9-Mavzu: O‘zgaruvchan tok zanjiri. Sinusoidlar va fazalar.

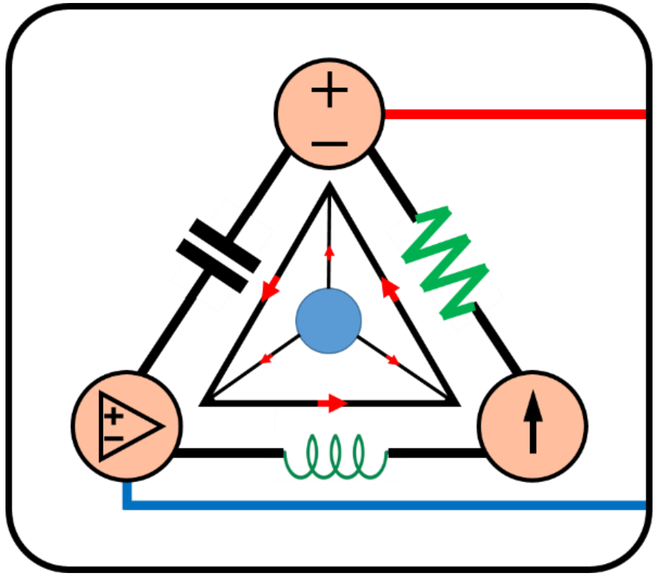
(9th Topic: AC Circuit. Sinusoids and Phasors)

9-Mavzuning 2-qismi

(2nd part of the 9th Topic)

11-hafta uchun

For the 11th week



Lecturer: Ph.D., Yusupov Sarvarbek

Toshkent Kimyo Xalqaro Universiteti
“Mashinasozlik texnologiyasi” kafedrasi
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9-Mavzu: O‘zgaruvchan tok zanjiri. Sinusoidlar va fazalar.

(9th Topic: AC Circuit. Sinusoids and Phasors.)

O‘quv rejasi:

9.1. Umumiy tushunchalar.

9.2. Sinusoidlar.

9.3. Fazalar.

9.4. Zanjir elementlari uchun fazaning bog‘liqligi.

9.3. Fazalar.

Sinusoidlar *sin* va *cos* funksiyalaridan ko‘ra ular bilan ishlash qulayroq bo‘lgan fazalar bilan osongina ifodalanadi.

Faza - bu sinusoidning amplitudasi va fazasini (*phase*) ifodalovchi kompleks son.

Fazalar sinusoidal manbalar tomonidan qo‘zg‘atilgan chiziqli elektr zanjirlarni tahlil qilishning oddiy vositalarini ta’minlaydi. Aks holda bunday elektr zanjirlarning yechimlari qiyin bo‘lar edi.

Fazalar yordamida o‘zgaruvchan tok zanjirlarini yechish tushunchasi birinchi marta 1893 yilda Charlz Shtaynmets tomonidan kiritilgan.

Fazalarni to‘liq aniqlashdan va ularni elektr zanjirlar tahliliga qo‘llashdan oldin, biz kompleks sonlarni yaxshilab tanishishimiz kerak.

Kompleks z sonni to‘rtburchaklar shaklida yozish mumkin.

$$z = x + jy \tag{9.14a}$$

bu yerda: $j = \sqrt{-1}$; x - z ning haqiqiy qismi; y - z ning xayoliy qismi.

Shu nuqtai nazardan, x va y o‘zgaruvchilari ikki o‘lchovli vektor tahlilidagi kabi joyni emas, balki murakkab tekislikdagi ***z*** ning haqiqiy va hayoliy qismlarini ifodalaydi.

Kompleks son z qutbli yoki eksponensial shaklda ham yozilishi mumkin.

$$z = r\angle\phi = re^{j\phi} \tag{9.14b}$$

bu yerda: r - z ning kattaligi, ϕ - z ning fazasi.

