

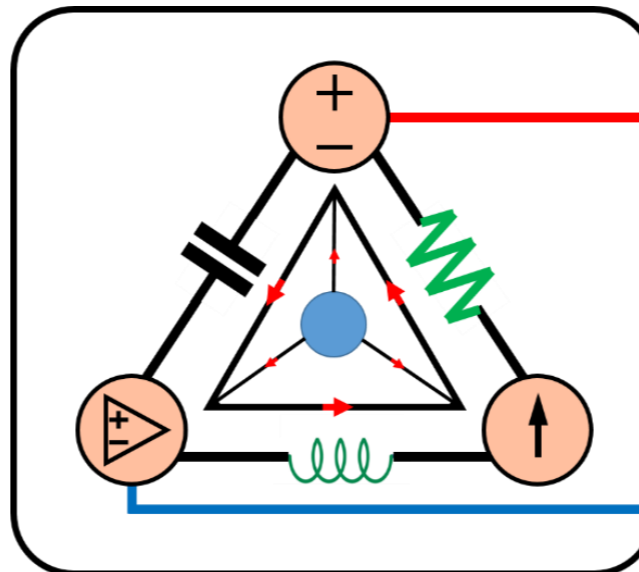
8-Mavzu: Ikkinchi tartibli elektr zanjiri (*davomi*).

(8th Topic: Second-Order Circuit)

8-Mavzuning 4-qismi

(4th part of the 8th Topic)

10-hafta uchun
For the 10th week



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8-Mavzu: Ikkinchi tartibli elektr zanjiri (*davomi*).

(8th Topic: Second-Order Circuit)

O'quv rejasi:

8.5. Ketma-ket ulangan qarshilik, induktor va kondensator (*RLC*) zanjirining qadam reaksiyasi.

8.6. Parallel ulangan qarshilik, induktor va kondensator (*RLC*) zanjirining qadam reaksiyasi.

8.7. Umumiy ikkinchi tartibli zanjirlar.

8.8. Ikkinchi tartibli operatsion kuchaytirgich zanjirlari.

8.9. Qo'llanilishi.

8.7. Umumiy ikkinchi tartibli zanjirlar.

Ketma-ket va parallel *RLC* zanjirlarini tahlil qilish usuli va bosqichlarini o'zgarmas qiymatlarga ega bir yoki bir nechta mustaqil manbalari bo'lgan har qanday ikkinchi tartibli elektr zanjirda qo'llashimiz mumkin.

Garchi ketma-ket va parallel *RLC* zanjirlari ikkinchi tartibli elektr zanjirlari bo'lsa-da, ikkinchi tartibli zanjirlar, shu jumladan operatsion kuchaytirgichlar ham foydalidir.

Ikkinchi tartibli zanjirlar, biz uning $x(t)$ qadam reaksiyasini (kuchlanish yoki tok kuchi bo'lishi mumkin) quyidagi to'rt bosqichni bajarish orqali aniqlaymiz:

1. Birinchi navbatda dastlabki shartlar bo'yicha $x(0)$ va $dx(0)/dt$ hamda yakuniy qiymat $x(\infty)$ larni aniqlaymiz.
2. Biz mustaqil manbalarni o'chiramiz va KCL va KVLni qo'llash orqali vaqtinchalik reaksiya shakli $x_t(t)$ ni topamiz.

Ikkinchi tartibli differensial tenglama olingandan keyin uning xarakterli ildizlarini aniqlaymiz.

Reaksiyaning yuqori, kritik yoki quyi so'ndirilganligiga qarab, oldingi bo'limlarda bo'lgani kabi, ikkita noma'lum konstanta bilan $x_t(t)$ ni olamiz.

3. Barqaror holat reaksiyasini quyidagicha aniqlaymiz,

$$x_{SS}(t) = x(\infty) \quad (8.51)$$

bu yerda: $x(\infty)$ x ning yakuniy qiymati, bu birinchi bosqaichda aniqlanadi.

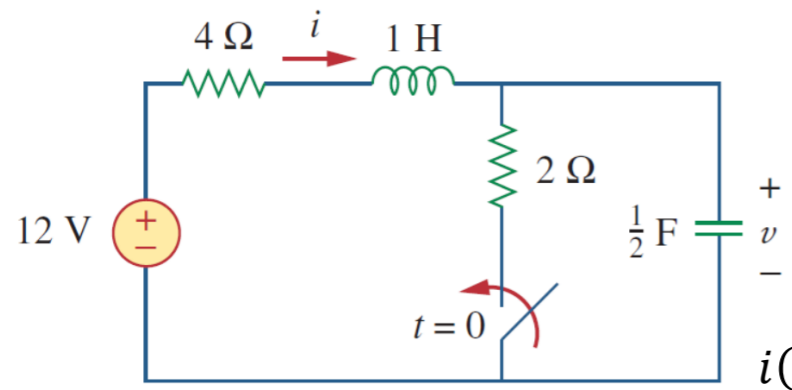
4. Endi umumiy reaksiya vaqtinchalik reaksiya va barqaror holatdagi reaksiyaning yig'indisi sifatida topiladi.

$$x(t) = x_t(t) + x_{ss}(t) \quad (8.52)$$

1-bosqichda aniqlangan dastlabki shartlarni $x(0)$ va $dx(0)/dt$ qo'yish orqali vaqtinchalik reaksiya bilan bog'liq konstantalarni aniqlaymiz.

