8-Mavzu: Ikkinchi tartibli elektr zanjiri.

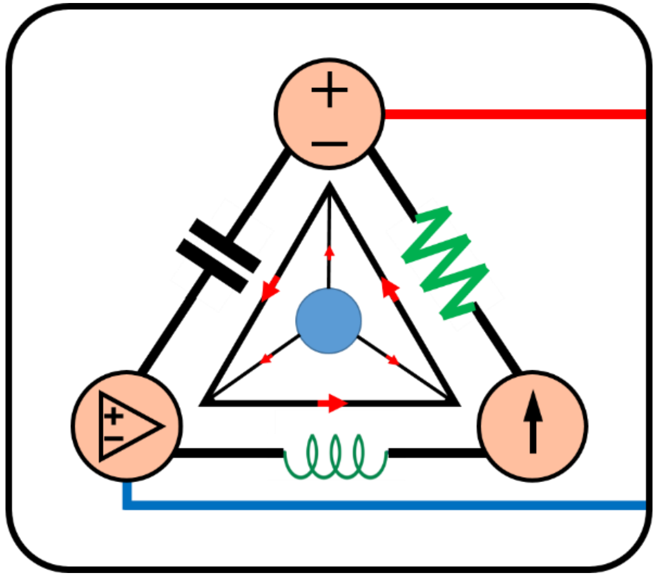
(8th Topic: Second-Order Circuit)

8-Mavzuning 2-qismi

(2nd part of the 8th Topic)

9-hafta uchun

For the 9th week



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8-Mavzu: Ikkinchi tartibli elektr zanjiri.

(8th Topic: Second-Order Circuit)

O'quv rejasi:

8.1. Umumiy tushunchalar.

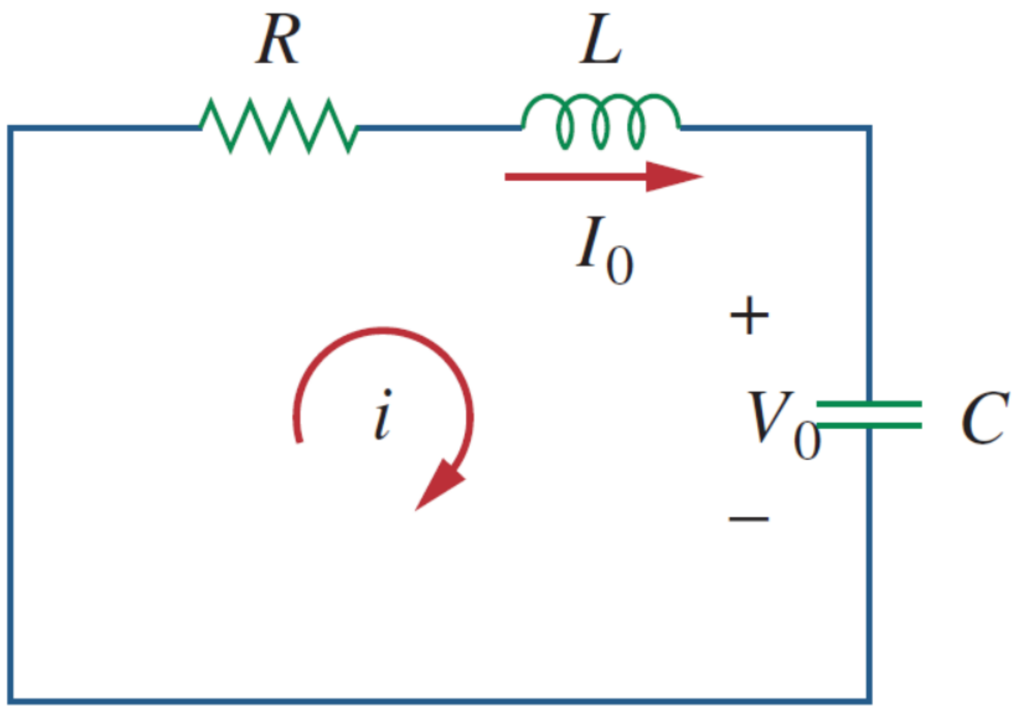
8.2. Boshlang'ich va yakuniy qiymatlarni topish.

