8-Mavzu: Ikkinchi tartibli elektr zanjiri *(davomi)*.

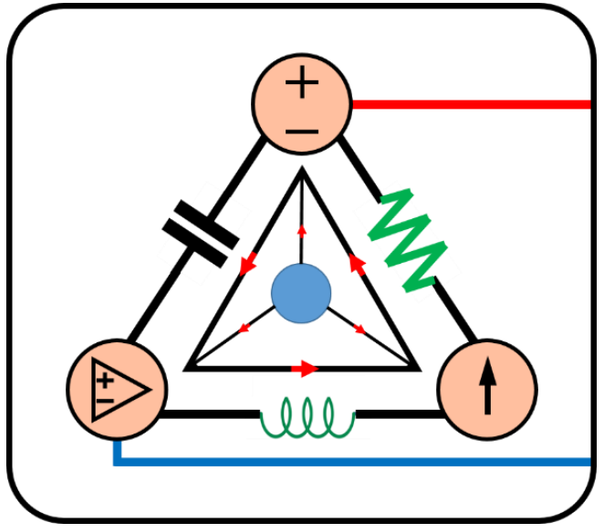
(8th Topic: Second-Order Circuit)

8-Mavzuning 3-qismi

(3rd part of the 8th Topic)

10-hafta uchun

For the 10th week



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8-Mavzu: Ikkinchi tartibli elektr zanjiri *(davomi)*.

(8th Topic: Second-Order Circuit)

O‘quv rejasi:

8.5. Ketma-ket ulangan qarshilik, induktor va kondensator (*RLC*) zanjirining qadam reaksiyasi.

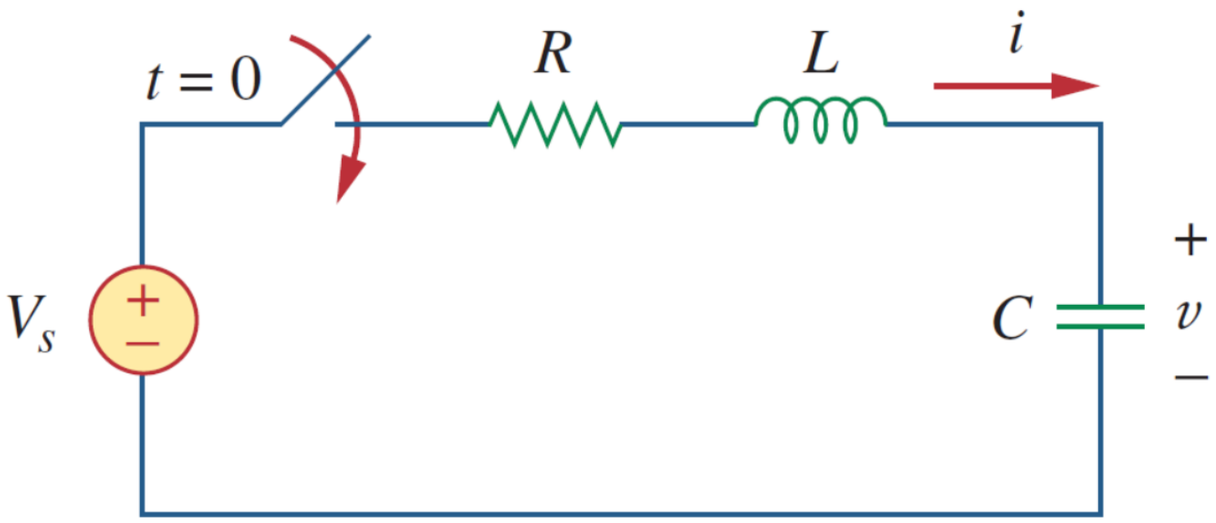
8.6. Parallel ulangan qarshilik, induktor va kondensator (*RLC*) zanjirining qadam reaksiyasi.

8.7. Umumiy ikkinchi tartibli zanjirlar.

8.8. Ikkinchi tartibli operatsion kuchaytirgich zanjirlari.

8.9. Qo‘llanilishi.

8.5. Ketma-ket ulangan qarshilik, induktor va kondensator (*RLC*) zanjirining qadam reaksiyasi.



8.8-rasm. Ketma-ket ulangan *RLC* zanjiriga bosqichli kuchlanishni qo‘llash.

Photo source: [12] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 331.

Qadamli reaksiya o‘zgarmas tok manbasini birdan qo‘llash orqali olinadi.