Biz ushbu umumiy tartibni har qanday ikkinchi tartibli elektr zanjirlarini, shu jumladan operatsion kuchaytirgich qurilmalarni ham topish uchun qo'llashimiz mumkin.

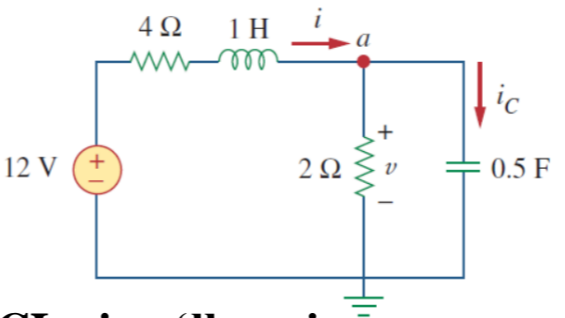
8.7.1-masala: 8.13-rasmdagi zanjir $t > 0$ bo'lganda to'liq $u(t)$ reaksiyani va keyin $i(t)$ ni toping.



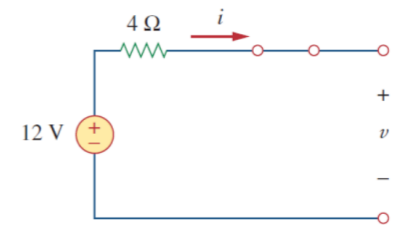
8.13-rasm.

Yechish: 1. $t < 0$, $u(0)$ va $i(0)$.

2. $t > 0$, $C \frac{du}{dt} = i_C$
yoki $\frac{du}{dt} = \frac{i_C}{C}$.



$t = 0^-$ da zanjir barqaror holatga erishadi.



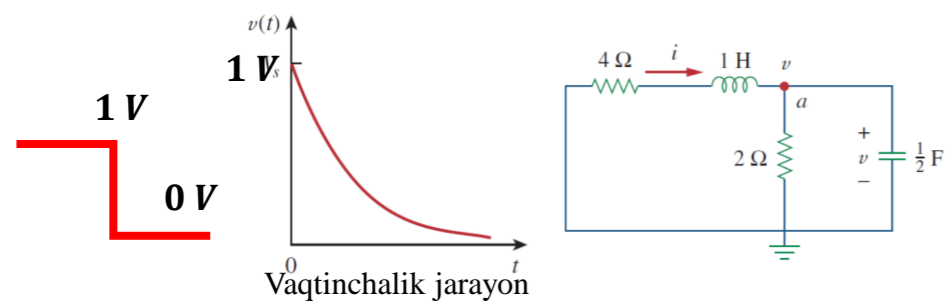
$u(0^-) = 12 V;$
 $i(0^-) = 0 A.$

$i(0^+) = i(0^-) = 0 A;$
 $u(0^+) = u(0^-) = 12 V.$ **KCL ni qo'llaymiz:**
 $i = i_C + i_R;$

$0 = i_C(0^+) + \frac{12}{2} \rightarrow i_C(0^+) = -6 A;$

$\frac{du(0^+)}{dt} = -\frac{6}{0,5} = -12 \frac{V}{s}.$

3. $t > 0$, Vaqtinchalik reaksiya topiladi.



$i(0^+) = i_C(0^+) + \frac{u(0^+)}{2};$

$i(\infty) = \frac{12}{4+2} = 2 A; \quad u(\infty) = 2i(\infty) = 4 V.$

$\frac{d^2u}{dt^2} + 5 \frac{du}{dt} + 6u = 0$

4. Tabiiy reaksiyani olamiz. $u_n(t) = Ae^{-2t} + Be^{-3t}$

$s^2 + 5s + 6 = 0$

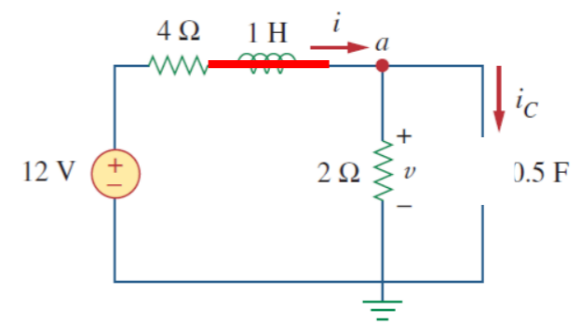
5. $t > 0$, $u(\infty)$ va $i(\infty)$. $u_{SS}(t) = u(\infty) = \frac{12}{4+2} 2 = 4 V$

