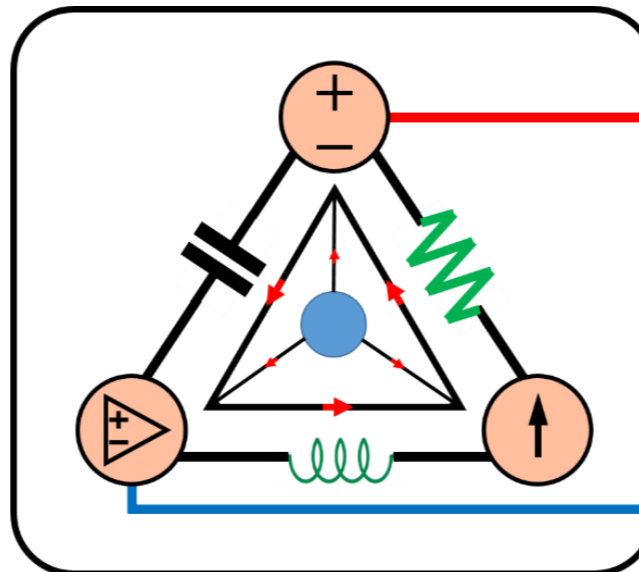


7-Mavzu: Birinchi tartibli elektr zanjiri (*davomi*).
(7th Topic: First-Order Circuit)

7-Mavzuning 3-qismi
(3rd part of the 7th Topic)

8-hafta uchun
For the 8th week



Lecturer: Ph.D., Yusupov Sarvarbek

Toshkent Kimyo Xalqaro Universiteti
“Mashinasozlik texnologiyasi” kafedrası
Toshkent shahri, Usmon Nosir, 156-uy.

7-Mavzu: Birinchi tartibli elektr zanjiri (*davomi*).

(7th Topic: First-Order Circuit)

O'quv rejası:

7.5. RC zanjirining qadamli reaksiyasi.

7.6. RL zanjirining qadamli reaksiyasi.

7.7. Birinchi tartibli operatsion kuchaytirgich zanjirlari.

7.8. Qo'llanilishi.

7.5. RC zanjirining qadamli reaksiyasi.

RC zanjirining doimiy manbai qo‘llanilganda, kuchlanish yoki tok kuchi manbai qadam funksiyasi sifatida modellashtiriladi va bu jarayonda reaksiya qadamli reaksiya (*step response*) ni anglatadi.

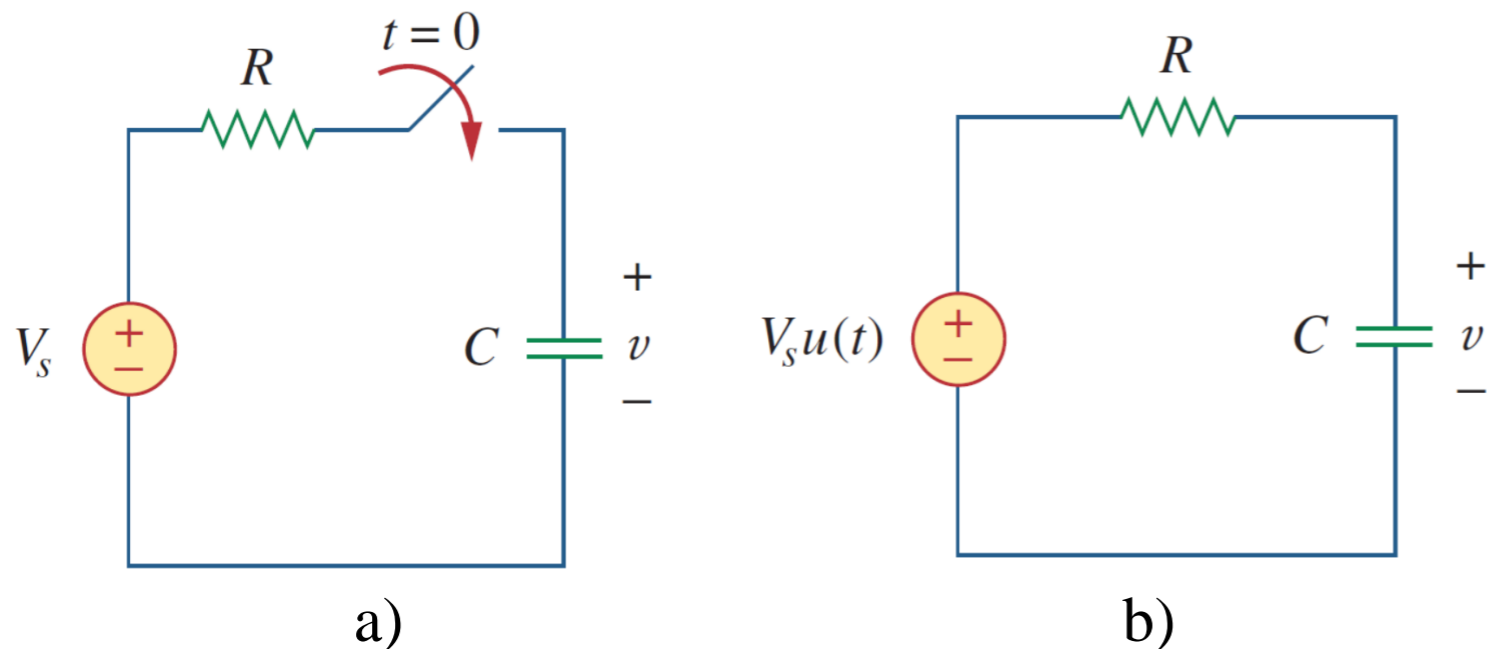
Zanjirning *qadamli reaksiyasi* qo‘zg‘aluvchi (*uyg‘atuvchi*) kuchlanish yoki tok kuchi manbalarining qadam funksiyasi bo‘lganda uning munosabati tushuniladi.

Qadamli reaksiyasi – o‘zgarmas kuchlanish yoki tok kuchi manbasining birdaniga qo‘llanilishi tufayli zanjirning reaksiyasini bildiradi.

Bu yerda U_s o'zgarmas kuchlanish manbai. Zanjirning reaksiyasini aniqlash uchun kondensator kuchlanishi tanlaymiz.

Kondensator da dastlabki kuchlanishni U_0 deb qabul qilamiz.

Kondensatorning kuchlanishi bir zumda o'zgarmasligi sababli,



$$u(0^-) = u(0^+) = U_0 \quad (7.40)$$

bu yerda: $u(0^-)$ - kalit berkilishidan oldingi kondensator dagi kuchlanishi;

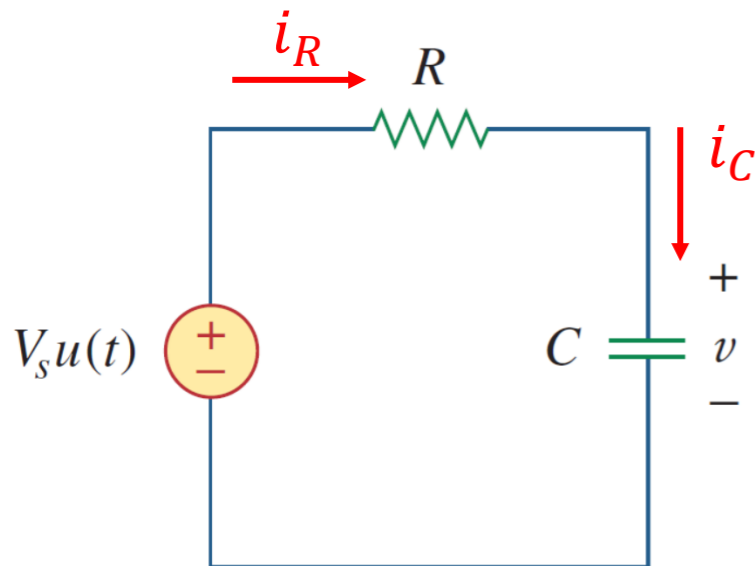
$u(0^+)$ - kalit berkilishidan keyingi kuchlanish.

7.26-rasm. Bosqichli kirish kuchlanishiga ega RC zanjiri.

Photo source: [9] - Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 274.

