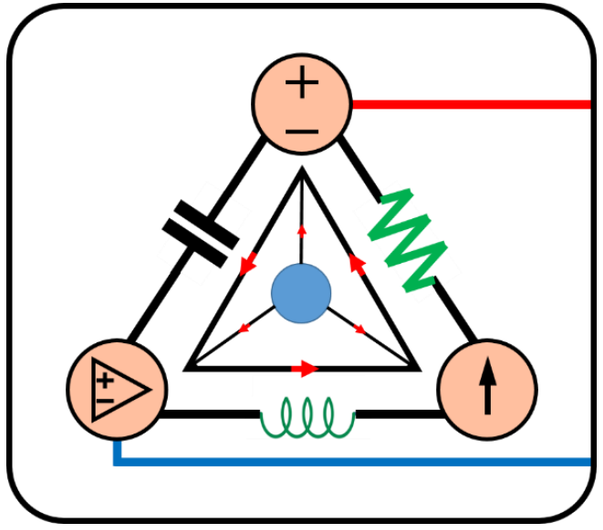
3-Mavzu: Zanjirlarni tahlil qilish usullari.

(Lecture-3: Methods of Analysis)

3-Mavzuning 1-qismi

(Part 1 of the Lecture-3)



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3-Mavzu: Zanjirlarni tahlil qilish usullari.

(Lecture-3: Methods of Analysis)

O‘quv rejasi:

3.1. Tugun tahlili.

3.2. Kuchlanish manbalari bilan tugunlarni tahlil qilish.

3.3. Mesh (tarmoq) tahlili.

3.4. Tok kuchi manbalari bilan tarmoqni tahlil qilish.

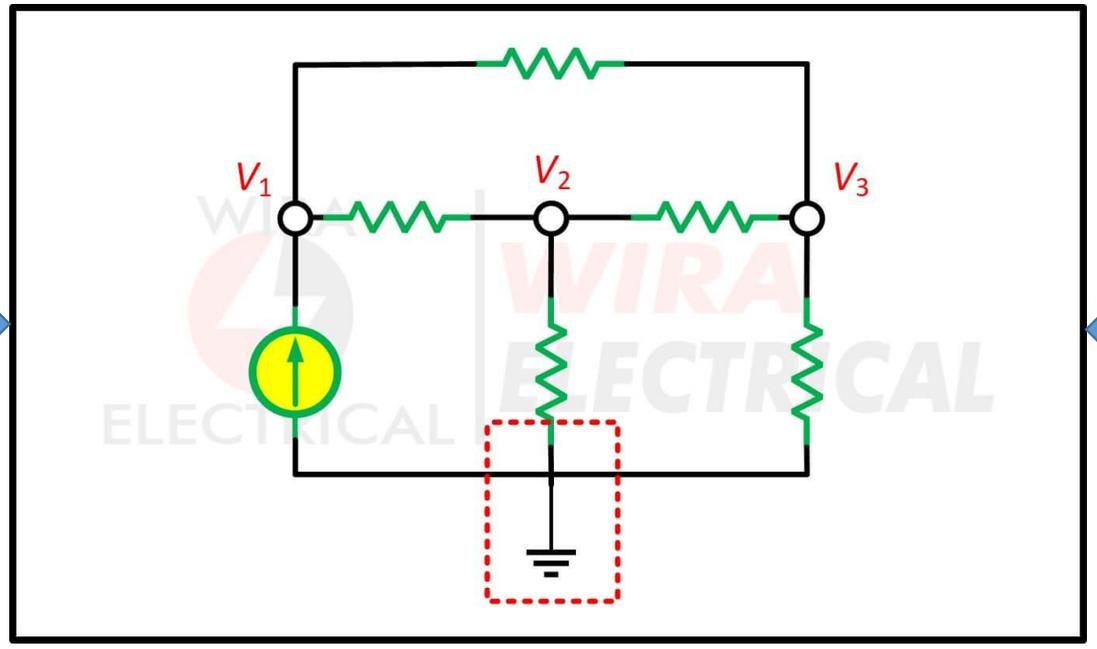
3.5. Tekshiruv yo‘li bilan tugun va mesh tahlillari.