$s_1 = -2$ va $s_2 = -3.$

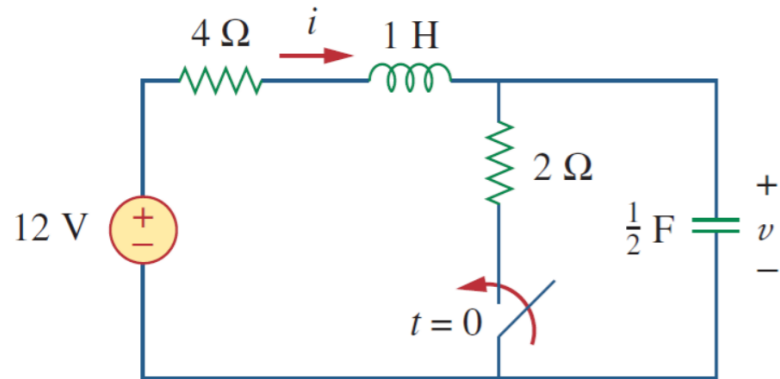
KCL ni qo'llaymiz: **KVLni chap to'rga qo'llaymiz:**

$i = \frac{u}{2} + \frac{1}{2} \frac{du}{dt};$ $4i = 1 \frac{du}{dt} + u = 0$

$2u + 2 \frac{du}{dt} + \frac{1}{2} \frac{du}{dt} + \frac{1}{2} \frac{d^2u}{dt^2} + u = 0$



8.7.1-masala: 8.13-rasmdagi zanjir $t > 0$ bo'lganda to'liq $u(t)$ reaksiyani va keyin $i(t)$ ni toping.



8.13-rasm.

7. A va $B = ?$

Yechish:

6. $t > 0$, $u(\infty)$ va $i(\infty)$.

$$u(t) = u_t + u_{ss} = 4 + Ae^{-2t} + Be^{-3t}$$

$$t = 0 \rightarrow 12 = 4 + A + B \rightarrow A + B = 8$$

$$u(t) = u_t + u_{ss} = 4 + Ae^{-2t} + Be^{-3t}$$

tenglamadagi u ning hosilasini olamiz.

$$\frac{du}{dt} = -2Ae^{-2t} - 3Be^{-3t}$$

$$t = 0$$

$$-12 = -2A - 3B \rightarrow 2A + 3B = 12$$

$$A = 12, B = -4$$

$$u(t) = 4 + 12e^{-2t} - 4e^{-3t}, t > 0$$

$$i = \frac{u}{2} + \frac{1}{2} \frac{du}{dt};$$

$$i(t) = \frac{(4 + 12e^{-2t} - 4e^{-3t})}{2} + \frac{1}{2} \frac{d}{dt} (4 + 12e^{-2t} - 4e^{-3t}) =$$

$$= 2 + 6e^{-2t} - 2e^{-3t} - 12e^{-2t} + 6e^{-3t} \rightarrow$$

$$i(t) = 2 - 6e^{-2t} + 4e^{-3t} A, t > 0$$

$$i(0^+) = i(0^-) = 0 A \text{ ga asosan } t = 0 \rightarrow$$

$$i(0) = 0$$

8.8. Ikkinchi tartibli operatsion kuchaytirgich zanjirlari.

Bitta ekvivalent elementga birlashtirib bo'lmaydigan ikkita saqlash elementiga ega bo'lgan operatsion kuchaytirgich zanjiri ikkinchi tartibli.

Induktorlar katta va og'ir bo'lganligi sababli, ular operatsion kuchaytirgich zanjirlarida kamdan-kam qo'llaniladi.