KCLni qo‘llash orqali quyidagi tenglamaga ega bo‘lamiz.

$$i_R + i_C = 0$$



$$C \frac{du}{dt} + \frac{u - U_s u(t)}{R} = 0 \quad \text{yoki,} \quad \frac{du}{dt} + \frac{u}{RC} = \frac{U_s}{RC} u(t) \quad (7.41)$$

$$t > 0 \text{ uchun:} \quad \frac{du}{dt} + \frac{u}{RC} = \frac{U_s}{RC} \quad (7.42)$$

Shartlarni qayta tartibga solamiz.

$$\frac{du}{dt} = -\frac{u - U_s}{RC} \quad \text{yoki,} \quad \frac{du}{u - U_s} = -\frac{dt}{RC} \quad (7.43)$$

Ikkala tomonni boshlang‘ich funksiyasini hosil qilamiz va dastlabki shartlarni qo‘llaymiz.

$$\ln(u - U_s) \Big|_{U_0}^{u(t)} = -\frac{t}{RC} \Big|_0^t \quad \text{yoki,}$$

$$\ln(u(t) - U_s) - \ln(U_0 - U_s) = -\frac{t}{RC} + 0$$

$$\ln \frac{u - U_s}{U_0 - U_s} = -\frac{t}{RC} \quad (7.44)$$

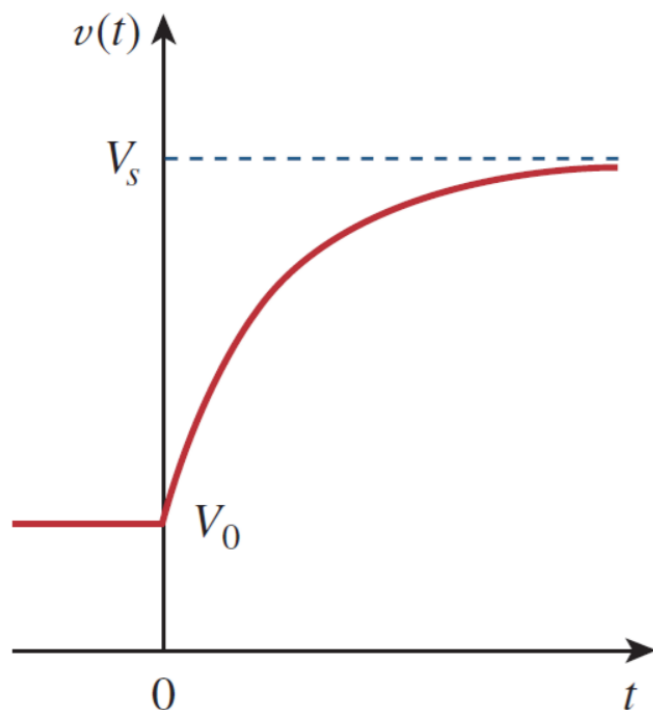
Ikkala tomonning eksponensialini olamiz.

$$\frac{u - U_s}{U_0 - U_s} = e^{-\frac{t}{\tau}}, \quad \tau = RC$$

yoki,

$$u - U_s = (U_0 - U_s)e^{-\frac{t}{\tau}}$$

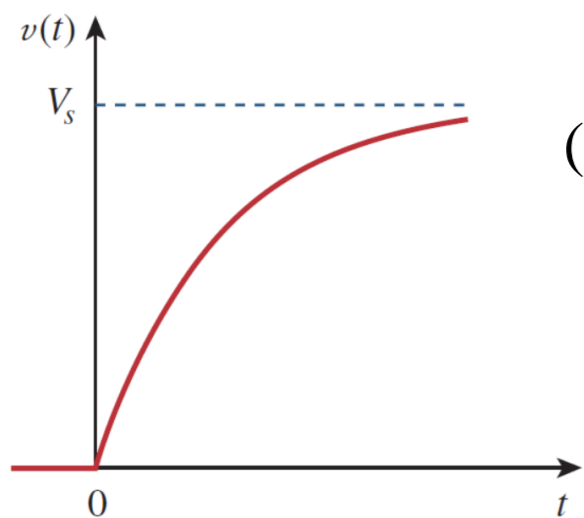
$$u(t) = U_s + (U_0 - U_s)e^{-\frac{t}{\tau}}, \quad t > 0 \quad (7.45)$$



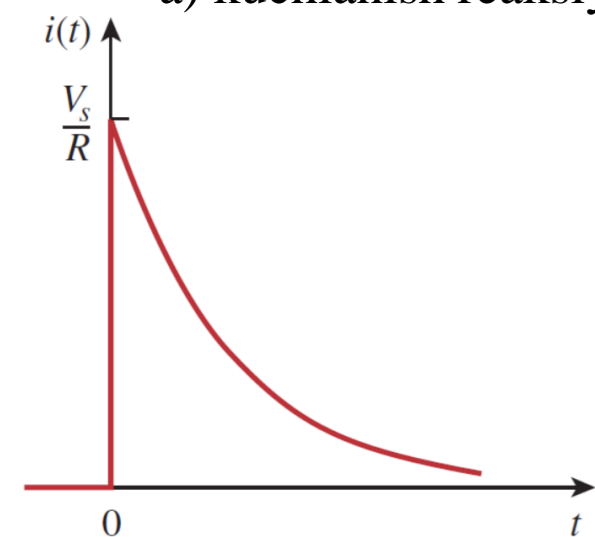
$$u(t) = \begin{cases} U_0, & t < 0 \\ U_s + (U_0 - U_s)e^{-\frac{t}{\tau}}, & t > 0 \end{cases} \quad (7.46)$$

Kondensator dastlab zaryadlangan bo'lsa, doimiy kuchlanish manbasining to'satdan qo'llanilishiga RC pallasining to'liq reaksiyasi (yoki umumiy reaksiyasi) deb nomlanadi.

kondensator bilan RC zanjirining reaksiyasi.



a) kuchlanish reaksiyasi,



b) tok kuchi reaksiyasi.

Agar kondensatorni dastlab zaryadsizlangan deb hisoblasak,

(7.46) tenglamada $U_0 = 0$ ni o'rnatib,

$$u(t) = \begin{cases} 0, & t < 0 \\ U_s(1 - e^{-\frac{t}{\tau}}), & t > 0 \end{cases} \quad (7.47)$$

$$u(t) = U_s(1 - e^{-\frac{t}{\tau}})u(t) \quad (7.48)$$

Bu kondensator dastlab zaryadsizlanganda RC zanjirining to'liq bosqichli (*qadamli*) reaksiya hisoblanadi.

Kondensator orqali tok kuchi $i(t) = C \frac{du}{dt}$ yordamida (7.47) \rightarrow

$$i(t) = C \frac{du}{dt} = \frac{C}{\tau} U_s e^{-\frac{t}{\tau}}, \quad \tau = RC, \quad t > 0$$

$$i(t) = \frac{U_s}{R} e^{-\frac{t}{\tau}} u(t) \quad (7.49)$$

7.28-rasm. Dastlab zaryadsizlangan kondensator RC zanjirining bosqichli reaksiyasi.

