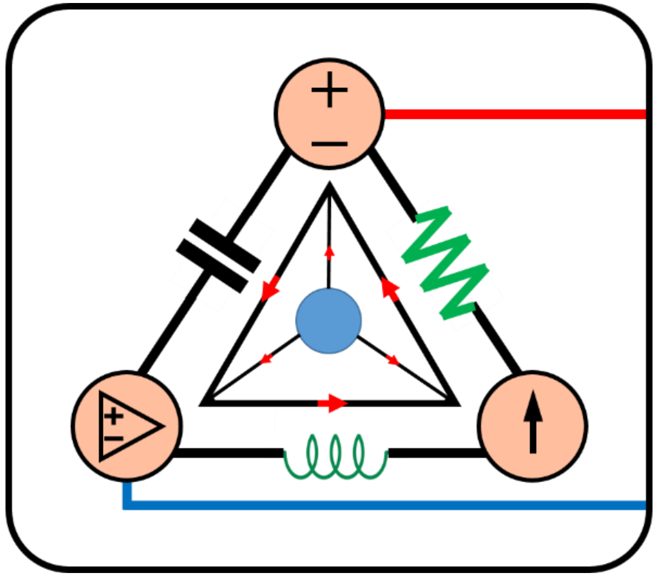
7-Mavzu: Birinchi tartibli elektr zanjiri.

(7th Topic: First-Order Circuit)

7-Mavzuning 2-qismi

(2nd part of the 7th Topic)



7-hafta uchun

For the 7th week

Lecturer: Ph.D., Yusupov Sarvarbek

Toshkent Kimyo Xalqaro Universiteti
“Mashinasozlik texnologiyasi” kafedrasi
Toshkent shahri, Usmon Nosir, 156-uy.

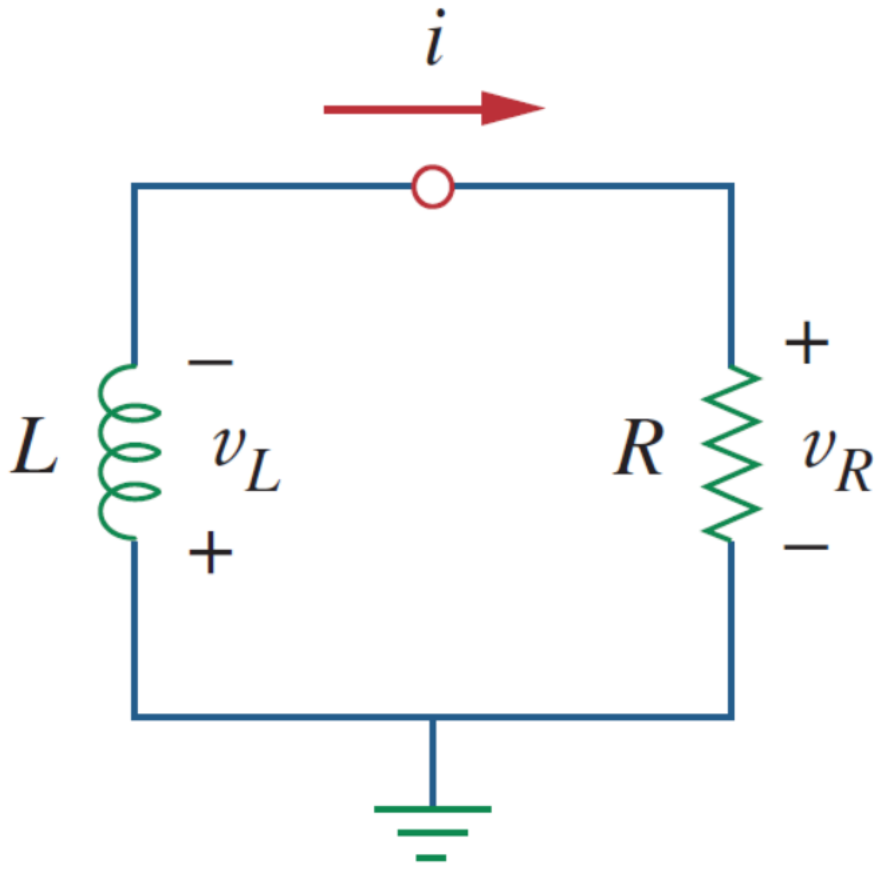
7-Mavzu: Birinchi tartibli elektr zanjiri.

(7th Topic: First-Order Circuit)

O‘quv rejasi:

7.3. Manbadan holi qarshilik va induktor (*RL*) zanjiri.

Rezistor bilan induktorning ketma-ket ulangan zanjir reaksiyasini aniqlash uchun induktor orqali o‘tayotgan tok kuchini $i(t)$ deb hisoblaymiz.



7.7-rasm. Manbadan holi *RL* zanjiri.

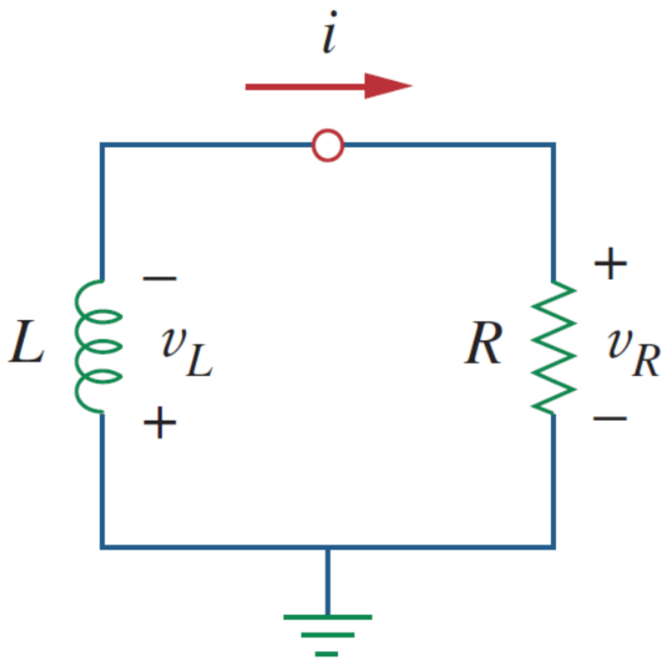
Induktordagi tok kuchi bir zumda o‘zgartirmaydi.

$t = 0$ bo‘lganda, biz induktorning boshlang‘ich tokini I_0 deb qaraymiz, ya’ni

$$\boldsymbol{i(0) = I_0} \tag{7.13}$$

Induktorda saqlanadigan energiya,

$$\boldsymbol{W(0) = \frac{1}{2} L I_0^2} \tag{7.14}$$



Zanjir uchun KVLni qo‘llaymiz. $$\boldsymbol{U_L + U_R = 0} \qquad (7.15)$$

$$U_L = L\frac{di}{dt} \qquad U_R = iR$$

$$L\frac{di}{dt} + Ri = 0 \quad \text{yoki,} \quad \boldsymbol{\frac{di}{dt} + \frac{R}{L}i = 0} \qquad (7.16)$$

Shartlarni qayta tartibga solib, har ikkala tomonni boshlang‘ich funksiyasini hosil qilamiz.

$$\int_{I_0}^{i(t)} \frac{di}{i} = \int_0^t -\frac{R}{L}dt$$

$$\ln i\Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Big|_0^t \quad \rightarrow \quad \ln i(t) - \ln I_0 = -\frac{R}{L}(t-0), \quad i(0) = I_0$$

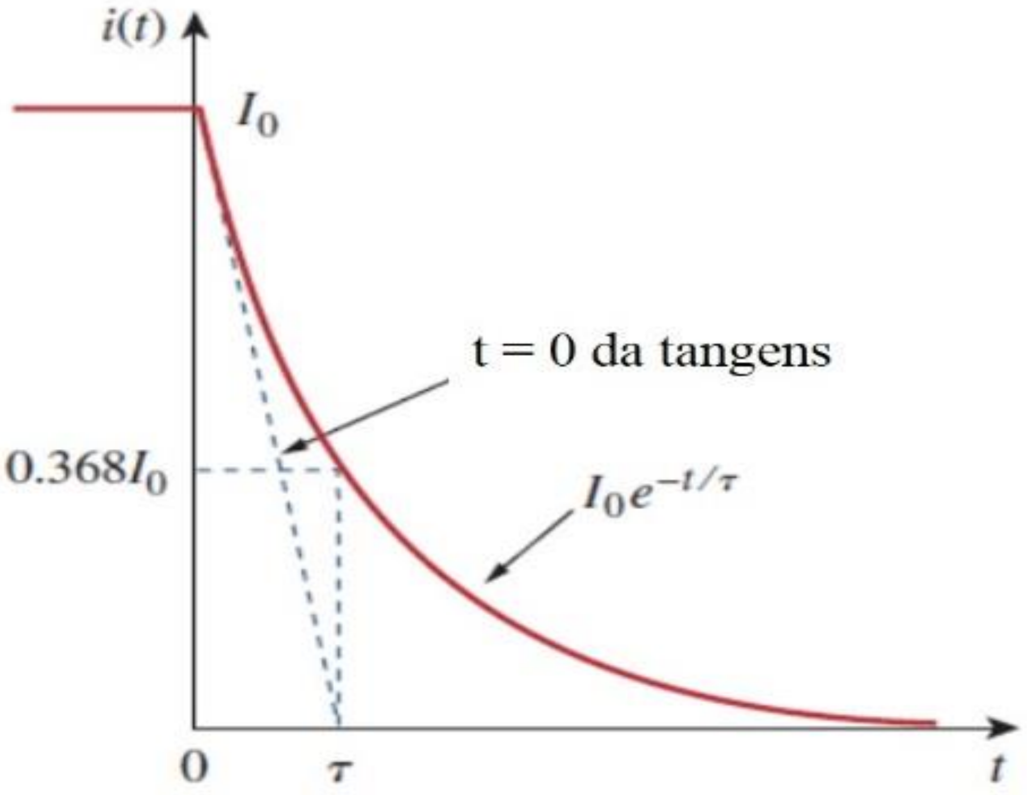
yoki,

$$\boldsymbol{\ln\frac{i(t)}{I_0} = -\frac{Rt}{L}} \quad \rightarrow \quad \boldsymbol{\frac{i(t)}{I_0} = e^{-\frac{Rt}{L}}} \qquad (7.17)$$

