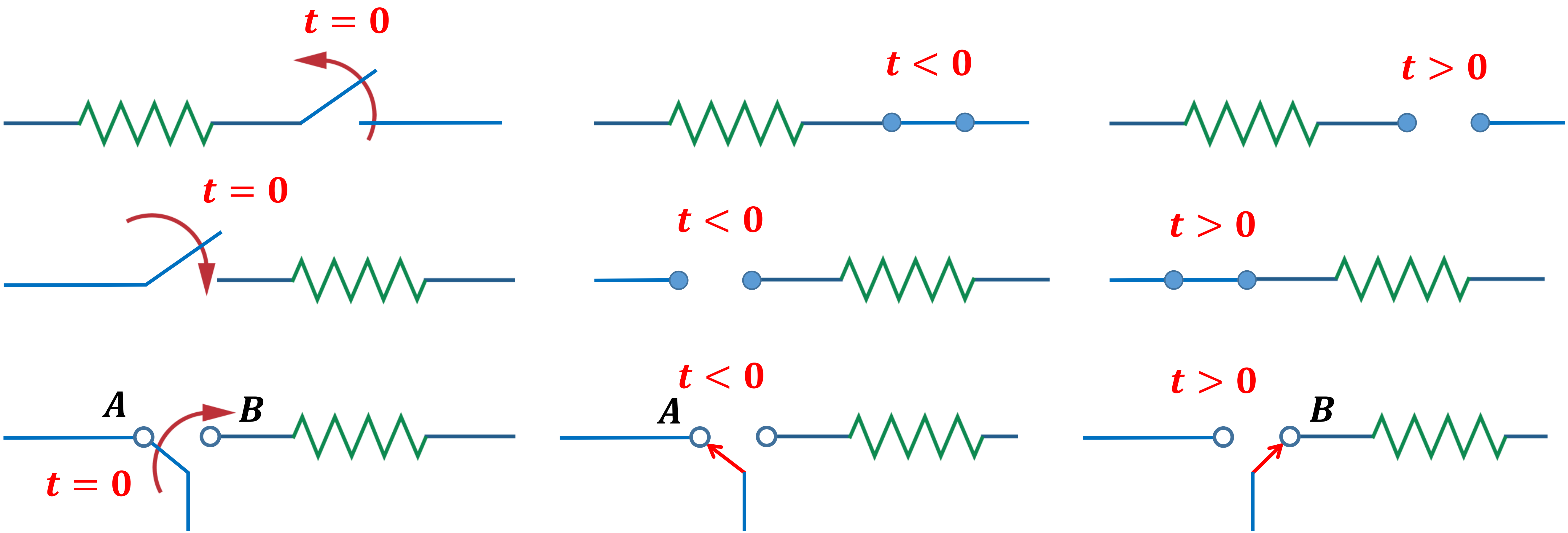
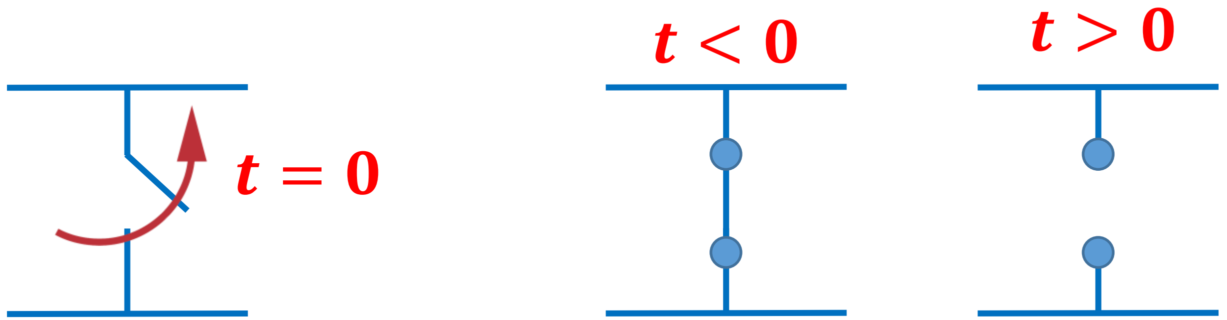


Photo source: [5] - https://i0.wp.com/studentlesson.com/wp-content/uploads/2020/10/ignition-system.png?fit=512%2C457&ssl=1

Kalitning zanjirdagi ahamiyati:



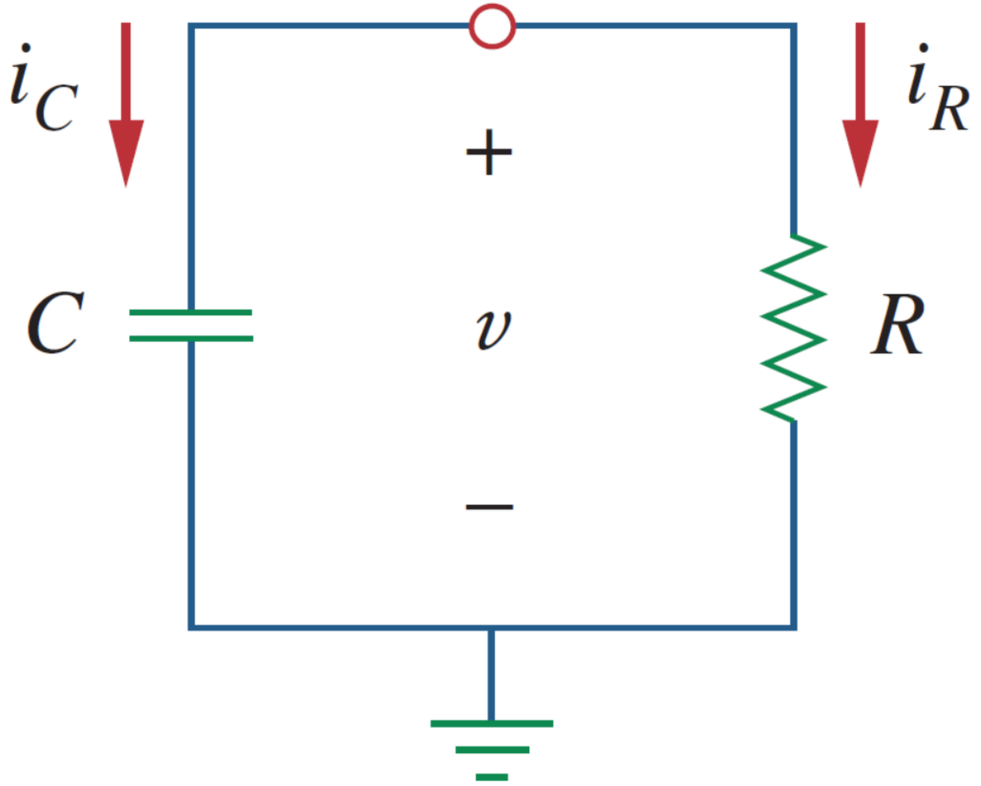


$U_0(t = 0) -$ *boshlang‘ich kuchlanish.*

$U_0(t = \infty) -$ *oxirgi kuchlanish.*

7.2. Manbadan holi qarshilik va kondensator (*RC*) zanjiri.

Manbasiz ***RC*** zanjiri uning o‘zgarmas tok manbai to‘satdan uzilganda paydo bo‘ladi. Bu holatda kondensatordagi saqlangan energiya rezistorlarga o‘tkaziladi.



7.1-rasm. Manbadan holi *RC* zanjiri.

Bizning maqsadimiz kondensator bo‘ylab o‘tuvchi kuchlanishni $u(t)$ deb hisoblab, zanjir reaksiyasini aniqlashdir.

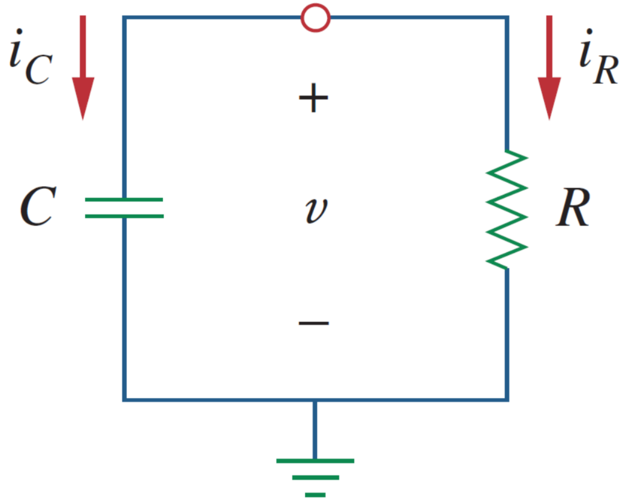
Kondensator dastlab zaryadlanganligi sababli, *t=0* vaqtda boshlang'ich kuchlanishni taxmin qilishimiz mumkin.

$$u(0) = u_0 \qquad (7.1)$$

Kondensatorda saqlanadigan energiyaning qiymati.

