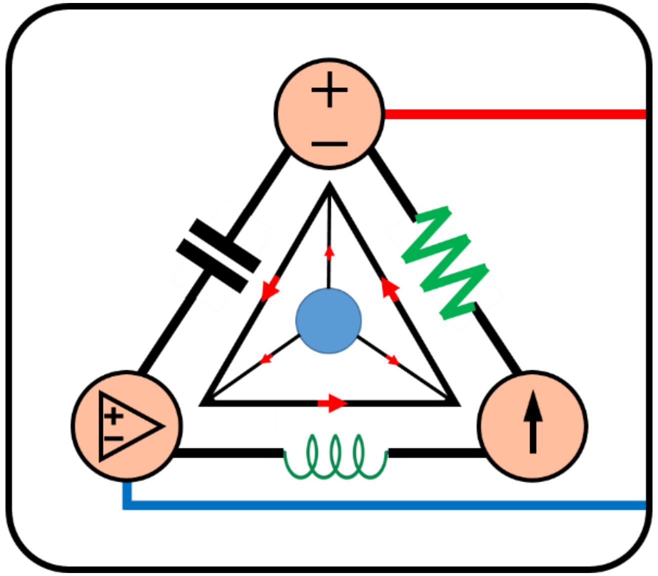
10-Mavzu: Sinusoidal barqaror holat tahlili.

(10th Topic: Sinusoidal Steady-State Analysis.)

10-Mavzuning 1-qismi

(1st part of the 10th Topic)

13-hafta uchun
For the 13th week



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10-Mavzu: Sinusoidal barqaror holat tahlili.

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O‘quv rejasi:

10.1. Umumiy tushunchalar.

10.2. Tugun potensiallar tahlili.

10.3. Mesh (to‘r) tahlili.

10.4. Superpozitsiya (ustma-ustlash) teoremasi.

10.5. Manba transformatsiyasi.

10.6. Tevenin va Norton ekvivalent zanjirlari.

10.7. O‘zgaruvchan tok zanjirlarida operatsion kuchaytirgichlar.

10.8. Qo‘llanilishi.

10.1. Umumiy tushunchalar

Oldingi mavzuda biz zanjirlarning sinusoidal kirishlarga majburiy yoki barqaror holatdagi reaksiyani fazalar yordamida olish mumkinligini bilib oldik.

Om va Kirxgof qonunlari o‘zgaruvchan tok zanjirlarida ham qo‘llaniladi.

Ushbu mavzuda o‘zgaruvchan tok zanjirlarini tahlil qilishda tugun potensiallar tahlili, to‘r tahlili, Thevenin va Norton teoremalari, superpozitsiya va manba transformatsiyasi qanday qo‘llanilishini ko‘rib chiqamiz.

Ushbu usullar orqali o‘zgarmas tok zanjirlarini tahlil qilish odatda uchta bosqichni talab qiladi.

Ushbu bosqichlar:

1. Zanjirni faza yoki chastota sohasiga aylantirish.

2. Masalani zanjir usullari (tugun tahlili, to‘r tahlili, superpozitsiya va boshqalar) yordamida yechish.

3. Olingan fazaviy natijani vaqt sohasiga aylantirish.

Muammo chastota sohasida ko‘rsatilgan bo‘lsa, 1-bosqich kerak emas. 2-bosqichda tahlil o‘zgarmas tok zanjirini tahlili bilan bir xil tarzda amalga oshiriladi. Bundan tashqari kompleks sonlar ishtirok etadi.

Fazalar orqali o‘zgaruvchan tok zanjirining chastota sohasini tahlil qilish, vaqt sohasidagi zanjirni tahlil qilishdan ko‘ra osonroqdir.

10.2. Tugun potensiallar tahlili.

Bu usulga ko‘ra Kirxgofning 1 - qonuniga asoslanib elektr zanjir tugunlaridagi potensiallar zanjirining tayanch tuguniga nisbatan aniqlanadi.

Bunda tayanch *tugun potensiali nolga teng* deb qabul qilinadi.