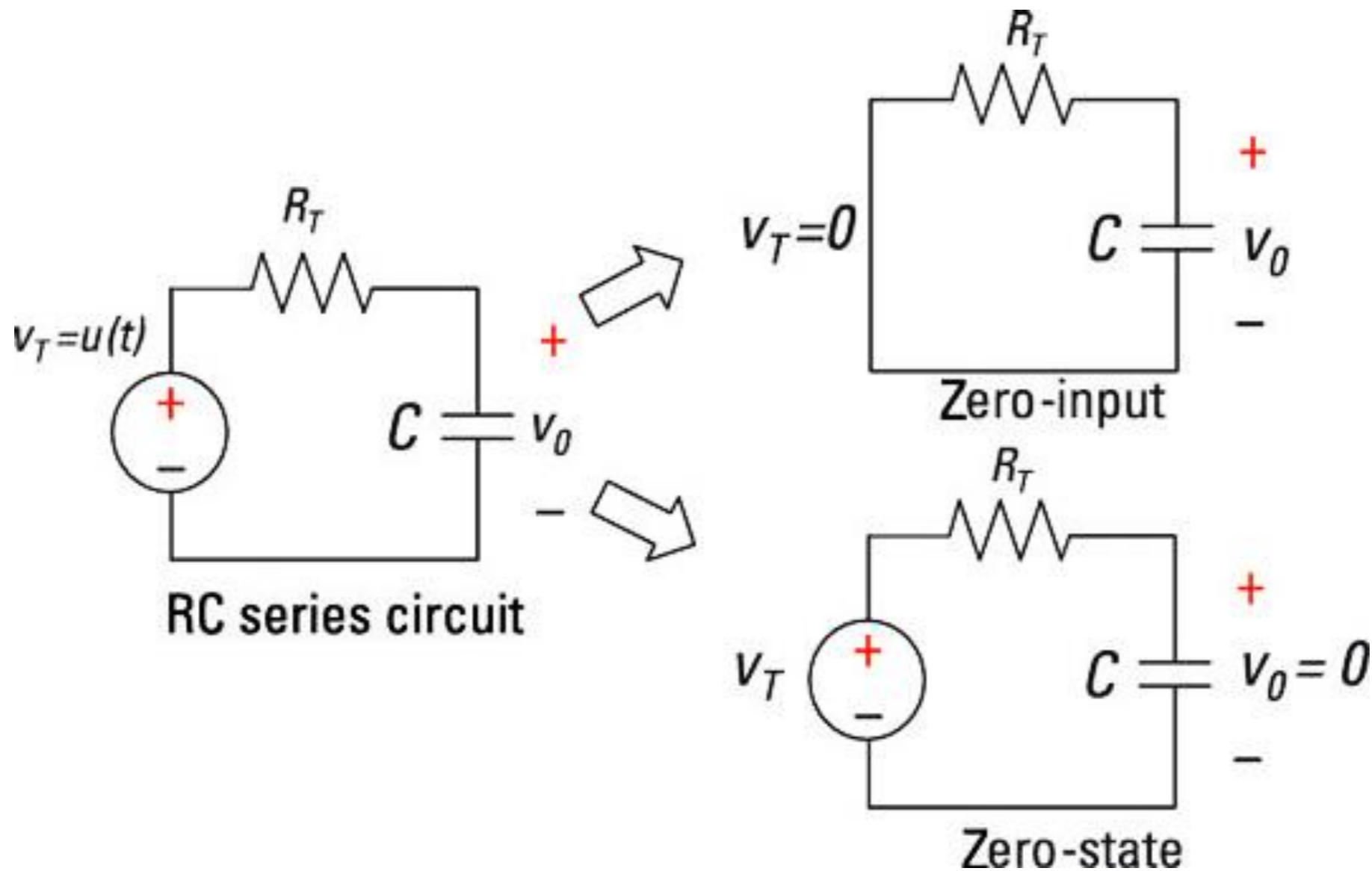


Photo source: [10] - <https://www.dummies.com/wp-content/uploads/375985.image4.jpg>

$$u(t) = U_s + (U_0 - U_s)e^{-\frac{t}{\tau}}, t > 0$$

$u(t)$ ikkita komponentdan iborat.

Klassik ravishda uni ikkita komponentga ajratishning ikkita usuli mavjud.

Birinchisi - *tabiiy reaksiyasi* (*natural response*) va *majburiy reaksiyasi* (*forced response*);

Ikkinchisi - *o'tish jarayoni reaksiya* (*transient response*) va *barqaror holatdagi reaksiyasi* (*steady-state response*).

To'liq reaksiya =

= tabiiy reaksiya (*saqlangan energiya*) + **majburiy reaksiya** (*mustaqil manba*)

$$u = u_n + u_f$$

(7.50)

$$u_n = U_0 e^{-\frac{t}{\tau}}$$

$$u_f = U_s (1 - e^{-\frac{t}{\tau}})$$

To‘liq reaksiyani aniqlashning yana bir usuli - ikkita komponentga bo‘linishi - biri vaqtinchalik va ikkinchisi doimiy, ya’ni.

To‘liq reaksiya =
= o‘tish jarayoni reaksiya (temporary part) + barqaror holatdagi reaksiya (permanent part)

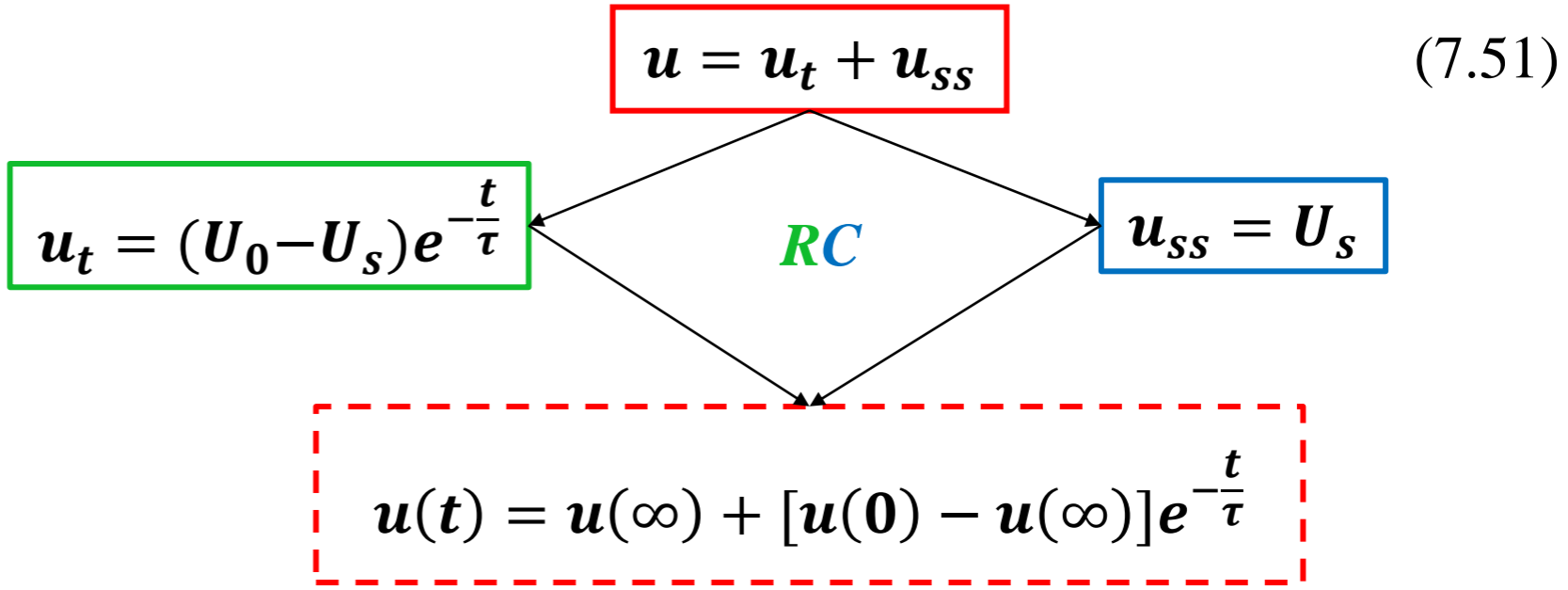
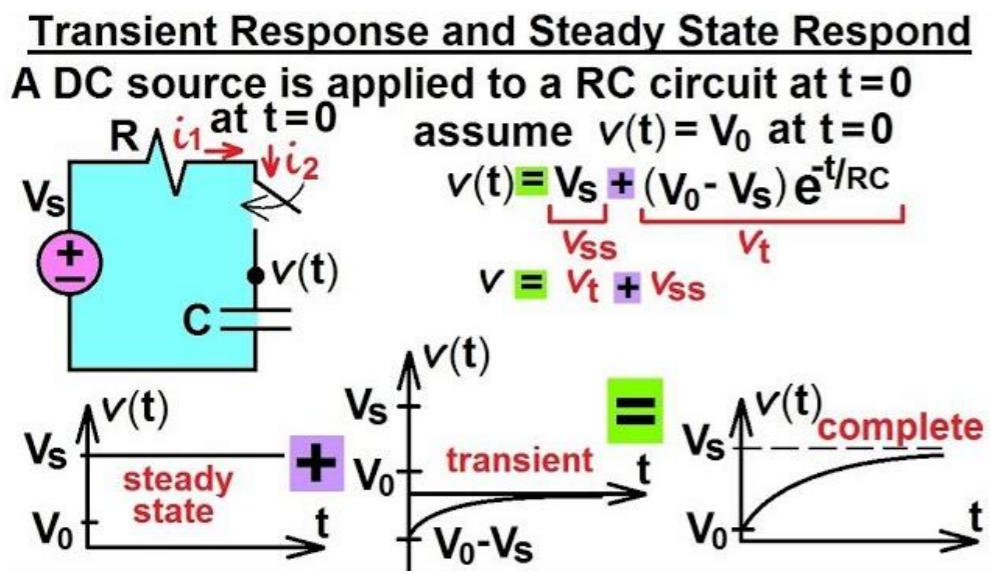


Photo source: [11] - <https://i.ytimg.com/vi/KPOQ-cdGbjQ/sddefault.jpg>

O'tish jarayoni reaksiyasi u_t - vaqtinchalikdir; bu to'liq reaksiyaning vaqt cheksizlikka yaqinlashganda nolga tushadigan qismidir. U vaqt o'tishi bilan yo'qoladi.