Ikkala tomonlarni *e* darajaga keltirib, quyidagiga ega bo‘lamiz,

$$\boldsymbol{i(t) = I_0 e^{-\frac{Rt}{L}}} \qquad (7.18)$$

Bu shuni ko‘rsatadiki, ***RL*** zanjirining tabiiy reaksiyasi dastlabki tok kuchining eksponensial parchalanishini hosil qiladi.



$t = 0$ da,	$i(0) = I_0$
$t = 1$ da,	$i(1) = I_0 e^{-\frac{1}{\tau}}$
$t = 5$ da,	$i(5) = I_0 e^{-\frac{5}{\tau}}$

7.8-rasm. *RL* zanjirining tok kuchi reaksiyasi.

Bu (7.18) tenglamadan ko‘rinib turibdi ***RL*** zanjiri uchun vaqt doimiysi τ yana soniya birligiga ega bo‘ladi.

$$\boldsymbol{\tau = \frac{L}{R}} \qquad (7.19)$$

$$\boldsymbol{i(t) = I_0 e^{-\frac{Rt}{L}}} \qquad (7.18) \quad \rightarrow \quad \boldsymbol{i(t) = I_0 e^{-\frac{t}{\tau}}} \qquad (7.20)$$

Zanjirning vaqt doimiysi qanchalik kichik bo‘lsa, reaksiyaning turg‘unlik tezligi shunchalik tez bo‘ladi.

Vaqt doimiysi qanchalik katta bo‘lsa, reaksiyaning turg‘unlik tezligi shunchalik sekin bo‘ladi.

Har qanday holatda, reaksiya 5τ dan keyin boshlang‘ich qiymatining 1 foizidan kam bo‘ladi (ya’ni, barqaror holatga yetadi).

$i(t) = I_0 e^{-\frac{t}{\tau}}$ tenglamadagi tok kuchi bilan rezistordan o‘tayotgan kuchlanishni quyidagicha topamiz,

$$U_R(t) = iR = I_0 R e^{-\frac{t}{\tau}} \qquad (7.21)$$

Rezistorda sarflangan quvvat:

$$p(t) = U_R i = I_0^2 R e^{-\frac{2t}{\tau}} \qquad (7.22)$$

Rezistor tomonidan yutilgan energiya:

$$W_R(t) = \int_0^t p(\lambda)d\lambda = \int_0^t I_0^2 R e^{-\frac{2t}{\tau}} d\lambda = I_0^2 R \int_0^t e^{-\frac{2\lambda}{\tau}} d\lambda = I_0^2 R \cdot \left. \frac{e^{-\frac{2\lambda}{\tau}}}{-\frac{2}{\tau}} \right|_0^t =$$

$$= -\frac{\tau I_0^2}{2R} \left. \left(e^{-\frac{2\lambda}{\tau}}\right) \right|_0^t = -\frac{\frac{L}{R} I_0^2}{2R} \left(e^{-\frac{2\lambda}{\tau}} - e^0\right) = \frac{1}{2} L I_0^2 \left(e^{-\frac{2\lambda}{\tau}} - 1\right), J \qquad \tau = \frac{L}{R} \qquad (7.23)$$

$$W(0) = \frac{1}{2} L I_0^2 \qquad (7.14)$$

$t \to \infty$, $W_R(\infty) \to \frac{1}{2} L I_0^2$ kabi $W_L(0)$ bilan bir xil bo‘lgan, boshlang‘ich energiya (7.14) tenglamadagi kabi induktorda saqlanadi. Dastlab induktorda saqlangan energiya oxirida rezistorda tarqaladi.

Qisqacha xulosa:

Manbasiz *RL* zanjir bilan ishlashning kaliti topiladi:

1. Induktordagi dastlabki tok kuchi $i(0) = I_0$.

2. Zanjirning vaqt doimiysi τ.

$$i_L(t) = i(t) = i(0)e^{-\frac{t}{\tau}}$$

Ushbu ikki element bilan biz reaksiyani induktorning tok kuchi sifatida olamiz.

Induktorning tok kuchi i_L ni aniqlaganimizdan so‘ng, boshqa o‘zgaruvchilar (U_L, U_R va i_R) aniqlanishi mumkin.

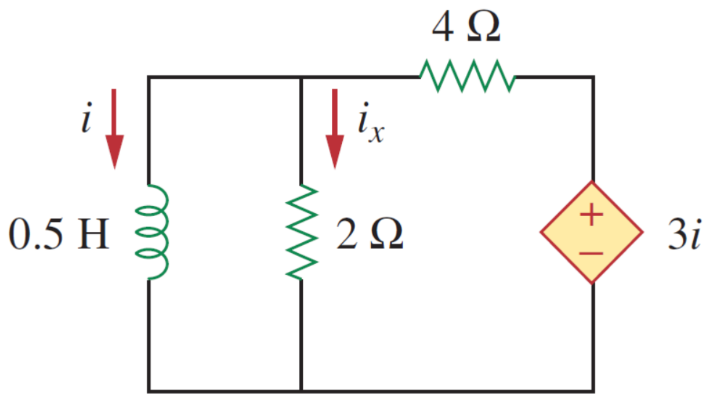
$$\boldsymbol{\tau = \frac{L}{R}}$$

R - induktorning terminallaridagi Tevenin qarshiligidir.

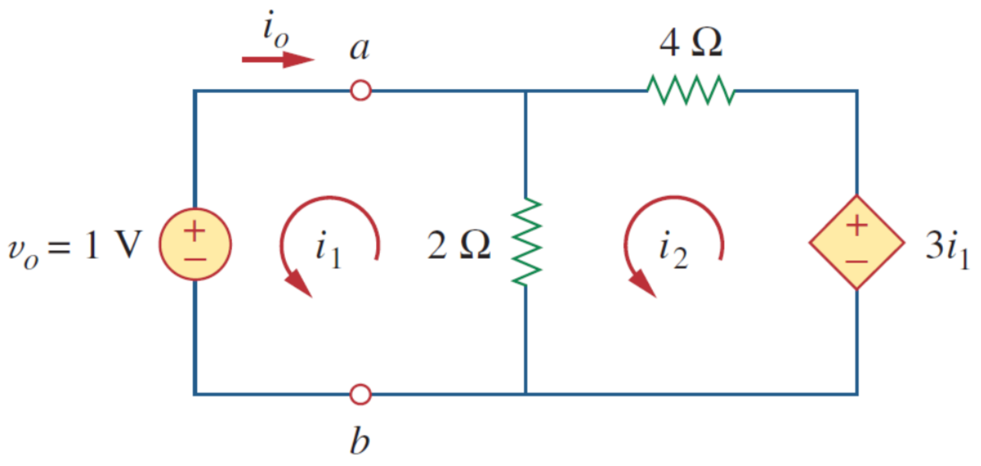
Izoh: Zanjirda bitta induktor va bir nechta rezistorlar va bog‘liq manbalar bo‘lsa, oddiy ***RL*** zanjirini hosil qilish uchun Tevenin ekvivalentini induktorning terminallarida topish mumkin.

Bundan tashqari, Tevenin teoremasidan bir nechta induktorlarni bir ekvivalent induktor hosil qilish uchun birlashtirganda foydalanish mumkin.

7.3.1-masala: 7.9-rasmda zanjirdagi tok kuchi $i(0) = 10\ A$ deb faraz qilsak, $i(t)$ va $i_x(t)$ larni hisoblang.



7.9-rasm.



a)

7.10-rasm.