$t > 0$ uchun halqa bo‘ylab KVL qo‘llanilsa,

$$L\frac{di}{dt} + Ri + u = U_s \qquad (8.39)$$

$$i = C\frac{du}{dt}$$

(8.39) tenglamadagi i ni o‘rniga qaytadan almashtiramiz.

$$\frac{d^2u}{dt^2} + \frac{R}{L}\frac{du}{dt} + \frac{u}{LC} = \frac{U_s}{LC} \qquad (8.40)$$

$$\frac{d^2u}{dt^2} + \frac{R}{L}\frac{du}{dt} + \frac{u}{LC} = \frac{U_s}{LC} \qquad (8.40) \qquad = \qquad \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \qquad (8.4)$$

Koeffitsientlar bir xil (*va bu chastota parametrlarini aniqlashda muhim*), lekin o‘zgaruvchi boshqacha.

Demak, *RLC* ketma-ket ulangan zanjir uchun xarakteristik tenglamaga doimiy tok manbai mavjudligi ta’sir qilmaydi.

(8.40) tenglamaning yechimi ikkita komponentdan iborat: vaqtinchalik reaksiya $u_t(t)$ va barqaror holatdagi reaksiya $u_{ss}(t)$; ya’ni,

$$u(t) = u_t(t) + u_{ss}(t) \qquad (8.41)$$

Vaqtinchalik reaksiya $u_t(t)$ vaqt o‘tishi bilan o‘chib ketadigan umumiy reaksiyaning tarkibiy qismidir.

Vaqtinchalik reaksiya $u_t(t)$ yuqori so‘ndirilgan, quyi so‘ndirilgan va kritik so‘ndirilgan holatlar uchun:

$$u(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(Yuqori so‘ndirilgan)} \quad (8.42a)$$

$$u(t) = (A_1 + A_2 t) e^{-\alpha t} \quad \text{(Kritik so‘ndirilgan)} \quad (8.42b)$$

$$u(t) = (A_1 cos\omega_d t + A_2 sin\omega_d t) e^{-\alpha t} \quad \text{(Quyi so‘ndirilgan)} \quad (8.42c)$$

Stabil holatdagi reaksiya $u(t)$ ning yakuniy qiymati hisoblanadi.

8.8-rasmdagi zanjirda kondensator kuchlanishining yakuniy qiymati manba kuchlanishi U_S bilan bir xil.

$$u_{ss}(t) = u(\infty) = U_S \quad (8.43)$$

Shunday qilib, yuqori, quyi va kritik so‘ndirilgan holatlar uchun to‘liq yechimlar sifatida quyidagilarni keltirishimiz mumkin:

$$u(t) = \boldsymbol{U_S} + A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(Yuqori so‘ndirilgan)} \qquad (8.44a)$$

$$u(t) = \boldsymbol{U_S} + (A_1 + A_2 t) e^{-\alpha t} \qquad \text{(Kritik so‘ndirilgan)} \qquad (8.44b)$$

$$u(t) = \boldsymbol{U_S} + (A_1 cos\omega_d t + A_2 sin\omega_d t) e^{-\alpha t} \qquad \text{(Quyi so‘ndirilgan)} \qquad (8.44c)$$

A_1 va A_2 konstantalarining qiymatlari dastlabki shartlardan olinadi: $u(0)$ va $du(0)/dt$.

u va i mos ravishda kondensatordagi kuchlanish va induktor orqali o‘tadigan tok kuchidir.

Shuning uchun (8.44) tenglama faqat u ni topish uchun qo‘llaniladi.

Lekin kondensator kuchlanishi $u_C = u$ ma’lum bo‘lgandan so‘ng, kondensator, induktor va qarshilik orqali bir xil tok bo‘lgan $i = C\ du/dt$ ni aniqlashimiz mumkin.

Demak, rezistordagi kuchlanish $u_R = iR$, induktiv kuchlanish esa $u_L = L\ di/dt$ ga teng.

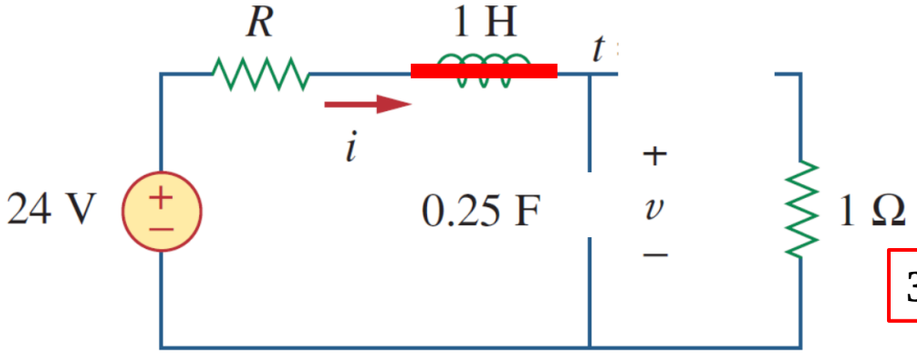
Shu bilan bir qatorda, har qanday $x(t)$ o‘zgaruvchisi uchun to‘liq reaksiyani bevosita topish mumkin, chunki u umumiy shaklga ega.