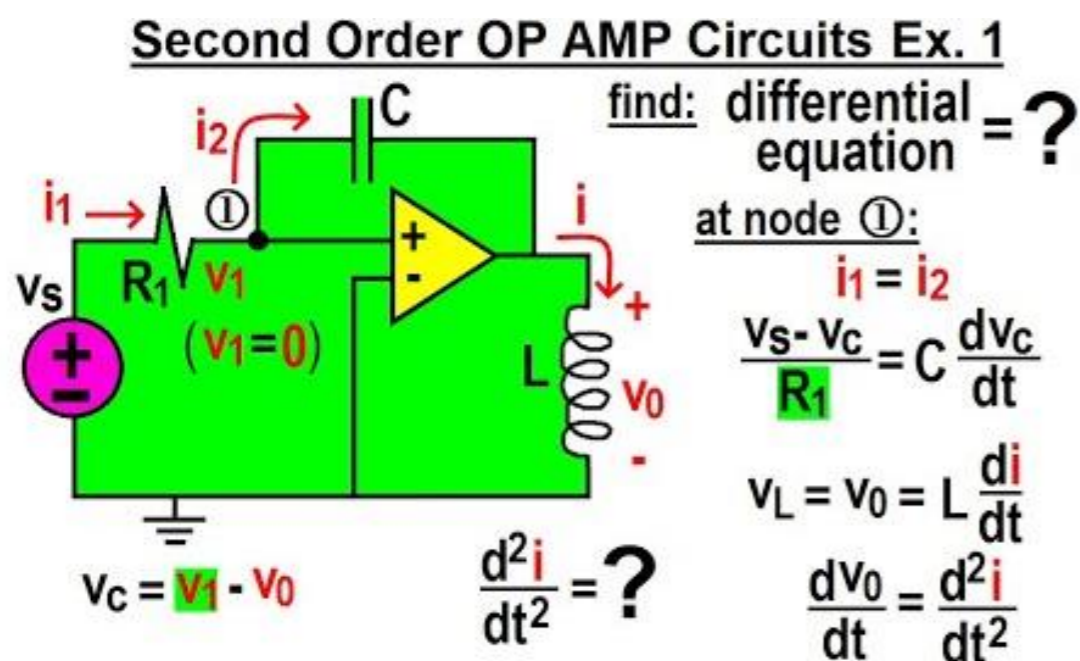


Photo source: [14] - <https://i.ytimg.com/vi/ZRQO4yupxqw/hqdefault.jpg>

Shu sababli, biz bu yerda faqat RC ikkinchi darajali operatsion kuchaytirgich zanjirlarini ko‘rib chiqamiz.