8.3. Manbadan holi ketma-ket ulangan qarshilik, induktor va kondensator (RLC) zanjiri.

8.4. Manbadan holi parallel ulangan qarshilik, induktor va kondensator (RLC) zanjiri.

8.3. Manbadan holi ketma-ket ulangan qarshilik, induktor va kondensator (RLC) zanjiri.

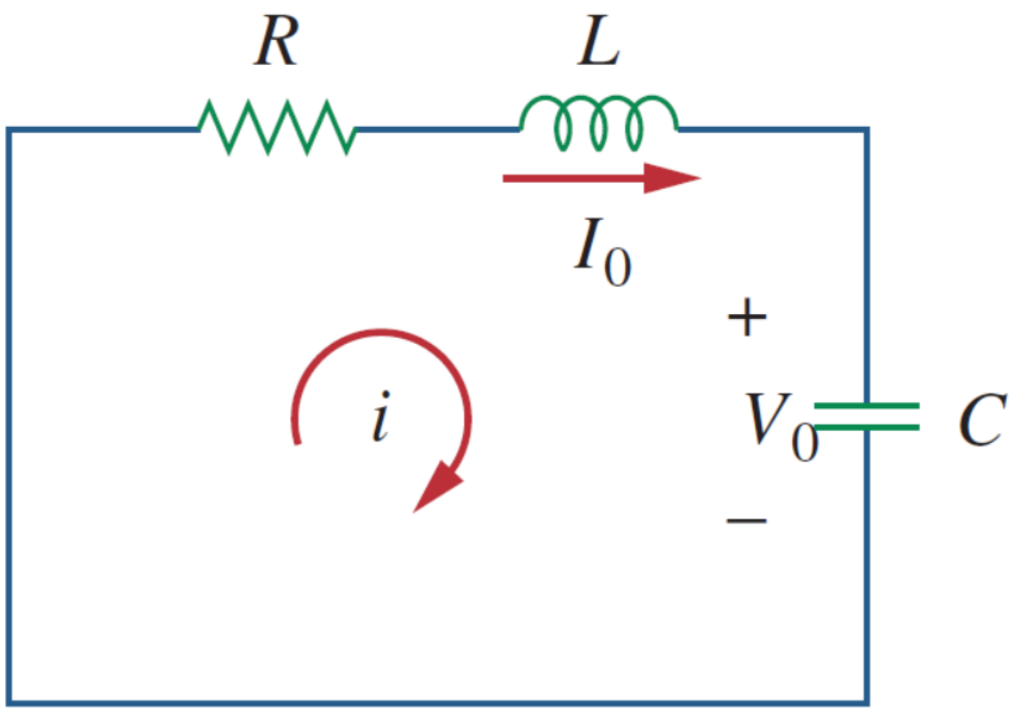
RLC ketma-ketligidan iborat bo‘lgan janjirning tabiiy reaksiyasini tushunish elektrotexnika muhandislari tomonidan kelajakdagi tadqiqotlarni olib borishda fundament hisoblanadi.



8.4-rasm. Manbadan holi ketma-ket ulangan *RLC* zanjiri.

Zanjir kondensator va induktorda dastlabki saqlangan energiya bilan uyg‘otiladi.

Energiya kondensatorning boshlang‘ich kuchlanishi U_0 va induktorning boshlang‘ich toki kuchi I_0 bilan belgilanadi.



8.4-rasm.

Shunday qilib, $t = 0$ da,

$$u(0) = \frac{1}{C}\int_{-\infty}^{0} i\,dt = U_0 \qquad (8.2\text{ a})$$

$$i(0) = I_0 \qquad (8.2\text{ b})$$

8.4-rasmdagi halqaga KVL ni qo‘llaymiz.

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = 0 \qquad (8.3)$$

Integralni yo‘q qilish uchun biz t ga nisbatan differensiallaymiz va hadlarni o‘zgartiramiz.

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \qquad (8.4)$$

i ning boshlang‘ich qiymati (8.2b) tenglamada berilgan. (8.2a) va (8.3) tenglamalardan i hosilasining boshlang‘ich qiymatini olamiz. Ya’ni,

$$Ri(0) + L\frac{di(0)}{dt} + U_0 = 0 \qquad \text{yoki,} \qquad \frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + U_0) \qquad (8.5)$$

(8.2b) va (8.5) tenglamalardagi ikkita boshlang‘ich shart bilan biz (8.4) tenglikni yechamiz. Birinchi tartibli zanjirlar bo‘yicha oldingi mavzudagi tajribamiz shuni ko‘rsatadiki, yechim eksponensial shaklda. Shunday qilib,

$$i = Ae^{st} \qquad (8.6)$$

bu yerda: *A* va *s* aniqlanishi kerak bo‘lgan doimiylar.

Shunday qilib, (8.6) tenglamani (8.4) tenglamaga almashtiramiz.

$$As^2e^{st} + \frac{AR}{L}se^{st} + \frac{A}{LC}e^{st} = 0 \quad \text{yoki,} \quad Ae^{st}(s^2 + \frac{R}{L}s + \frac{1}{LC}) = 0 \qquad (8.7)$$

$i = Ae^{st}$ biz topmoqchi bo‘lgan taxminiy yechim bo‘lgani uchun, faqat qavs ichidagi ifoda nolga teng bo‘lishi mumkin:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad (8.8)$$

Bu kvadrat tenglama (8.4) differensial tenglamaning xarakteristik tenglamasi sifatida tanilgan, chunki tenglamaning ildizlari *i* ning xarakterini belgilaydi. (8.8) tenglamaning ikkita ildizi quyidagicha yoziladi.

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad (8.9\ a) \qquad s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad (8.9\ b)$$

Ildizlarni ifodalashning yanada ixcham usuli.

$$s_{1,2} = -\alpha \pm \sqrt{(\alpha)^2 - \omega_0^2} \quad (8.10)$$

bu yerda:

$$\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}} \quad (8.11)$$

s_1 va s_2 ildizlar sekundiga neperlar (Np/s) bilan o‘lchanadigan tabiiy (natural) chastotalar deb ataladi, 1Np=0,868 bel = 8,68 dB. **Data source:** [8] - https://en.wikipedia.org/wiki/Neper

ω_0 - rezonans chastotasi yoki qat’iy ravishda sekundiga radyanlarda (rad/s) ifodalangan so‘nmagan tabiiy chastota sifatida tanilgan;

α sekundiga neperlarda ifodalangan neper chastotasi yoki so‘ndiruvchi (damping) omili.

α va ω_0 (8.8) tenglamani ko‘rinishida yozish mumkin.

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \rightarrow \qquad s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \rightarrow \qquad s^2 + 2\alpha s + \omega_0^2 = 0 \qquad (8.8\ a)$$

s va ω_0 o‘zgaruvchilar biz mavzuning qolgan qismida muhokama qiladigan muhim miqdorlardir.