$$x(t) = x_{ss}(t) + x_t(t) \qquad (8.45)$$

bu yerda: $x_{ss} = x(\infty)$ yakuniy qiymat va $x_t(t)$ vaqtinchalik reaksiyadir.

8.5.1-masala: 8.9-rasmdagi zanjir uchun $t > 0$ bo‘lganda $\boldsymbol{u(t)}$ va $\boldsymbol{i(t)}$ toping.

$\boldsymbol{R = 5\ \Omega}$, $R = 4\ \Omega$ va $R = 1\ \Omega$ holatlarni ko‘rib chiqing.



8.9-rasm.

Yechish:

1-holat uchun $\boldsymbol{R = 5\ \Omega}$ bo‘lganda.

1. $t < 0$, $u(0)$ va $i(0)$.

$$i(0) = \frac{24}{5+1} = 4\ A$$

$$u(0) = i(0) \cdot 1 = 4\ V$$

2. $t > 0$, $\frac{du(0)}{dt}$.

$$C\frac{du_C}{dt} = i_C = i(0);$$

$$\frac{du_C(0)}{dt} = \frac{i(0)}{C} = \frac{4}{0{,}25} = 16\frac{V}{s}.$$

3. $t > 0$, α, ω_0

$$\alpha = \frac{R}{2L} = \frac{5}{2\cdot 1} = 2{,}5;$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\cdot 0{,}25}} = 2.$$

$\alpha > \omega_0$ zanjirning tabiiy reaksiyasi yuqori so‘ndirilgan.

4. $u(t)$=?, $s_{1,2}$ yoki ω_d v.h.k.

$$\boldsymbol{u(t) = U_S + A_1 e^{s_1 t} + A_2 e^{s_2 t};}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1;\ -4.$$

5. A_1 va A_2 =?

$$u(t) = 24 + A_1 e^{-1t} + A_2 e^{-4t};$$

$$t = 0 \rightarrow u(0) = 24 + A_1 + A_2;$$

$$4 = 24 + A_1 + A_2;$$

$$\boldsymbol{-20 = A_1 + A_2.}$$

$$u(t) = 24 + A_1 e^{-1t} + A_2 e^{-4t};$$

tenglamani differensiallaymiz.

$$\frac{du(t)}{dt} = -A_1 e^{-t} - 4A_2 e^{-4t}$$

$$\frac{du(0)}{dt} = -A_1 e^{-t} - 4A_2 e^{-4t}$$

$$t = 0 \downarrow$$

$$\boldsymbol{16 = -A_1 - 4A_2}$$

$$+\begin{cases} \boldsymbol{-20 = A_1 + A_2} \\ \boldsymbol{16 = -A_1 - 4A_2} \end{cases}$$

$$A_1 = -\frac{64}{3} \text{ va } A_2 = \frac{4}{3}$$

$$\boldsymbol{u(t) = 24 + \frac{4}{3}(-16e^{-1t} + e^{-4t}), V}$$

$$t > 0, i(t) = i_C(t) = C\frac{du}{dt}$$

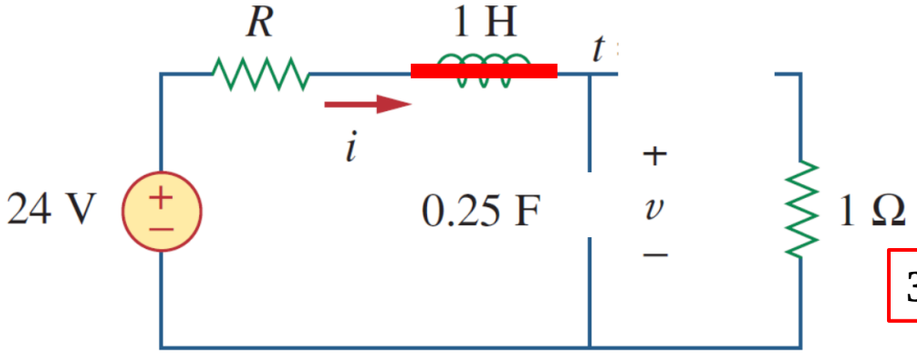
$$\boldsymbol{i(t)} = 0{,}25\frac{d}{dt}[24 + \frac{4}{3}(-16e^{-1t} + e^{-4t}0] =$$

$$= 0{,}25[\frac{4}{3}(16e^{-1t} - 4e^{-4t})]$$

$$\boldsymbol{i(t) = \frac{4}{3}\left(4e^{-1t} - e^{-4t}\right), A}$$

8.5.1-masala: 8.9-rasmdagi zanjir uchun $t > 0$ bo‘lganda $\boldsymbol{u(t)}$ va $\boldsymbol{i(t)}$ toping.