Barqaror holatdagi reaksiya u_{ss} - bu o'tkazuvchi reaksiya yo'q bo'lgandan keyin qolgan to'liq reaksiyaning bir qismidir. U uzoq vaqt davomida zanjirning holatini bildiradi.

To'liq reaksiyaning birinchi parchalanishi reaksiyalar manbai nuqtai nazaridan, ikkinchi parchalanish esa reaksiyalarning doimiyligi nuqtai nazaridan yuzaga keladi.

Muayyan sharoitlarda tabiiy reaksiya va vaqtinchalik reaksiya bir xil bo'ladi. Majburiy reaksiya va barqaror holat haqida ham shunday deyish mumkin.

Bu to‘liq reaksiya vaqtinchalik reaksiya va barqaror holatdagi reaksiyaning yig‘indisi degan tushuncha bilan bir xil.

Qaysi tomondan qarajak, (7.45) tenglamadagi to‘liq reaksiya quyidagicha yozilishi mumkin.

$$\mathbf{u}(t) = \mathbf{u}(\infty) + [\mathbf{u}(0) - \mathbf{u}(\infty)]e^{-\frac{t}{\tau}} \quad (7.52)$$

bu yerda: $u(0)$ - $t = 0^+$ da dastlabki kuchlanish va $u(\infty)$ - yakuniy yoki barqaror holat qiymati.

Shunday qilib, RC zanjirining qadam reaksiyasini topish uchun uchta narsa talab qilinadi:

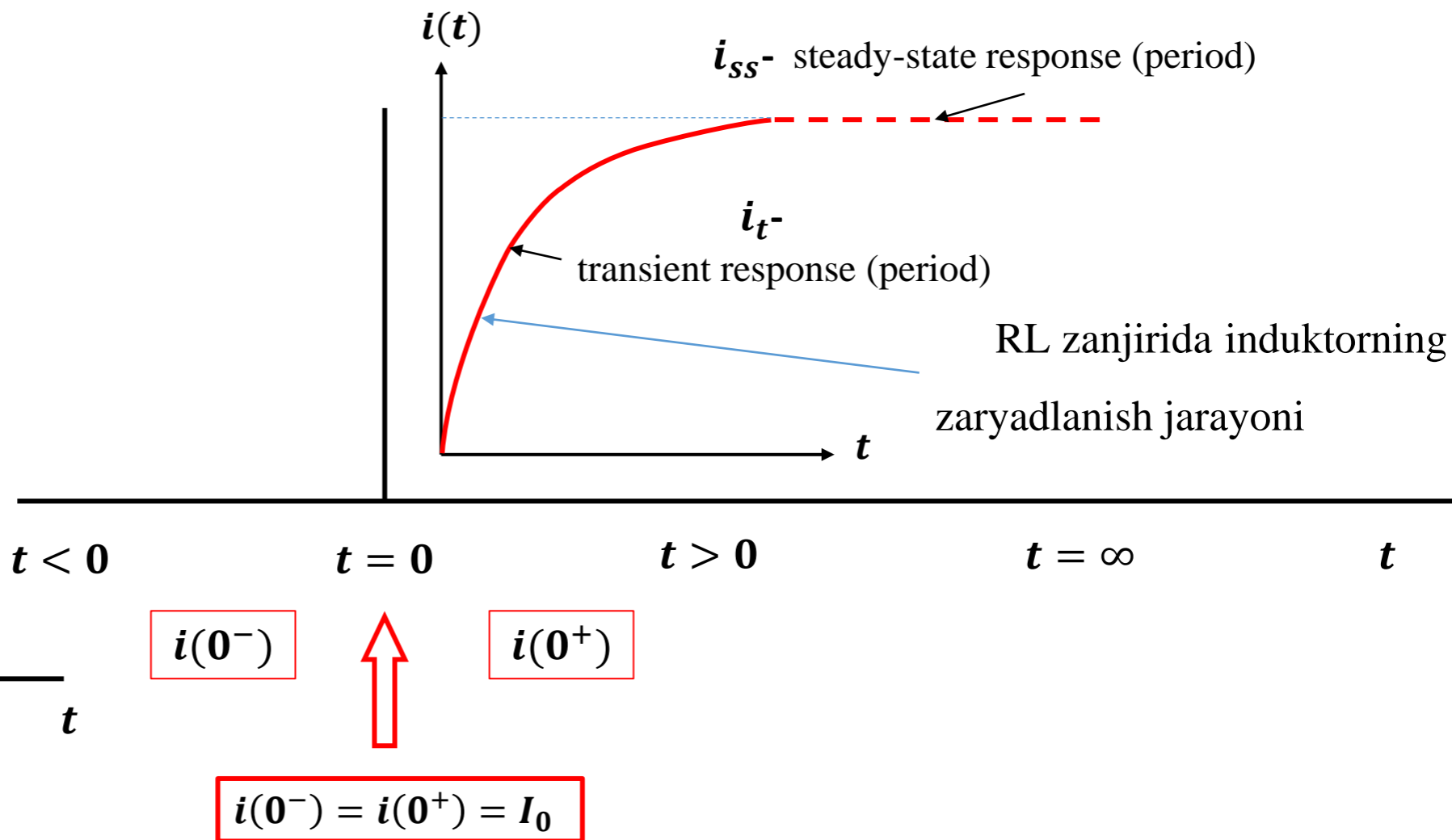
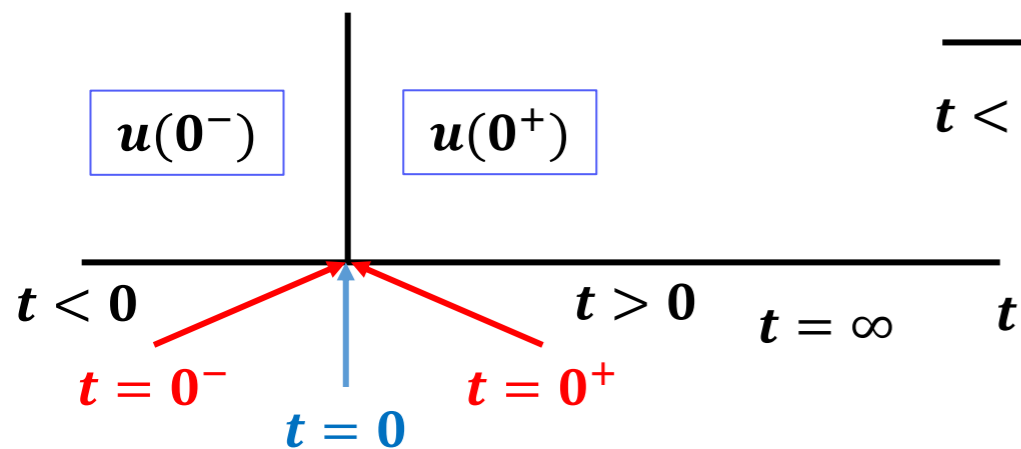
1. $t = 0$ bo‘lganda kondensatorning dastlabki kuchlanishi $u(0)$;