(8.10) tenglamadagi s ning ikkita qiymati i uchun ikkita mumkin bo‘lgan yechim mavjudligini ko‘rsatadi, ularning har biri (8.6) tenglamada qabul qilingan yechim shaklida bo‘ladi. Ya’ni,

$$s_{1,2} = -\alpha \pm \sqrt{(\alpha)^2 - \omega_0^2} \rightarrow$$

$$i = Ae^{st} \rightarrow \qquad i_1 = A_1 e^{s_1 t}, \qquad i_2 = A_2 e^{s_2 t} \qquad (8.12)$$

Shunday qilib, ketma-ket ulangan *RLC* zanjirining tabiiy reaksiyasi.

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad (8.13)$$

bu yerda A_1 va A_2 konstantalar tenglamadagi $i(0)$ va $di(0)/dt$ boshlang‘ich qiymatlardan aniqlanadi.

$$i(0) = I_0 \qquad \frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + U_0)$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

α va ω_0 yechimlarning uch turi mavjud:

1. Agar $\alpha > \omega_0$ bo‘lsa, bizda yuqori so‘ndirilgan holat mavjud;
2. Agar $\alpha = \omega_0$ bo‘lsa, bizda kritik so‘ndirilgan holat mavjud;
3. Agar $\alpha < \omega_0$ bo‘sa, bizda quyi so‘ndirilgan holat mavjud.

Ketma-ket ulangan *RLC* zanjirining tabiiy reaksiyasi:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \downarrow$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \downarrow$$

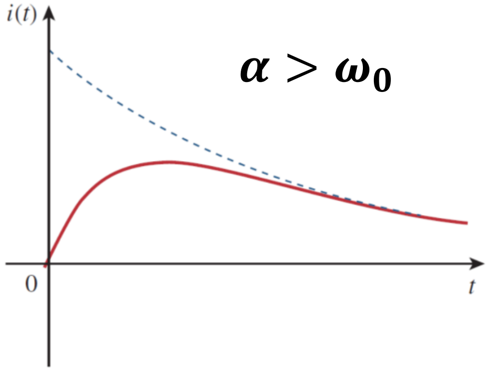
$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

1. Yuqori so'ndirilgan holat ($\alpha > \omega_0$) uchun:

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

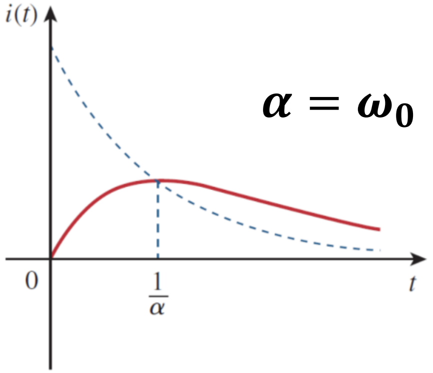
$$s_{1,2} = -\alpha \pm \sqrt{(\alpha^2 - \omega_0^2)}$$



2. Kritik so'ndirilgan holat ($\alpha = \omega_0$) uchun:

$$i(t) = (A_2 + A_1 t)e^{-\alpha t}$$

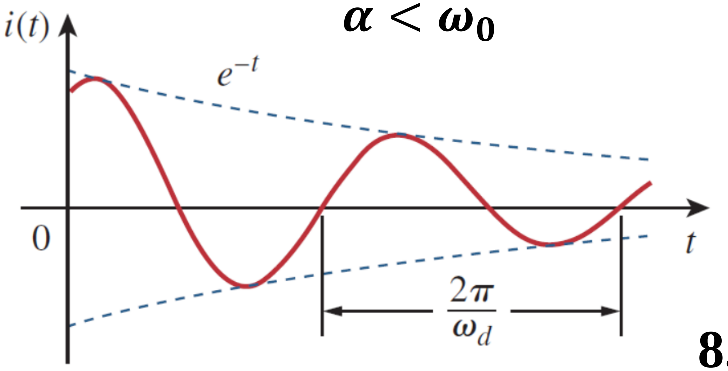
$$s_{1,2} = -\alpha$$



3. Quyi so'ndirilgan holat ($\alpha < \omega_0$) uchun:

$$i(t) = e^{-\alpha t}[B_1 \cos \omega_d t + B_2 \sin \omega_d t]$$

$$j = \sqrt{-1} \text{ va } \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

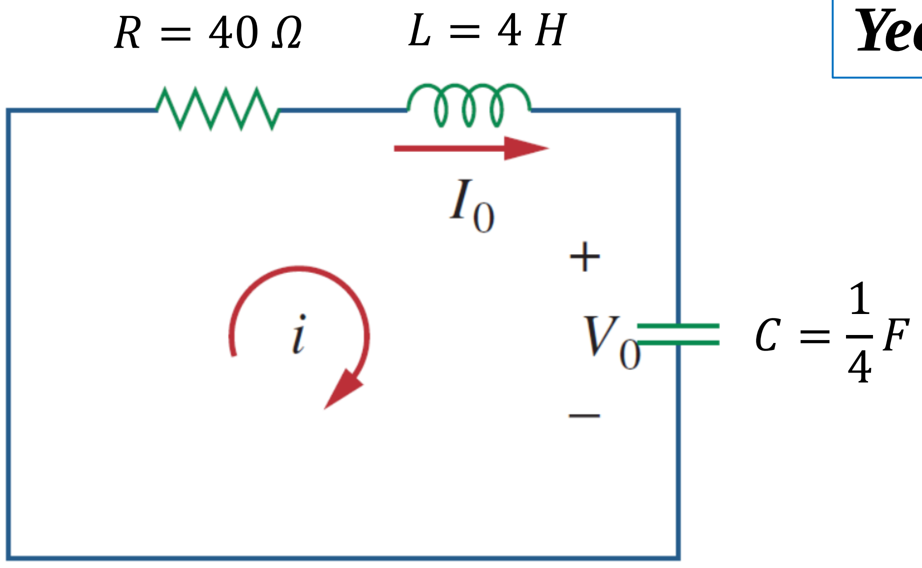


8.5-rasm.