$R = 5\ \Omega$, $\boldsymbol{R = 4\ \Omega}$ va $R = 1\ \Omega$ holatlarni ko‘rib chiqing.



8.9-rasm.

Yechish:

1-holat uchun $\boldsymbol{R = 4\ \Omega}$ bo‘lganda.

1. $t < 0$, $u(0)$ va $i(0)$.

$$i(0) = \frac{24}{4+1} = 4{,}8\ A$$

$$u(0) = i(0) \cdot 1 = 4{,}8\ V$$

2. $t > 0$, $\frac{du(0)}{dt}$.

$$C\frac{du_C}{dt} = i_C = i(0);$$

$$\frac{du_C(0)}{dt} = \frac{i(0)}{C} = \frac{4{,}8}{0{,}25} = 19{,}2\frac{V}{s}.$$

3. $t > 0$, α, ω_0

$$\alpha = \frac{R}{2L} = \frac{4}{2\cdot 1} = 2;$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\cdot 0{,}25}} = 2.$$

$\alpha = \omega_0$ zanjirning tabiiy reaksiyasi kiritik so‘ndirilgan.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2.$$

4. $u(t)$=?, $s_{1,2}$ yoki ω_d v.h.k.

$$\boldsymbol{u(t) = U_S + (A_1 + A_2 t)e^{-\alpha t}};$$

$$u(t) = 24 + (A_1 + A_2 t)e^{-2t};$$

tenglamani differensiallaymiz.

$$\frac{du(t)}{dt} = (A_1 + A_2 t)\frac{d}{dt}e^{-2t} + e^{-2t}\frac{d}{dt}(A_1 + A_2 t) = (-2A_1 - 2tA_2 + A_2)e^{-2t}$$

$$t = 0 \rightarrow \frac{du(0)}{dt} = -2A_1 + A_2;$$

5. A_1 va A_2 =?

$$u(t) = 24 + (A_1 + A_2 t)e^{-2t};$$

$$t = 0 \rightarrow u(0) = 24 + A_1;$$

$$4{,}8 = 24 + A_1;$$

$$\boldsymbol{-19{,}2 = A_1.}$$

$$\boldsymbol{19{,}2 = -2A_1 + A_2;}$$

$$\boldsymbol{A_1 = -19{,}2;}$$

$$\boldsymbol{A_2 = -19{,}2.}$$

$$\boldsymbol{u(t) = 24 - 19{,}2(1 + t)e^{-2t}, V}$$

$t > 0, i(t) = i_C(t) = C\frac{du}{dt}$

$$\boldsymbol{i(t)} = 0{,}25\frac{d}{dt}[24 - 19{,}2(1 + t)e^{-2t}] =$$

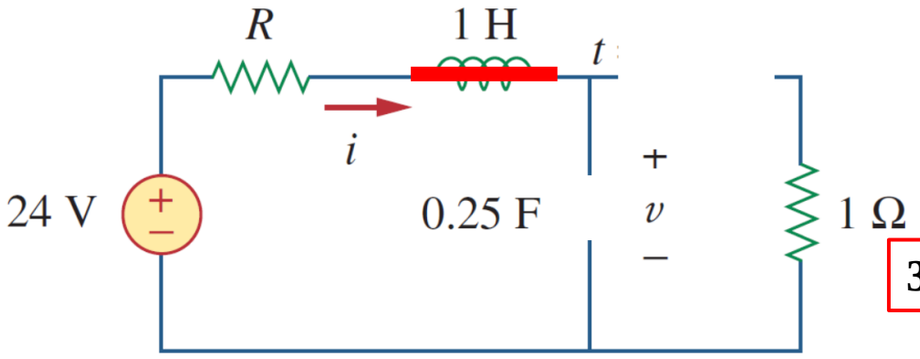
$$= 0{,}25[-19{,}2(1 + t)\frac{d}{dt}e^{-2t} + e^{-2t}\frac{d}{dt}(-19{,}2(1 + t)] =$$

$$= 0{,}25[-19{,}2(1 + t)(-2)e^{-2t} + e^{-2t}(-19{,}2)]$$

$$\boldsymbol{i(t) = (4{,}8 + 9{,}6t)e^{-2t}, A}$$

8.5.1-masala: 8.9-rasmdagi zanjir uchun $t > 0$ bo‘lganda $\boldsymbol{u(t)}$ va $\boldsymbol{i(t)}$ toping.

