Mathematics For Information Technology

Week 13: Set theory : Applications (Mutually exclusive, independent events)

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outline

- Intended learning outcome
- Mutually Exclusive Events:
- Independent Events:

Learning outcomes

- Understand the concept of mutually exclusive events and be able to identify them in various contexts.
- Apply the concept of mutually exclusive events to calculate probabilities..
- Understand the concept of independent events and be able to identify them in various scenarios.
- Distinguish between mutually exclusive events and independent events.
- Solve Problems involving Both Mutually Exclusive and Independent Events:

Events in probability

- A probability event can be defined as a set of outcomes of an experiment.
- In other words, an event in probability is the subset of the respective sample space.
- A sample space is an entire possible set of outcomes of a random experiment

- The sample space for the tossing three coins simultaneously is given by:

$$S = \{(T,T,T),(T,T,H),(T,H,T),(T,H,H),$$
$$(H,T,T),(H,T,H),(H,H,T),(H,H,H)\}$$

- Suppose, if we want to find only the outcomes which have at least two heads;
- then the set of all such possibilities can be given as:

$$E = \{(H,T,H),(H,H,T),(H,H,H),(T,H,H)\}$$

Thus, an event is a subset of the sample space, i.e., E is a subset of S.

Types of Events

Events Associated with "OR"

- If two events E_1 and E_2 are associated with **OR** then it means that either E_1 or E_2 or both.
- The union symbol (∪) is used to represent OR in probability.
- Thus, the event $E_1 \cup E_2$ denotes $E_1\ OR\ E_2$.
- If we have mutually exhaustive events

$E_1, E_2, E_3 \ldots \ldots \ldots E_n$ associated with sample space S then,

$$E_1 \cup E_2 \cup E_3 \cup \cdots \ldots \ldots En = S$$

Events Associated with “AND”

- If two events E_1 and E_2 are associated with **AND** then it means the intersection of elements which is common to both the events.
- The intersection symbol (∩) is used to represent AND in probability.
- Thus, the event $E_1 \cap E_2$ denotes $E_1\ and\ E_2$.

Mutually Exclusive Events

- **Definition: Mutually exclusive events** are those events that do not occur at the same time i.e. when a coin is tossed then the result will be either head or tail, but we cannot get both the results.
- Such events are also called disjoint events since they do not happen simultaneously.
- If A and B are mutually exclusive events then its probability is given by $P(A\ Or\ B)$ or $P(A \cup B)$

- Two events are considered disjoint events, if the probability of both events occurring at the same time is zero.
- If A and B are the two events, then the probability of disjoint of event A and B is written by:

$$\boldsymbol{Probability\ of\ Disjoint\ (or) Mutually\ Exclusive\ Event}$$

$$\boldsymbol{P(\,A\ and\ B) = \ 0}$$

How to Find Mutually Exclusive Events?

- In probability, the specific addition rule is valid when two events are mutually exclusive.
- It states that the probability of either event occurring is the sum of probabilities of each event occurring.
- If A and B are said to be mutually exclusive events then the probability of an event A occurring or the probability of event B occurring that is P (A∪B) formula is given by P(A) + P(B), i.e.,

$$\boldsymbol{P\ (A\ Or\ B) = P(A) + P(B)}$$

$$\boldsymbol{P\ (A \cup B) = P(A) + P(B)}$$

- Some of the examples of the mutually exclusive events are:
- When tossing a coin, the event of getting head and tail are mutually exclusive. Because the probability of getting head and tail simultaneously is 0.
- In a six-sided die, the events “2” and “5” are mutually exclusive. We cannot get both the events 2 and 5 at the same time when we threw one die.

Rules for Mutually Exclusive Events

- In probability theory, two events are mutually exclusive or disjoint if they do not occur at the same time i.e.
- While tossing the coin, both outcomes are collectively exhaustive, which suggests that at least one of the consequences must happen, so these two possibilities collectively exhaust all the possibilities.
- For example, the outcomes 1 and 4 of a six-sided die, when we throw it, are mutually exclusive but not collectively exhaustive

Conditional Probability for Mutually Exclusive Events

- Conditional probability is stated as the probability of an event A, given that another event B has occurred.
- Conditional Probability for two independent events B has given A is denoted by the expression $P(B/A)$ and it is defined using the equation

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

- For mutually exclusive events, $P\ (A \cap B)\ =\ 0$

$$P(B/A) = \frac{0}{P(A)}$$

$$\boldsymbol{P\ (B/A)\ =\ 0}$$

Example

What is the probability of a die showing a number 3 or number 5?