$$W(0) = \frac{1}{2}Cu_0^2 \qquad (7.2)$$

KCLni qo'llaymiz.



$$i_C + i_R = 0 \qquad (7.3)$$

$$i_C = C\frac{du}{dt} \qquad i_R = \frac{u}{R}$$

$$C\frac{du}{dt} + \frac{u}{R} = 0 \qquad (7.4.a)$$

$$\frac{du}{dt} + \frac{u}{RC} = 0 \quad \text{yoki,} \quad \frac{du}{dt} = -\frac{u}{RC} \qquad (7.4.b)$$

Bu birinchi tartibli differensial tenglama, chunki u ning faqat birinchi hosilasi ishtirok etadi. Buni hal qilish uchun biz atamalarni quyidagicha tartibga solamiz.

$$du = -\frac{u}{RC}dt \quad \text{yoki,} \quad \frac{du}{u} = -\frac{1}{RC}dt \qquad (7.5)$$

Ikkala tomonni integrallab, quyidagini olamiz, $\int \frac{1}{U}du = -\frac{1}{RC}\int dt \;\rightarrow\; \ln u = -\frac{t}{RC} + \ln A$

bu yerda $\ln A$ integratsiya doimiysi (*konstantasi*). Shunday qilib,

$$\ln u = -\frac{t}{RC} + \ln A \qquad (7.6)$$

$$\ln u - \ln A = -\frac{t}{RC} \;\rightarrow\; \ln u - \ln A = \ln(\frac{u}{A})$$

Ikkala tomonlarni e darajaga keltirib, quyidagiga ega bo‘lamiz,

$$\ln\left(\frac{u}{A}\right) = -\frac{t}{RC} \;\rightarrow\; \frac{u}{A} = e^{-\frac{t}{RC}}$$

$$\boxed{u(t) = Ae^{-\frac{t}{RC}}}$$

Dastlabki shartlardan kelib chiqib, $u(0) = A = u_0$. Demak,

$$\boldsymbol{u(t) = U_0 e^{-\frac{t}{RC}}} \tag{7.7}$$

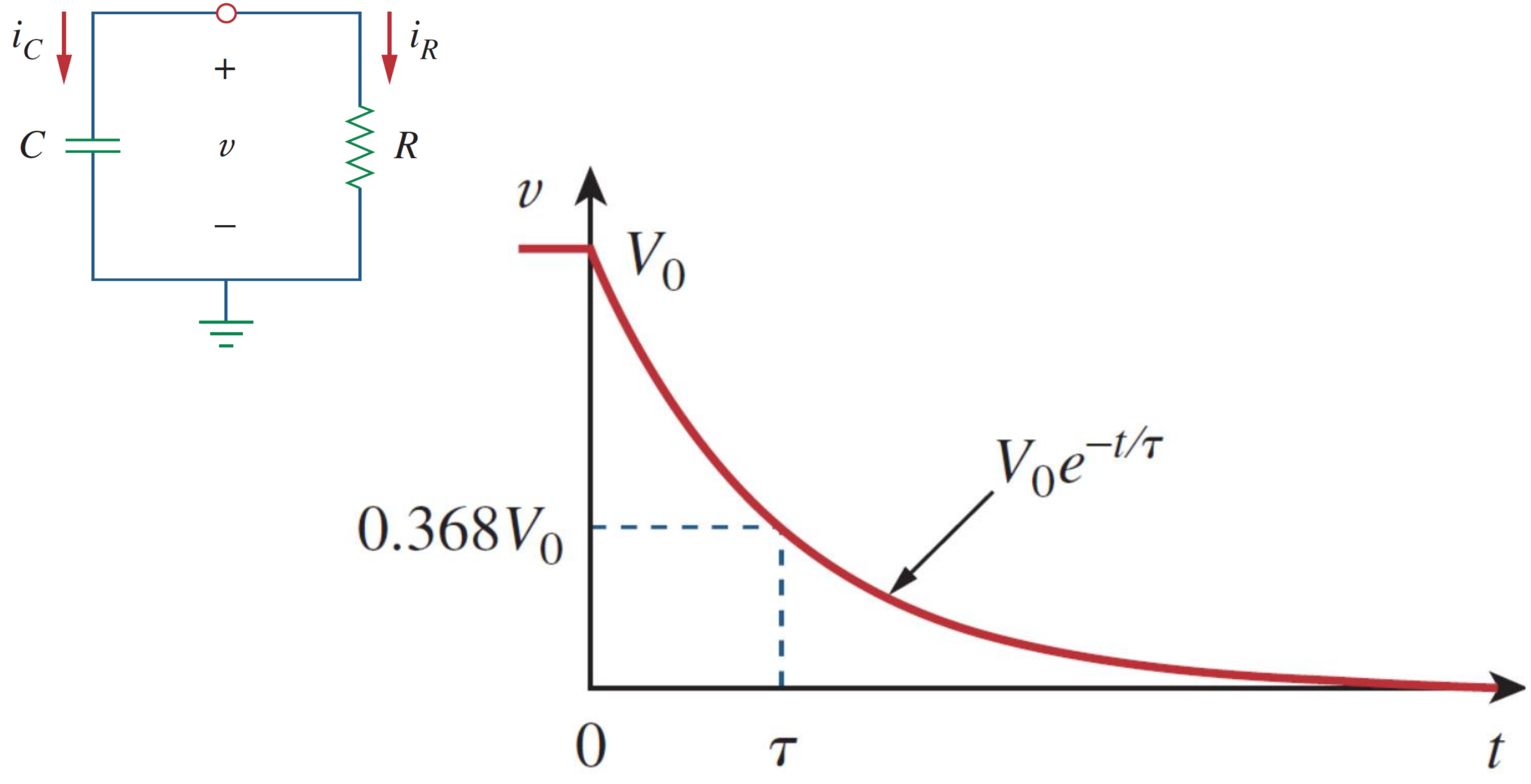
Bu shuni ko‘rsatadiki, ***RC*** qutblarida kuchlanish reaksiyasi dastlabki kuchlanishning eksponensial parchalanishini hosil qiladi.