$R = 5\ \Omega$, $R = 4\ \Omega$ va $\boldsymbol{R = 1\ \Omega}$ holatlarni ko‘rib chiqing.



8.9-rasm.

Yechish:

1-holat uchun $\boldsymbol{R = 1\ \Omega}$ bo‘lganda.

1. $t < 0$, $u(0)$ va $i(0)$.

$$i(0) = \frac{24}{1+1} = 12\ A$$

$$u(0) = i(0) \cdot 1 = 12\ V$$

2. $t > 0$, $\frac{du(0)}{dt}$.

$$C\frac{du_C}{dt} = i_C = i(0);$$

$$\frac{du_C(0)}{dt} = \frac{i(0)}{C} = \frac{12}{0{,}25} = 48\frac{V}{s}.$$

3. $t > 0$, α, ω_0

$$\alpha = \frac{R}{2L} = \frac{1}{2\cdot 1} = 0{,}5;$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\cdot 0{,}25}} = 2.$$

$\alpha < \omega_0$ zanjirning tabiiy reaksiyasi quyi so‘ndirilgan.

4. $u(t)$=?, $s_{1,2}$ yoki ω_d v.h.k.

$$\boldsymbol{u(t) = U_S + (A_1 cos\omega_d t + A_2 sin\omega_d t)e^{-\alpha t}};$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{2^2 - 0{,}5^2} = 1{,}936;$$

$$t > 0, i(t) = i_C(t) = C\frac{du}{dt}$$

5. A_1 va A_2 =?

$t = 0 \downarrow$

$$u(t) = 24 + (A_1 cos1{,}936t + A_2 sin1{,}936t)e^{-0{,}5t};$$

$$t = 0 \rightarrow u(0) = 24 + A_1 \rightarrow 12 = 24 + A_1 \rightarrow$$

$$\boldsymbol{A_1 = -12.}$$

$$u(t) = 24 + (A_1 cos1{,}936t + A_2 sin1{,}936t)e^{-0{,}5t};$$

tenglamani differensiallaymiz.

$$\frac{du(t)}{dt} = e^{-0{,}5t}(-1{,}936A_1 sin1{,}936t + 1{,}936cos1{,}936t) - 0{,}5e^{-0{,}5t}(A_1 cos1{,}936t + A_2 sin1{,}936t)$$

$$\frac{du(0)}{dt} = (-0 + 1{,}936A_2) - 0{,}5(A_1 + 0);$$

$$48 = 1{,}936A_2 - 0{,}5A_1;$$

$$\boldsymbol{A_2 = 21{,}694;}$$

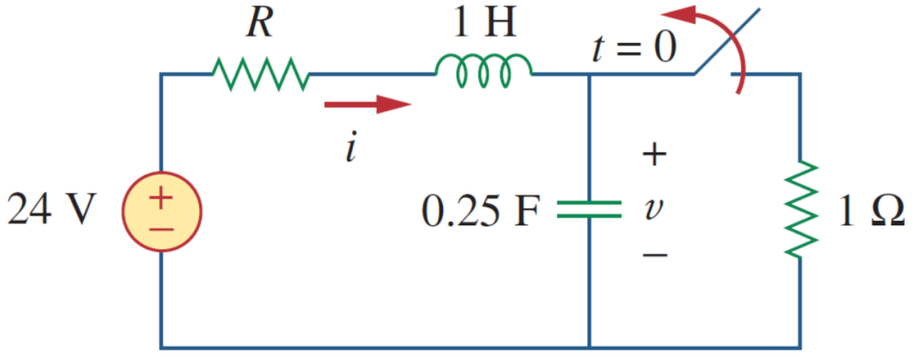
$$\boldsymbol{u(t) = 24 + (21{,}694 sin1{,}936t - -12 cos1{,}936t)e^{-0{,}5t}\ V;}$$

$$\boldsymbol{i(t)} = 0{,}25\frac{d}{dt}[24 + (21{,}694sin1{,}936t - -12cos1{,}936t)e^{-0{,}5t} - 19{,}2(1+t)e^{-2t}]$$

