Mathematics For Information Technology

Week 5: Trigonometry : Trigonometric curves

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outline

- Intended learning outcome
- The general angle
- Trigonometric curves

Intended learning outcomes

- Understanding the basic properties of sine, cosine, and tangent curves.
- Identifying the key features of curves, such as their maxima, minima, and using these features to graph and interpret the curves.

The general angle

- Consider a wheel which is free to rotate about a fixed axis and suppose that one spoke is marked with a thin line of paint.

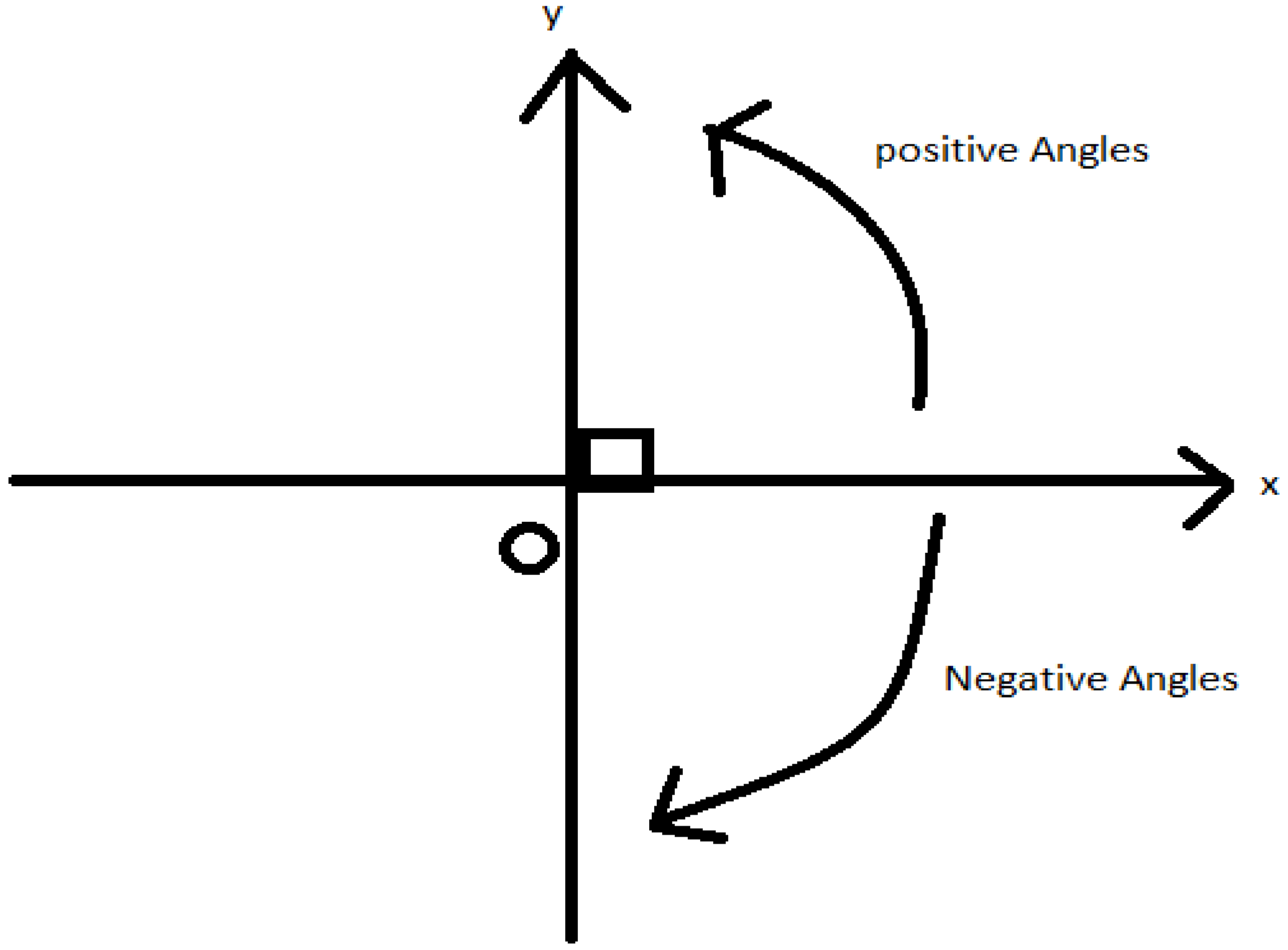
- If the wheel starts from rest and marks one revolution , it turns through 360^0 and also when makes another revolution it turns through 360^0 again.

- Thus we may say that the wheel has turned through a total of 720^0 and by using angles greater than 360° the number of revolutions may be specified, as well as the position of the marked spoke.