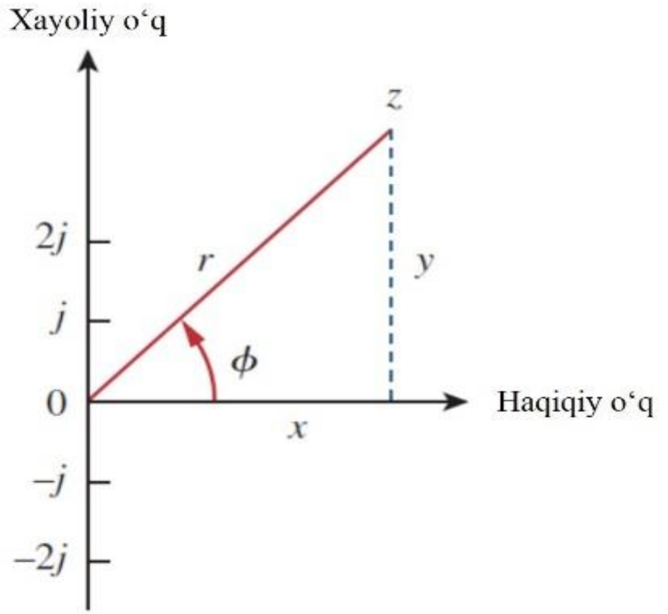
Biz z ni uchta usulda ifodalash mumkinligini ko‘ramiz:

$\boldsymbol{z = x + jy}$ To‘rtburchaklar shakli

$\boldsymbol{z = r\angle\phi}$ Qutb shakli (9.15)

$\boldsymbol{z = re^{j\phi}}$ Eksponensial shakli

To‘g‘ri to‘rtburchak shakl va qutb shakli o‘rtasidagi munosabat 9.6-rasmda ko‘rsatilgan.



9.6-rasm. Kompleks sonning $z = x + jy = r\angle\phi$ ifodalanishi.

Photo source: [6] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 377.

bu yerda: *x* o‘qi haqiqiy qismni, *y* o‘qi esa kompleks sonning hayoliy qismini ifodalaydi.

x va *y* berilgan, biz *r* va ϕ sifatida olishimiz mumkin.

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x} \tag{9.16a}$$

Boshqa tomondan, agar biz *r* va ϕ ni bilsak, biz *x* va *y* ni olishimiz mumkin.

$$x = r\cos\phi, \qquad y = r\sin\phi \tag{9.16b}$$

Demak, *z* quyidagicha yozilishi mumkin.

$$z = x + jy = r\angle\phi = r(\cos\phi + \sin\phi) \tag{9.17}$$

Kompleks sonlarni qo‘shish va ayirish to‘rtburchaklar shaklida yaxshiroq bajariladi; ko‘paytirish va bo‘linish qutbli shaklda yaxshiroq amalga oshiriladi.

$$\boldsymbol{z = x + jy = r\angle\phi:} \qquad \boldsymbol{z_1 = x_1 + jy_1 = r_1\angle\phi_1;} \qquad \boldsymbol{z_2 = x_2 + jy_2 = r_2\angle\phi_2.}$$

Kompleks sonlarni hisobga olgan holda quyidagi operatsiyalar muhim ahamiyatga ega.

Qo‘shish:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \qquad (9.18a)$$

Ayirish:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \qquad (9.18b)$$

Ko‘paytirish:

$$z_1 \cdot z_2 = r_1 \cdot r_2\angle\phi_1 + \phi_2 \qquad (9.18c)$$

Bo‘lish:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\phi_1 - \phi_2 \qquad (9.18d)$$

O‘zaro bog‘liqlik:

$$\frac{1}{z} = \frac{1}{r}\angle - \phi \qquad (9.18e)$$

Kvadrat ildiz:

$$\sqrt{z} = \sqrt{r}\angle\phi/2 \qquad (9.18f)$$

Kompleks konjugat (*conjugate*):

$$z^* = x - jy = r\angle - \phi = re^{-j\phi} \qquad (9.18g)$$

(9.18e) tenglamada

$$\frac{1}{j} = -j \qquad (9.18h)$$

Bular bizga kerak bo‘lgan kompleks sonlarning asosiy xususiyatlari.