2. Kondensatorning oxirgi kuchlanishi $u(\infty)$;

3. Vaqt doimiysi (konstantasi) τ .

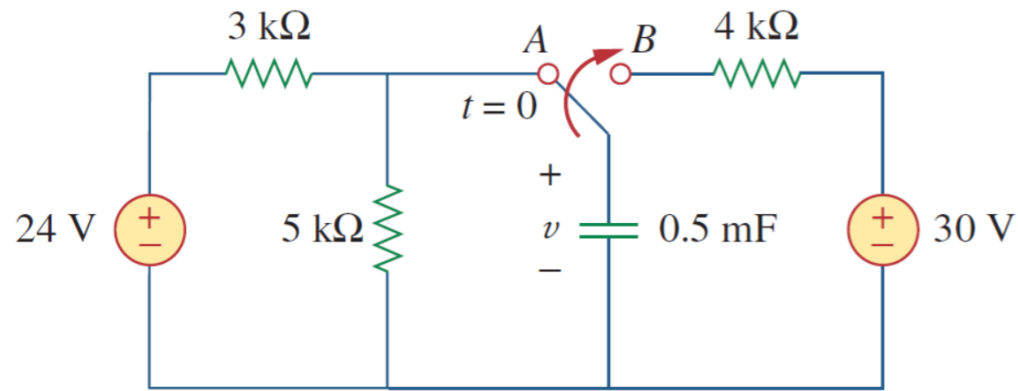
$t=0$ vaqtda RC zanjirida kondensatorning kuchlanishi bir zumda o'zgarmaydi. Shuning uchun

$$u(0^-) = u(0^+) = U_0$$



7.5.1-masala: 7.29-rasmdagi uzgich uzoq vaqt davomida A holatida bo'lgan.

$t = 0$ da uzgich B ga o'tadi. $t > 0$ uchun $u(t)$ ni aniqlang va uning qiymatini $t = 1$ s va 4 soniyada hisoblang.

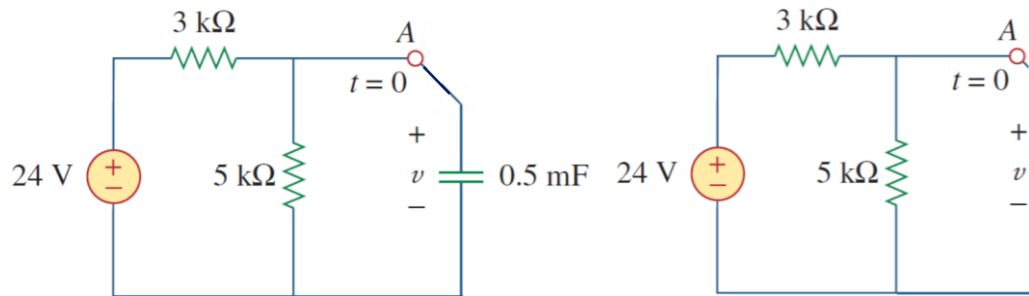


7.29-rasm.

$t < 0$ uchun:

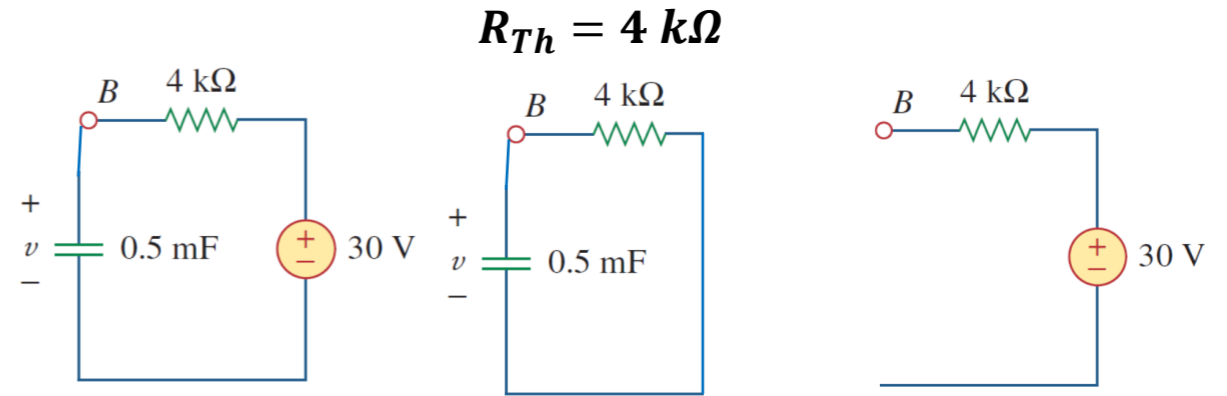
$$u(0^-) = \frac{5}{5+3}(24) = 15 \text{ V}$$

$$u(0) = u(0^-) = u(0^+) = 15 \text{ V}$$



Yechish:

$t > 0$ uchun:



$$R_{Th} = 4 \text{ k}\Omega$$

$$\tau = R_{Th}C = 4 \cdot 10^3 \cdot 0,5 \cdot 10^{-3} = 2 \text{ s}$$

$$u(\infty) = 30 \text{ V}$$

$$u(t) = u(\infty) + [u(0) - u(\infty)]e^{-\frac{t}{\tau}}$$

$$u(t) = 30 + (15 - 30)e^{-\frac{t}{2}} = (30 - 15e^{-0,5t}) \text{ V}$$

$t = 1$ da,

$$u(1) = 30 - 15e^{-0,5} = 20,9 \text{ V}$$

$t = 4$ da,

$$u(4) = 30 - 15e^{-2} = 27,97 \text{ V}$$

7.6. RL zanjirining qadamli reaksiyasi.

RL zanjir reaksiyasi sifatida induktordagi tok kuchi i ni aniqlashni o‘rganamiz.

$$u = u_n + u_f$$

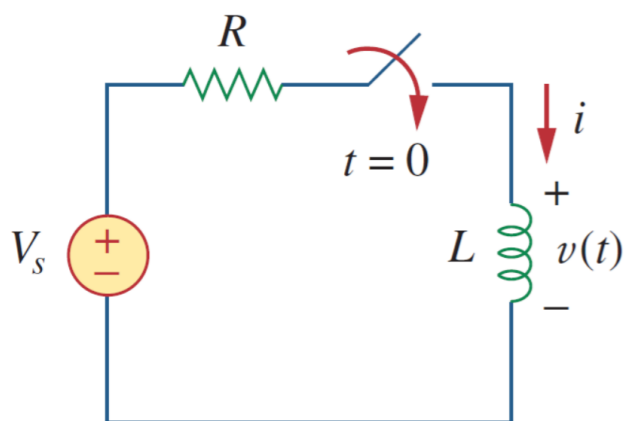
$$u(t) = u(\infty) + [u(0) - u(\infty)]e^{-\frac{t}{\tau}}$$

$$i = i_t + i_{ss} \quad (7.54)$$

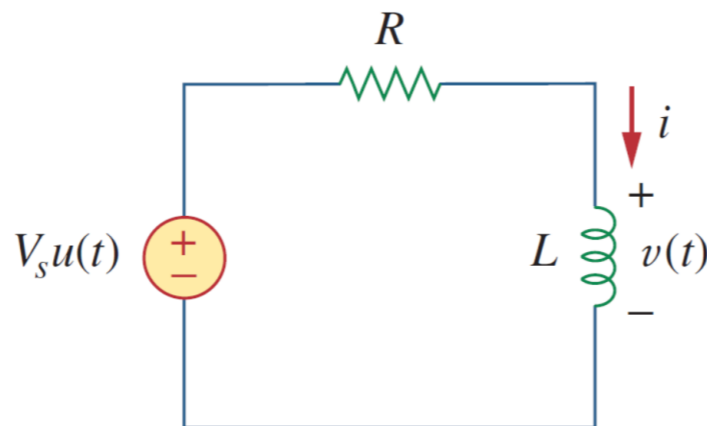
Vaqtinchalik reaksiya har doim pasayuvchi eksponensialdir.

$$i_t = Ae^{-\frac{t}{\tau}}, \quad \tau = \frac{L}{R} \quad (7.55)$$

bu yerda: $A = const.$



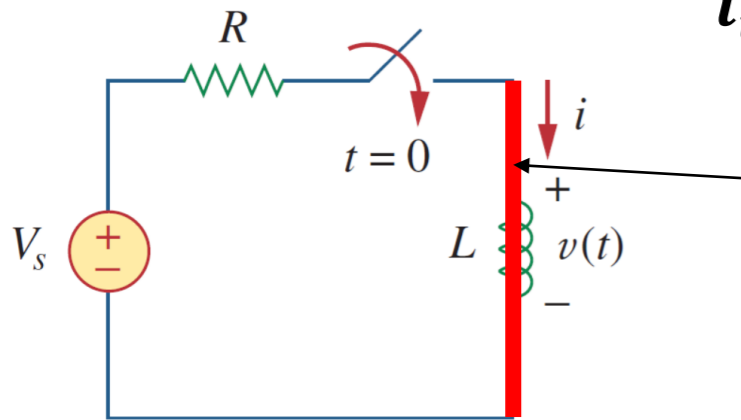
a)



b)

7.30-rasm. Bosqichli kirish kuchlanishiga ega *RL* zanjiri.