Reaksiya ba’zi bir tashqi kuchlanish yoki tok manbasiga bog‘liq emas, balki saqlangan boshlang‘ich energiya va zanjirning fizik xususiyatlariga bog‘liq bo‘lganligi sababli, bu zanjirning ***tabiiy reaksiyasi*** deb ataladi.

Zanjirning ***tabiiy reaksiyasi*** tashqi ta’sir manbalarisiz zanjirning o‘zini tutishini (kuchlanish va tok kuchlari nuqtai nazaridan) bildiradi.

Tabiiy reaksiya tashqi manbalarsiz, faqat zanjirning tabiatiga bog‘liq.

Aslida, zanjir faqat kondensatorda saqlanadigan energiya tufayli reaksiya beradi.



$t = 0$ **da,**	$u(0) = u_0$
$t = 1$ **da,**	$u(1) = u_0 e^{-\frac{1}{RC}}$
$t = 5$ **da,**	$u(5) = u_0 e^{-\frac{5}{RC}}$

7.2-rasm. *RC* zanjirining kuchlanish reaksiyasi.

t=0 da $\boldsymbol{u(0) = u_0}$ boshlang‘ich shartga egamiz. *t* ortishi bilan kuchlanish nolga tushadi.

Kuchlanishning pasayish tezligi $\boldsymbol{\tau}$ - *vaqt doimiysi* bilan ifodalanadi.

Zanjirning ***vaqt doimiysi*** - $\frac{1}{e}$ koeffitsientiga yoki uning boshlang‘ich qiymatining 36,8 foiz reaksiyaning turg‘unligi uchun zarur bo‘lgan vaqt.

Bu shuni anglatadiki, *t*= τ da (7.7) tenglama quyidagi qiymatga ega bo‘ladi.

$$\boldsymbol{u(t) = U_0 e^{-\frac{t}{RC}}} \qquad \boldsymbol{u_o e^{-\frac{t}{RC}} = u_o e^{-1} = 0,368 u_0}$$

yoki,

$$\boldsymbol{\tau = RC} \tag{7.8}$$

Vaqt doimiysi nuqtai nazaridan, (7.7) tenglama quyidagicha yozilishi mumkin.

$$u(t) = U_0 e^{-\frac{t}{RC}} \longrightarrow u(t) = u_o e^{-\frac{t}{\tau}} \qquad (7.9)$$

7.1-jadval

$\frac{u(t)}{u_o} = e^{-\frac{t}{\tau}}$ **ning qiymati**	
t	$u(t)/u_o$
τ	0,36788
2τ	0,13534
3τ	0,04979
4τ	0,01832
5τ	0,00674

7.1-jadvaldan ko‘rinib turibdiki, $u(t)$ kuchlanish 5τ dan keyin u_0 ning 1 foizidan kam ekanligini ko‘rsatadi.

Demak, kondensatorning besh vaqt doimiysidan keyin to‘liq zaryadsizlangan (yoki zaryadlangan) deb taxmin qilish mumkin.