Photo source: [9] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 322.

8.3.1-masala: 8.4-rasmda zanjirning xarakterli ildizlarini hisoblang.

Tabiiy reaksiya yuqori so‘ndirilganmi, quyi so‘ndirilganmi yoki kritik so‘ndirilganmi?



8.4-rasm.

Yechish:

$$\alpha = \frac{R}{2L} = \frac{40}{2(4)} = 5 \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \cdot \frac{1}{4}}} = 1$$

$\boldsymbol{\alpha > \omega_0}$ - reaksiya yuqori so‘ndirilgan

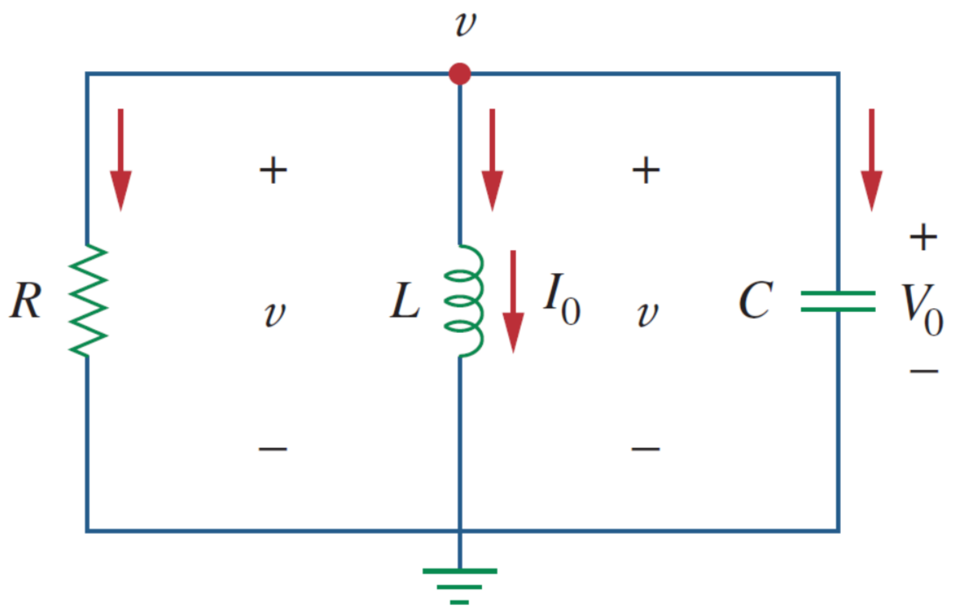
$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{(\alpha)^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1}$$

$$s_1 = -0{,}101, \qquad s_2 = -9{,}899$$

8.4. Manbadan holi parallel ulangan qarshilik, induktor va kondensator (RLC) zanjiri.

Parallel *RLC* zanjirlari ko‘plab amaliy dasturlarda ayniqsa aloqa tarmoqlarida keng qo‘llaniladi.



8.6-rasm. Manbadan holi parallel ulangan *RLC* zanjiri.

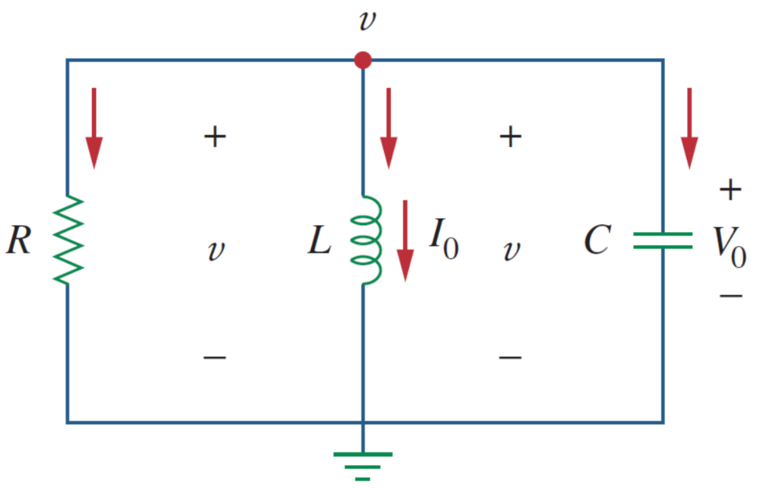
Dastlabki induktor tok kuchi I_0 va dastlabki kondensator kuchlanishi U_0 deb faraz qilaylik,

$$i(0) = I_0 = \frac{1}{L}\int_{\infty}^{0} u(t)\,dt \qquad (8.27\text{ a})$$

$$u(0) = U_0 \qquad (8.27\text{ b})$$

Uchta element parallel bo‘lganligi sababli, ular bo‘ylab bir xil u kuchlanishga ega.

Photo source: [10] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 326.



Passiv belgi konvensiyasiga ko‘ra, tok kuchi har bir elementga kiradi.