Yechish:

Birinchi usuli: Ikkala kontur uchun KVLni qo‘llaymiz.

loop-1: $2(i_1 - i_2) + 1 = 0 \ \rightarrow\ i_1 - i_2 = -\frac{1}{2}$ (7.3.1)

loop-2: $-3i_1 + 4i_2 + 2(i_2 - i_1) = 0 \ \rightarrow\ -5i_1 + 6i_2 = 0 \ \rightarrow\ i_2 = \frac{5}{6} i_1$ (7.3.2)

$$i_1 - \frac{5}{6} i_1 = -\frac{1}{2} \ \rightarrow\ \frac{1}{6} i_1 = -\frac{1}{2} \ \rightarrow\ i_1 = -\frac{1}{2} \cdot 6 = -3\ A$$

$$i_1 = -3\ A, \quad i_0 = -i_1 = 3\ A \qquad R_{um} = R_{Th} = \frac{u_o}{i_o} = \frac{1}{3} \Omega$$

$$\tau = \frac{L}{R_{um}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}\ s$$

Ekvivalent qarshilik = Tevenin qarshiligi

Bog‘liq manba

a-b terminal

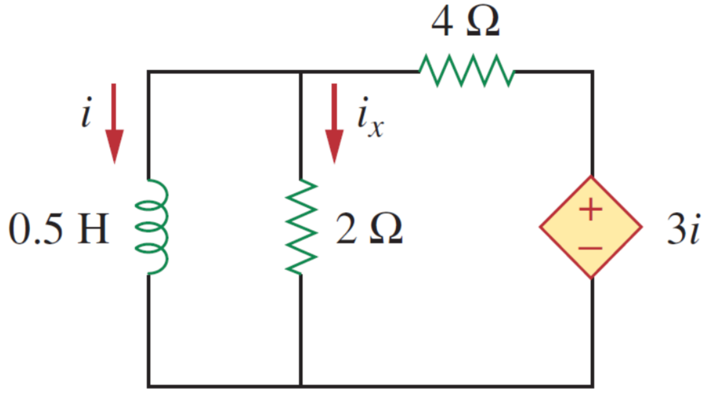
$u(0) = 1\ V$

$$\boldsymbol{i(t) = i(0)e^{-\frac{t}{\tau}} = 10e^{-\left(\frac{2}{3}\right)t}\ A, \quad t > 0}$$

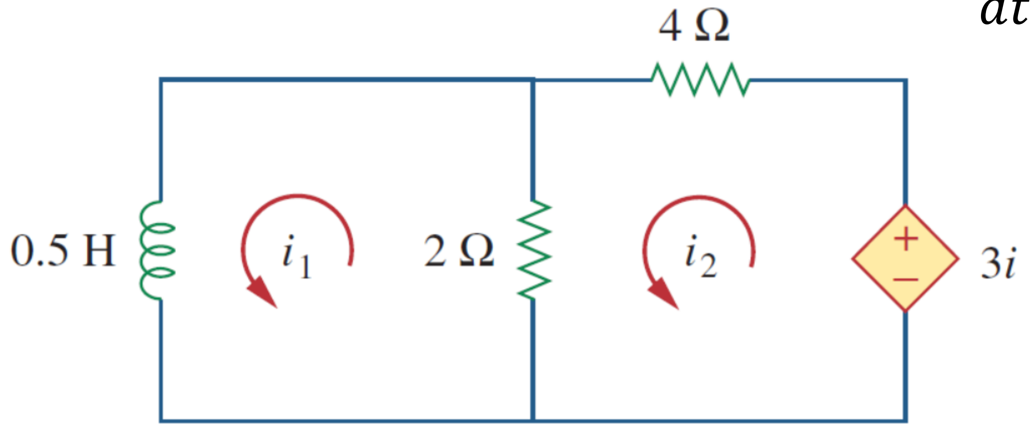
$$u = L\frac{di}{dt} = 0{,}5(10)\left(-\frac{2}{3}\right)e^{-\left(\frac{2}{3}\right)t} = -\frac{10}{3}e^{-\left(\frac{2}{3}\right)t}\ V$$

$$\boldsymbol{i_x(t) = \frac{u}{2} = -1{,}6667e^{-\left(\frac{2}{3}\right)t}\ A, \qquad t > 0}$$

7.3.1-masala: 7.9-rasmda zanjirdagi tok kuchi $i(0) = 10\ A$ deb faraz qilsak, $i(t)$ va $i_x(t)$ larni hisoblang.



7.9-rasm.



b)
7.10-rasm.

Yechish: ***Ikkinchi usuli:*** KVLni qo'llaymiz.

1-kontur uchun: $\frac{1}{2}\frac{di_1}{dt} + 2(i_1 - i_2) = 0$ yoki, $\frac{di_1}{dt} + 4i_1 - 4i_2 = 0$ (7.3.3)

2-kontur uchun: $\frac{di_1}{dt} + 4i_1 - 4i_2 = 0$ (7.3.3)

$$6i_2 - 2i_1 - 3i_1 = 0 \quad \rightarrow \quad i_2 = \frac{5}{6}i_1 \qquad (7.3.4)$$

$$\frac{di_1}{dt} + \frac{2}{3}i_1 = 0 \qquad \frac{di_1}{i_1} = -\frac{2}{3}dt \qquad \ln i\Big|_{i(0)}^{i(t)} = -\frac{2}{3}t\Big|_0^t \qquad \ln\frac{i(t)}{i(0)} = -\frac{2}{3}t$$

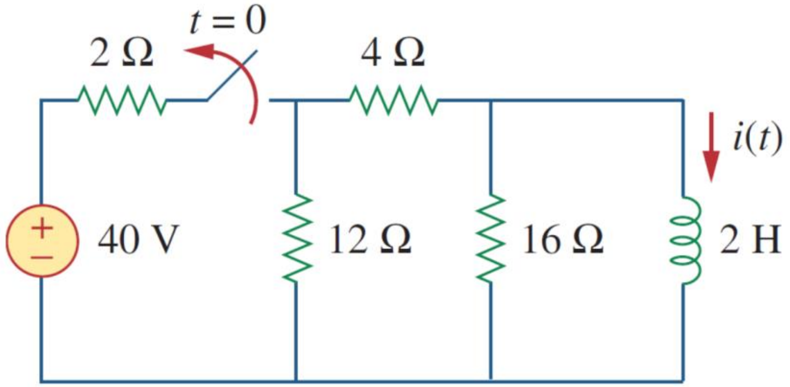
$$i(t) = i(0)e^{-\left(\frac{2}{3}\right)t} = 10e^{-\left(\frac{2}{3}\right)t}\ A, \qquad t > 0$$

$$u = L\frac{di}{dt} = 0{,}5(10)\left(-\frac{2}{3}\right)e^{-\left(\frac{2}{3}\right)t} = -\frac{10}{3}e^{-\left(\frac{2}{3}\right)t}\ V$$

$$\boldsymbol{i_x(t) = \frac{u}{2} = -1{,}6667e^{-\left(\frac{2}{3}\right)t}\ A, \qquad t > 0}$$

7.3.2-masala: 7.11-rasmda zanjirdagi kalit uzoq vaqt davomida yopiq turgan.

$t = 0$ da kalit ochiladi. $t > 0$ uchun $i(t)$ ni hisoblang.



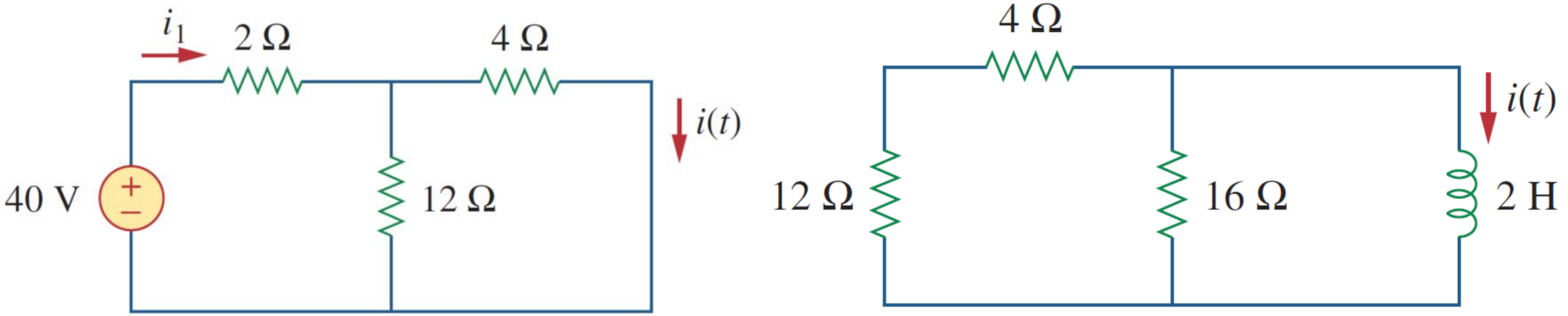
7.11-rasm.