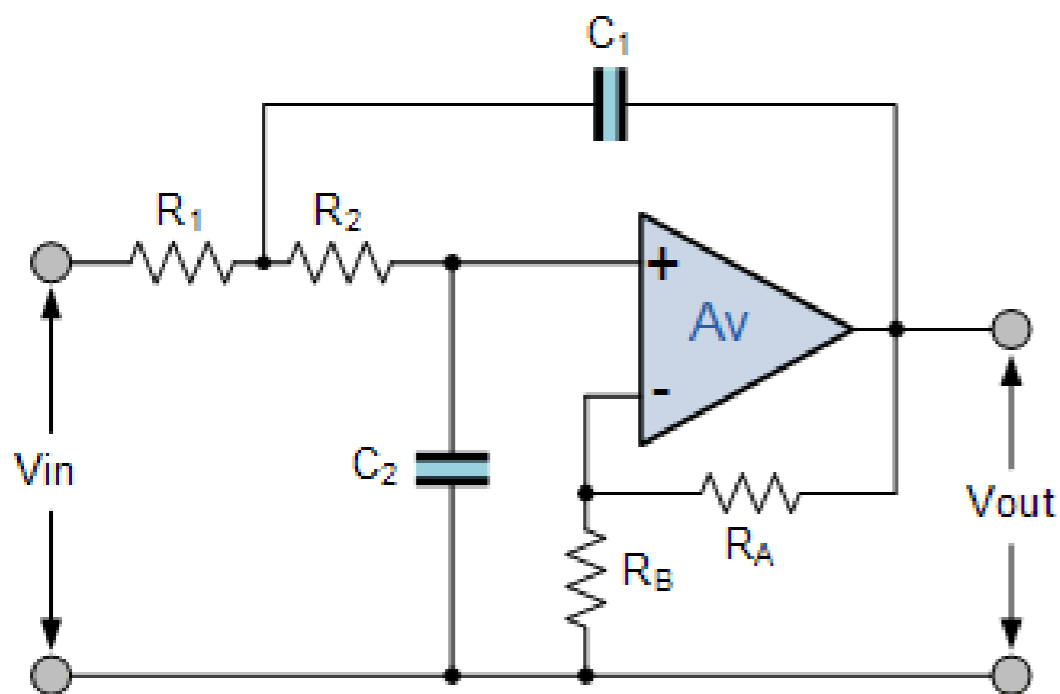


Photo source: [15] - <https://i.stack.imgur.com/byytP.png>

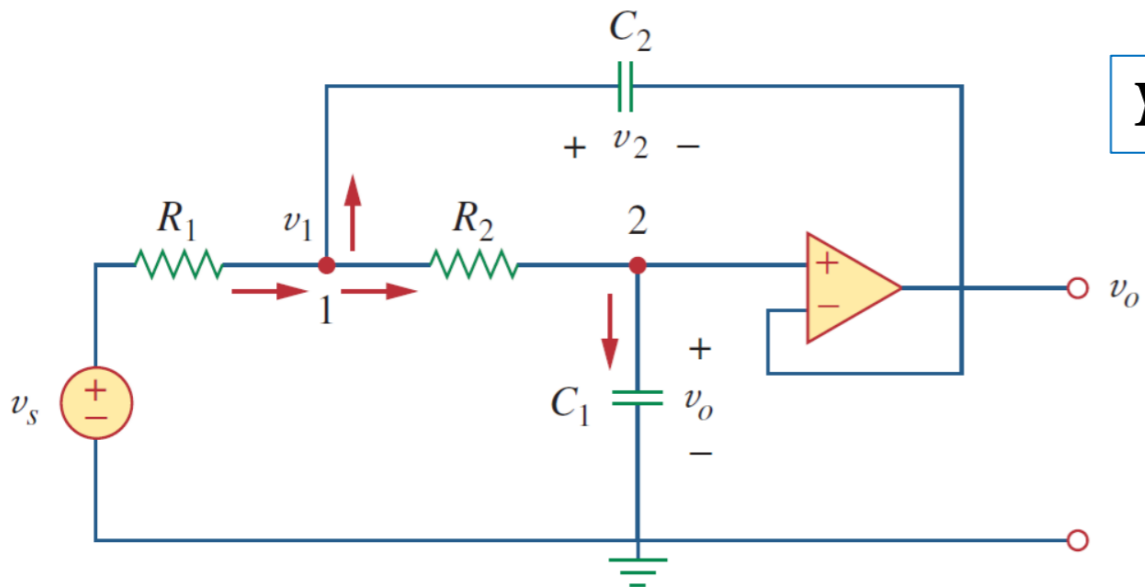
Bunday zanjirlar filtrlar va otsilatorlar kabi qurilmalarda keng ko‘lamli tadbig‘ini topadi.

Ikkinchi tartibli operatsion kuchaytirgich zanjirini tahlil qilish oldingi bo‘limda berilgan va ko‘rsatilgan to‘rtta qadamni bajaradi.

8.8.1-masala: 8.16-rasmdagi operatsion kuchaytirgichning kuchlanish manbai

$u_s = 10u(t)mV$ bo'lganda $t > 0$ uchun $u_o(t)$ ni aniqlang. $R_1 = R_2 = 10 k\Omega$,

$C_1 = 20 \mu F$, va $C_2 = 100 \mu F$ deb qabul qiling.



Yechish:

1-tugunda KCLni qo'llaymiz: $\frac{u_s - u_1}{R_1} = C_2 \frac{du_2}{dt} + \frac{u_1 - u_o}{R_2}$

2-tugunda KCLni qo'llaymiz: $\frac{u_1 - u_o}{R_2} = C_1 \frac{du_o}{dt}$

$$u_2 = u_1 - u_o$$

$$\frac{u_s - u_1}{R_1} = C_2 \frac{du_1}{dt} - C_2 \frac{du_o}{dt} + C_1 \frac{du_o}{dt}$$

$$\frac{u_1 - u_o}{R_2} = C_1 \frac{du_o}{dt} \rightarrow u_1 = u_o + R_2 C_1 \frac{du_o}{dt}$$

8.16-rasm.

$$\frac{u_s - u_1}{R_1} = C_2 \frac{du_1}{dt} - C_2 \frac{du_o}{dt} + C_1 \frac{du_o}{dt}$$

$$u_1 = u_o + R_2 C_1 \frac{du_o}{dt}$$

$$\frac{u_s}{R_1} = \frac{u_o}{R_1} + \frac{R_2 C_1}{R_1} \frac{du_o}{dt} + C_2 \frac{du_o}{dt} + R_2 C_1 C_2 \frac{d^2 u_o}{dt^2} - C_2 \frac{du_o}{dt} + C_1 \frac{du_o}{dt}$$

yoki,

$$\frac{d^2 u_o}{dt^2} + \left(\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2} \right) \frac{du_o}{dt} + \frac{u_o}{R_1 R_2 C_1 C_2} = \frac{u_s}{R_1 R_2 C_1 C_2}$$



8.8.1-masala: 8.16-rasmdagi operatsion kuchaytirgichning kuchlanish manbai

$u_s = 10u(t)mV$ bo'lganda $t > 0$ uchun $u_o(t)$ ni aniqlang. $R_1 = R_2 = 10 k\Omega$,

$C_1 = 20 \mu F$, va $C_2 = 100 \mu F$ deb qabul qiling.

Yechish:

Berilgan R_1, R_2, C_1 va C_2 qiymatlari bilan tenglama quyidagicha hosil bo'ladi.

$$\frac{d^2 u_o}{dt^2} + 2 \frac{du_o}{dt} + 5u_o = 5u_s$$

$$s^2 + 2s + 5 = 0 \rightarrow s_{1,2} = -1 \pm j2.$$

vaqtinchalik reaksiya shakli,

$$u_{ot} = e^{-t}(A \cos 2t + B \sin 2t)$$

$t \rightarrow \infty$, sifatida zanjir barqarorlik holatiga etadi

$$u_o(\infty) = u_1(\infty) = u_s$$

$$u_{oss} = u_o(\infty) = u_s = 10 mV, \quad t > 0$$

To'liq reaksiya,

$$u_o(t) = u_{ot} + u_{oss} = 10 + e^{-t}(A \cos 2t + B \sin 2t) mV$$

A va B ni aniqlash uchun biz boshlang'ich sharoitni bilishimiz kerak.