Shunday qilib, yuqori tugunda KCLni qo‘llaymiz:

$$i_R + i_L + i_C = 0$$

$$\frac{u}{R} + \frac{1}{L}\int_{-\infty}^{t} u(\tau)\, d\tau + C\frac{du}{dt} = 0 \qquad (8.28)$$

Differensiallash orqali quyidagi ifodaga ega bo‘lamiz.

$$C\frac{d^2u}{dt^2} + \frac{1}{R}\frac{du}{dt} + \frac{1}{L}u = 0 \qquad (8.29)$$

Xarakteristik tenglamani birinchi hosilani s ga, ikkinchi hosilani s^2 ga almashtirib olamiz.

$$u = Ae^{st}, \qquad CAs^2e^{st} + \frac{1}{R}Ase^{st} + \frac{1}{L}Ae^{st} = 0, \; Ae^{st}(Cs^2 + \frac{1}{R}s + \frac{1}{L}) = 0$$

$$Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0 \qquad (8.29a)$$

Hosil bo‘lgan (8.29a) tenglamani C ga bo‘lish natijasida quyidagi tenglama hosil bo‘ladi.

$$Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0 \;/\; C \rightarrow \qquad s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \tag{8.30}$$

Xarakteristik tenglamaning ildizlari,

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad \text{yoki,} \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \tag{8.31}$$

bu yerda:

$$\alpha = \frac{1}{2RC}, \qquad \omega_0 = \frac{1}{\sqrt{LC}} \tag{8.32}$$

Parallel ulangan *RLC* zanjirining tabiiy reaksiyasi:

1. Yuqori so‘ndirilgan holat ($\alpha > \omega_0$) uchun:

$$u(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{(\alpha^2 - \omega_0^2)}$$

2. Kritik so‘ndirilgan holat ($\alpha = \omega_0$) uchun:

$$u(t) = (A_1 + A_2 t)e^{-\alpha t}$$

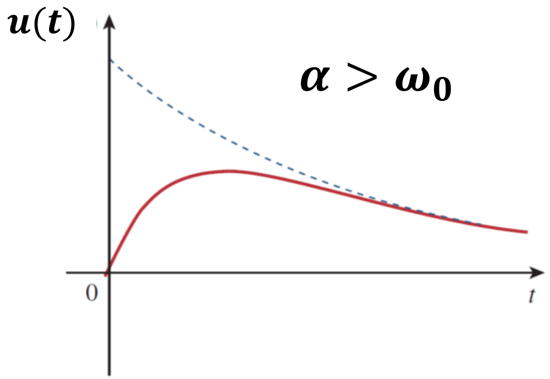
$$s_{1,2} = -\alpha$$

3. Quyi so‘ndirilgan holat ($\alpha < \omega_0$) uchun:

$$u(t) = e^{-\alpha t}[A_1 \cos \omega_d t + A_2 \sin \omega_d t]$$

$$s_{1,2} = -\alpha \pm j\omega_d$$

$$j = \sqrt{-1} \text{ va } \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

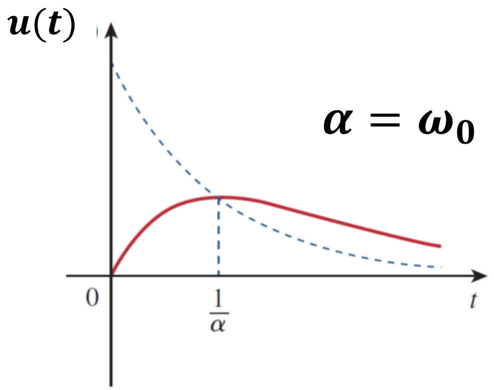


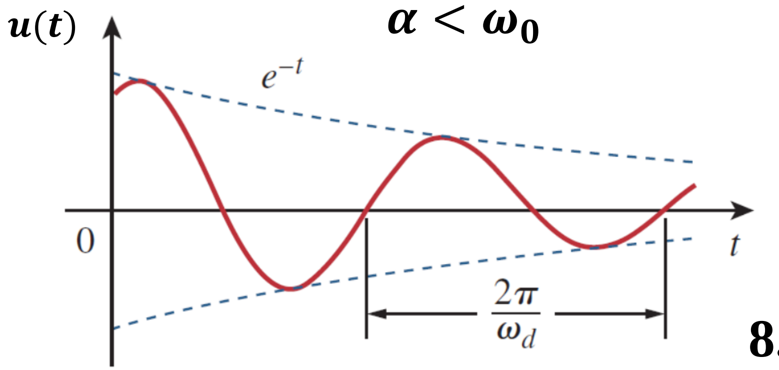
$$\frac{d^2u}{dt^2} + \frac{1}{RC}\frac{du}{dt} + \frac{1}{LC}u = 0 \downarrow$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \downarrow$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$



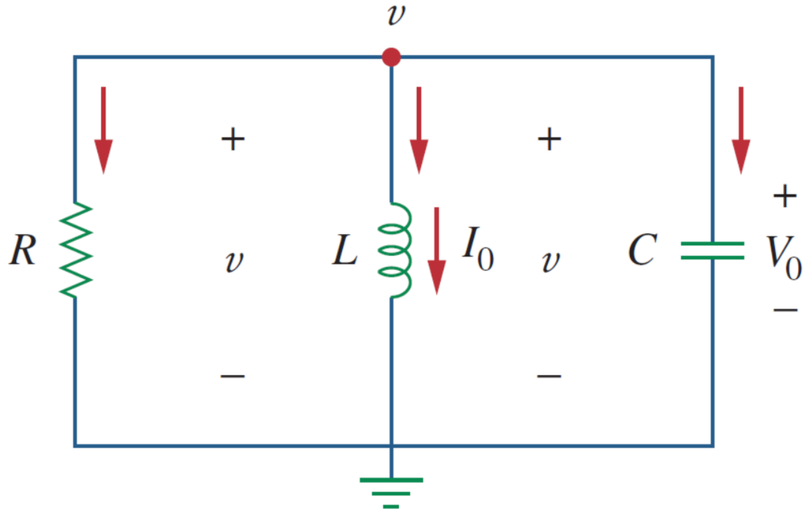


8.7-rasm.

8.4.1-masala: 8.6-rasmning parallel zanjiridagi $t > 0$ uchun $u(t)$ toping.