Barqaror holatdagi reaksiyasi kalit yopilgandan keyin uzoq vaqt davomida tok kuchining qiymatini bildiradi.



$$i_t \rightarrow 5\tau \quad u(\mathbf{0}) \rightarrow 1\% \downarrow$$

Barcha manba kuchlanishi U_s rezistor

bo‘ylab paydo bo‘ladi.

Shu vaqtda inductor
va $u = 0$

$$i_{ss} = \frac{U_s}{R} \quad (7.56)$$

$$i = i_t + i_{ss}$$

$$i_t = Ae^{-\frac{t}{\tau}}$$

$$i_{ss} = \frac{U_s}{R}$$

$$i = Ae^{-\frac{t}{\tau}} + \frac{U_s}{R}$$

(7.57)

$A=?$

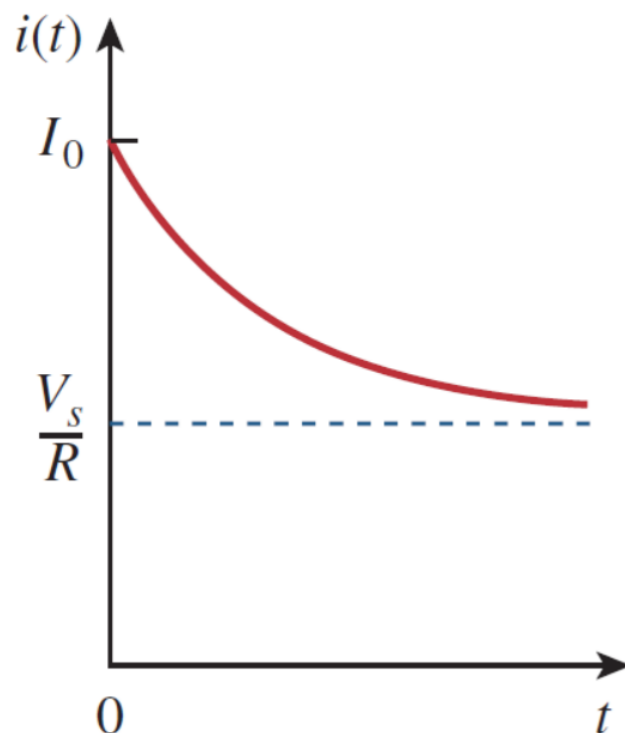
I_0 induktor orqali o‘tadigan dastlabki tok kuchi

$$i(\mathbf{0}^-) = i(\mathbf{0}^+) = I_0 \quad (7.58)$$

Induktor orqali tok kuchi bir zumda o‘zgarmasligi sababli,

Demak, $t = 0$ bo'lganda $\rightarrow i = Ae^{-\frac{t}{\tau}} + \frac{U_s}{R} \rightarrow I_0 = A + \frac{U_s}{R} \rightarrow A = I_0 - \frac{U_s}{R}$

$$i(t) = \frac{U_s}{R} + (I_0 - \frac{U_s}{R})e^{-\frac{t}{\tau}} \quad (7.59)$$



Bu RL zanjirining to'liq reaksiyasidir.

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}} \quad (7.60)$$

bu yerda: $i(0)$ - i ning boshlang'ich qiymati;
 $i(\infty)$ - i ning yakuniy qiymati.

7.31-rasm. Dastlabki induktor tok kuchi I_0 bilan RL zanjirining to'liq reaksiyasi.

To'liq reaksiya =

= o'tish jarayoni reaksiya (temporary part) + barqaror holatdagi reaksiya (permanent part)

Step Response on an RL Circuit

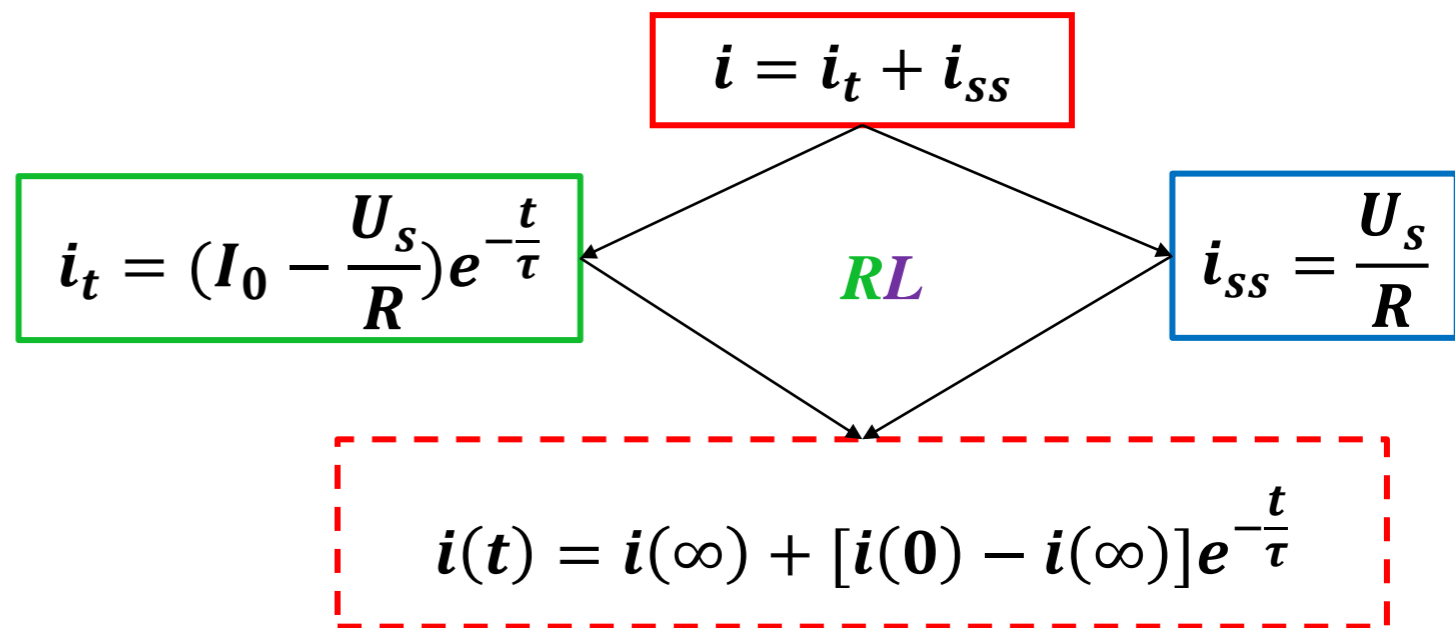
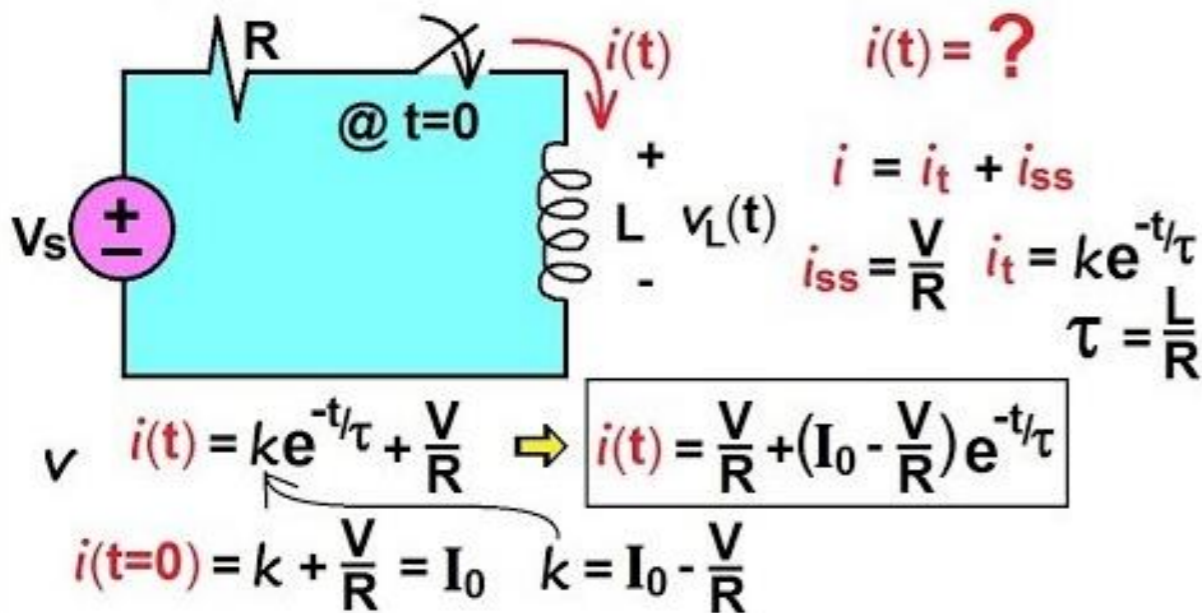