Faza tushunchasi Eyler tengligi asosida tavsiflanadi.

$$e^{\pm j\phi} = cos\phi \pm j\,sin\phi \qquad (9.19)$$

Umuman olganda, bu $e^{j\phi}$ ning haqiqiy $cos\phi$ va hayoliy $sin\phi$ qismlarini ko'rsatadi. Ya'ni,

$$cos\phi = Re(e^{j\phi}) \quad (9.20a)$$

$$sin\phi = Im(e^{j\phi}) \quad (9.20b)$$

bu yerda: *Re* (real part) va *Im* (imaginary part) haqiqiy qismini va xayoliy qismini bildiradi.

Sinusoid tenglamasi $u(t) = U_m \cos(\omega t + \phi)$ berilgan. $u(t)$ ni quyidagicha ifodalash uchun (9.20a) tenglamadan foydalanamiz.

$$u(t) = U_m \cos(\omega t + \phi) = \mathrm{Re}(U_m e^{j(\omega t + \phi)}) \quad (9.21)$$

yoki,

$$u(t) = \mathrm{Re}(U_m e^{j\phi} e^{j\omega t}) \quad (9.22)$$

Demak,

$$\boldsymbol{u(t) = \mathrm{Re}(U e^{j\omega t})} \quad (9.23)$$

bu yerda:

$$U = U_m e^{j\phi} = U_m \angle \phi \quad (9.24)$$

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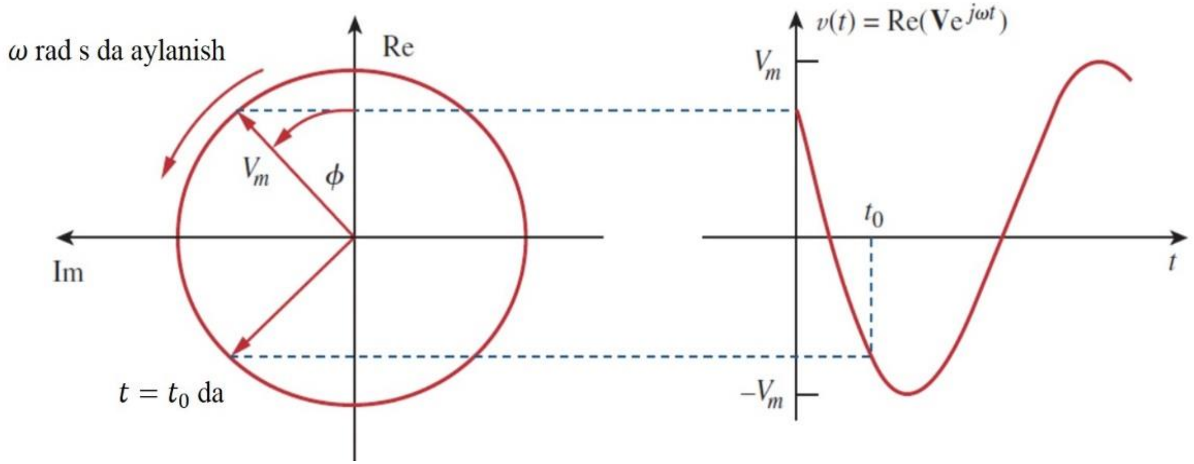
Shunday qilib, U yuqorida aytganimizdek sinusoid $u(t)$ ning fazaviy ifodasidir. Boshqacha qilib aytganda, faza sinusoidning tasavvuri va fazasining murakkab tasviridir.

$$\boldsymbol{u(t)} = \mathbf{Re}(\boldsymbol{U e^{j\omega t}}) \qquad (9.23) \qquad\qquad U = U_m e^{j\phi} = U_m \angle \phi \qquad (9.24)$$

tenglamalarni ko‘rib chiqishning usullaridan biri kompleks tekislikdagi sinor $Ue^{j\omega t} = U_m e^{j(\omega t+\phi)}$ ning grafigini ko‘rib chiqishdir.



a) sinor soat miliga teskari yo‘nalishda aylanadi,

b) uning real o‘qga proyeksiyasi, vaqt funksiyasi sifatida.