$u(0) = 5\ V$, $i(0) = 0\ A$, $L = 1\ H$ va $C = 10\ mF$ deb faraz qiling.

$\boldsymbol{R = 1,923\ \Omega}$, $R = 5\ \Omega$ va $R = 6{,}25\ \Omega$ holatlarni ko‘rib chiqing.



8.6-rasm.

Yechish:

1-holat uchun $\boldsymbol{R = 1,923\ \Omega}$ bo‘lganda.

1. $t < 0$, $u(0)$ va $i(0)$.

$u(0) = 5\ V$, $i(0) = 0\ A$

2. $\frac{du(0)}{dt}$ ni KCL qo ‘llanadi.

$i_R(t) + i(t) + i_C(t) = 0;$

$i_R(0) + i(0) + i_C(0) = 0;$ $\frac{U_0}{R} + I_0 + C\frac{du(0)}{dt} = 0;$ $\frac{5}{1{,}923} + 0 + 10\ mF\frac{du(0)}{dt} = 0;$

$10\ mF\frac{du(0)}{dt} = -2{,}6;$ $\frac{du(0)}{dt} = -260\frac{V}{s}.$

3. α, ω_0

$\alpha = \frac{1}{2RC} = \frac{1}{2\cdot1{,}923\cdot10\cdot10^{-3}} = 26;$ $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\cdot10\cdot10^{-3}}} = 10;$

$\alpha > \omega_0$ bu holda reaksiya yuqori so‘ndirilgan.

4. $u(t)$=?, $s_{1,2}$ yoki ω_d v.h.k.

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2};$

$s_{1,2} = -26 \pm \sqrt{26^2 - 10^2} = -2;\ -50$

$u(t) = A_1e^{-2t} + A_2e^{-50t}$

5. A_1 va A_2 =?

$t = 0\ \downarrow$

$u(0) = A_1 + A_2$

$\boldsymbol{5 = A_1 + A_2}$

$u(t) = A_1e^{-2t} + A_2e^{-50t}$ tenglamani differensiallaymiz.

$\frac{du}{dt} = -2A_1e^{-2t} - 50A_2e^{-50t};$

$t = 0\ \downarrow$

$-260 = -2A_1 - 50A_2$

$\begin{cases} 5 = A_1 + A_2 & /(\cdot 2) \\ -260 = -2A_1 - 50A_2 \end{cases} \rightarrow$

$+\begin{cases} 10 = 2A_1 + 2A_2 \\ -260 = -2A_1 - 50A_2 \end{cases}$

$-250 = -48A_2\ \downarrow$

$\boldsymbol{A_2 = 5,208}$

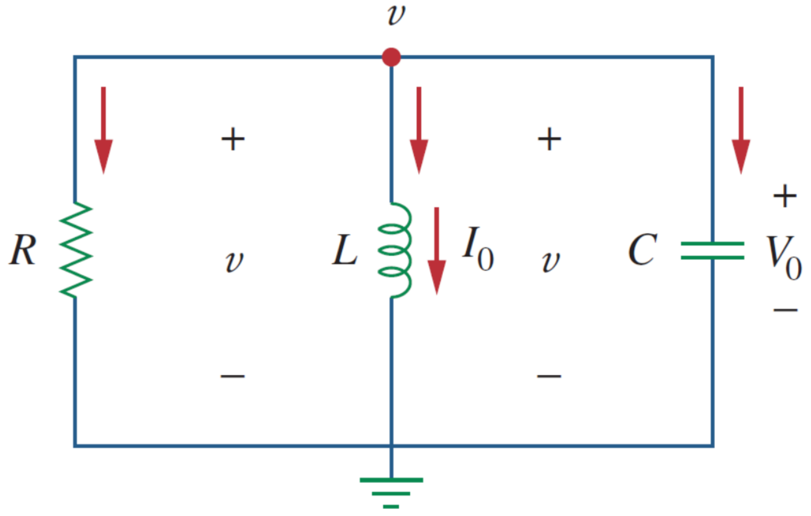
$\boldsymbol{A_1 = -0,2083}$

$\boldsymbol{u(t) = -0,2083e^{-2t} + 5,208e^{-50t}\ V}$

8.4.1-masala: 8.6-rasmning parallel zanjiridagi $t > 0$ uchun $u(t)$ toping.

$u(0) = 5\ V$, $i(0) = 0\ A$, $L = 1\ H$ va $C = 10\ mF$ deb faraz qiling.

$R = 1{,}923\ \Omega$, $\boldsymbol{R = 5\ \Omega}$ va $R = 6{,}25\ \Omega$ holatlarni ko‘rib chiqing.



8.6-rasm.

Yechish:

1-holat uchun $\boldsymbol{R = 5\ \Omega}$ bo‘lganda.

1. $t < 0$, $u(0)$ va $i(0)$.

$u(0) = 5\ V,\ i(0) = 0\ A$

2. $\frac{du(0)}{dt}$ ni KCL qo ‘llanadi.

$i_R(t) + i(t) + i_C(t) = 0;$

$i_R(0) + i(0) + i_C(0) = 0;\quad \frac{U_0}{R} + I_0 + C\frac{du(0)}{dt} = 0;\quad \frac{5}{5} + 0 + 10\ mF\frac{du(0)}{dt} = 0;$

$10\ mF\frac{du(0)}{dt} = -1;\quad \frac{du(0)}{dt} = -100\frac{V}{s}.$

3. α, ω_0

$\alpha = \frac{1}{2RC} = \frac{1}{2\cdot 5\cdot 10\cdot 10^{-3}} = 10;\quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\cdot 10\cdot 10^{-3}}} = 10;$

$\alpha = \omega_0$ bu holda reaksiya kritik so‘ndirilgan.