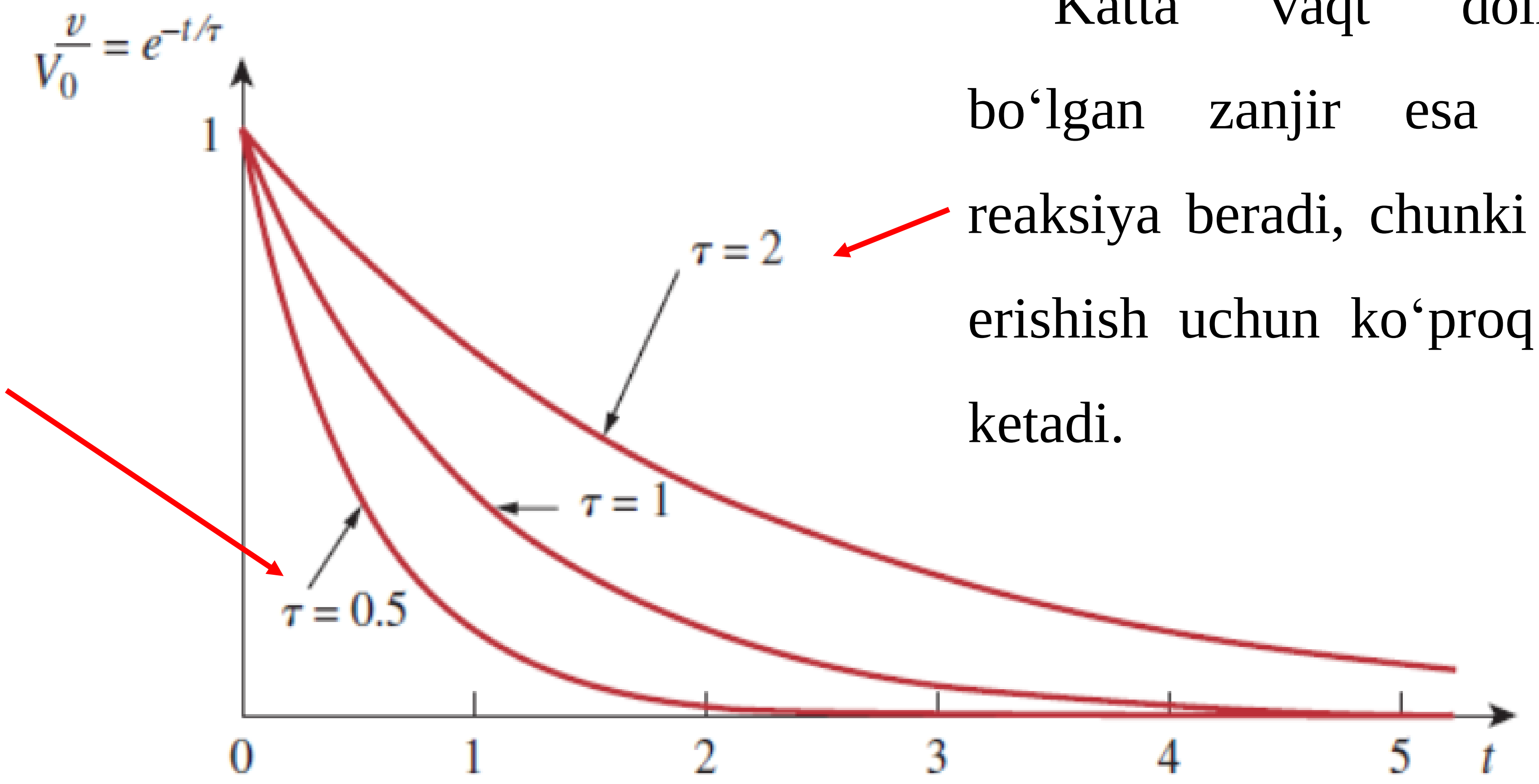
Boshqacha qilib aytadigan bo‘lsak, vaqt o‘tishi bilan hech qanday o‘zgarishlar ro‘y bermasa, zanjir uchun 5τ kerak bo‘ladi.

$\boldsymbol{\tau}$ ning har bir vaqt oralig‘ida $\boldsymbol{t}$ qiymatidan qat’iiy nazar kuchlanish avvalgi qiymatidan 36,8 foizga kamayadi,

$$\boldsymbol{u(t+\tau) = \frac{u(t)}{e} = 0,368\, u(t)}$$

(7.8) tenglamani tahlil qilsak, vaqt doimiysi qanchalik kichik bo‘lsa, kuchlanish shunchalik tez pasayadi, ya’ni reaksiya tezroq bo‘ladi.