9.7-rasm. $\boldsymbol{U e^{j\omega t}}$ ning tasvirlanishi.

Vaqt o‘tishi bilan sinor 9.7-rasm, a da radius U_m aylana bo‘ylab burchak tezlikda ω soat miliga teskari yo‘nalishda aylanadi.

9.7-rasm, b da $u(t)$ ni sinorning $Ue^{j\omega t}$ haqiqiy o‘qdagi proyeksiyasi deb hisoblashimiz mumkin.

Photo source: [7] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 379.

Sinorning $t = 0$ vaqtdagi qiymati $u(t)$ sinusoidning U fazasidir. Sinorni davriy faza deb hisoblash mumkin. Shunday qilib, sinusoid faza sifatida ifodalanganda, $e^{j\omega t}$ atamasi bevosita mavjud bo‘ladi.

Shuning uchun fazalar bilan ishlashda fazaning chastotasini ω yodda tutish kerak; aks holda jiddiy xatolarga yo‘l qo‘yishimiz mumkin.

$\boldsymbol{u(t) = \mathrm{Re}\left(Ue^{j\omega t}\right)} \rightarrow$ U fazaga mos keladigan sinusoidni olish uchun fazani vaqt koeffitsientiga $e^{j\omega t}$ ko‘paytirib, haqiqiy qismini aniqlashimiz mumkin.

Murakkab miqdor sifatida faza to‘rtburchaklar shaklida, qutb shaklida yoki eksponensial shaklida ifodalanishi mumkin.

Faza kattalik (*magnitude*) va davrga (“yo‘nalish”) ega bo‘lganligi sababli, u vektor sifatida ishlaydi va qalin shrift bilan bosiladi.

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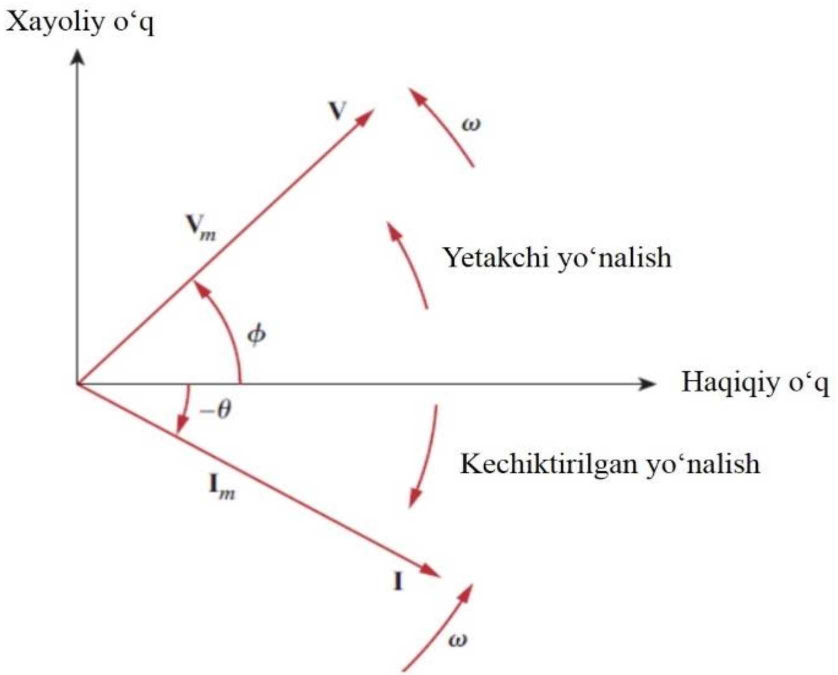
Masalan, $U = U_m \angle \phi$ va $I = I_m \angle \phi$ fazalar 9.8-rasmda grafik ko‘rsatilgan. Fazalarning bunday grafik tasviri *faza diagrammasi* deb nomlanadi.