Photo source: [13] - <https://i.ytimg.com/vi/tEmksld8sCO/hqdefault.jpg>

Shunday qilib, RL zanjirining qadamli reaksiyasini topish uchun uchta narsa talab qilinadi:

1. $t = 0$ bo'lganda induktorning dastlabki tok kuchi $i(0)$;

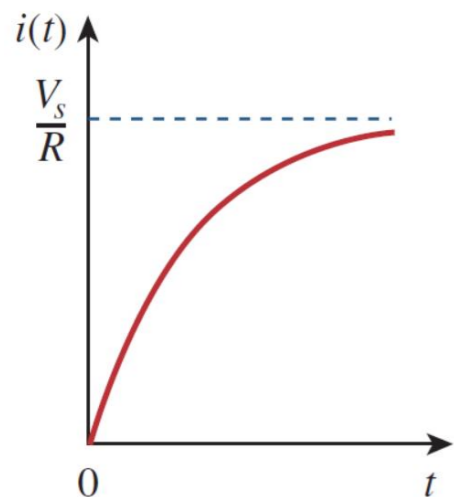
2. Induktorning oxirgi tok kuchi $i(\infty)$;

3. Vaqt doimiysi (*konstantasi*) τ .

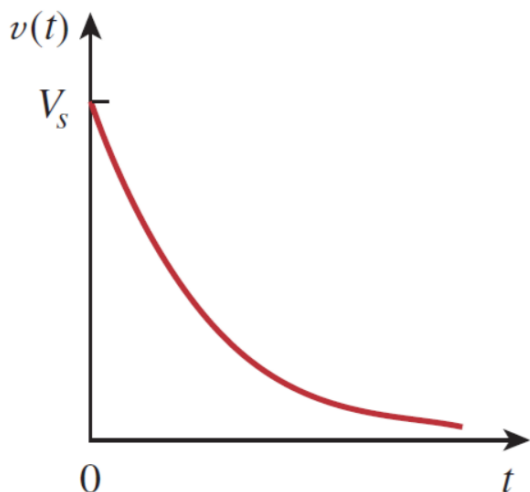
Agar almashtirish $t = 0$ o'rniga $t = t_0$ o'rnini o'zgartirsa, (7.60) tenglama quyidagi ko'rinishga ega bo'ladi.

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}} \quad (7.60)$$

$$i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-\frac{t-t_0}{\tau}} \quad (7.61)$$



a) tok kuchi reaksiyasi



b) kuchlanish reaksiyasi

Agar, $I_0 = 0$ bo'lsa, u holda

$$i(t) = \begin{cases} 0, & t < 0 \\ \frac{U_s}{R} (1 - e^{-\frac{t}{\tau}}), & t > 0 \end{cases} \quad (7.62a)$$

yoki,

$$i(t) = \frac{U_s}{R} (1 - e^{-\frac{t}{\tau}}) U(t) \quad (7.62b)$$

Bu boshlang'ich induktor tok kuchi bo'lmagan RL zanjirining qadamli reaksiyasidir.

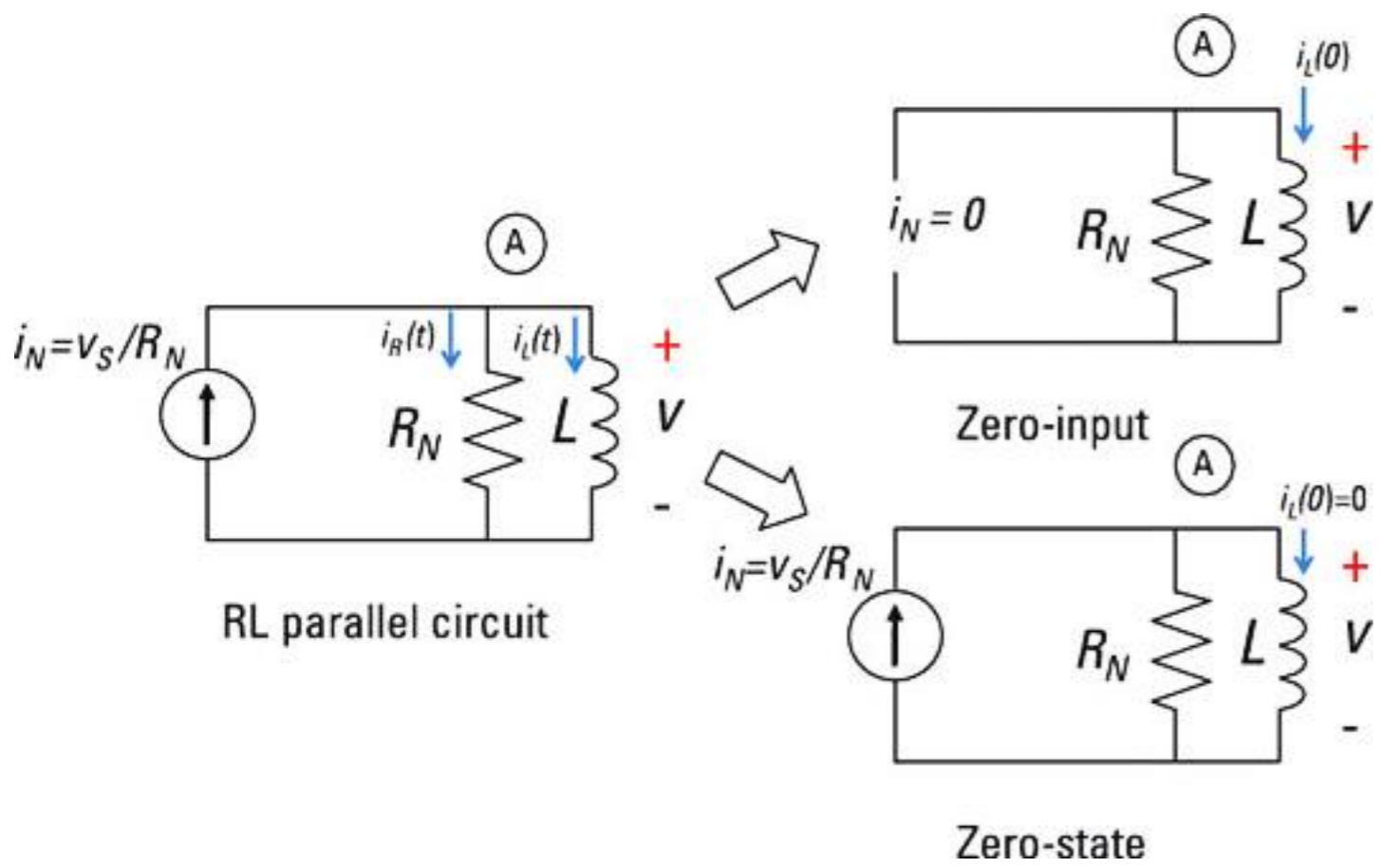
Induktordagi kuchlanish $U = L \frac{di}{dt}$ yordamida (7.62) \rightarrow

$$u(t) = L \frac{di}{dt} = U_s \frac{L}{\tau R} e^{-\frac{t}{\tau}}, \quad \tau = \frac{L}{R}, \quad t > 0$$

yoki,

$$u(t) = U_s e^{-\frac{t}{\tau}} U(t) \quad (7.63)$$

7.32-rasm. Boshlang'ich induktor tok kuchi bo'lmagan RL zanjirining bosqichli reaksiyalari.