Ma’lumki, har qanday shaxobchadagi kuchlanish shu shaxobcha ulangan tugunlar potensiallarining ayirmasiga teng.

Bu kuchlanishni shu shaxobcha o‘tkazuvchanligiga ko‘paytmasi esa shaxobcha tokiga teng bo‘ladi.

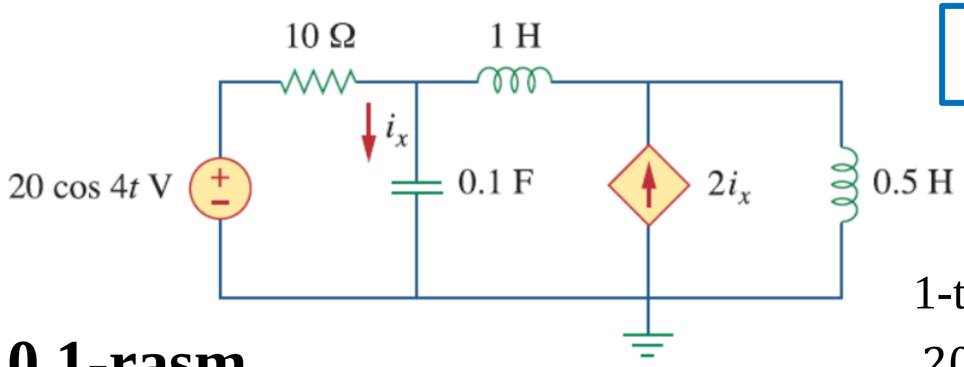
Shunday qilib, tugun potensiallarini aniqlab har bir shaxobchadagi tok qiymatini topishimiz mumkin.

Ushbu usul noma’lum toklarni topishda tugun potensiallarini aniqlashga asoslanganligi uchun ***tugun potensiallar usuli*** deb ataladi.

Demak, tugun potensiallar usulining asosi Kirxgofning tok kuchiga oid (KCL) bo‘lgan birinchi qonuni hisoblanar ekan.

KCL fazalar uchun amal qilganligi sababli, biz o‘zgaruvchan tok zanjirlarini tugundagi potensiallarni farqi orqali tahlil qilishimiz mumkin.

10.2.1-masala: 10.1-rasmda ko‘rsatilgan zanjirdagi i_x ni tugun tahlilidan foydalanib aniqlang.



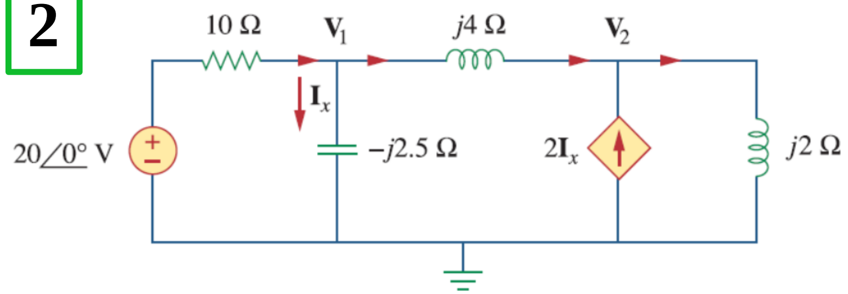
10.1-rasm.

Yechish:

1

$$20\cos 4t \rightarrow 20\angle 0^\circ \qquad \omega = 4\, rad/s$$
$$1\,H \rightarrow j\omega L = j4$$
$$0{,}5\,H \rightarrow j\omega L = j2$$
$$0{,}1\,F \rightarrow \frac{1}{j\omega C} = \frac{-j}{\omega C} = -j2{,}5$$

2



1-tugunga KCL ni qo‘llaymiz.

$$\frac{20 - V_1}{10} = \frac{V_1}{-j2{,}5} + \frac{V_1 - V_2}{j4}$$

$$20 - V_1 = -(-j4V_1) - j2{,}5\,(V_1 - V_2);$$

$$20 = V_1 + j4V_1 - j2{,}5V_1 + j2{,}5V_2$$

$$\mathbf{(1 + j1,5)V_1 + j2,5V_2 = 20} \qquad (10.2.1)$$

2-tugunga KCL ni qo‘llaymiz.