Yechish:

t<0 uchun: $\frac{4 \cdot 12}{4+12} = 3\ \Omega$ $\qquad i_1 = \frac{40}{2+3} = 8\ A$

$$i(t) = \frac{12}{12+4} i_1 = 6\ A, \quad t < 0$$

$$i(0) = i(0^-) = 6\ A$$



a) b)

7.12-rasm. 7.11-rasmning zanjirini yechimi.

a) t < 0 uchun, b) t > 0 uchun.

t>0 uchun:

$$\boldsymbol{R_{um} = (12+4)||16 = 8\ \Omega}$$

$$\tau = \frac{L}{R_{um}} = \frac{2}{8} = \frac{1}{4} s$$

$$\boldsymbol{i(t) = i(0)e^{-t/\tau} = 6e^{-4t}\ A}$$

7.4. Yakkalik funksiyalari.

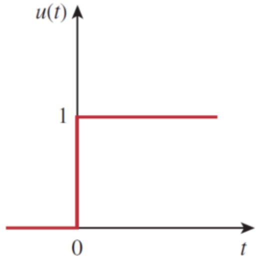
Yakkalik funksiya (***Singularity function***) larning asosiy tushunchasi bizga birinchi tartibli elektr zanjirlarining mustaqil o'zgarmas kuchlanish yoki tok kuchi manbasining birdaniga qo‘llanilish munosabatini tushunishga yordam beradi.

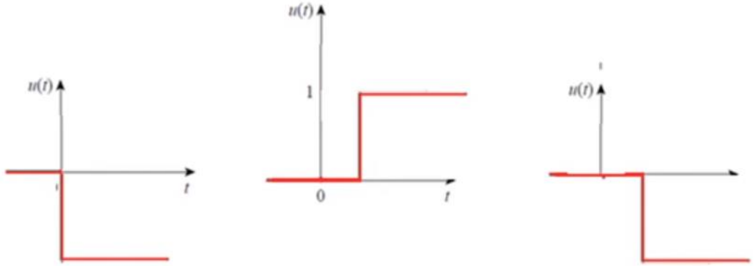
Yakkalik funksiyalari (*shuningdek, switching functions (almashtirish) funksiyalari deb ham ataladi*) zanjirni tahlil qilishda juda foydali hisoblanadi.

Ular kommutatsiya jarayonlarida sodir bo‘ladigan kommutatsiya signallariga mos xizmat qiladi.

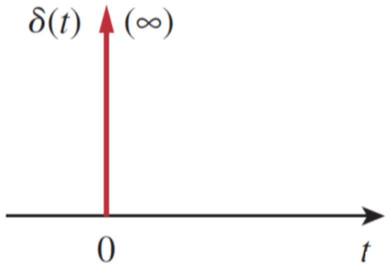
Zanjir tahlilida eng ko‘p ishlatiladigan uchta yakkalik funksiyalari bo‘lib, ular:

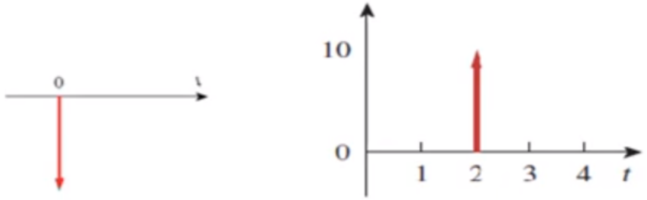
- **Birlik qadam (*unit step*);**



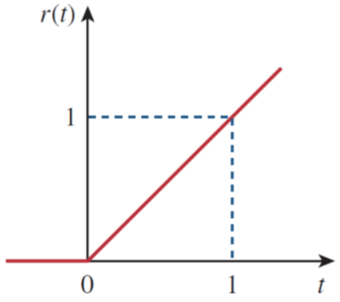


- **Birlik impuls (*unit impulse*);**





- **Birlik rampa yoki chiziqli birlik (*unit ramp*) funksiyalaridir.**



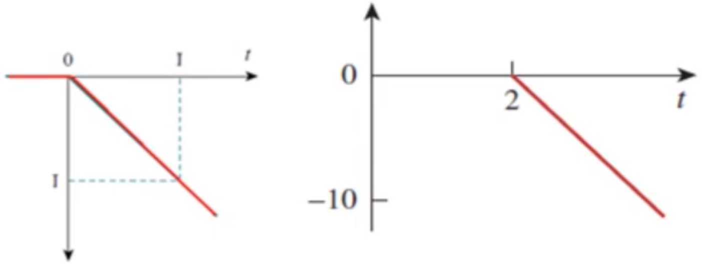
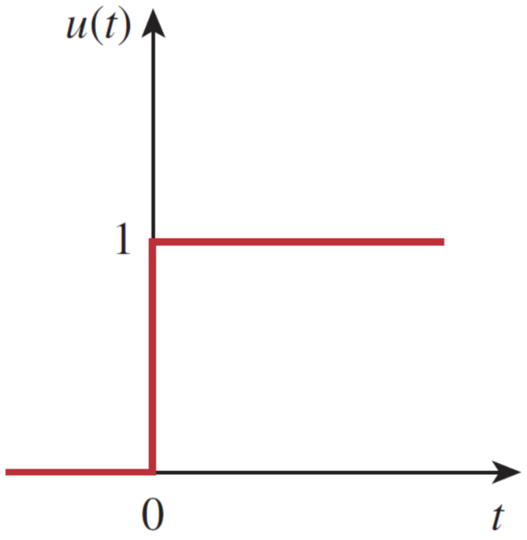


Photo source: [6] - Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 266-268.

Birlik qadam funksiyasi:



7.13-rasm. Birlik qadam funksiyasi.

Birlik qadam funksiyasi $u(t)$ t ning manfiy qiymatlari uchun 0 ga, *t* ning musbat qiymatlari uchun esa 1 ga tengdir.

Matematik nuqtai nazardan,
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \qquad (7.24)$$

Birlik qadam funksiyasi *t=0* da aniqlanmagan, u 0 dan 1 gacha keskin o‘zgaradi.

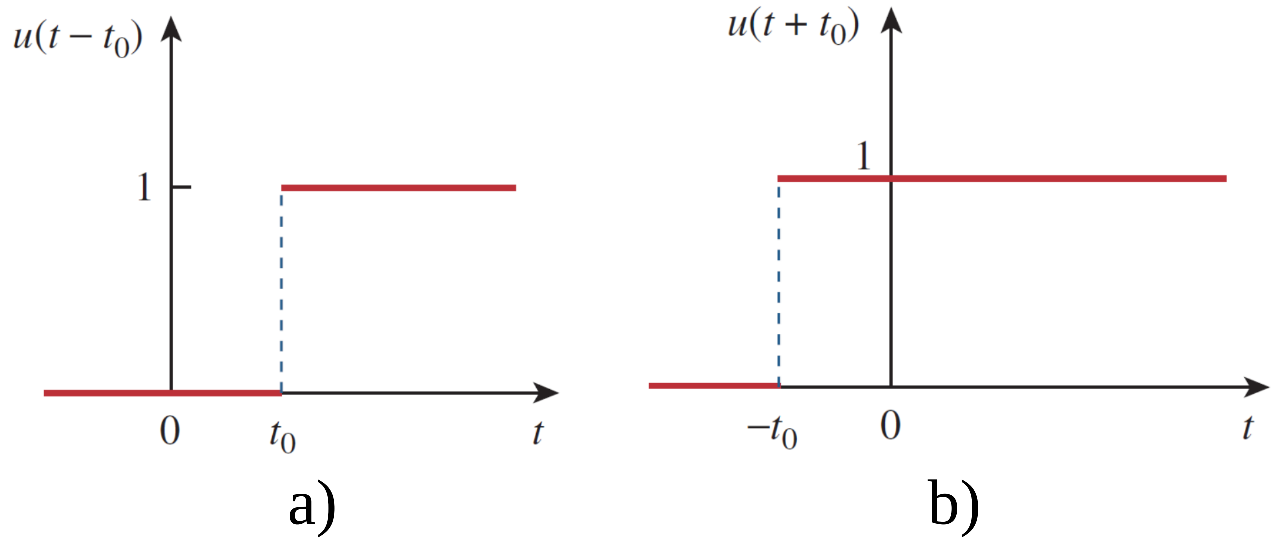
U *sin* va *cos* singari boshqa matematik funksiyalar kabi o‘lchovsizdir.

Agar keskin o‘zgarish $t = 0$ o‘rniga $t = t_0$ (*bu yerda* $t_0 > 0$) da sodir bo‘lsa, birlik qadam funksiyasi bo‘ladi.

$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases} \qquad (7.25)$$

Bu 7.14-rasm, a da $u(t)$ t_0 sekundlarga kechikadi. (7.24) tenglamadan (7.25) tenglamani olish uchun har bir t ni $t - t_0$ ga almashtiramiz. Agar, o‘zgarish $t = -t_0$ bo‘lsa, birlik qadam funksiyasi bo‘ladi.

$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases} \qquad (7.26)$$



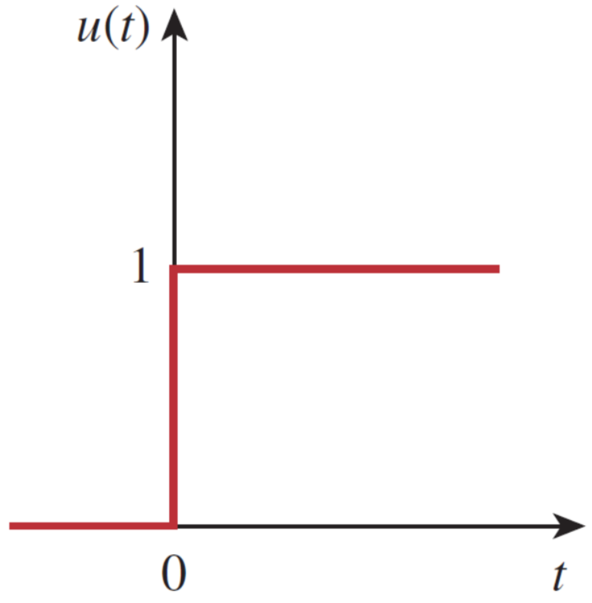
7.14-rasm.

a) t_0 ga kechiktirilgan birlik qadam funksiyasi,

b) t_0 ga oshirilgan birlik qadam.