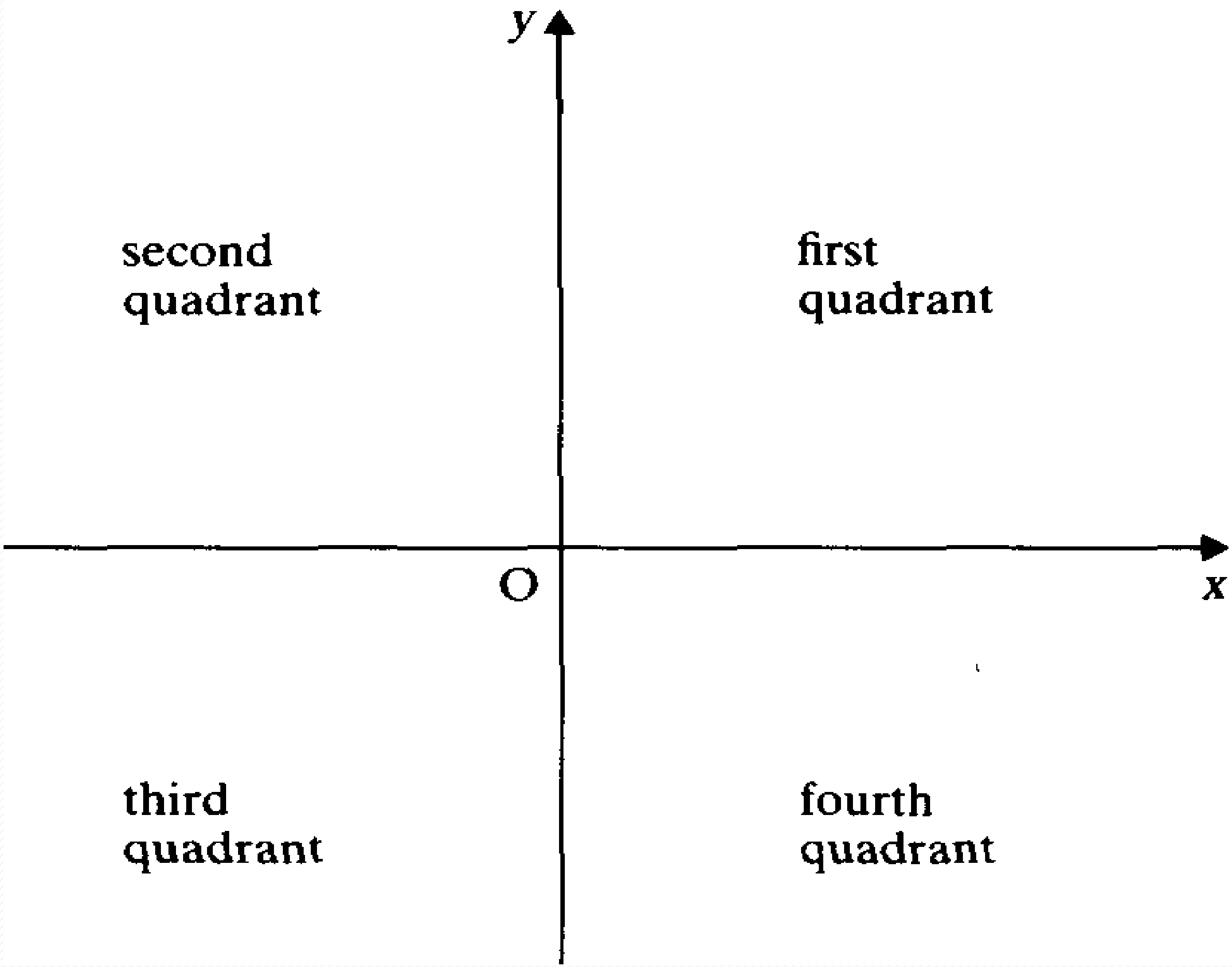
- The positive direction is usually taken to the right and the negative direction is opposite to this as seen in the figure below.

- Similarly, if the wheel rotates anti-clockwise, we take that to be positive, and then a clockwise rotation is considered negative.

- Angles measured from the x-axis in an anti-clockwise sense are positive, and those measured in a clockwise sense are negative


y
positive Angles
x
O
Negative Angles

The axes divide the plane into four quadrants and as angles are measured in an anticlockwise direction from the axis, the quadrants are numbered as in the figure below.