$$2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2} \qquad I_x = \frac{V_1}{-j2{,}5}$$

$$\frac{2V_1}{-j2{,}5} + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2}$$

$$-8V_1 + 2{,}5\,(V_1 - V_2) = 5V_2$$
$$-8V_1 + 2{,}5V_1 - 2{,}5V_2 = 5V_2$$
$$-5{,}5V_1 = 7{,}5V_2$$

$$\mathbf{11V_1 + 15V_2 = 0} \qquad (10.2.2)$$

$$\begin{bmatrix} 1 + j1{,}5 & j2{,}5 \\ 11 & 15 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix} \qquad \Delta_1 = \begin{bmatrix} 20 & j2{,}5 \\ 0 & 15 \end{bmatrix} = 300; \qquad \Delta_2 = \begin{bmatrix} 1 + j1{,}5 & 20 \\ 11 & 0 \end{bmatrix} = -220.$$

$$\Delta = \begin{bmatrix} 1 + j1{,}5 & j2{,}5 \\ 11 & 15 \end{bmatrix} = (1 + j1{,}5)15 - 11(j2{,}5) = 15 - j5;$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13{,}91\angle 198{,}43^\circ\, V.$$

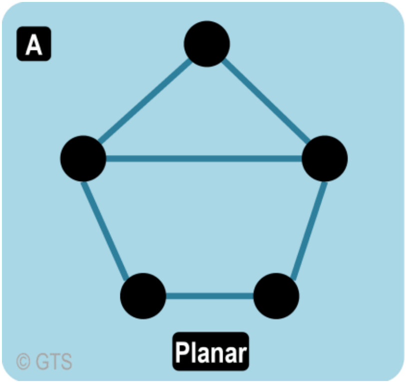
$$V_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = \frac{300(15 + j5)}{(15 - j5)(15 + j5)} = 18 + j6 = 18{,}97\angle 18{,}43^\circ\, V;$$

$$\boldsymbol{I_x} = \frac{V_1}{-j2{,}5} = \frac{18{,}97\angle 18{,}43^\circ}{2{,}5\angle -90^\circ} = \mathbf{7,59\angle 108,43^\circ\, A}$$

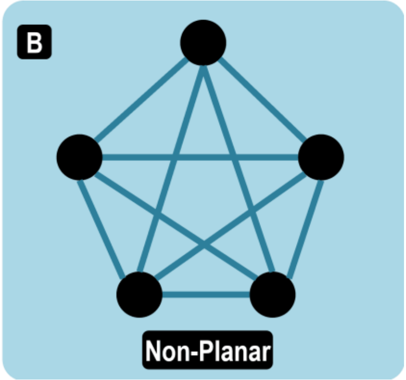
3

$$i_x = 7{,}59\cos(4t + 108{,}43^\circ)\, A.$$

10.3. Mesh (to‘r) tahlili.



A) Planar (planar) zanjir,



B) Planar bo‘lmagan (non-planar) zanjir.

10.3-rasm.

Kirxgofning kuchlanishga oid qonuni (KVL) to‘r (mesh) tahlilining asosini tashkil qiladi.

AC zanjirlari uchun KVLning qo‘llanilishi quyidagi misolda keltirilgan.