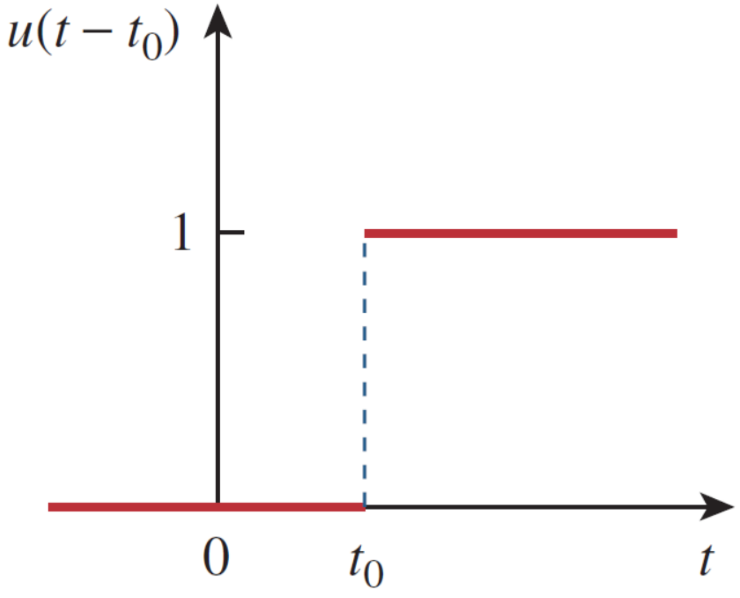
$u(t)$ 7.14-rasm, b da t_0 soniyalarda oldinga siljiydi.

Birlik qadam funksiyasi



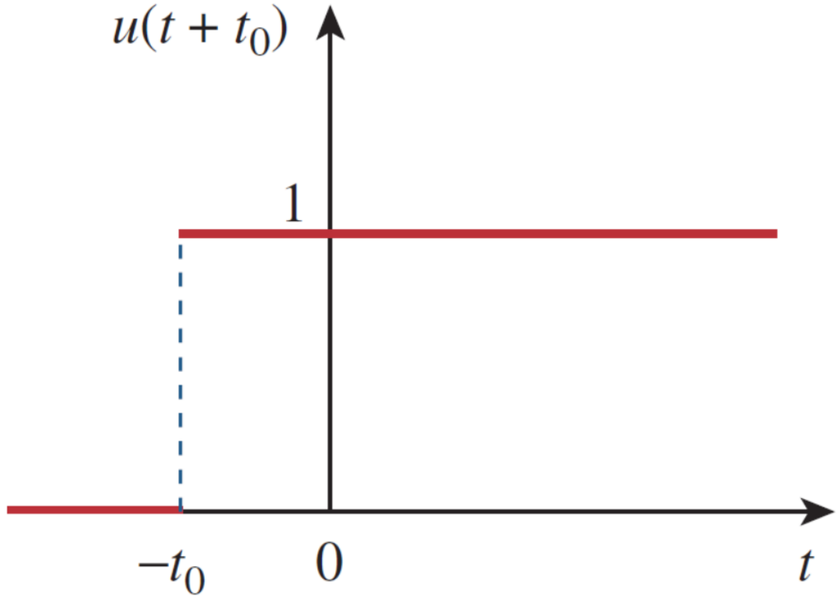
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

Kechikkan birlik qadam funksiyasi



$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

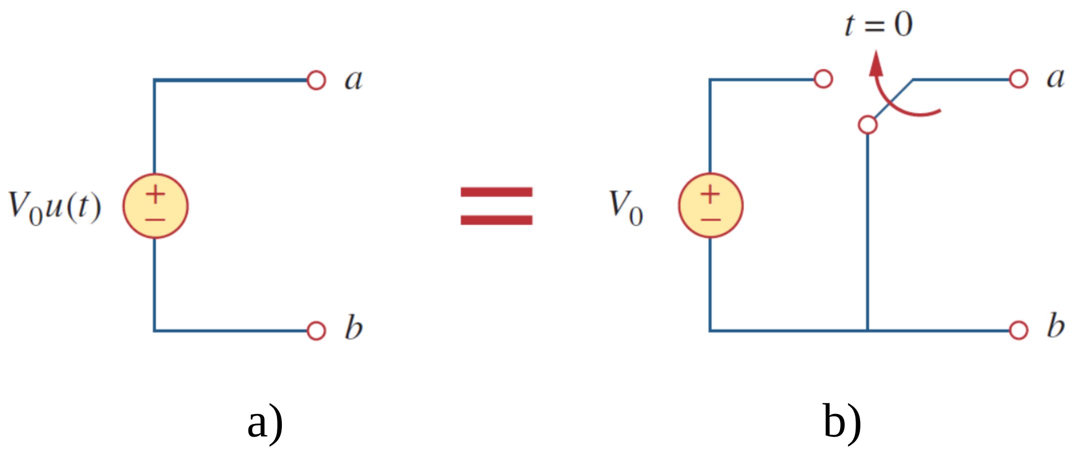
Ilgarilangan birlik qadam funksiyasi



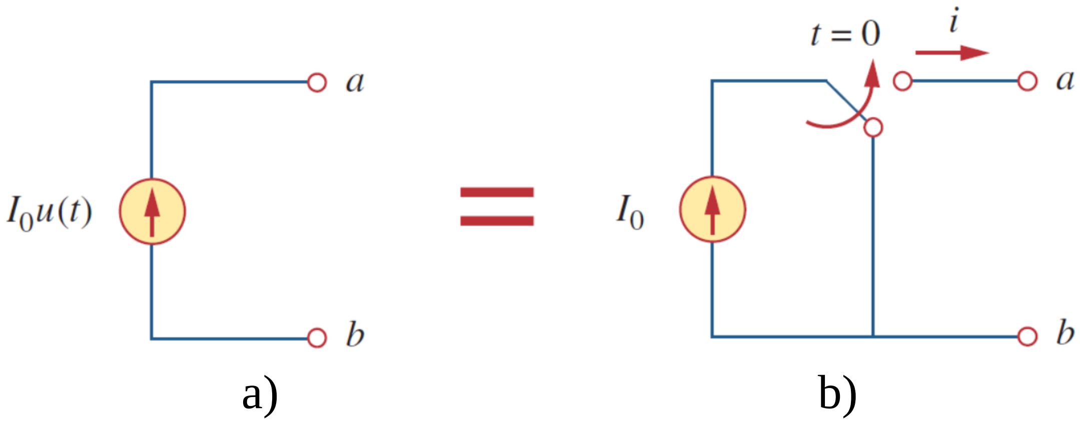
$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$

Kuchlanish yoki tok kuchining keskin o‘zgarishini ifodalash uchun qadam funksiyasidan foydalaniladi.

Bu orqali boshqaruv tizimlari va raqamli kompyuterlarning elektr zanjirlarida sodir bo‘ladigan o‘zgarishlar tahlil qilinadi.



7.15-rasm.
a) $U_ou(t)$ ning kuchlanish manbai, b) uning ekvivalent zanjiri.



7.16-rasm.
a) $I_0u(t)$ ning kuchlanish manbai, b) uning ekvivalent zanjiri.