y
second quadrant
first quadrant
O
x
third quadrant
fourth quadrant

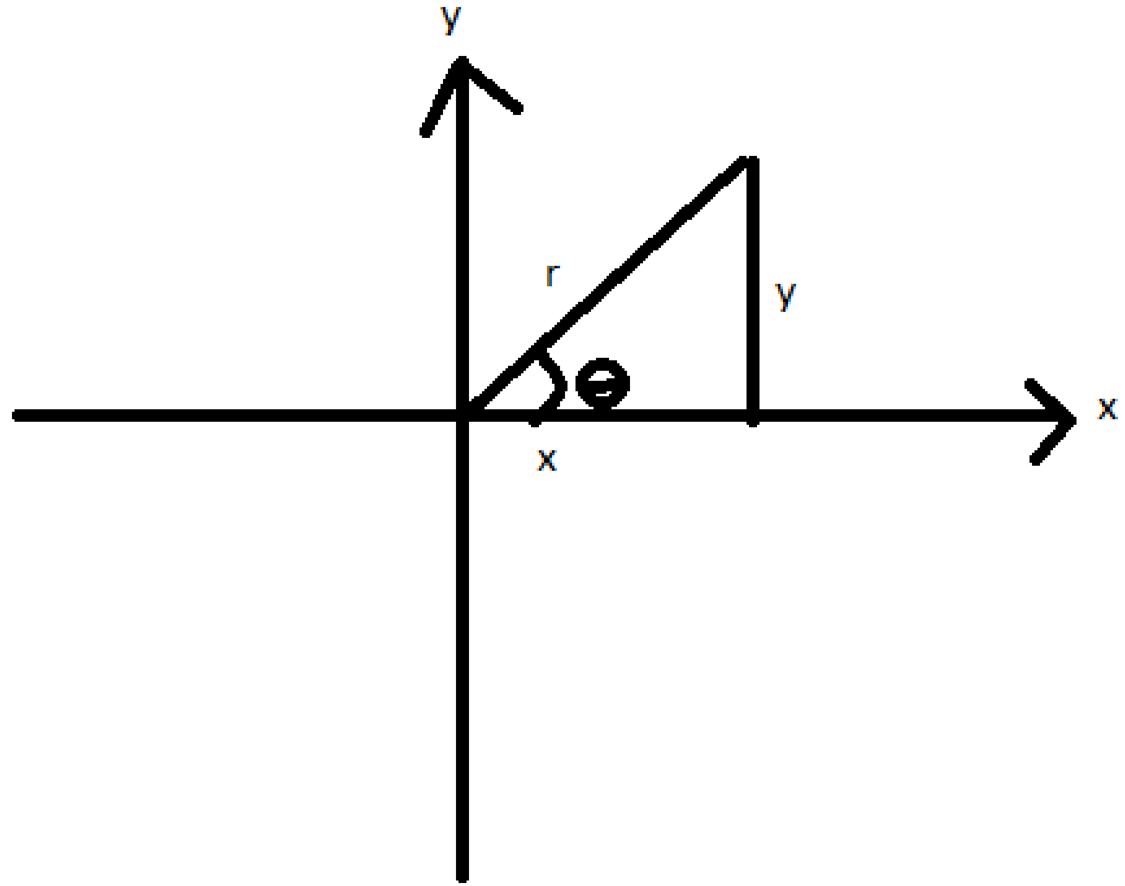
First quadrant

- The first quadrant covers angles ranging from 0^0 *to* 90^0.
- Such angles are called acute angles and all their trigonometric functions and their reciprocals are positive.

$$sin\theta = \frac{opposite}{hypotunus}$$

$$cos\theta = \frac{adjacent}{hypotunus}$$

$$tan\theta = \frac{opposite}{adjacent}$$


y
r
y
θ
x
x

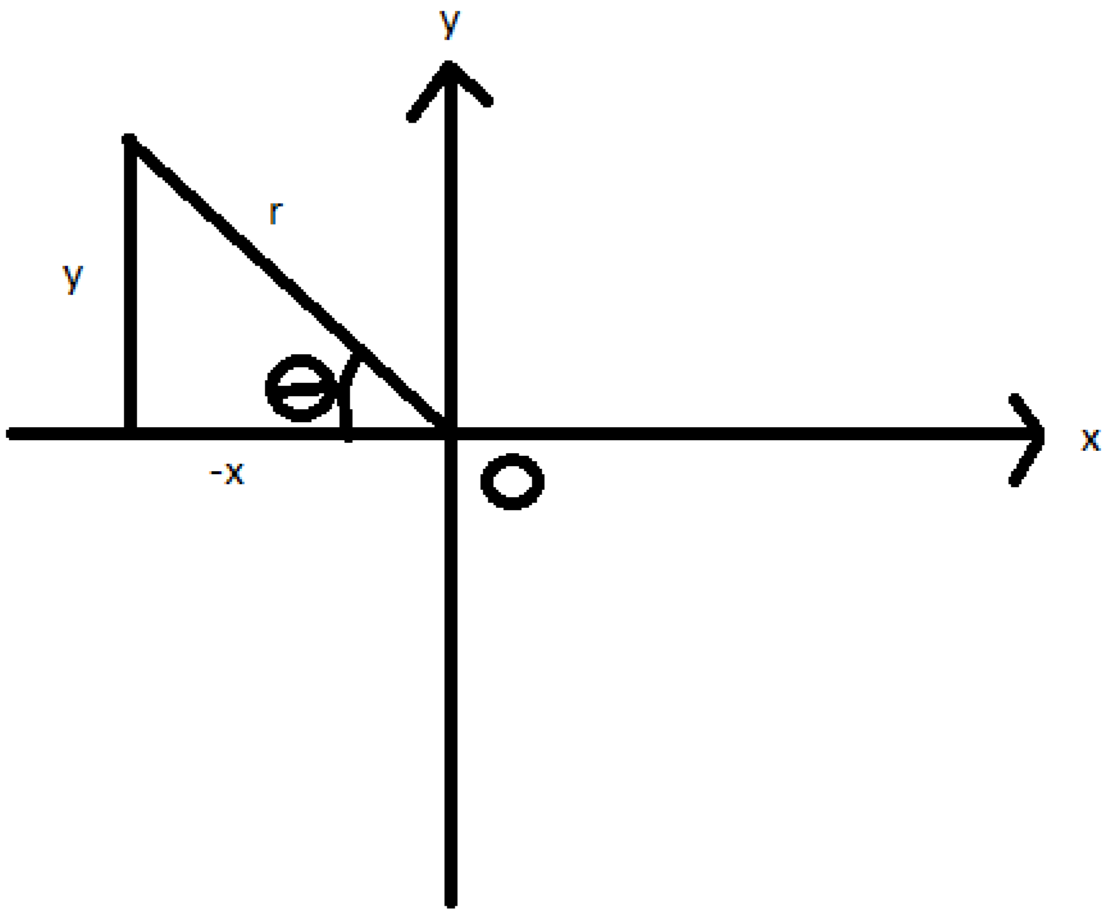
$$sin\theta = \frac{y}{r}, \qquad cosec\theta = \frac{1}{sin\theta} = \frac{r}{y}$$