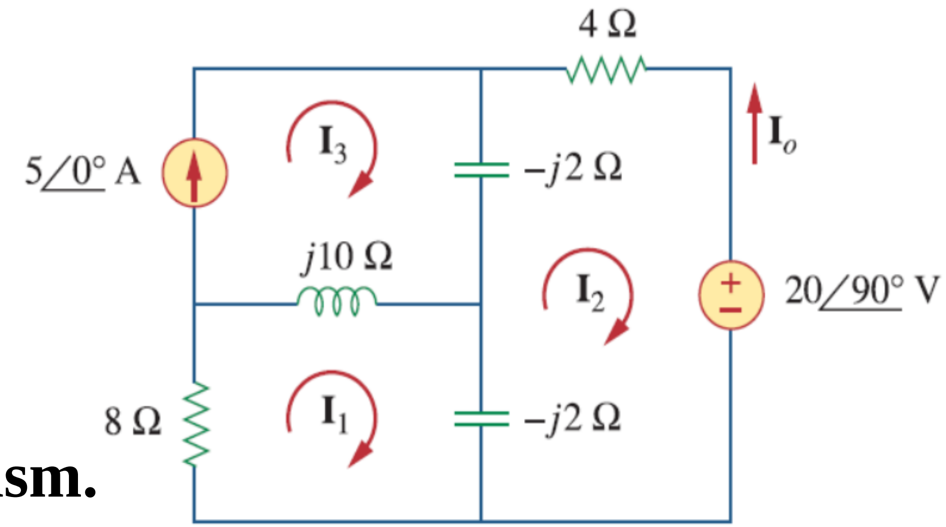
Yodda tutingki, mesh tahlilidan foydalanishning o‘ziga xos xususiyati shundaki, u planar zanjirlarida qo‘llaniladi.

Planar zanjirlar - bir-birini kesib o‘tmagan simlarsiz tekislik yuzasida chizilgan zanjirlar.

Planar bo‘lmagan zanjirar - bu ba’zi simlar bir-birini kesib o‘tgan holda tekislik yuzasida chizilishi mumkin bo‘lgan zanjirlar hisoblanadi (10.3-rasm).

Photo source: [1] -https://transportgeography.org/wp-content/uploads/planar_non_planar_graphs.png

10.3.1-masala: 10.4-rasmda ko‘rsatilgan zanjirdagi I_0 ni mesh tahlilidan foydalanib aniqlang.



10.4-rasm.

Yechish: 1-mesh uchun KVL ni qo‘llaymiz.

$$8I_1 + j10(I_1 - I_3) + (-j2)(I_1 - I_2) = 0$$
$$8I_1 + j10I_1 - j10I_3 - j2I_1 + j2I_2 = 0$$
$$(8 + j10 - j2)I_1 + j2I_2 - j10\, I_3 = 0 \qquad (10.3.1)$$

2-mesh uchun KVL ni qo‘llaymiz.

$$4I_2 + (-j2)(I_2 - I_3) + (-j2)(I_2 - I_1) + 20\angle 90° = 0$$
$$4I_2 - j2I_2 + j2I_3 - j2I_2 + j2I_1 + 20\angle 90° = 0$$
$$j2I_1 + (4 - j2 - j2)I_2 + j2I_3 + 20\angle 90° = 0 \qquad (10.3.2)$$

3-mesh uchun $I_3 = 5$ ekanligini hisobga olib, (10.3.1) va (10.3.2) tenglamalarga qo‘ysak, quyidagilar hosil bo‘ladi.

$$(8 + j8)I_1 + j2I_2 j - j10 \cdot 5 = 0$$
$$(8 + j8)I_1 + j2I_2 = j50 \qquad (10.3.3)$$
$$j2\, I_1 + (4 - j4)I_2 + j2 \cdot 5 + j20 = 0$$
$$j2\, I_1 + (4 - j4)I_2 = -j30 \qquad (10.3.4)$$

(10.3.3) va (10.3.4) tenglamalarni matritsa shaklida ifodalaymiz.

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

Quyidagi determinantlarni olamiz.

$$\Delta = \begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} = 32(1 + j)(1 - j) + 4 = 68;$$
$$\Delta_2 = \begin{bmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{bmatrix} = 340 - j240 = 416{,}17\angle -35{,}22° .$$
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{416{,}17\angle -35{,}22°}{68} = 6{,}12\angle -35{,}22°\, A.$$

Kerakli tok kuchi quyidagicha aniqlanadi. $I_0 = -I_2$ bo‘lganligi sababli π qo‘shiladi:

$$\boldsymbol{I_0 = -I_2 = 6{,}12\angle -35{,}22° + 180° = 6{,}12\angle 144{,}78°\ A}$$

10.4. Superpozitsiya (ustma-ustlash) teoremasi.

Ustma-ustlash teoremasi: “Chiziqli elektr zanjirlarida o‘zaro, bog‘liq bo‘lmagan bir nechta manbalarning umumiy ta’siri alohida olingan har bir manba ta’siri natijalarining algebraik yig‘indisiga teng”.