9.8-rasm. $U = U_m \angle \phi$ va $I = I_m \angle \phi$ ko‘rsatilgan fazalar diagrammasi.

$$u(t) = U_m \cos(\omega t + \phi) \quad \leftrightarrow \quad U = U_m \angle \phi \qquad (9.25)$$

(Vaqt sohasini ifodalanishi) *(Faza sohasini ifodalanishi)*

9.1-jadval

Sinusoid faza transformatsiyasi

Vaqt sohasini ifodalanishi *(Time domain representation)*	**Faza sohasini ifodalanishi** *(Phasor domain representation)*
$U_m \cos(\omega t + \phi)$	$U_m \angle \phi$
$U_m \sin(\omega t + \phi)$	$U_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90°$

Photo source: [8] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 380.

$$\boldsymbol{u(t) = \mathrm{Re}(Ue^{j\omega t})} \qquad (9.23)$$

$$U = U_m e^{j\phi} = U_m \angle \phi \qquad (9.24)$$

$$\boldsymbol{u(t) = \mathrm{Re}\left(Ue^{j\omega t}\right) = U_m \cos(\omega t + \phi)}$$

$$\boldsymbol{\frac{du}{dt} = -\omega U_m \sin(\omega t + \phi) = \omega U_m \cos(\omega t + \phi + 90°) =}$$

$$\boldsymbol{= \mathrm{Re}\left(\omega U_m e^{j\omega t} e^{j\phi} e^{j90°}\right) = \mathrm{Re}\left(j\omega U_m e^{j\omega t}\right)} \qquad (9.26)$$

Bu shuni ko‘rsatadiki, $u(t)$ hosilasi $j\omega U$ kabi faza sohasiga aylanadi.

$$\frac{du}{dt} \quad \leftrightarrow \quad \boldsymbol{j\omega} U \qquad (9.27)$$

(Vaqt sohasi) *(Faza sohasi)*

Sinusoidni farqlash unga mos fazasini $\boldsymbol{j\omega}$ ga ko‘paytirish bilan topiladi.

Xuddi shunday, $u(t)$ ning integrali $\frac{U}{j\omega}$ sifatida faza sohasiga aylanadi.

$$\int u\ dt \quad \leftrightarrow \quad \frac{U}{\boldsymbol{j\omega}} \qquad (9.28)$$

(Vaqt sohasi) *(Faza sohasi)*

Sinusoidni integrallash unga mos keladigan fazasini $\boldsymbol{j\omega}$ ga bo‘lish bilan topiladi.

9.3.1-masala: $i_1(t) = 4\cos(\omega t + 30°)\ A$ va $i_2(t) = 5\sin(\omega t - 20°)\ A$ ularni yig‘indisini toping.

Yechish:

$I_1 = 4\angle 30°;$ $i_2(t) = 5\cos(\omega t - 20° - 90°) = 5\ \cos(\omega t - 110°);$

$I_2 = 5\angle - 110°;$ $\boldsymbol{i = i_1 + i_2}$

$$I = I_1 + I_2 = 4\angle 30° + 5\angle - 110° = 3{,}464 + j2 - 1{,}71 - j4{,}698 =$$

$$= 1{,}754 - j2{,}698 = 3{,}218\angle - 56{,}97°\ \mathrm{A}$$

Vaqt domeniga aylantirsak,

$$\boldsymbol{i(t) = 3{,}218\, cos(\omega t - 56{,}97°)\ A}$$

9.3.2-masala: Ushbu murakkab sonlarni baholang.

a) $(40\angle 50° + 20\angle -30°)^{1/2}$, b) $\frac{10\angle -30° + (3-4j)}{(2+j4)(3-j5)^*}$.