Vaqt doimiysi kichik bo‘lgan zanjirda tez reaksiya beradi, chunki u saqlangan energiyaning tez tarqalishi tufayli barqaror holatga tezda erishadi,



Katta vaqt doimiysi bo‘lgan zanjir esa sekin reaksiya beradi, chunki unga erishish uchun ko‘proq vaqt ketadi.

7.3-rasm. Vaqt doimiysining turli qiymatlari uchun $\frac{u(t)}{u_o} = e^{-\frac{t}{\tau}}$ grafigi.

Har qanday tezlikda, vaqt doimiysi kichik yoki katta bo‘ladimi, zanjir besh vaqt doimiysida barqaror holatga erishadi.

(7.9) tenglamadagi $u(t)$ kuchlanish bilan tok kuchi $i_R(t)$ ni topishimiz mumkin.

$$\boldsymbol{i_R(t) = \frac{u(t)}{R} = \frac{u_o}{R} e^{-\frac{t}{\tau}}} \qquad (7.10)$$

Rezistorda sarflangan quvvat:

$$\boldsymbol{p(t) = u i_R = \left(u_o e^{-\frac{t}{\tau}}\right)\left(\frac{u_o e^{-\frac{t}{\tau}}}{R}\right) = \frac{u_o^2}{R} e^{-\frac{2t}{\tau}}} \qquad (7.11)$$

Rezistor tomonidan *t* vaqtgacha yutilgan energiya:

$$W_R(t) = \int_0^t p(\lambda)d\lambda = \int_0^t \frac{U_o^2}{R} e^{-\frac{2\lambda}{\tau}} d\lambda = \frac{U_o^2}{R} \int_0^t e^{-\frac{2\lambda}{\tau}} d\lambda = \frac{U_o^2}{R} \cdot \left. \frac{e^{-\frac{2\lambda}{\tau}}}{-\frac{2}{\tau}} \right|_0^t =$$

$$= -\frac{\tau U_o^2}{2R} \left. \left(e^{-\frac{2\lambda}{\tau}}\right) \right|_0^t = -\frac{RCU_o^2}{2R} \left(e^{-\frac{2\lambda}{\tau}} - e^0\right) = \frac{1}{2} CU_o^2 \left(1 - e^{-\frac{2\lambda}{\tau}}\right), \qquad \tau = RC$$

$$W_R(t) = \frac{1}{2} CU_o^2 \left(1 - e^{-\frac{2\lambda}{\tau}}\right),\ J \qquad (7.12)$$

E'tibor bering, $t \to \infty$, $W_R(\infty) \to \frac{1}{2} CU_o^2$, kondensatorda dastlab saqlanadigan energiya $W_C(0)$ bilan bir xil. Dastlab kondensatorda saqlangan energiya oxirida rezistorda tarqaladi.

Qisqacha xulosa:

Manbasiz *RC* zanjir bilan ishlashning kaliti topiladi:

1. Kondensatordagi dastlabki kuchlanish $\boldsymbol{U(0) = U_o}$.

2. Vaqt doimiysi τ.

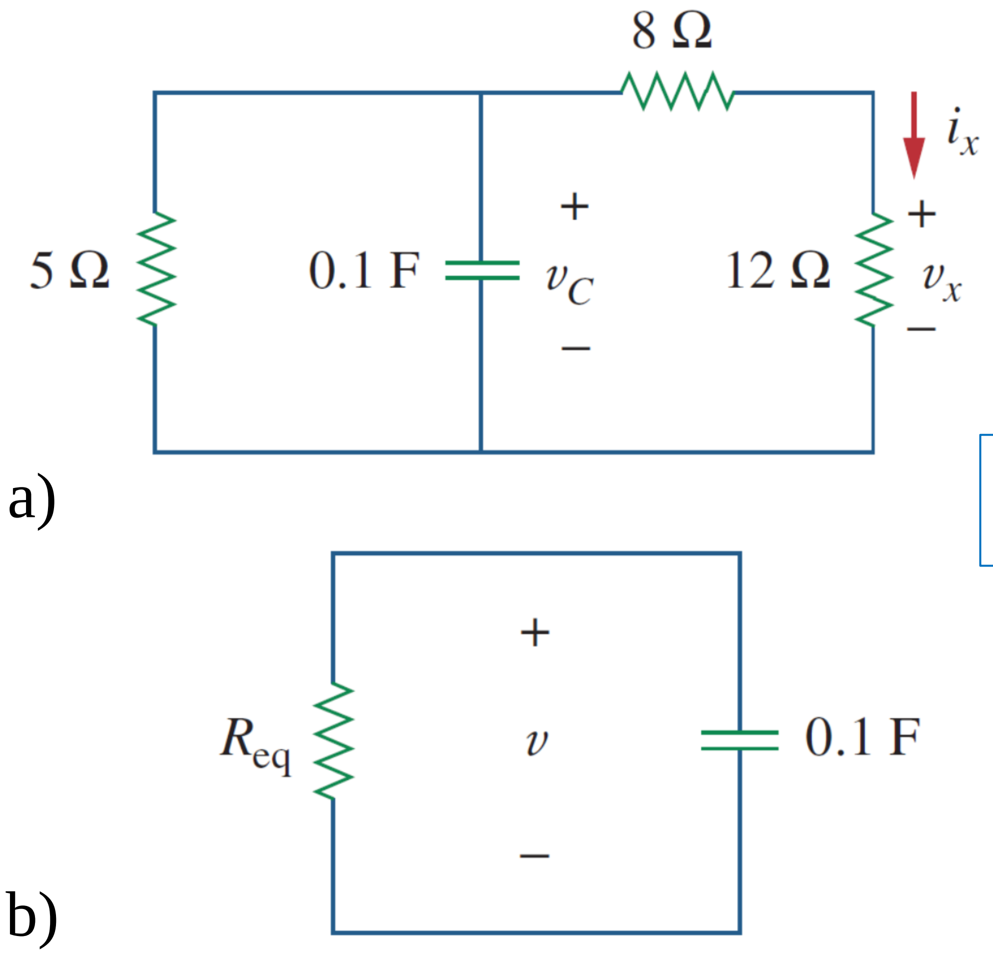
Izoh: Vaqt doimiysi chiqish nima bo‘lishidan qat’iy nazar bir xil bo‘ladi.

Ushbu ikkita element bilan biz reaksiyani kondensator kuchlanishi $\boldsymbol{U_C(t) = U(t) = U(0)e^{-\frac{t}{\tau}}}$ sifatida olamiz.

$$\boldsymbol{\tau = RC}$$

****Izoh:*** Zanjirda bitta kondensator va bir nechta rezistorlar va bog‘liq manbalar bo‘lsa, oddiy *RC* zanjirini hosil qilish uchun Tevenin ekvivalentini kondensator terminallarida topish mumkin.

7.2.1-masala: 7.4-rasm, *a* da $u_C(0) = 15\ V$ bo‘lsin. *t>0* uchun u_C, u_x va i_x ni toping.



7.4-rasm.

a) asl zanjir, b) a uchun ekvivalent zanjiri.

Yechish:

$$R_{um} = \frac{20 \cdot 5}{20 + 5} = 4\ \Omega$$

$$\boldsymbol{\tau = R_{um}C} = 4 \cdot 0{,}1 = 0{,}4\ s$$

$$\boldsymbol{u(t) = U_0 e^{-\frac{t}{RC}}}$$

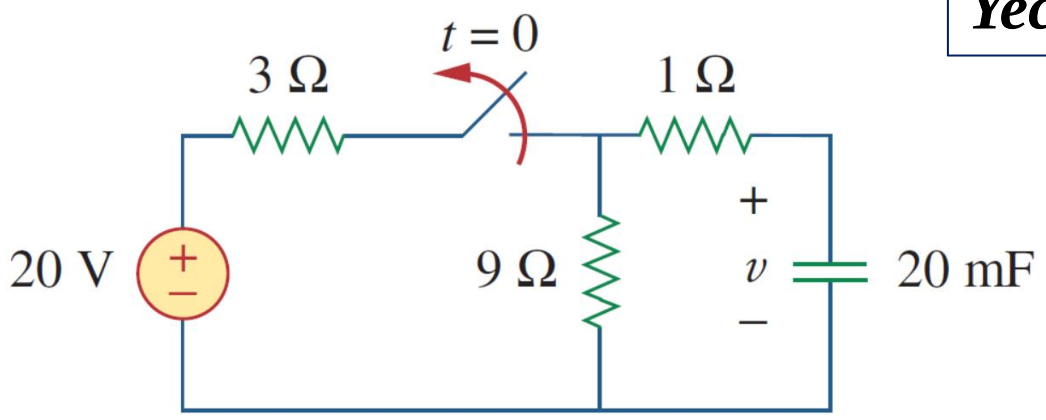
$$\boldsymbol{u = u(0)e^{-\frac{t}{\tau}}} = 15e^{-\frac{t}{0{,}4}}\ V$$