$$\boldsymbol{i(t) = (3{,}1 sin1{,}936t + +12 cos1{,}936t)e^{-0{,}5t}, A}$$

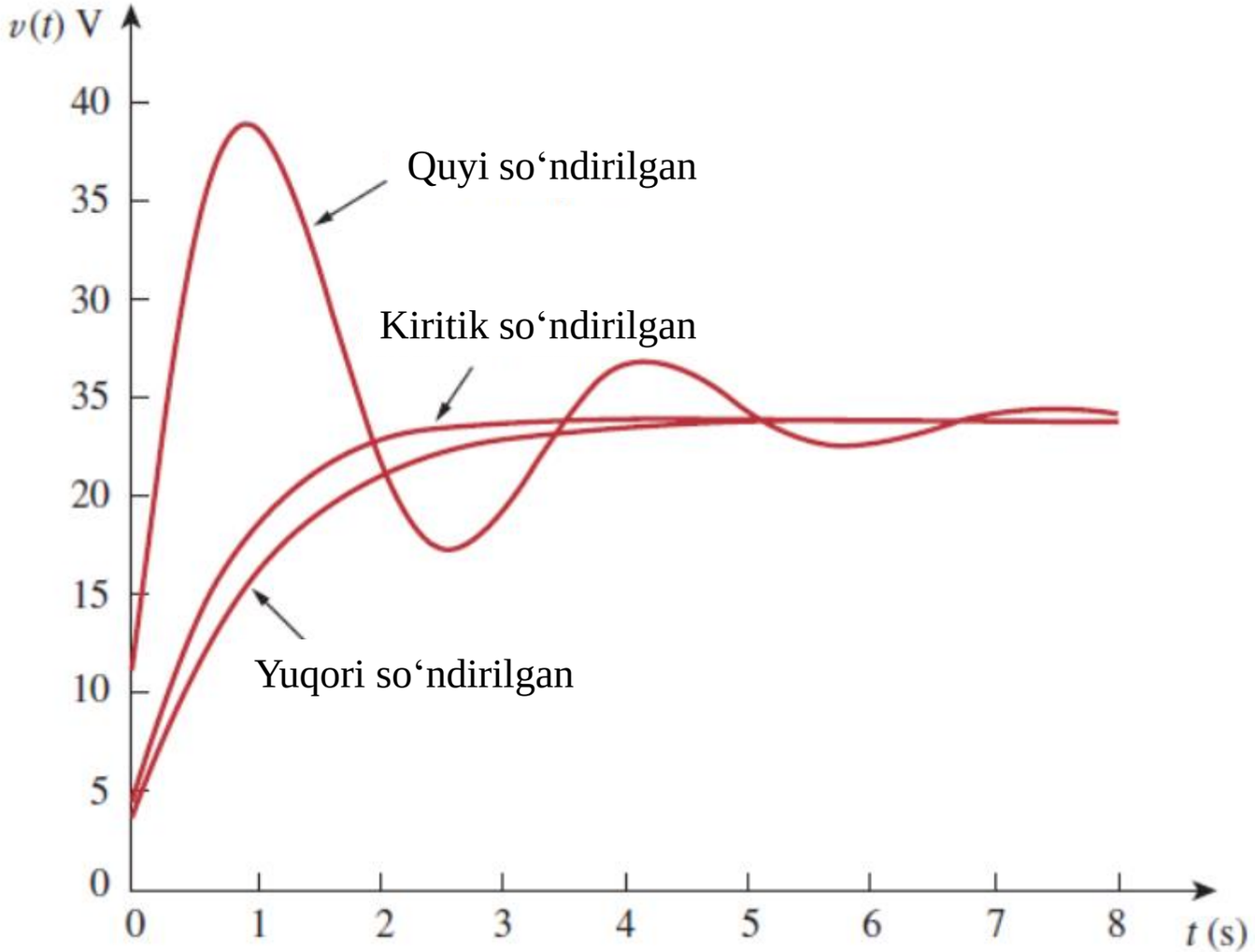
Kutilganidek,
$i(0) = 12\ A$ ekanligi ma’lum bo‘ldi.

R qiymatini oshirib, so‘ndirish darajasi pasayadi va reaksiyalar farqlanadi.



8.9-rasm.

Ushbu rasmdan biz kritik so‘ndirilgan reaksiya 24 V ning qadam kiritishiga eng tez yaqinlashishini kuzatamiz.