Yechish:

a) Qutb shaklidan to‘rtburchak shakliga transformatsiya qilishdan foydalaniladi.

$$40\angle 50° = 40(\cos 50° + j\sin 50°) = 25{,}71 + j30{,}64$$

$$10\angle -30° = 20[\cos(-30°) + j\sin(-30°)] = 17{,}32 - j10$$

Ularni qo‘shish orqali quyidagi hosil bo‘ladi.

$$40\angle 50° + 20\angle -30° = \mathbf{43,03 + j20,64} = \mathbf{47,72\angle 25,63°}$$

$$\boldsymbol{Z = R + jX}$$

$$\boldsymbol{|Z| = \sqrt{R^2 + X^2}}; \qquad \boldsymbol{\theta = tan^{-1}\frac{X}{R}}.$$

b) Qutb shaklidan to‘rtburchak shakliga transformatsiya qilish, qo‘shish, ko‘paytirish va bo‘lishdan foydalaniladi.

$$\frac{10\angle -30° + (3-4j)}{(2+j4)(3-j5)^*} = \frac{8{,}66 - j5 + (3-4j)}{(2+j4)(3-j5)^*} = \frac{11{,}66 - 4j}{-14 + j22} =$$

$$= \frac{(11{,}66 - 4j)(-14 - j22)}{(-14 + j22)(-14 - j22)} =$$

$$= \frac{14{,}73\angle -37{,}66°}{26{,}08\angle 122{,}47°} = \mathbf{0,565\angle -160,13°}$$

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9.4. Zanjir elementlari uchun fazaning bog‘liqligi.

O‘zgarmas tok zanjirlaridan farqli o‘laroq, sinusoidal tok zanjirlarida rezistordan tashqari induktiv g‘altak va kondensator kabi elementlardan keng foydalaniladi.

Sinusoidal tok zanjirlarida energiyani issiqlik energiyasiga aylantiruvchi elementlar ***aktiv qarshiliklar*** deb ataladi.

Elektr zanjirlarida energiya davriy ravishda elektr yoki magnit maydoni ko‘rinishida to‘planib turuvchi elementlari **reaktiv elementlar**, ularni o‘zgaruvchan tokka ko‘rsatadigan qarshiliklari esa ***reaktiv qarshiliklar*** deb ataladi.

Induktiv g‘altak va kondensator zanjirning reaktiv elementlari hisoblanadi. Ushbu elementlardan tashkil topgan sinusoidal tok zanjirlarini hisoblashdan oldin bu elementlarda tok va kuchlanishlar o‘rtasidagi munosabatlarni o‘rganib chiqamiz.

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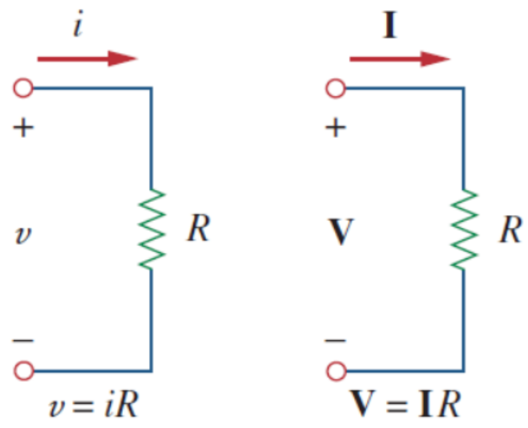
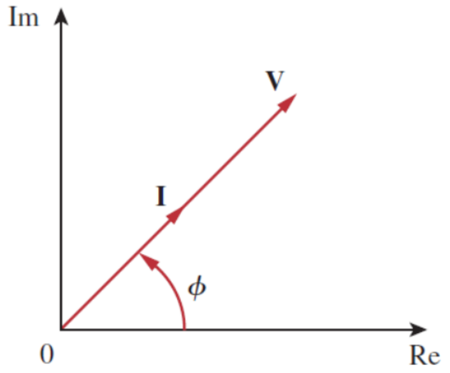
Rezistor uchun kuchlanish va tok kuchi munosabati.



a) vaqt sohasi; b) chastota sohasi.

9.9-rasm. Rezistor uchun kuchlanish-tok kuchi munosabatlari.



9.10-rasm. Rezistor uchun faza diagrammasi.