Solution

Let,

$P(3)$ is the probability of getting a number 3

$P(5)$ is the probability of getting a number 5

$$P(3) = \frac{1}{6} \text{ and } P(5) = \frac{1}{6}$$

- So,

$$P(3 \; or \; 5) = P(3) + P(5)$$

$$P(3 \; or \; 5) = \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) = \frac{2}{6}$$

$$P(3 \; or \; 5) = \frac{1}{3}$$

❖ Therefore, the probability of a die showing 3 or 5 is $\frac{1}{3}$.

Example

Three coins are tossed at the same time. What is the event of receiving at least 2 heads and the event of getting no heads and event of getting heads on the second coin. Which of these is mutually exclusive?

Solution:

Let A represent 2 heads

Let B represent no head

Let C represent head on second coin

Firstly, let us create a sample space for each event.

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

- Therefore, we have to include all the events that have two or more heads.

$$A = \{HHT, HTH, THH, HHH\}.$$

- This set A has 4 elements or events in it i.e. $n(A) = 4$
- In the same way, for event B, we can write the sample as:

$$B = \{TTT\}\ and\ n(B) = 1$$

$$C = \{THT, HHH, HHT, THH\}\ and\ n(C) = 4$$

- So B & C and A & B are mutually exclusive since they have nothing in their intersection.

Example:

The likelihood of the 3 teams A, B, C winning a football match are $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{9}$ respectively. Find the probability that

a) out of the three teams, either team A or team B will win

b) either team A or team B or team C will win

c) none of the teams will win the match

d) neither team A nor team B will win the match

solution

a) P (A or B will win) $= \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

b) P (A or B or C will win) $= \frac{1}{3} + \frac{1}{5} + \frac{1}{9} = \frac{29}{45}$

c) P (none will win) = $1 - P\,(A \text{ or } B \text{ or } C \text{ will win})$

$$= 1 - \frac{29}{45}$$

$$= \frac{16}{45}$$

d) P(neither A nor B will win)

$$= 1 - P(\text{either } A \text{ or } B \text{ will win})$$

$$= 1 - \frac{8}{15}$$

$$= \frac{7}{15}$$

Example

- If $P\ (A) = \frac{1}{3}, P\ (B) = \frac{1}{2}$ and $P\ (A \cup B) = \frac{5}{6}$ then events A and B are:

Solution

$$P\ (A \cup B)\ =\ P(A)\ +\ P(B)\ -\ P(A \cap B)$$

$$\frac{5}{6} = \left(\frac{1}{3}\right) + \left(\frac{1}{2}\right) - P\ (A \cap B)$$

$$\Rightarrow P\ (A \cap B)\ =\ 0$$

The events A and B are mutually exclusive.

Independent Events

- **Independent events** are those events whose occurrence is not dependent on any other event.
- For example, if we flip a coin in the air and get the outcome as Head, then again if we flip the coin but this time we get the outcome as Tail.
- In both cases, the occurrence of both events is independent of each other.

- In Probability, the set of outcomes of an experiment is called events.
- If the probability of occurrence of an event A is not affected by the occurrence of another event B, then A and B are said to be independent events.
- Consider an example of rolling a die. If A is the event 'the number appearing is odd' and B be the event 'the number appearing is a multiple of 3', then

$$P(A) = \frac{3}{6} = \frac{1}{2} \; and \; P(B) = \frac{2}{6} = \frac{1}{3}$$

❖Also A and B is the event 'the number appearing is odd and a multiple of 3' so that

$$P(A \cap B) = \frac{1}{6}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\left(\frac{1}{6}\right)}{\left(\frac{1}{3}\right)}$$

$$P(A) = P(A/B) = \frac{1}{2}$$

- which implies that the occurrence of event B has not affected the probability of occurrence of the event A .
- If A and B are independent events, then

$$P(A/B) = P(A)$$

- Using the Multiplication rule of probability,

$$P(A \cap B) = P(B).P(A/B)$$

$$P(A \cap B) = P(B).P(A)$$

Note: A and B are two events associated with the same random experiment, then A and B are known as independent events if

$$P(A \cap B) = P(B).P(A)$$

Example:

If A and B are two independent events, then A and B' is:

solution

$A \cap B'$ and $A \cap B$ are mutually exclusive events such that;

$$A = (A \cap B') \cup (A \cap B)$$

$$P(A) = P(A \cap B') + P(A \cap B)$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