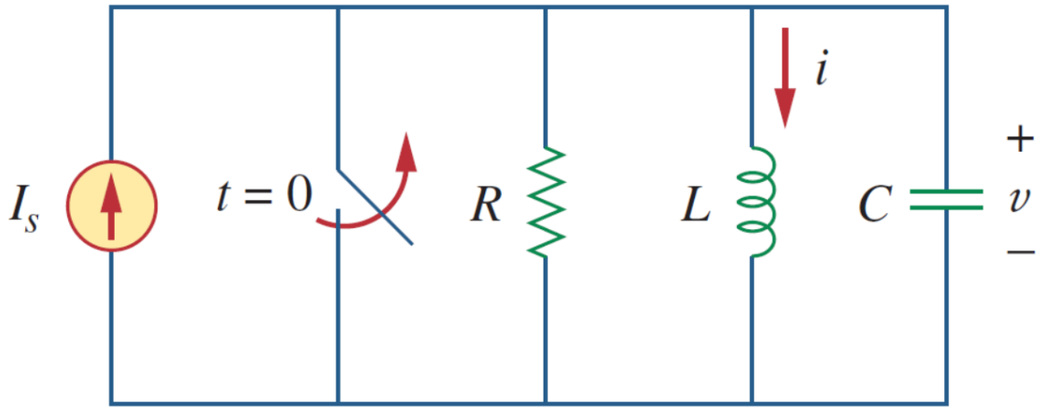
8.10-rasm. So‘ndirishning uchta darajalari uchun reaksiyalar.

Photo source: [13] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 336.

8.6. Parallel ulangan qarshilik, induktor va kondensator (*RLC*) zanjirining qadam reaksiyasi.

RLC elementlari zanjirda parallel ulanganligini hisobga olib, zanjirda o‘zgarmas tok birdaniga qo‘llanilishi tufayli ***i*** ni topish uchun $t > 0$ holatda tugunlarga KCL qo‘llaymiz,

$$\frac{u}{R} + i + C\frac{du}{dt} = I_S \qquad (8.46)$$



8.11-rasm. Parallel *RLC* zanjiri bilan tok manbaini qo‘llanilishi.

Lekin, $u = L\frac{di}{dt}$

(8.46) tenglamadagi u ni o‘rniga qo‘yib, LC ga bo‘lamiz.

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_S}{LC} \qquad (8.47)$$

(8.47) tenglamaning yechimi vaqtinchalik reaksiya $i_t(t)$ va barqaror holatdagi reaksiya $i_{ss}(t)$ dan iborat; ya'ni,

$$i(t) = i_t(t) + i_{ss}(t) \qquad (8.48)$$

Vaqtinchalik reaksiya oldingi bo'limda ko'rib chiqqanimiz bilan bir xil. Barqaror holat reaksiya i ning yakuniy qiymati hisoblanadi. 8.11-rasmdagi zanjirda induktor orqali tok kuchining yakuniy qiymati manba toki I_S bilan bir xil. Shunday qilib,

$$i(t) = I_S + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(Yuqori so'ndirilgan)} \qquad (8.49a)$$

$$i(t) = I_S + (A_1 + A_2 t) e^{-\alpha t} \quad \text{(Kritik so'ndirilgan)} \qquad (8.49b)$$

$$i(t) = I_S + (A_1 cos\omega_d t + A_2 sin\omega_d t) e^{-\alpha t} \quad \text{(Quyi so'ndirilgan)} \qquad (8.49c)$$

Har bir holatda A_1 va A_2 konstantalarini i va di/dt uchun dastlabki shartlardan aniqlash mumkin.

(8.49) tenglama faqat induktor toki i ni topish uchun amal qiladi.

Ammo induktor toki $i_L = i$ ma'lum bo'lgach, induktor, kondensator va rezistorlardan o'tuvchi bir xil kuchlanish $u = L\ di/dt$ ni topishimiz mumkin.

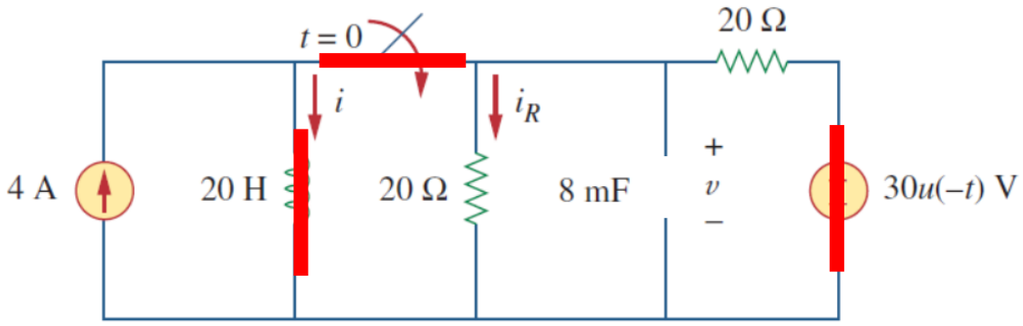
Demak, rezistordan o'tuvchi tok $i_R = u/R$, kondensator toki esa $i_C = C\ du/dt$.