Agar R rezistordan o‘tadigan tok kuchi $i = I_m \cos(\omega t + \phi)$ bo‘lsa, undagi kuchlanish Om qonuni bilan belgilanadi.

$$u = iR = RI_m \cos(\omega t + \phi) \qquad (9.29)$$

Ushbu kuchlanishning fazaviy shakli,

$$U = RI_m \angle \phi \qquad (9.30)$$

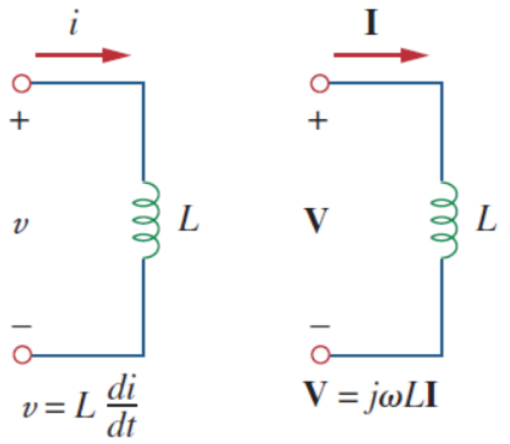
Lekin tokning fazaviy ifodasi $I = I_m \angle \phi$ ga teng.

$$U = RI \qquad (9.31)$$

Demak, faza sohasidagi rezistor uchun kuchlanish-tok kuchi munosabati vaqt sohasida bo‘lgani kabi Om qonuni bo‘lib qolishini ko‘rsatadi.

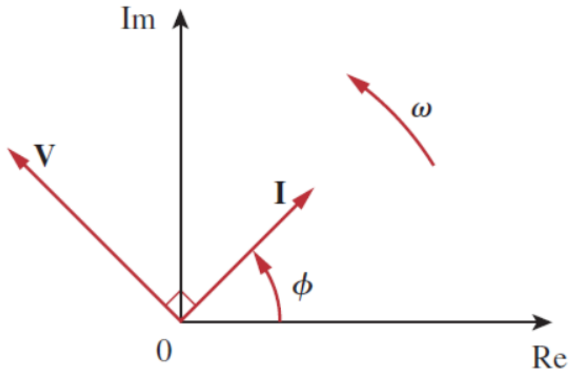
Photo source: [9] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 385.

L induktor uchun kuchlanish va tok kuchi munosabati.



a) vaqt sohasi; b) chastota sohasi.

9.11-rasm. Induktor uchun kuchlanish-tok kuchi munosabatlari:



9.12-rasm. Induktor uchun faza diagrammasi, I ni U dan kechikishi.

Induktor orqali o'tadigan tok kuchi $i = I_m \cos(\omega t + \phi)$ deb faraz qilaylik. Induktordagi kuchlanish

$-\sin A = \cos(A + 90°)$

$$u = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi) \qquad (9.32)$$

$$u = \omega L I_m \cos(\omega t + \phi + 90°) \qquad (9.33)$$

$$(U = \omega L I_m e^{j(\phi+90°)} = \omega L I_m e^{j\phi} e^{j90°} = \omega L I_m \angle \phi + 90° \qquad (9.34)$$

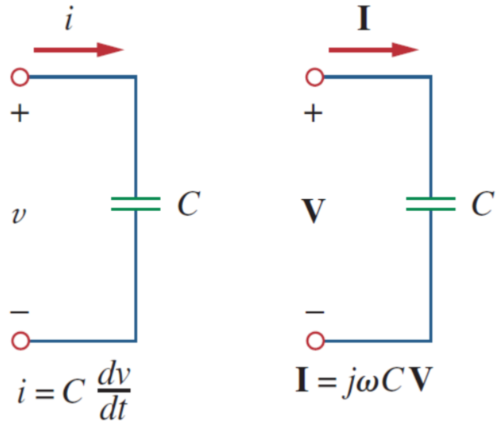
Lekin $I_m \angle \phi = I$. $e^{j90°} = j$

$$U = j\omega L I \qquad (9.35)$$

Kuchlanishning $\omega L I_m$ kattaligiga va $\phi + 90°$ fazaga ega ekanligini ko'rsatadi. Kuchlanish va tok kuchi fazadan 90° ga teng. Xususan, tok kuchi kuchlanishdan 90° ga ortda qoladi.