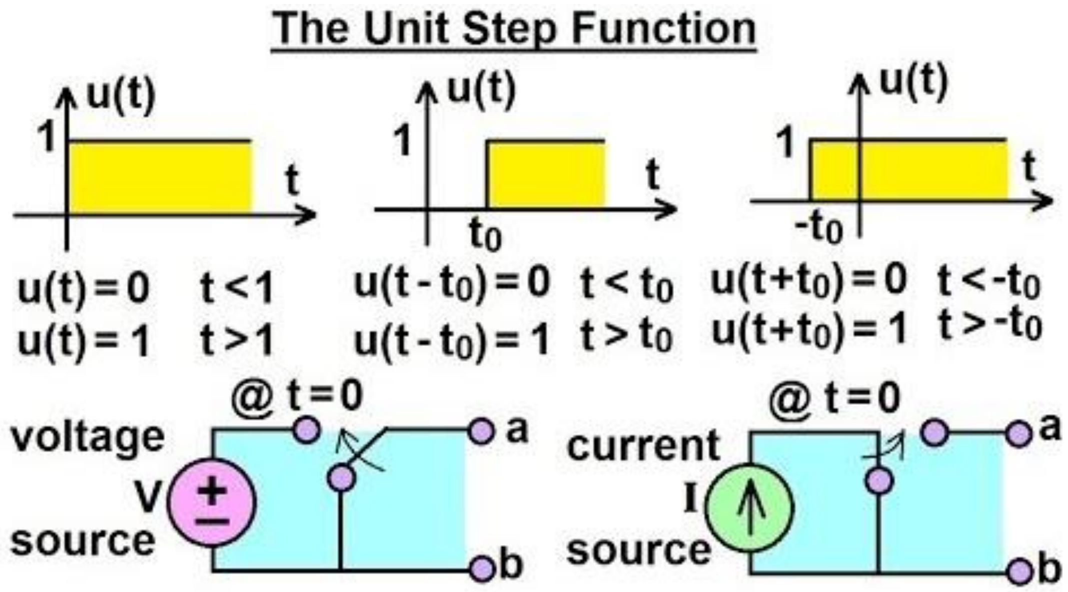


Photo source: [7] - https://i.ytimg.com/vi/LphJ1DqPfjo/hqdefault.jpg

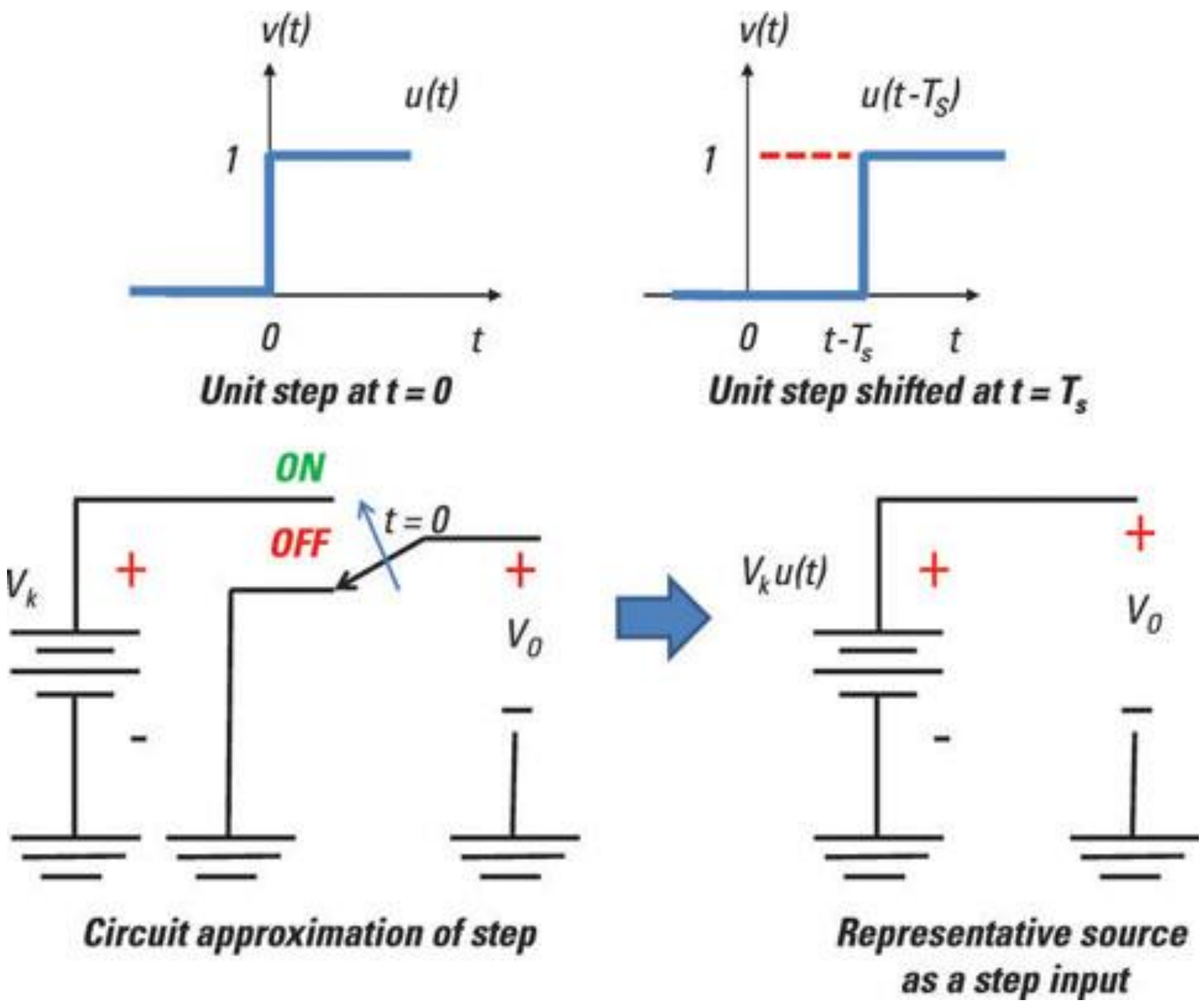


Photo source: [8] - https://www.dummies.com/wp-content/uploads/376127.image1.jpg

Birlik qadam (***Heavyside***) funksiyasi kalitning harakatini modellashtiradi (o‘chirish/yoqish). Qadam funksiyasi zanjirdagi tok kuchi yoki kuchlanishning to‘satdan o‘zgarishini tasvirlashi mumkin. Birlik qadam funksiyasi qadam kabi ko‘rinadi. Amaliy qadam funksiyalari har kuni, masalan, mobil qurilmalar, stereolar va chiroqlarni yoqish va o‘chirish kabi har kuni amalga oshiriladi. Shuningdek,

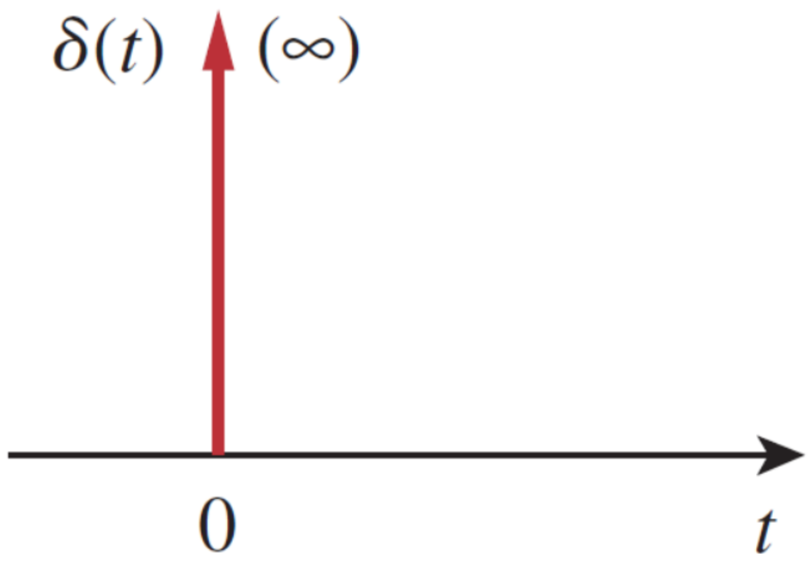
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

$$U_k u(t - T_s) = \begin{cases} 0, & T_s < t_0 \\ U_o, & T_s > t_0 \end{cases} \qquad U_k u(t) = \begin{cases} 0, & T_s < t_0 \\ U_o, & T_s > t_0 \end{cases}$$

Birlik impuls funksiyasi:

Birlik qadam funksiyasining hosilasi $u(t)$, birlik impuls funksiyasi $\delta(t)$ bo‘lib, biz uni quyidagicha yozamiz.

$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t < 0 \\ Aniqlanmagan, & t = 0 \\ 1, & t > 0 \end{cases} \quad (7.29)$$



7.17-rasm. Birlik impuls funksiyasi.

Birlik impuls funksiyasi $\delta(t)$ hamma joyda nolga teng, $t = 0$ dan tashqari, u noaniq.

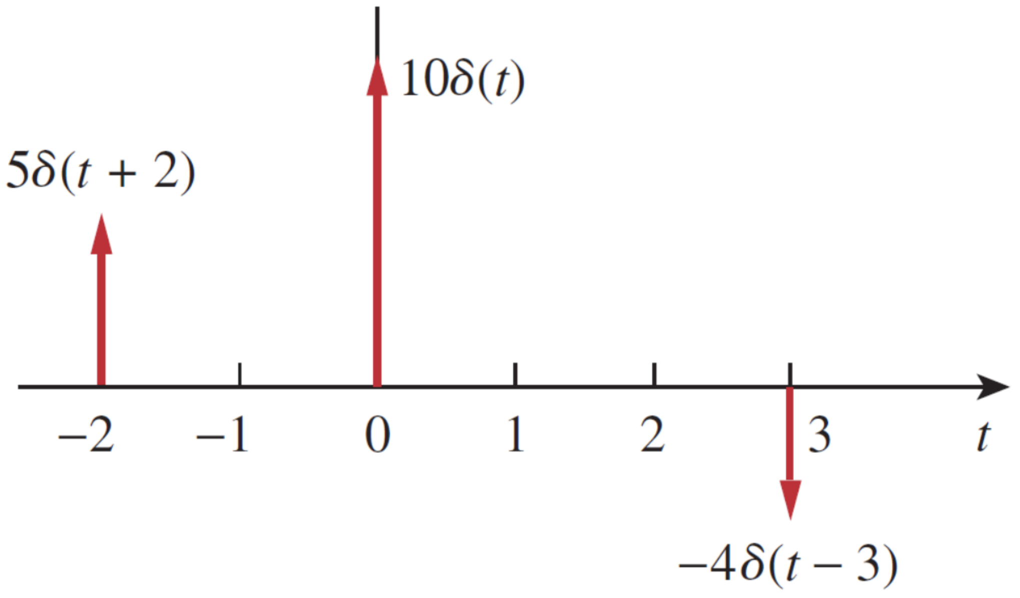
Elektr zanjirlarida kommutatsiya operatsiyalari yoki impulsiv manbalar natijasida impuls tok kuchlari va kuchlanishlar paydo bo‘ladi.