Ma’lumki, kontur toklar usuliga binoan elektr zanjirida EYuK manbai ta’sirida shoxobchalardan o‘tuvchi kontur toklar kontur EYuK larining chiziqli funksiyasidir.

Murakkab chiziqli elektr zanjirining har bir shoxobchasidagi tok alohida olingan har bir EYuK manbaining ta’siridan hosil bo‘lgan toklarning algebraik yig‘indisiga teng.

Bu prinsip ***superpozitsiya*** (*ustma - ustlash*) prinsipi deyiladi. Ushbu prinsipga asosan kontur yoki shoxobchadagi toklarni aniqlash usuli ustma-ustlash usuli deb ataladi. Murakkab elektr zanjirni ustma - ustlash usulida hisoblash quyidagi ketma-ketlikda bajariladi:

a) har bir EYuK manbai ta’siridan shoxobchalarda hosil bo‘lgan xususiy toklar aniqlanadi, bunda fikran zanjirda yagona EYuK manbai qoldirilib, boshqa EYuK lar olib tashlanadi va ularning ichki qarshiliklari zanjirga EYuK lar o‘rniga ulangan deb qabul qilinadi;

b) shoxobchalardagi haqiqiy toklar esa alohida hisoblangan xususiy toklarning algebraik yig‘indisiga teng bo‘ladi.

Agar chiziqli zanjirda tok manbalari ulangan bo‘lsa, tugunlardagi potensiallar yoki shoxobchalardagi kuchlanishlar har bir tok manbai toklarining chiziqli funksiyasi bo‘ladi.

Shunday qilib, o‘zgaruvchan tok zanjirlari chiziqli bo‘lganligi sababli, superpozitsiya teoremasi o‘zgarmas tok zanjirlarda qo‘llanilgan kabi o‘zgaruvchan tok zanjirlariga ham bir xil qo‘llaniladi.

Agar zanjirda turli chastotalarda ishlaydigan manbalar bo‘lsa, teoremani qo‘llash kerak bo‘ladi.

Bunday holda, impidenslar chastotaga bog‘liq bo‘lganligi sababli, biz har bir chastota uchun boshqa chastotali soha zanjiriga ega bo‘lishimiz kerak.

Jami reaksiyalar vaqt sohasidagi individual reaksiyalarni qo‘shish orqali olinishi kerak.

Reaksiyalarni faza yoki chastota sohasiga qo‘shish noto‘g‘ri. Nega? Chunki, sinusoidal analizda eksponensial omil $e^{j\omega t}$ mavhum bo‘ladi va bu omil har bir burchak chastotasi ω uchun o‘zgaradi.

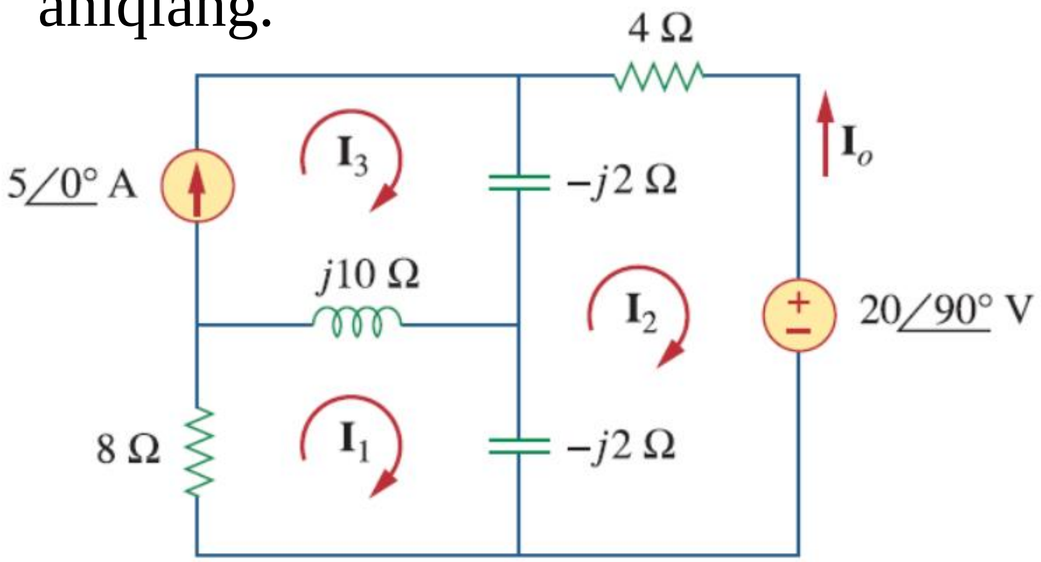
Shuning uchun faza sohasida turli chastotalarda reaksiyalarini qo‘shish mantiqiy bo‘lmaydi.