$$cos\theta = \frac{x}{r}, \qquad sec\theta = \frac{1}{cos\theta} = \frac{r}{x}$$

$$tan\theta = \frac{y}{x}, \qquad cot\theta = \frac{1}{tan\theta} = \frac{x}{y}$$

Second quadrant

- This covers angles between 90^0 *and* 180^0 and such angles are called obtuse angles
- The sine trigonometric ratio and its reciprocal is the only positive ratio.


y
r
y
θ
-x
x
O

$$\sin\theta = \frac{y}{r}, \qquad cosec\theta = \frac{r}{y}$$

$$\cos\theta = \frac{-x}{r}, \qquad \sec\theta = \frac{-r}{x}$$

$$\tan\theta = \frac{y}{-x}, \qquad \cot\theta = \frac{-x}{y}$$

- To obtain obtuse angles in this quadrant we use the following expressions

$$sin\alpha = \sin(180 - \theta)$$

$$cos\alpha = -\cos(180 - \theta)$$

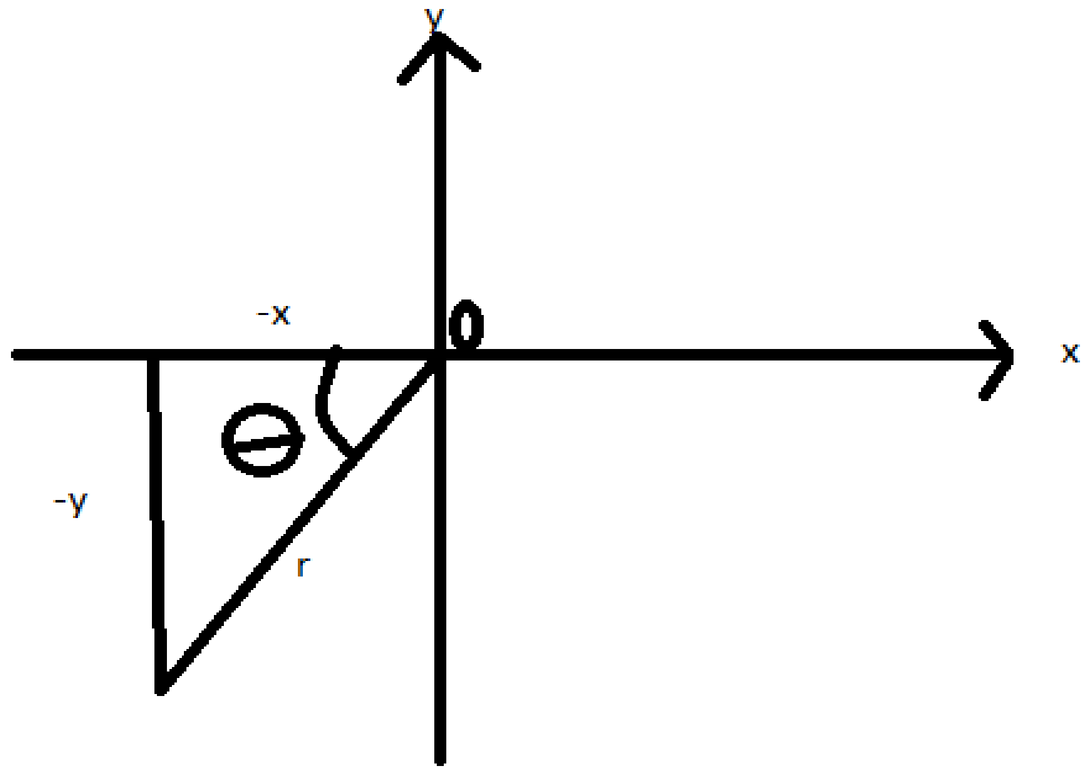
$$tan\alpha = -\tan(180 - \theta)$$

- Where α being an obtuse angle and θ being an acute angle

Third quadrant

- This covers angles in the range of 180^0 *to* 270^0 and such angles are called reflex angles

- $\tan\theta$ and its reciprocal are positive in the third quadrant.