= P(A) – P(A).P(B) (Since A and B are independent)

$$= P(A \cap B')$$

$$= P(A)\,(1 - P(B)) = P(A)\,P(B')$$

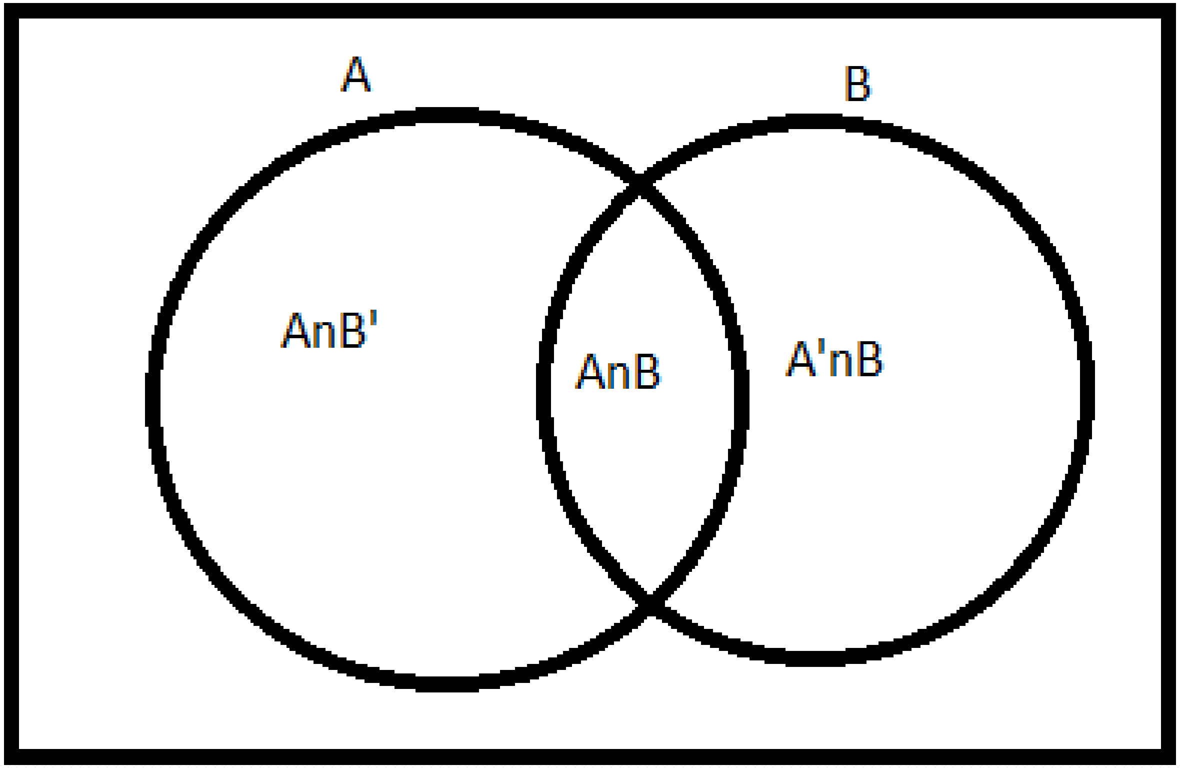
- Thus, A and B' are also independent.

Independent Events Venn Diagram

- Let us proof the condition of independent events using a Venn diagram.
- **Theorem:** If A and B are independent events, then the events A and B' are also independent.
- **Proof:** The events A and B are independent, so,

$$P(A \cap B) = P(A)\,P(B).$$

- Let us draw a Venn diagram for this condition:


A
B
AnB'
AnB
A'nB

- From the Venn diagram, we see that the events $A \cap B$ and $A \cap B'$ are mutually exclusive, and together they form the event A.

$$A = (A \cap B) \cup (A \cap B')$$

Also, $P(A) = P[(A \cap B) \cup (A \cap B')]$

or,

$$P(A) = P(A \cap B) + P(A \cap B')$$

or,

$$P(A) = P(A)\,P(B) + P(A \cap B')$$

Example

Let X and Y are two independent events such that

$P(X) = 0.3$ and $P(Y) = 0.7$.

Find

1) P(X and Y),
2) P(X or Y),
3) P(Y not X), and
4) P(neither X nor Y).

Solution:

Given $P(X) = 0.3$ and $P(Y) = 0.7$ and events X and Y are independent of each other.

$$P(X\ and\ Y) = P(X \cap Y)$$

$$= P(X)P(Y) = 0.3 \times 0.7$$

$$= 0.21$$

$$P(X\ or\ Y) = P(X \cup Y)$$

$$= P(X) + P(Y) - P(X \cap Y)$$

$$= 0.3 + 0.7 - 0.21$$

$$= 0.79$$

$$P(Y\ not\ X) = P(Y \cap X')$$

$$= P(Y) - P(X \cap Y)$$

$$= 0.7 - 0.21$$

$$= 0.49$$

$$P(neither\ X\ nor\ Y)$$

$$= P(X' \cap Y')$$

$$= 1 - P(X \cup Y)$$

$$= 1 - 0.79$$

$$= 0.21$$

References

- Jech, T. (2002). Set Theory (3rd ed.). Springer
- Enderton, H. B. (1977). Elements of Set Theory. Academic Press.
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End of lecture 13

Next topic: Matrices

Thank you