Birlik impuls funksiyasi fizik jihatdan amalga oshirilmasa ham (*huddi ideal manbalar, ideal rezistorlar va boshqalar kabi*), bu juda foydali matematik vositadir.

Birlik impuls qo‘llaniladigan yoki natijaviy holat sifatida tahlil qilinadi. U birlik maydonining juda qisqa muddatli impulsi sifatida tasvirlanadi. Buni matematik tarzda quyidagicha ifodalaymiz.

$$\int_{0_-}^{0^+} \boldsymbol{\delta(t)\,dt = 1} \qquad (7.30)$$

bu yerda: $t = 0^-$ $t = 0$ dan oldingi vaqtni va $t = 0^+$ $t = 0$ dan keyingi vaqtni bildiradi.



7.18-rasm. Uchta impuls funksiyalari.

$$\int_a^b f(t)\delta(t - t_0)\, dt \qquad (7.31)$$

Bu shuni ko‘rsatadiki, funksiya impuls funksiyasi bilan integrallashganda, biz impuls sodir bo‘lgan nuqtada funksiyaning qiymatini olamiz.

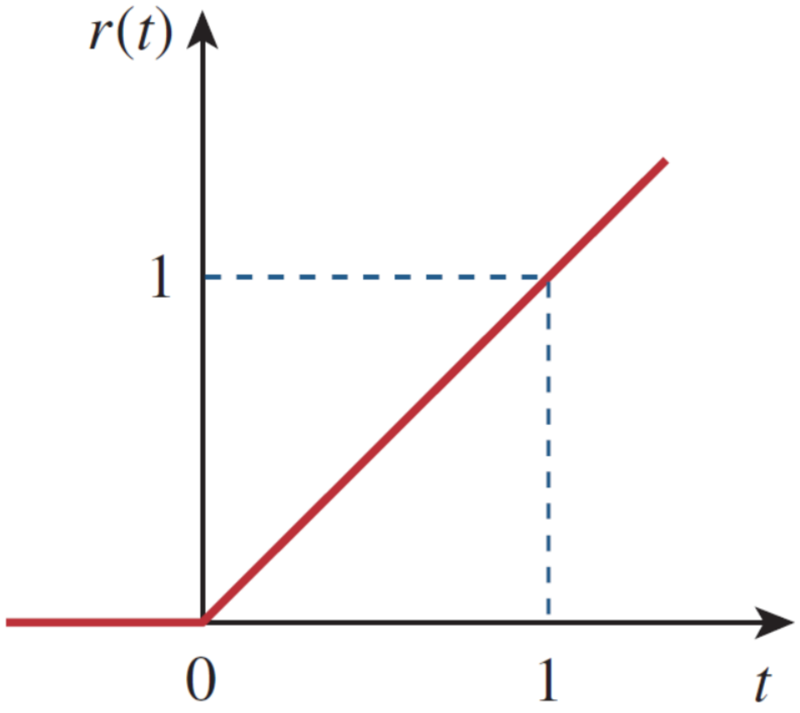
Bu impuls funksiyasining juda foydali xususiyati bo‘lib, ***namuna olish*** *(sampling)* yoki ***saralash*** *(sifting)* ***xususiyati*** deb nomlanadi.

Birlik rampa funksiyasi:

$u(t)$ birlik qadam funksiyasini integrallash natijasida birlik rampa funksiyasi $r(t)$ hosil bo‘ladi.

$$\boldsymbol{r(t) = \int_{-\infty}^{t} u(\lambda)\, d\lambda = tu(t)} \qquad (7.34)$$

$$\boldsymbol{r(t) = \begin{cases} 0, & t \leq t_0 \\ t & t \geq t_0 \end{cases}} \qquad (7.35)$$

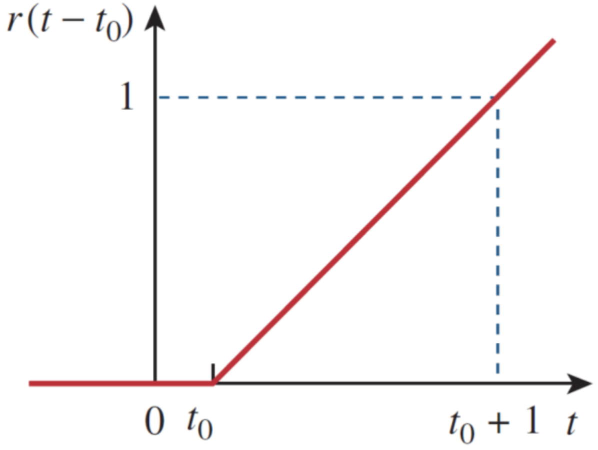


7.19-rasm. Birlik rampa funksiyasi.

Birlik rampa *(umumiy yoki chiziqli)* ***funksiyasi*** t ning manfiy qiymatlari uchun nolga teng va t ning musbat qiymatlari uchun birlik qiyalikka ega.

Umuman olganda, rampa doimiy tezlikda o‘zgarib turadigan funksiyadir.

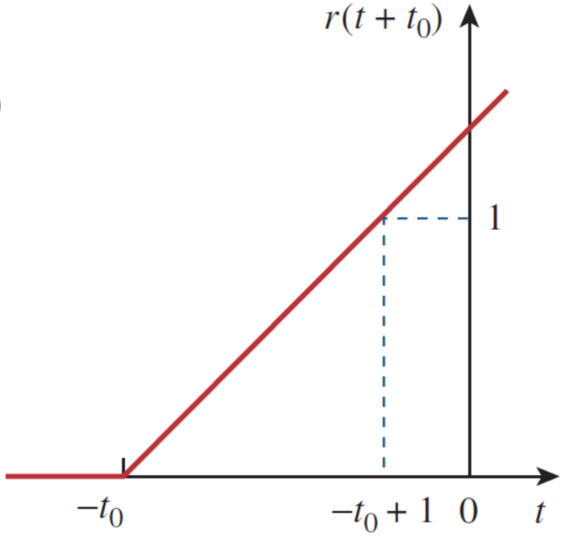
a)



Kechiktirilgan birlik rampa funksiyasi uchun,

$$r(t-t_0)=\begin{cases}0, & t\le t_0\\ t-t_0 & t\ge t_0\end{cases} \qquad (7.36)$$

b)



Ilgarilangan birlik rampa funksiyasi uchun,

$$r(t+t_0)=\begin{cases}0, & t\le -t_0\\ t+t_0 & t\ge -t_0\end{cases} \qquad (7.37)$$

7.20-rasm. Birlik rampa funksiyasi.

a) t_0 ga kechiktirilgan, b) t_0 ga ilgarilangan.

Uchta yakkalik funksiyasi (*qadam*, *impuls va rampa*) differensiallik bilan bog‘liq.

$$\boldsymbol{\delta(t) = \frac{du(t)}{dt}}, \qquad \boldsymbol{u(t) = \frac{dr(t)}{dt}} \qquad (7.38)$$

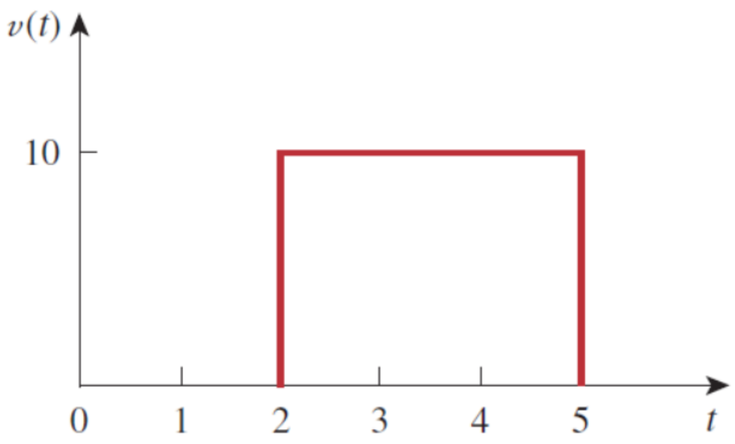
yoki integrallash orqali

$$\boldsymbol{u(t) = \int_{-\infty}^{t} \delta(\lambda)\, d\lambda,\ r(t) = \int_{-\infty}^{t} u(\lambda)\, d\lambda} \qquad (7.39)$$