y
-x
0
x
θ
-y
r

$$\sin\theta = \frac{-y}{r}, \qquad cosec\theta = \frac{r}{-y}$$

$$\cos\theta = \frac{-x}{r}, \qquad \sec\theta = \frac{-r}{x}$$

$$\tan\theta = \frac{y}{x}, \qquad \cot\theta = \frac{x}{y}$$

- To obtain angles in this quadrant we use the following expressions

$$sin\alpha = -\sin(180 + \theta)$$

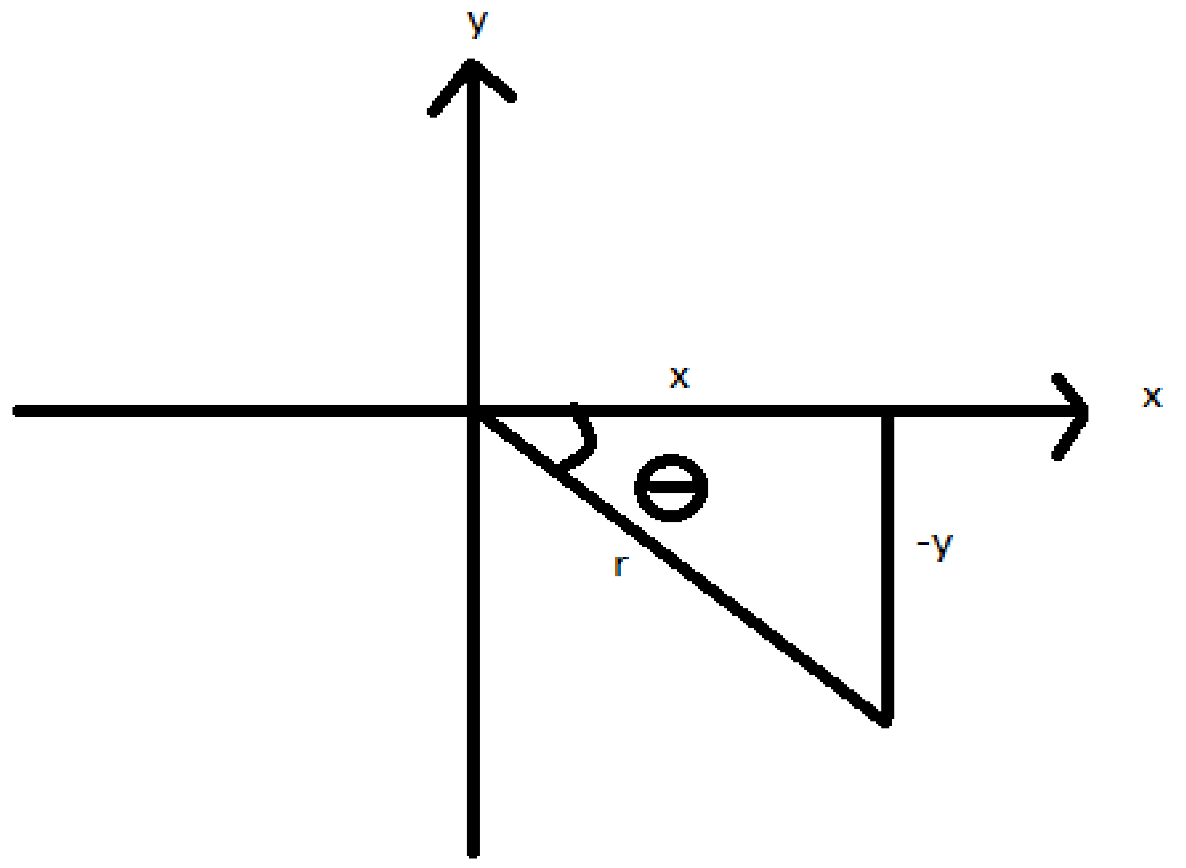
$$cos\alpha = -\cos(180 + \theta)$$

$$tan\alpha = \tan(180 + \theta)$$

- Where α being a reflex angle in the third quadrant and θ being an acute angle

Fourth quadrant

- This covers angles ranging between 270^0 *and* 360^0.
- $\cos\theta$ and its reciprocal are positive in this quadrant


y
x
x
θ
r
-y

$$sin\theta = \frac{-y}{r}, \qquad cosec\theta = \frac{r}{-y}$$

$$cos\theta = \frac{x}{r}, \qquad sec\theta = \frac{r}{x}$$

$$tan\theta = \frac{-y}{x}, \qquad cot\theta = \frac{x}{-y}$$

- To obtain angles in this quadrant we use the following expressions

$$sin\alpha = -\sin(360 - \theta)$$

$$cos\alpha = \cos(360 - \theta)$$

$$tan\alpha = -\tan(360 - \theta)$$

- Where α being a reflex angle in the fourth quadrant and θ being an acute angle

Example:

Express the following Trigonometrical ratios into acute angles

- $sin170$

$$\alpha = 170 = 180 - \theta$$

Introducing sin on both sides

$$= \sin(180 - 170)$$

$$= sin10$$

- $tan300$

$$\alpha = 300 = 360 - \theta$$

$$\theta = 360 - 300$$

Introducing tan on both sides

$$tan\theta = tan60$$

$$tan300 = -tan60$$

- $sin - 50$

For negative angles, we simply pull the negative sign out

$$sin - 50 = -sin50$$

- $cos200$

Its known that 200 lies in the third quadrant

where cos is negative

$$200 = 180 + \theta$$

$$\theta = 200 - 180$$

Introducing cos on both sides

$$cos\theta = \cos(20)$$

$$\therefore\ cos200 = -cos20$$

Solve the following equations for values of θ from 0° to 360°, inclusive:

- $\sin^2 \theta = \frac{1}{4}$

Taking the square root on both sides

$$sin\theta = \pm\frac{1}{2}$$

$$\theta = \pm \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 30^0, 150^0$$

$$\theta = -30^0, 210^0, 330^0$$

$$\therefore\ \theta = 30^0, 150^0, 210^0, 330^0$$

- $4cos2\theta = 1$

$$cos2\theta = \frac{1}{4}$$

$$2\theta = \cos^{-1}(\frac{1}{4})$$

$$2\theta = 75.5^0, 284.5^0, 435.5^0, 644.5^0$$

$$\theta = 37.75^0, 142.25^0, 217.75^0, 322.25^0$$

Trigonometric curves

- Note that for any angle θ, a single value of $sin\theta \; or \; cos\theta$ can be found.
- The same applies for $tan\theta$ with the exception of $\theta = \pm 90^0, \pm 270^0, \ldots$ for values of $tan\theta$ are not defined.

- Thus $sin\theta$ and $cos\theta$ are functions which are defined for all positive and negative values of θ, $tan\theta$ is a function which is defined for all positive and negative values of θ except $\pm 90, \pm 270$

- It is therefore possible to construct tables of values giving ordered pairs for these functions and hence to plot their graphs.

- Such graphs can then be used to find solutions of trigonometric equations.

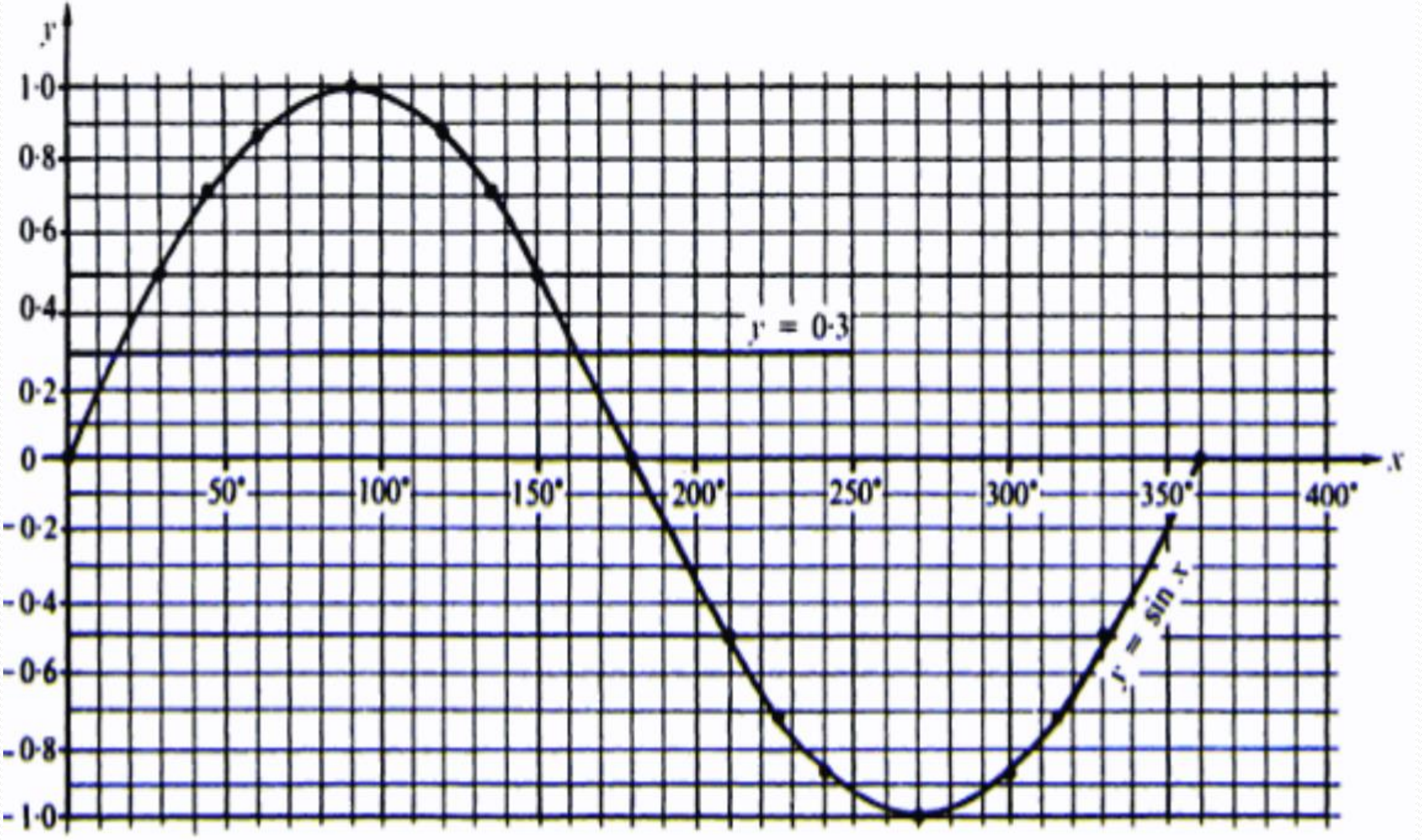
Examples

i. Copy and complete the following table for $y = sinx$

ii. Plot the graph of $y = sinx$ and use your graph to find approximate solutions to equation $sinx = 0.3, for\ 0 \le x \le 360$