$t < 0$ uchun $u_s = 0$ bo'lsa,

$$u_o(0^-) = u_2(0^-) = 0$$

$t > 0$ uchun, manba ishlaydi. Biroq, kondensator kuchlanishining

uzluksizligi tufayli,

$$u_o(0^+) = u_2(0^+) = 0$$

$$u_1(0^+) = u_2(0^+) + u_o(0^+) = 0$$

$$\frac{du_o(0^+)}{dt} = \frac{u_1 - u_o}{R_2 C_1} = 0; \quad t = 0 \rightarrow 0 = 10 + A \rightarrow A = -10$$

$$\frac{du_o(t)}{dt} = e^{-t}(-A \cos 2t - B \sin 2t - 2A \sin 2t + 2B \cos 2t)$$

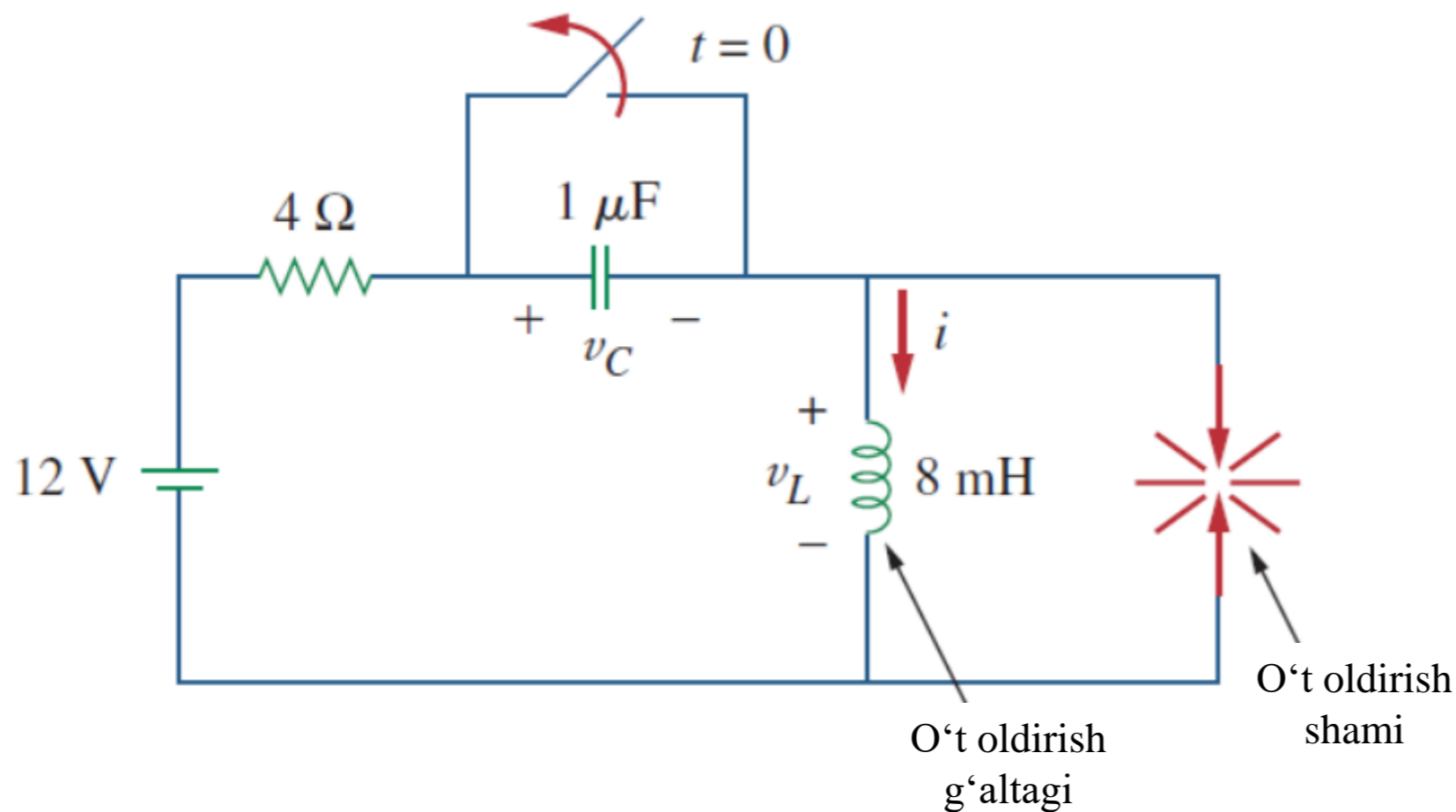
$$t = 0 \text{ qiymat bersak, } \quad \mathbf{0 = -A + 2B}$$

$$\mathbf{A = -10 \text{ va } B = -5}$$

$$\mathbf{u_o(t) = 10 - e^{-t}(10 \cos 2t + 5 \sin 2t) mV, \quad t > 0}$$

8.9. Qo'llanilishi.

Avtomobilni o't oldirish tizimini zaryadlash tizimi sifatida ko'rib chiqqan edik.