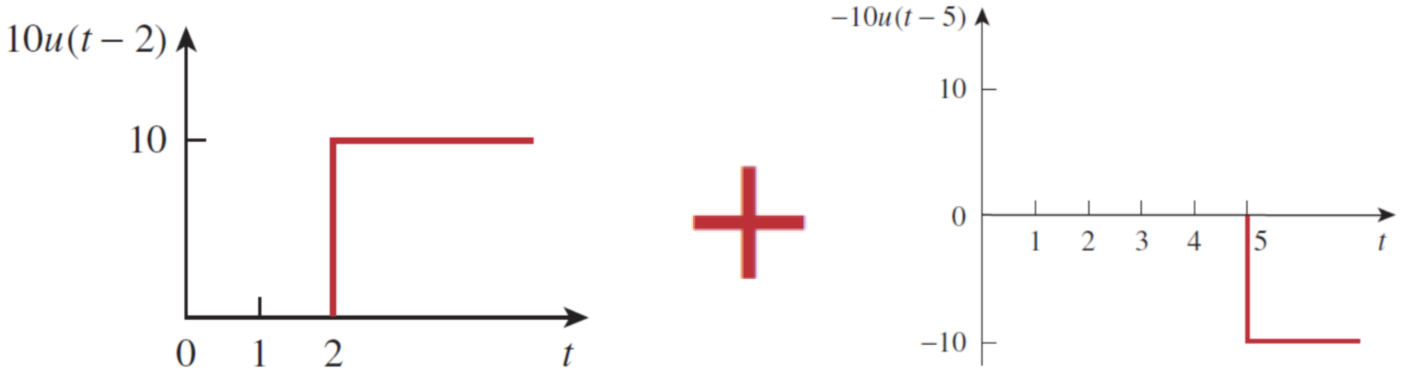
Yana ko‘p o‘ziga xoslik funksiyalari mavjud bo‘lsa-da, biz faqat ushu uchta (*qadam*, *impuls va rampa*) funksiyalarni tahlil qilib chiqdik.

7.4.1-masala: 7.21-rasmdagi kuchlanish impulsini birlik qadam bilan ifodalang. Uning hosilasini hisoblang va chizing.

Yechish: $\boldsymbol{u(t) = 10u(t-2) - 10u(t-5) = 10[u(t-2) - u(t-5)]}$

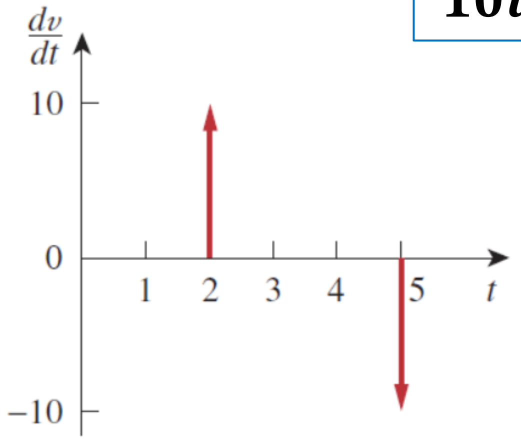


7.21-rasm.



$\boldsymbol{10u(t-2)}$ a) $\boldsymbol{-10u(t-5)}$

b)

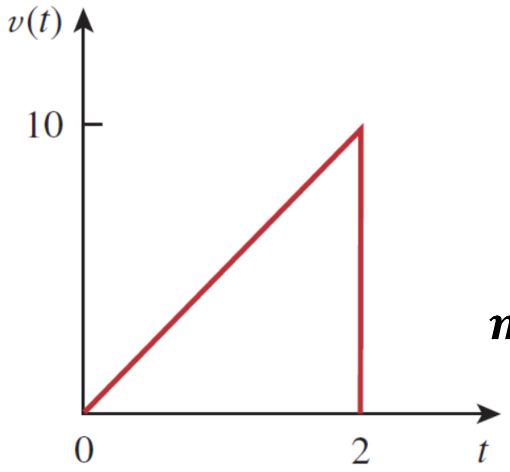


$$\boldsymbol{\frac{du}{dt} = 10[\delta(t-2) - \delta(t-5)]}$$

7.22-rasm.

a) 7.21-rasmdagi impulsning parchalanishi, b) 7.21-rasmdagi pulsning hosilasi.

7.4.2-masala: 7.23-rasmda ko‘rsatilgan arra tish funksiyasini yakkalik funksiyalari bilan ifodalang.

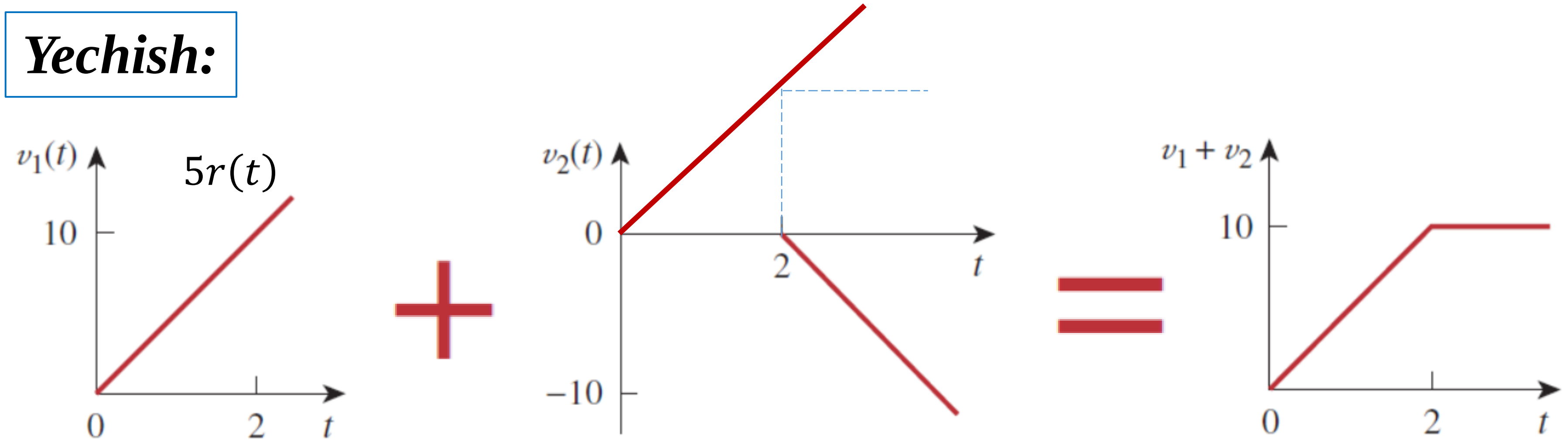


$$\boldsymbol{y = mx + c}$$

$$\boldsymbol{m = \frac{y}{x} = \frac{10}{2} = 5}$$

7.23-rasm.

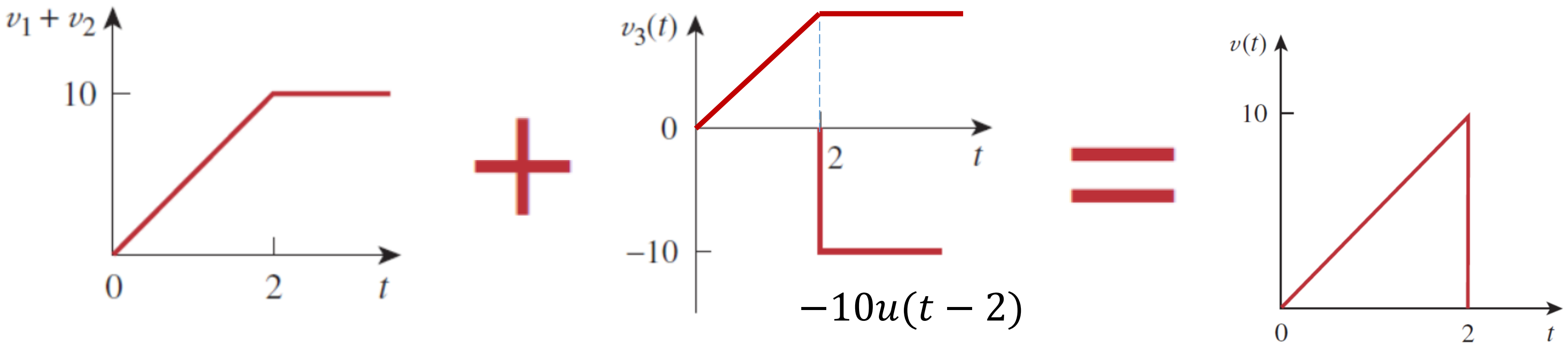
Yechish:



$$u_1(t) = 5r(t)$$

$$u_2(t) = -5r(t-2)$$

$$u_3 = -10u(t-2)$$



$$\boldsymbol{u(t) = 5r(t) - 5r(t-2) - 10u(t-2)}$$

FOYDALANILGAN MANBALAR:

6. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 266-268.
7. https://i.ytimg.com/vi/LphJ1DqPfjo/hqdefault.jpg
8. https://www.dummies.com/wp-content/uploads/376127.image1.jpg

E'TIBORINGIZ UCHUN RAHMAT!!!