solution

x	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
y	0	0.5	0.71	0.87	1.0	0.87	0.71	0.5	0	-0.5	-0.71	-0.87	-1.0	-0.87	-0.71	-0.5	0


y
1·0
0·8
0·6
0·4
0·2
0
-0·2
-0·4
-0·6
-0·8
-1·0
50°
100°
150°
200°
250°
300°
350°
400°
x
y = 0·3
y = sin x

- If $y = sinx$ and $y = 0.3$, solving these simultaneously gives the solution of the equation $sinx = 0.3$.
- We can achieve this on the graph by drawing the line $y = 0.3$
- The points of intersection are $x = 17^0$ and $x = 163^0$

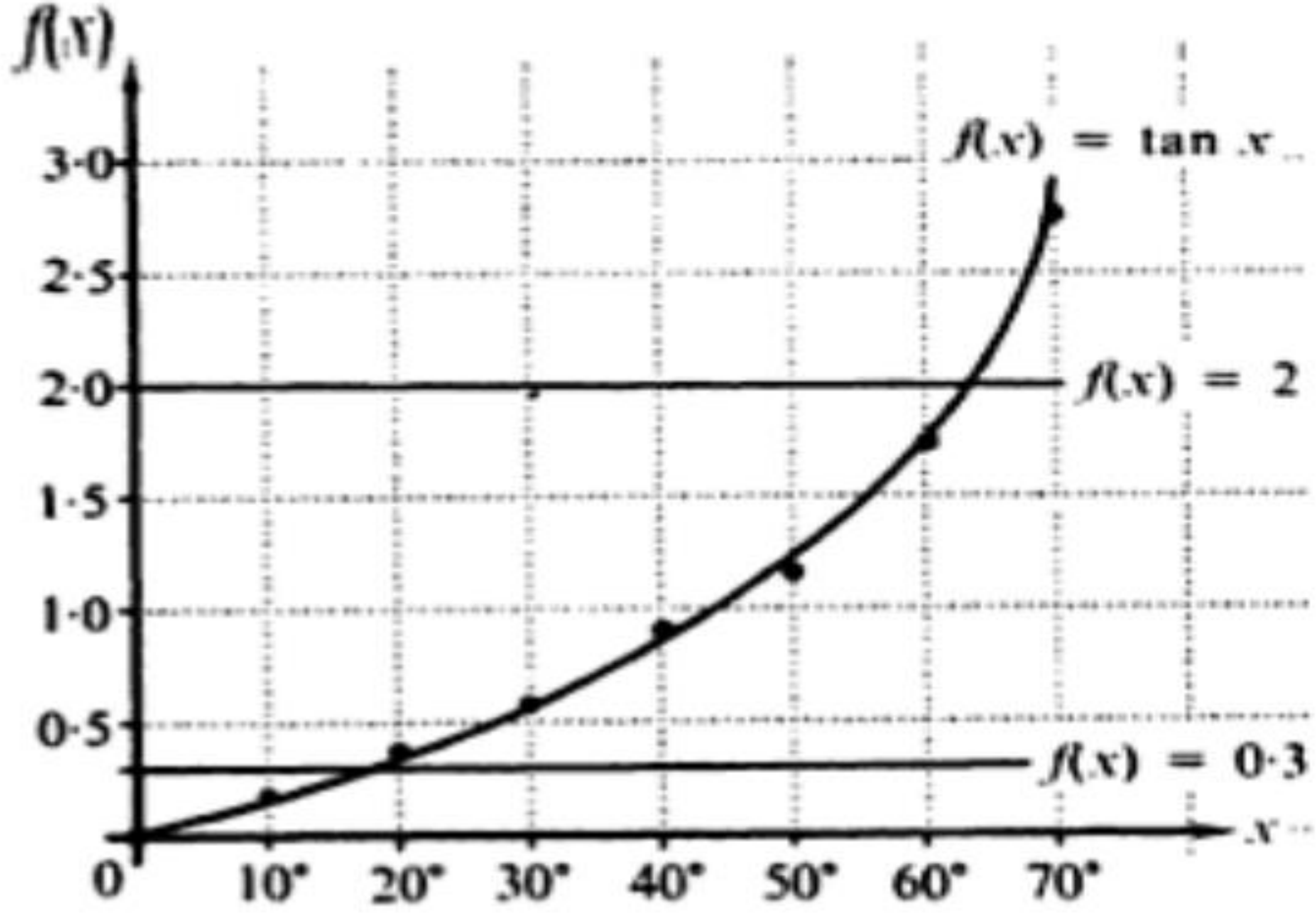
Copy and complete the following table of values for the function

$f: x \to tanx$

Plot the graph of the function $f: x \to tanx$ for the domain $0 \leq x \leq 70^0$ and use your graph to find approximate solution to the following equations for $0 \leq x \leq 70^0$

- $tanx = 0.3$
- $tanx = 2$

x	0	10	20	30	40	50	60	70
$f(x)$	0	0.18	0.36	0.58	0.84	1.19	1.73	2.75



(Sadler, A.J.& Thorning, D.W.S,(2004). *p*103).