Shu bilan bir qatorda, har qanday o'zgaruvchi $x(t)$ uchun to'liq reaksiyani bevosita topish mumkin.

$$x(t) = x_{ss}(t) + x_t(t) \qquad (8.50)$$

bu yerda: x_{ss} va x_t lar mos ravishda uning yakuniy qiymat va vaqtinchalik reaksiyadir.

8.6.1-masala: 8.12-rasmdagi zanjir $t > 0$ bo‘lganda $i(t)$ va $i_R(t)$ toping.

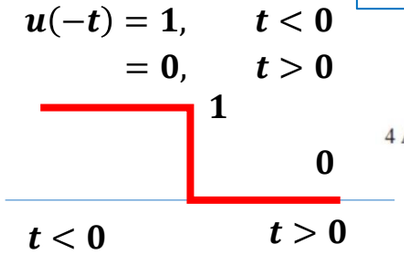


8.12-rasm.

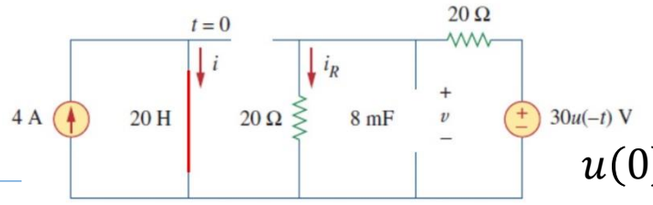
Yechish:

$u(-t) = 1, \quad t < 0$

$= 0, \quad t > 0$



1. $t < 0$, $u(0)$ va $i(0)$.



$$i(0) = 4\ A$$

$$u(0) = \frac{20}{20+20}(30) = 15\ V$$

2. $t > 0$, $\frac{di(0)}{dt}$.

$$u(0) = L\frac{di(0)}{dt} \rightarrow \frac{di(0)}{dt} = \frac{u(0)}{L} = \frac{15}{20} = 0{,}75$$

3. $t > 0$, $\alpha, \omega_0, s_{1,2}$

$$R_{um} = \frac{20 \cdot 20}{20+20} = 10\ \Omega; \quad \alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 10 \cdot 8 \cdot 10^{-3}} = 6{,}25; \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \cdot 8 \cdot 10^{-3}}} = 2{,}5.$$

$\alpha > \omega_0$ zanjirning tabiiy reaksiyasi yuqori so‘ndirilgan.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = 6{,}25 \pm \sqrt{6{,}25^2 - 2{,}5^2} = -6{,}25 \pm 5{,}7282; \quad s_1 = -11{,}978; \quad s_2 = -0{,}5218.$$

4. $i(t)$=?

$$i(t) = I_S + A_1 e^{s_1 t} + A_2 e^{s_2 t};\ \text{A} \quad I_S = 4\ A$$

$$\frac{di}{dt} = -11{,}978 A_1 e^{-11{,}978t} - 0{,}5218 A_2 e^{-0{,}5218t};$$

5. A_1 va A_2 =?

$$i(t) = 4 + A_1 e^{-11{,}978t} + A_2 e^{-0{,}5218t};\ A$$

$$t = 0 \rightarrow \frac{di(0)}{dt} = -11{,}978 A_1 - 0{,}5218 A_2;$$

$i(t)$ dan $u(t) = L\frac{di}{dt}$ ni topamiz.

$$t = 0 \rightarrow i(0) = 4 = 4 + A_1 + A_2; \quad A_2 = -A_1$$

$$0{,}75 = (11{,}978 - 0{,}5218) A_2 \rightarrow \boldsymbol{A_2 = 0{,}0655}$$

$$\boldsymbol{A_1 = -0{,}0655}$$

$$i(t) = 4 + A_1 e^{-11{,}978t} + A_2 e^{-0{,}5218t};\ \text{A}$$

$i(t)$ ning hosila olamiz..

$$\boldsymbol{i(t) = 4 + 0{,}0655\left(e^{-0{,}5218t} - e^{-11{,}978t}\right);\ A}$$

$$\boldsymbol{i_R(t) = \frac{u(t)}{20} = \frac{L}{20}\frac{di}{dt} =}$$

$$\boldsymbol{= 0{,}785 e^{-11{,}978t} - 0{,}0342 e^{-0{,}5218t}\ A}$$

FOYDALANILGAN MANBALAR:

12. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 331.
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