4. $u(t)$=?, $s_{1,2}$ yoki ω_d v.h.k.

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2};$

$\alpha = \omega_0 = 10;\quad s_1 = s_2 = -10;$

$u(t) = (A_1 + A_2 t)e^{-\alpha t}$

5. A_1 va A_2 =?

$u(t) = (A_1 + A_2 t)e^{-10t};$

$t = 0\ \downarrow$

$u(0) = A_1$

$\boldsymbol{5 = A_1}$

$u(t) = (A_1 + A_2 t)e^{-10t}$ tenglamani differensiallaymiz.

$\frac{du}{dt} = e^{-10t}(A_2) + (A_1 + A_2 t)(-10)e^{-10t} =$

$= (-10A_1 - 10A_2 t + A_2)e^{-10t}$

$t = 0\ \downarrow$

$-100 = -10A_1 + A_2$

$\begin{cases} 5 = A_1 & /(\cdot 10) \\ -100 = -10A_1 + A_2 \end{cases} \rightarrow$

$+\begin{cases} 50 = 10A_1 \\ -100 = -10A_1 + A_2 \end{cases}$

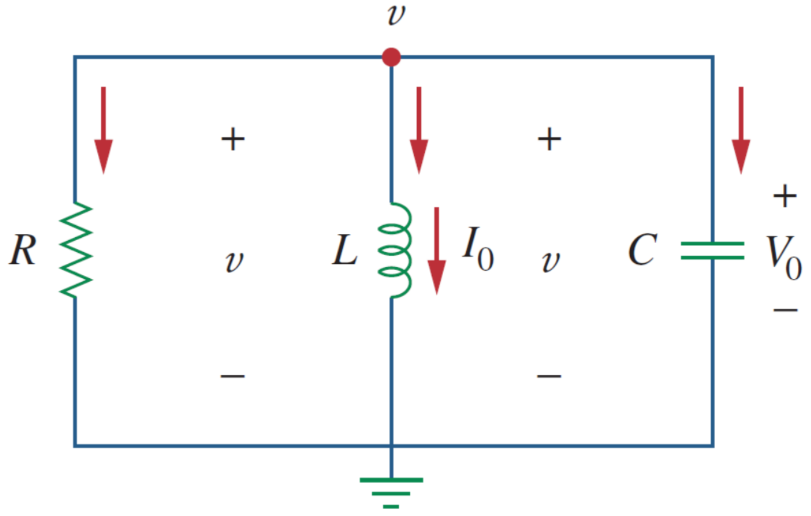
$\boldsymbol{A_2 = -50}$

$\boldsymbol{u(t) = (5 - 50t)e^{-10t}\ V}$

8.4.1-masala: 8.6-rasmning parallel zanjiridagi $t > 0$ uchun $u(t)$ toping.

$u(0) = 5\ V$, $i(0) = 0\ A$, $L = 1\ H$ va $C = 10\ mF$ deb faraz qiling.

$R = 1{,}923\ \Omega$, $\boldsymbol{R = 5\ \Omega}$ va $\boldsymbol{R = 6,25\ \Omega}$ holatlarni ko‘rib chiqing.



8.6-rasm.

Yechish:

1-holat uchun $\boldsymbol{R = 6,25\ \Omega}$ bo‘lganda.

1. $t < 0$, $u(0)$ va $i(0)$.

$u(0) = 5\ V,\ i(0) = 0\ A$

2. $\frac{du(0)}{dt}$ ni KCL qo ‘llanadi.

$i_R(t) + i(t) + i_C(t) = 0;$

$i_R(0) + i(0) + i_C(0) = 0;\quad \frac{U_0}{R} + I_0 + C\frac{du(0)}{dt} = 0;\quad \frac{5}{6{,}25} + 0 + 10\ mF\frac{du(0)}{dt} = 0;$

$10\ mF\frac{du(0)}{dt} = -0{,}8;\quad \frac{du(0)}{dt} = -80\frac{V}{s}.$

3. $\alpha,\ \omega_0$

$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 6{,}25 \cdot 10 \cdot 10^{-3}} = 8;\quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \cdot 10 \cdot 10^{-3}}} = 10;$

$\alpha = \omega_0$ bu holda reaksiya quyi so‘ndirilgan.

4. $u(t)$=?, $s_{1,2}$ yoki ω_d v.h.k.

$s_{1,2} = -\alpha \pm j\omega_d = -8 \pm j6;$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10^2 - 8^2 = 6};$

$u(t) = e^{-\alpha t}[A_1 \cos \omega_d t + A_2 \sin \omega_d t]$

5. A_1 va A_2 =?

$u(t) = e^{-8t}[A_1 \cos 6t + A_2 \sin 6t]$

$t = 0 \downarrow$

$u(0) = A_1$

$\boldsymbol{5 = A_1}$

$u(t) = e^{-8t}[A_1 \cos 6t + A_2 \sin 6t]$ tenglamani differensiallaymiz.