- To solve $tanx = 0.3$, we require the values of x for which $f(x) = 0.3$
- From the graph, $tanx = 0.3$ when $x = 17^0$

- Also to solve $tanx = 2$, we require the values of x for which $f(x) = 2$
- From the graph, $tanx = 2$ when $x = 63^0$

- Sketch the graph of $y = \cos(x + 30)$ for $-90^0 \leq x \leq 360^0$

Solution

- When the graph cuts the axes.
- It crosses the x-axis where $y = 0$

$$x + 30 = 90^0, 270^0..$$

$$x = 60^0, 240^0$$

- When it crosses the y-axis when x=o

$$y = cos30^0$$

$$y = 0.87$$

- Maximum value of y occurs where

$$\cos(x + 30) = 1$$

$$x + 30 = 0^0, 360^0$$

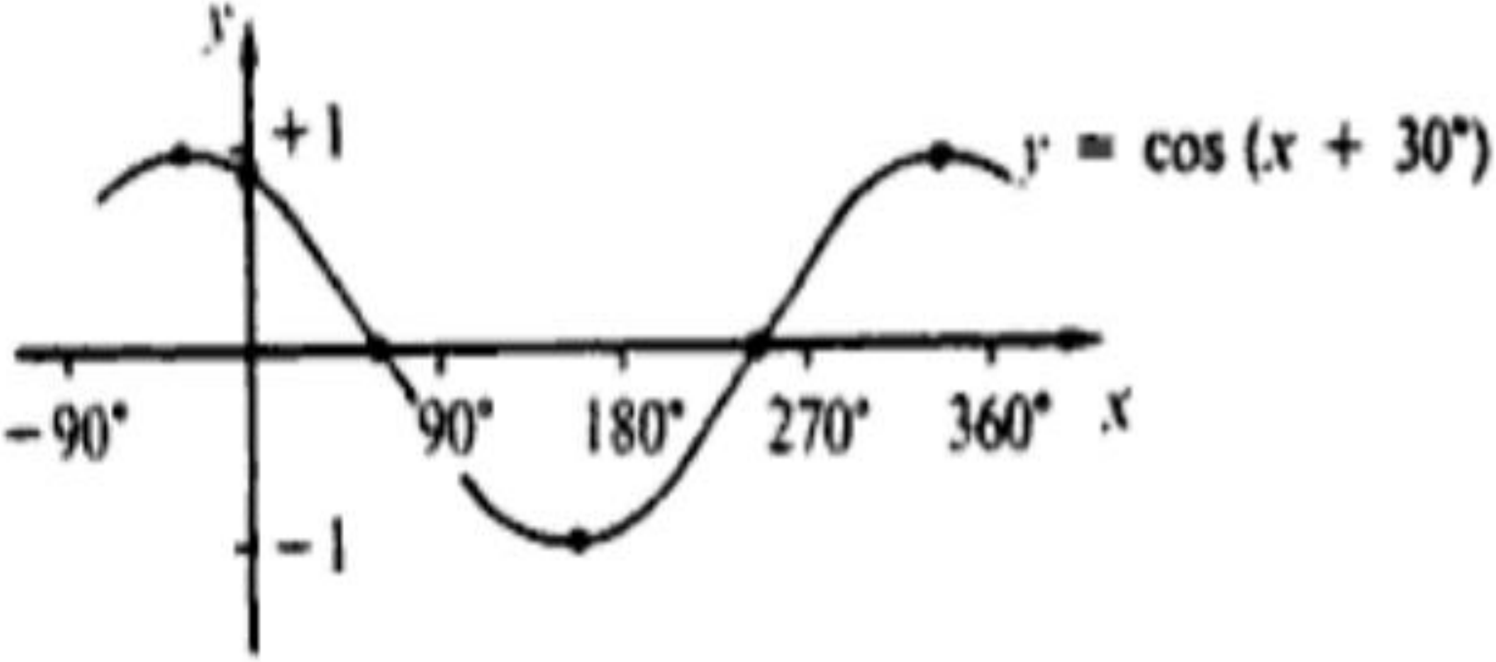
$$x = -30^0, 330^0$$

- Minimum value of y occurs where

$$\cos(x + 30) = -1$$

$$x + 30 = 180, ..$$

$$x = 150^0$$


y
+1
y = cos (x + 30°)
−90°
90°
180°
270°
360°
x
−1

References

- Sadler, A.J.& Thorning, D.W.S. (2004).Understanding pure mathematics. Oxford university press.

- Backhouse, J.K.& Houldsworth, S.P.T.(1985).Pure mathematics 1. PEARSON EDUCATION LIMITED.

Thank you for listening

Next lecture we shall look at compound angles, and half angle formulae