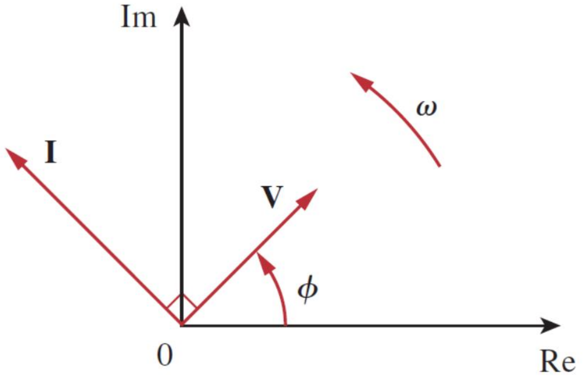
Photo source: [9] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 385.

C kondensator uchun kuchlanish va tok kuchi munosabati.



a) vaqt sohasi; b) chastota sohasi.

9.13-rasm. Kondensator uchun kuchlanish-tok kuchi munosabatlari:



9.14-rasm. Induktor uchun faza diagrammasi, I ni U dan oldindaligi.

Kondensator orqali o'tadigan kuchlanish $u = U_m \cos(\omega t + \phi)$ deb faraz qilaylik. Kondensatordagi tok kuchi

$$i = C\frac{du}{dt} \quad (9.36)$$

$$I = j\omega CU \quad \rightarrow \quad U = \frac{I}{j\omega C} \quad (9.37)$$

Tok kuchi kuchlanishdan 90° ga oldinda boradi.

9.2-jadval

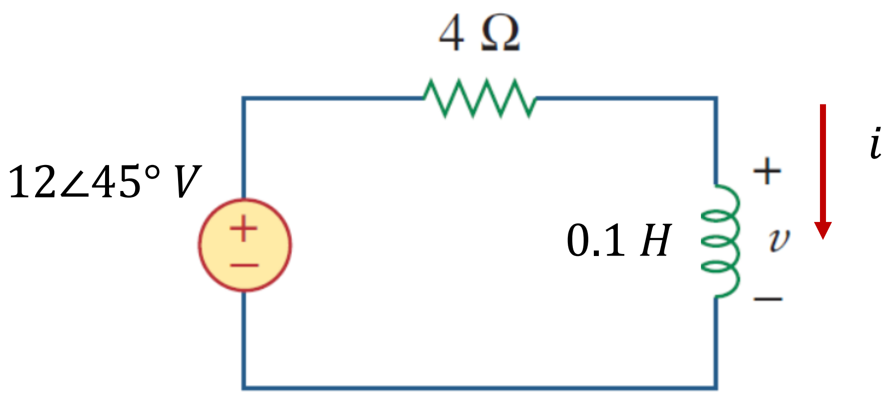
Kuchlanish-tok kuchi bog'liqligining qisqacha mazmuni

Element	**Vaqt sohasi** *(Time domain)*	**Faza sohasi** *(Phasor domain)*
R	$u = Ri$	$U = RI$
L	$u = L\frac{di}{dt}$	$U = j\omega LI$
C	$i = C\frac{du}{dt}$	$U = \frac{I}{j\omega C}$

Photo source: [10] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 386.

9.4.1-masala: $u = 12\cos(60t + 45°)$ kuchlanish 0,1 H induktorga qo‘llaniladi.

Induktordan o‘tgan barqaror holatdagi tok kuchini toping.



Yechish:

L induktor uchun $U = j\omega L I$

bu yerda:

$$\omega = 60\, rad/s;$$

$$U = 12\angle 45°\, V.$$

$$I = \frac{U}{j\omega L} = \frac{12\angle 45°}{j60 \cdot 0{,}1} = \frac{12\angle 45°}{6\angle 90°} = 2\angle -45°\, A$$

$$j = 1\angle 90°$$

Vaqt sohasiga aylantirsak,

$$\boldsymbol{i(t) = 2\cos(60t - 45°)\, A}$$

FOYDALANILGAN MANBALAR:

6. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 377.
7. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 379.
8. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 380.
9. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 385.
10. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 386.

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