Bu kuchlanish ishlab chiqaruvchi tizim 12 V manbai batareya va generatorga bog'liq.

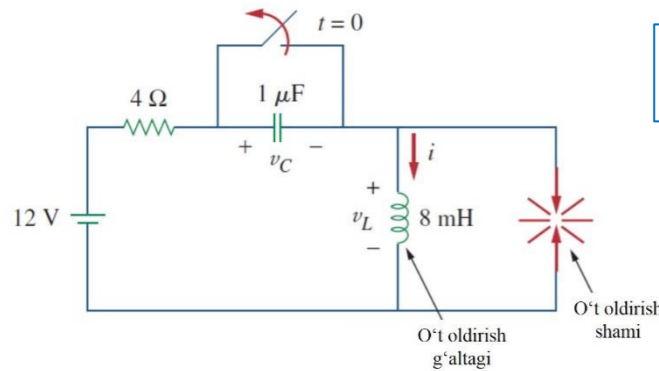
4 Ω rezistor simlarning qarshiligini ifodalaydi.

O't oldirish g'altagi 8 mH induktor tomonidan modellashtirilgan.

8.17-rasm. Avtomobilning o't oldirish zanjiri.

8.9.1-masala: 8.17-rasmdagi kalit $t = 0^-$ dan oldin yopilgan deb faraz qilib, $t > 0$ uchun

induktiv kuchlanish u_L ni toping.



8.17-rasm.

Yechish:

Agar kalit $t = 0^-$ dan oldin yopilgan bo'lsa va zanjir barqaror holatda bo'ladi, u holda

$$i(0^-) = \frac{12}{4} = 3 \text{ A}, \quad u_C(0^-) = 0$$

$t = 0^+$ da kalit ochiladi. O'zgarmas tok kuchi sharoiti talabi bo'yicha,

$$i(0^+) = 3 \text{ A}, \quad u_C(0^+) = 0$$

$\frac{du(0^+)}{dt}$ qiymatini $u_L(0^+)$ orqali olamiz. Meshga KVLni qo'llash orqali $t = 0^+$ da quyidagi tenglama hosil bo'ladi.

$$-12 + 4i(0^+) + u_L(0^+) + u_C(0^+) = 0$$

$$-12 + 4 \cdot 3 + u_L(0^+) + 0 = 0 \rightarrow u_L(0^+) = 0$$

Demak,

$$\frac{du(0^+)}{dt} = \frac{u_L(0^+)}{L} = 0$$

$t \rightarrow \infty$ da tizim barqarorlik holatiga erishib, kondensator ochiq zanjirda ishlaydi. U holda,

$$i(\infty) = 0$$

$t > 0$ uchun meshga KVL ni qo'llasak, quyidagi tenglama hosil bo'ladi.

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$

$$\alpha = \frac{R}{2L} = 250,$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 1,118 \cdot 10^4$$

$\alpha < \omega_o$ reaksiyasi quyi so'ndirilgan munosabatni berdi.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} \cong \omega_o = 1,118 \cdot 10^4$$

**Yechish:**

Vaqtinchalik reaksiyaning quyi so'ndirilgan sharti uchun,

$$i_t(t) = e^{-\alpha}(A \cos \omega_d t + B \sin \omega_d t)$$

Barqaror holat reaksiyasi,

$$i_{ss}(t) = i(\infty) = 0$$

To'la reaksiya,

$$i(t) = i_t(t) + i_{ss}(t) = e^{-250t}(A \cos 11,180 t + B \sin 11,180 t)$$

$$i(0) = 3 = A + 0 \quad \rightarrow \quad \mathbf{A = 3}$$

$$\begin{aligned} \frac{di(t)}{dt} &= -250e^{-250t}(A \cos 11,180 t + B \sin 11,180 t) + \\ &+ e^{-250t}(-11,180 A \sin 11,180 t + 11,180 B \cos 11,180 t) \end{aligned}$$

$$t = 0 \quad 0 = -250A + 11,180 B \quad \rightarrow \quad B = 0,0671$$

$$\mathbf{i(t) = e^{-250t}(3 \cos 11,180 t + 0,0671 \sin 11,180 t)}$$

$$u_L(t) = L \frac{di}{dt} = -268e^{-250t} \sin 11,180 t$$

$$11,180t_0 = \pi/2 \text{ yoki } t_0 = 140,5 \mu s.$$

$$\mathbf{u_L(t_0) = -268e^{-250t_0} = -259 V}$$

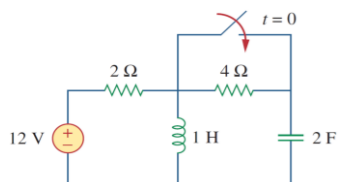


FOYDALANILGAN MANBALAR:

14. <https://i.ytimg.com/vi/ZRQO4yupxqw/hqdefault.jpg>
15. <https://i.stack.imgur.com/byytP.png>

TEKSHIRISH UCHUN SAVOLLAR!