General Strategy for Solving RL Circuits

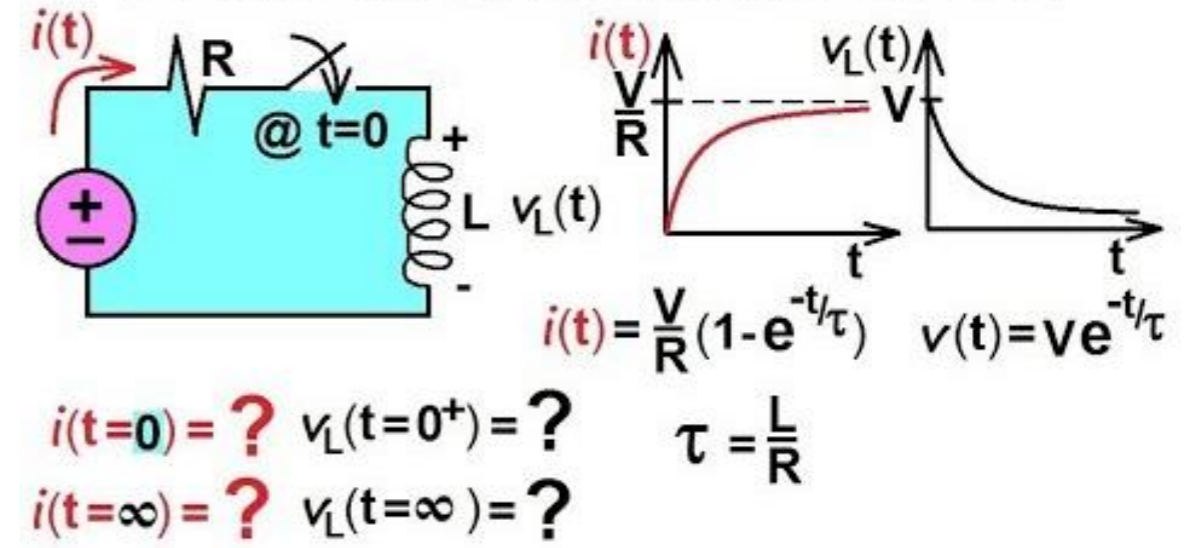


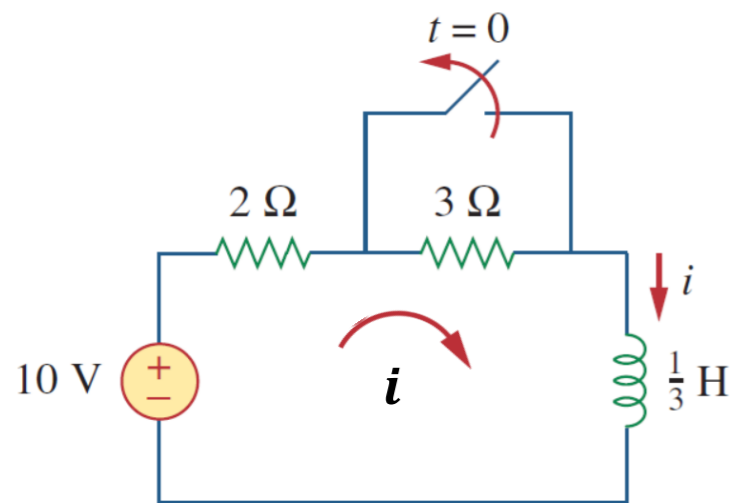
Photo source: [15] - <https://i.ytimg.com/vi/WEwTSFr3ux0/hqdefault.jpg>

Photo source: [14] - <https://www.dummies.com/wp-content/uploads/376026.image5.jpg>

7.6.1-masala: 7.33-rasmdagi RL zanjirida $t > 0$ uchun $i(t)$ ni toping.

Kalit uzoq vaqt davomida yopilgan deb hisoblang.

Yechish:



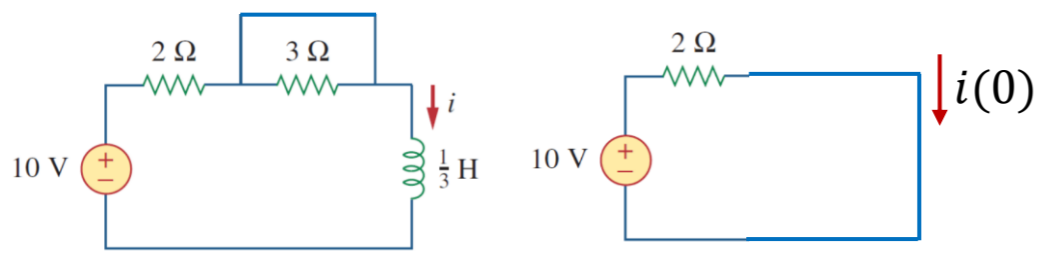
7.33-rasm.

$$i(0^-) = \frac{10}{2} = 5 \text{ A}$$

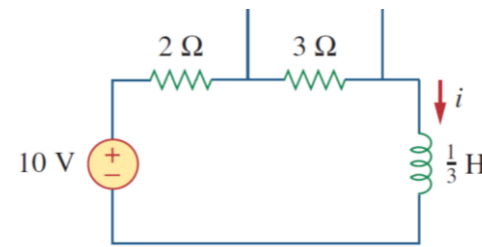
$t < 0$ uchun:

$$i(0) = i(0^-) = i(0^+) = 5 \text{ A}$$

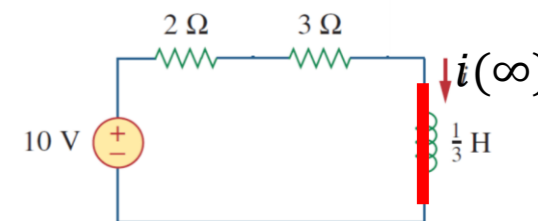
Induktor orqali tok kuchi $t = 0^-$ (ya'ni, $t = 0$ dan biroz oldin).



$t > 0$ uchun:



$$R_{Th} = 2 + 3 = 5 \Omega$$



$$i(\infty) = \frac{10}{2 + 3} = 2 \text{ A}$$

$$\tau = \frac{L}{R_{Th}} = \frac{1/3}{5} = \frac{1}{15} \text{ s}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}$$

$$i(t) = 2 + (5 - 2)e^{-15t} \text{ A}, \quad t > 0$$

Tekshirish: KVL bo'yicha $10 = 5i + L \frac{di}{dt}$

$$\begin{aligned} 5i + L \frac{di}{dt} &= 5(2 + (5 - 2)e^{-15t}) + \frac{1}{3} \frac{d}{dt} (2 + (5 - 2)e^{-15t}) = \\ &= [10 + 15e^{-15t}] + \left[\frac{1}{3} (3)(-15)e^{-15t} \right] = 10. \end{aligned}$$

FOYDALANILGAN MANBALAR:

9. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 274.
10. <https://www.dummies.com/wp-content/uploads/375985.image4.jpg>
11. <https://i.ytimg.com/vi/KPOQ-cdGbjQ/sddefault.jpg>
12. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 280.
13. <https://i.ytimg.com/vi/tEmksld8sC0/hqdefault.jpg>
14. <https://www.dummies.com/wp-content/uploads/376026.image5.jpg>
15. <https://i.ytimg.com/vi/WEwTSFr3ux0/hqdefault.jpg>



*E'TIBORINGIZ
UCHUN
RAHMAT!!!*