$\frac{du}{dt} = (-8A_1 \cos 6t - 8A_2 \sin 6t - 6A_1 \sin 6t + 6A_2 \cos 6t)e^{-8t}$

$t = 0 \downarrow$

$-80 = -8A_1 + 6A_2$

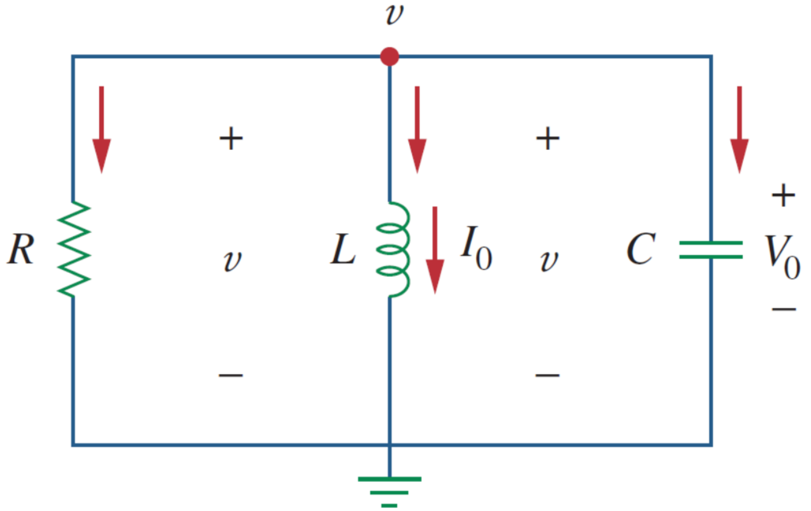
$\begin{cases} 5 = A_1 \quad /(\cdot 8) \\ -80 = -8A_1 + 6A_2 \end{cases} \rightarrow$

$+\begin{cases} 40 = 8A_1 \\ -80 = -8A_1 + 6A_2 \end{cases}$

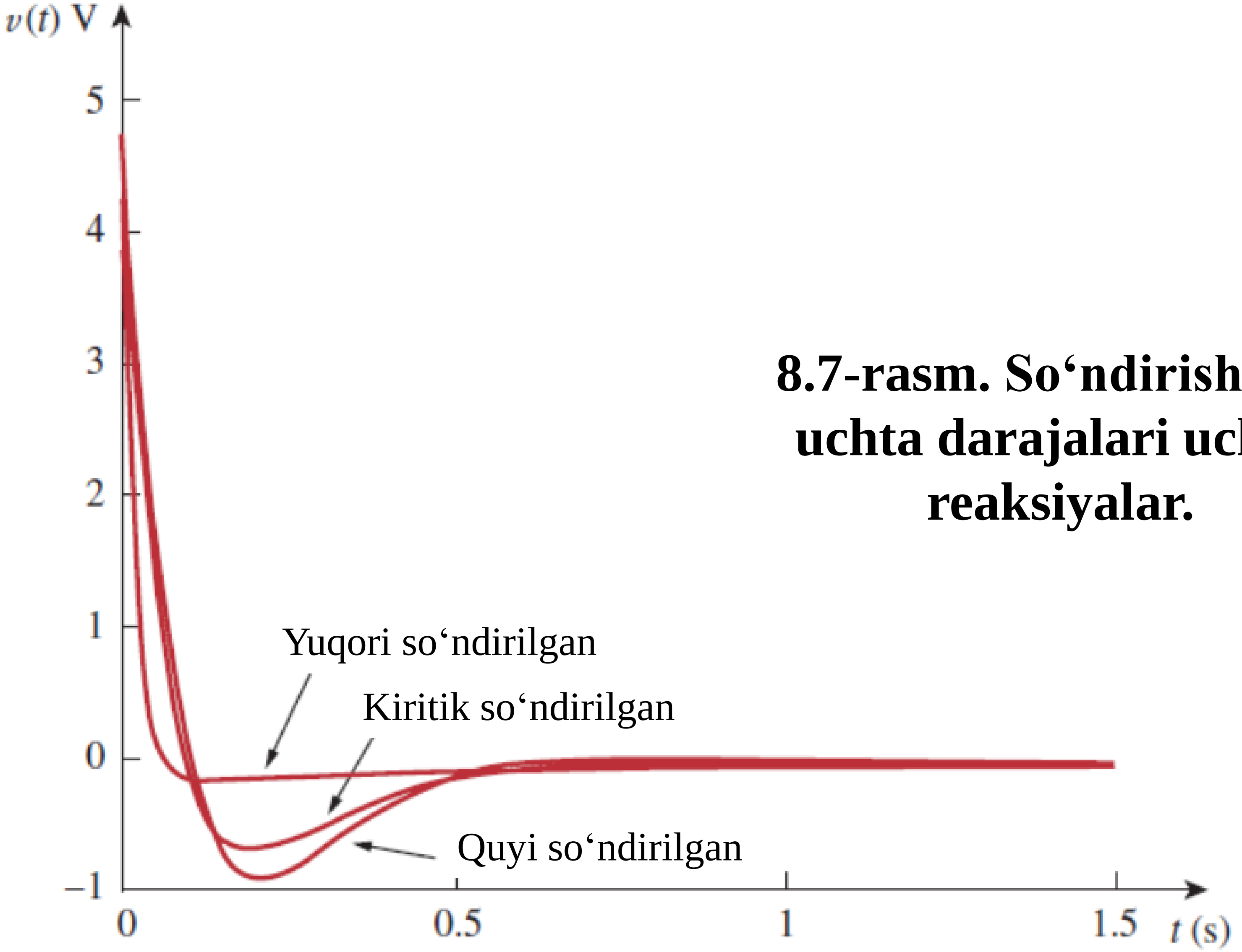
$\boldsymbol{A_2 = 6{,}67}$

$\boldsymbol{u(t) = (5\cos 6t - 6,667\ \sin 6t)e^{-8t}\ V}$

R qiymatini oshirib, so‘ndirish darajasi pasayadi va reaksiyalar farqlanadi.



8.6-rasm.



8.7-rasm. So‘ndirishning uchta darajalari uchun reaksiyalar.

Photo source: [11] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 329.

FOYDALANILGAN MANBALAR:

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9. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 322.
10. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 326.
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E'TIBORINGIZ UCHUN RAHMAT!!!