Zanjirda turli chastotalarda ishlaydigan manbalar mavjud bo‘lsa, vaqt sohasidagi individual chastotalar tufayli reaksiyalarni qo‘shish kerak.

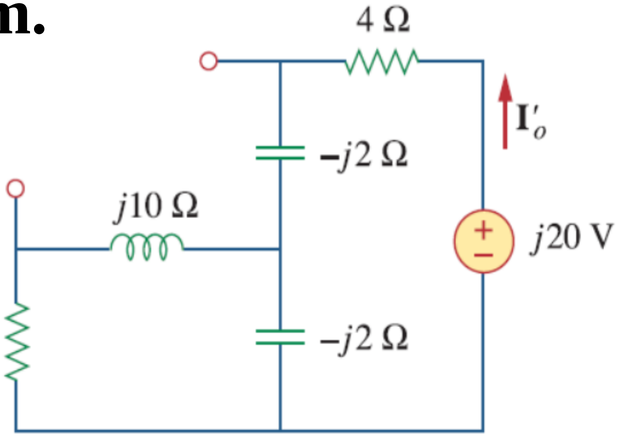
Ustma - ustlash usuliga binoan har bir tugun potensialini aniqlashda fikran bitta tok manbai qoldirilib, qolganlari zanjirdan chiqariladi, ularning o‘rniga esa tok manbalarining ichki o‘tkazuvchanliklari ulangan deb qaraladi.

Agar murakkab zanjirga bir vaqtda EYuK va tok manbalari ulangan bo‘lsa, bunda ham ustma - ustlash usulini qo‘llash mumkin.

10.4.1-masala: 10.4-rasmda ko‘rsatilgan zanjirdagi I_0 ni usta-ustlash teoremasidan foydalanib aniqlang.



10.5-rasm.



Yechish:

$$\boldsymbol{I_0 = I_0' + I_0''} \qquad (10.4.1)$$

Parallel kombinatsiyasili $\boldsymbol{Z}$ ni aniqlaymiz.

$$Z = \frac{-j2(8+j10)}{-2j+8+j10} = \frac{-j2(8+j10)(8-j8)}{(8+j8)(8-j8)}$$

$$= \frac{-j2(8+j10)(8-j8)}{64-j64+j64-j^2 64} =$$

$$= \frac{-j2(64-j64+j80-j^2 80)}{64-(-1)64} = \frac{-j2(64+j16-(-1)80)}{128} =$$

$$= \frac{-j2(144+j16)}{128} = \frac{-j288-j^2 32}{128} = \frac{32}{128} - \frac{j288}{128} = 0{,}25 - j2{,}25$$