8.1. 8.18-rasmdagi zanjirda $t = 0^-$ da (kalit yopilishidan oldin) kondensatoridagi kuchlanishi:



- A) 0 V.
- B) 4 V.
- C) 8 V.
- D) 12 V.

8.18-rasm.

8.2. 8.18-rasmdagi zanjirda $t = 0$ da induktordagi dastlabki tok kuchi:

- A) 0 A.
- B) 2 A.
- C) 6 A.
- D) 12 A.

8.3. Ikkinchi tartibli elektr zanjirida bosqichli kirish qo'llanilganda, zanjir o'zgaruvchilarining yakuniy qiymatlari quyidagicha topiladi:

- A) kondensatorlarni yopiq zanjir qilib, induktorlarni ochiq zanjir qilib almashtirish.
- B) kondensatorlarni ochiq zanjir qilib, induktorlarni esa yopiq zanjirga almashtirish.
- C) Yuqoridagilarning hech birini bajarmaydi.

8.4. RLC zanjirida xarakterli ildizlarning qiymatlari -2 va -3 bo'lsa, reaksiya qaysi tenglama bilan aniqlanadi?

- A) $(A \cos 2t + B \sin 2t)e^{-3t}$.
- B) $(A + B2t)e^{-3t}$.
- C) $Ae^{-2t} + Bte^{-3t}$.
- D) $Ae^{-2t} + Be^{-3t}$.

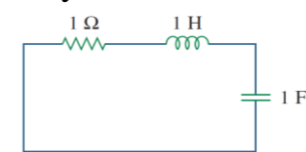
8.5. Ketma-ket ulangan RLC zanjirida $R = 0$ bo'lganda quyidagi reaksiya hosil bo'ladi:

- A) yuqori so'ndirilgan.
- B) kritik so'ndirilgan.
- C) quyi so'ndirilgan.
- D) so'ndirilmagan.
- E) yuqoridagilarning hech biri.

8.6. Parallel ulangan RLC zanjirida $L = 2H$ va $C = 0,25F$ qiymatga ega. Birlik so'ndirish omilini ishlab chiqaradigan R qiymati:

- A) $0,5 \Omega$.
- B) 1Ω .
- C) 2Ω .
- D) 4Ω .

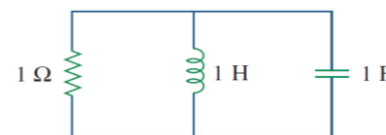
8.7. 8.19-rasmdagi RLC ketma-ket ulangan zanjirga qarang. U qanday reaksiya beradi?



8.19-rasm.

- A) yuqori so'ndirilgan.
- B) quyi so'ndirilgan.
- C) kritik so'ndirilgan.
- D) yuqoridagilarning hech biri.

8.8. 8.20-rasmdagi RLC parallel ulangan zanjirga qarang. U qanday reaksiya beradi?

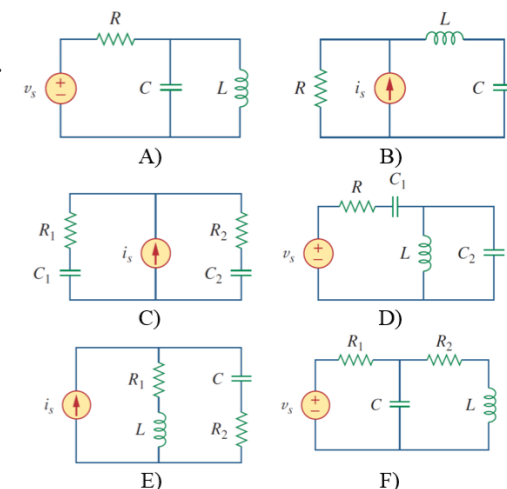


8.20-rasm.

- A) yuqori so'ndirilgan.
- B) quyi so'ndirilgan.
- C) kritik so'ndirilgan.
- D) yuqoridagilarning hech biri.

8.9. 8.21-rasmdagi zanjirlarni quyidagi elementlar bilan moslang:

- (I) birinchi tartibli zanjir.
- (II) ikkinchi tartibli ketma-ket ulangan zanjir.
- (III) ikkinchi tartibli parallel ulangan zanjir.
- (IV) yuqoridagilarning hech biri.



8.21-rasm.

8.10. Elektr zanjirida qarshilikning ikkiligi:

- A) o'tkazuvchanlik.
- B) induktivlik.
- C) sig'im.
- D) ochiq zanjir.
- E) qisqa tutashuv zanjiri.



*E'TIBORINGIZ
UCHUN
RAHMAT!!!*