$$\boldsymbol{u_C = u = 15e^{-2{,}5t}\ V}$$

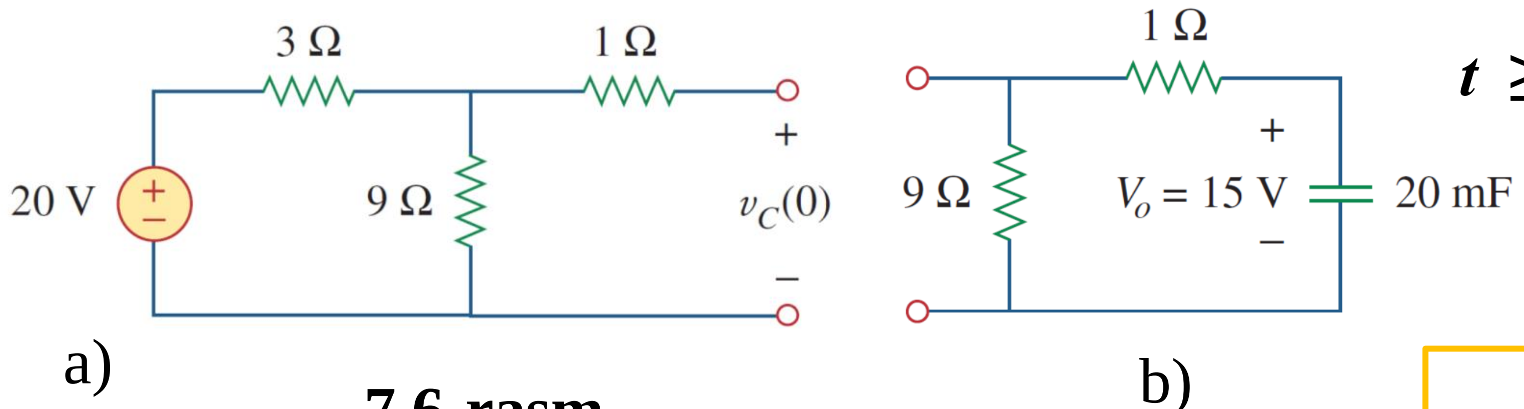
$$\boldsymbol{u_x} = \frac{12}{12 + 8}u = 0{,}6(15e^{-2{,}5t}) = \boldsymbol{9e^{-2{,}5t}\ V}$$

$$\boldsymbol{i_x = \frac{u_x}{12} = 0{,}75e^{-2{,}5t}\ A}$$

7.2.2-masala: 7.5-rasmda zanjirdagi kalit uzoq vaqt davomida yopilgan va u *t=0* da ochiladi. $t > 0$ uchun $u(t)$ ni toping. Shuningdek, kondensatorda saqlanadigan dastlabki energiyani hisoblang.



7.5-rasm.



7.6-rasm.
a) t < 0 uchun, b) t > 0 uchun.

Yechish:

***t < 0** uchun:* $$u_C(t) = \frac{9}{9+3}20 = 15\,V, \qquad t < 0$$

u_C bir zumda o'zgarmasligi sababli, $t = 0^-$ da kondensatordagi kuchlanish $t = 0$, yoki

$$u_C(0) = u_0 = 15\,V$$

$$R_{um} = 1 + 9 = 10\,\Omega$$

$$\tau = R_{um}C = 10 \cdot 20 \cdot 10^{-3} = 0{,}2\,s$$

t ≥ 0 uchun: $$u(t) = u_C(0)e^{-\frac{t}{\tau}} = 15e^{-t/0{,}2}\,V$$

$$u(t) = 15e^{-5t}\,V$$

$$W_C(0) = \frac{1}{2}Cu_C^2 = \frac{1}{2} \cdot 20 \cdot 10^{-3} \cdot 15^2 = 2{,}25\,J$$

FOYDALANILGAN MANBALAR:

1. https://i.stack.imgur.com/Zp36Jm.png
2. https://i.ytimg.com/vi/wgvo4aTATsU/maxresdefault.jpg
3. https://circuitdigest.com/sites/default/files/circuitdiagram/Simple-relay-switch-circuit-diagram_0.png
4. https://cdn11.bigcommerce.com/s-6eafq5t88w/images/stencil/500x659/products/33412/54192/743181__06345.1644440897.jpg?c=1
5. https://i0.wp.com/studentlesson.com/wp-content/uploads/2020/10/ignition-system.png?fit=512%2C457&ssl=1

KIUT

E'TIBORINGIZ UCHUN RAHMAT!!!