Tok kuchi I_0',

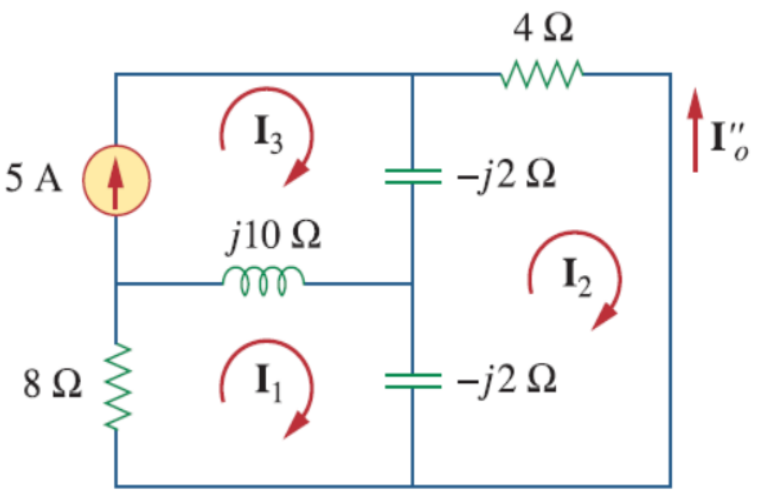
$$I_0' = \frac{j20}{4-j2+Z} = \frac{j20}{4-j2+(0{,}25-j2{,}25)} = \frac{j20(4{,}25+j4{,}25)}{(4{,}25-j4{,}25)(4{,}25+j4{,}25)} =$$

$$= \frac{j20(4{,}25+j4{,}25)}{4{,}25^2+4{,}25^2} = \frac{j85+j^2 85}{4{,}25^2+4{,}25^2} = \frac{-85+j85}{36{,}125} = \frac{-85}{36{,}125} + \frac{j85}{36{,}125}$$

$$\boldsymbol{I_0' = -2,353 + j2,353} \qquad \textbf{(10.4.2)}$$

Photo source: [2] -Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., - 2013. – p 421.

$I_0^{``}$ ni topish



Yechish:

1-mesh uchun,

$$(8+j8)I_1 + j10\, I_3 + j2\, I_2 = 0 \qquad (10.4.3)$$

2-mesh uchun,

$$(4-j4)I_2 + j2\, I_1 + j2\, I_3 = 0 \qquad (10.4.4)$$

3-mesh uchun, $I_3 = 5$ (10.4.5)

(10.4.4) tenglamaga (10.4.5) tenglamadagi qiymatni o‘rniga qo‘yamiz,

$$(4-j4)I_2 + j2\, I_1 + j10 = 0$$

$$-j2\, I_1 = (4-j4)I_2 + j10$$

$$I_1 = \frac{4I_2}{-j2} - \frac{j4I_2}{-j2} + \frac{j10}{-j2},$$

$$I_1 = \frac{4I_2}{-j2\cdot j} + j2I_2 - 5,$$

$$I_1 = \frac{4I_2}{-j^2\cdot 2} + j2I_2 - 5,$$

$$\boldsymbol{I_1 = (2+j2)I_2 - 5} \qquad (10.4.6)$$

(10.4.5) va (10.4.6) tenglamalarni (10.4.3) ga almashtiramiz,

$$(8+j8)[(2+j2)I_2 - 5] - j50 + j2\, I_2 = 0$$

$$(8+j8)(2I_2 + j2I_2 - 5) - j50 + j2\, I_2 = 0$$

$$16I_2 + j16I_2 - 40 + j16I_2 + j^2 16I_2 - 40j - j50 + j2\, I_2 = 0$$

$$j34I_2 - j90 - 40 = 0; \qquad j34I_2 = j90 + 40; \qquad I_2 = \frac{j90+40}{34j};$$

$$I_2 = \frac{j90}{j34} + \frac{40}{j34}; \qquad \frac{1}{j} = -j, \qquad I_2 = \frac{90}{34} - \frac{j40}{34} = 2{,}647 - j1{,}176$$

$$\boldsymbol{I_0^{``} = -I_2 = -2,647 + j1,176} \qquad \mathbf{(10.4.7)}$$

$$\boldsymbol{I_0 = I_0^{`} + I_0^{``} = -5 + j3,529 = 6,12\angle 144,78^\circ\ A}$$

FOYDALANILGAN MANBALAR:

1. https://transportgeography.org/wp-content/uploads/planar_non_planar_graphs.png
2. Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p 421.

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