3.6. Qo‘llanilishi.

3.1. Tugun tahlili.

Elektr zanjirlari nazariyasining asosiy qonunlarini (Om qonuni va Kirxgof qonunlari) tushunganimizdan so‘ng, endi biz ushbu qonunlarni zanjirni tahlil qilishda ishlab chiqilgan ikkita kuchli usullarni qo‘llashga harakat qilamiz.

Kirxgofning tok kuchiga oid qonuni (KCL)



Tugun tahlilida tizimli qo‘llashga asoslangan

Photo source: [1] - https://wiraelectrical.com/wp-content/uploads/2021/06/Nodal-voltage-analysis-1.jpg

Tugun tahlili asosan elektr zanjir tugunidagi noma’lum kuchlanishlarni topish uchun ishlatiladi.

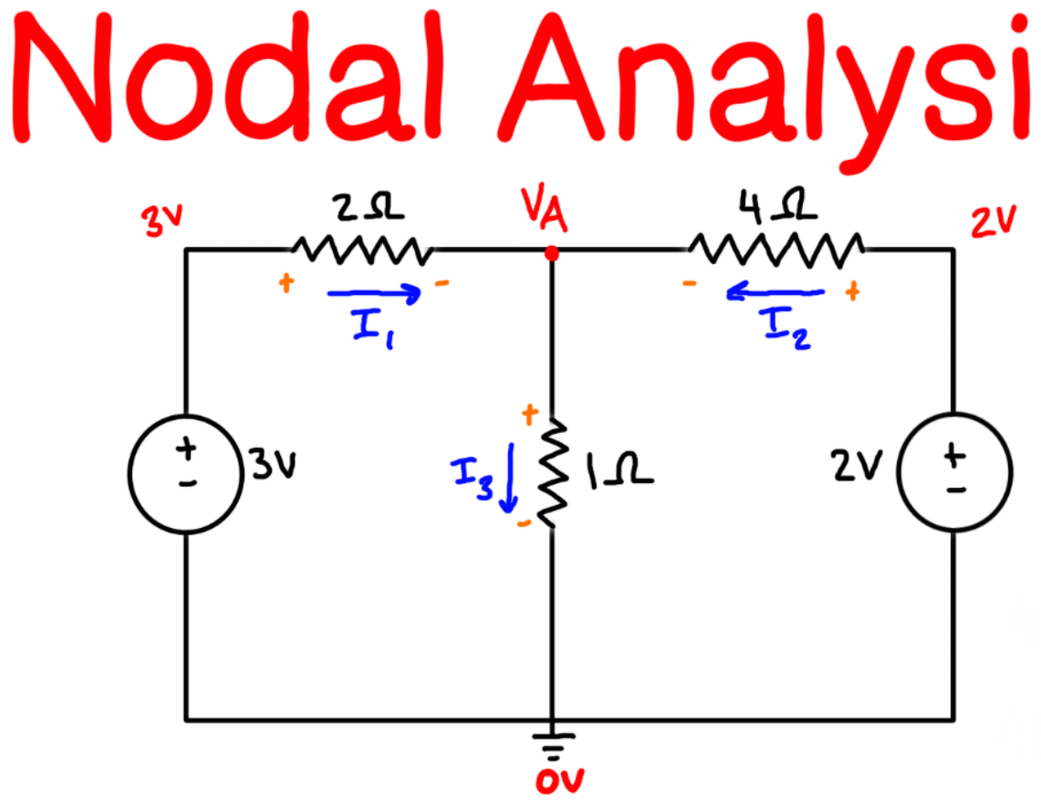
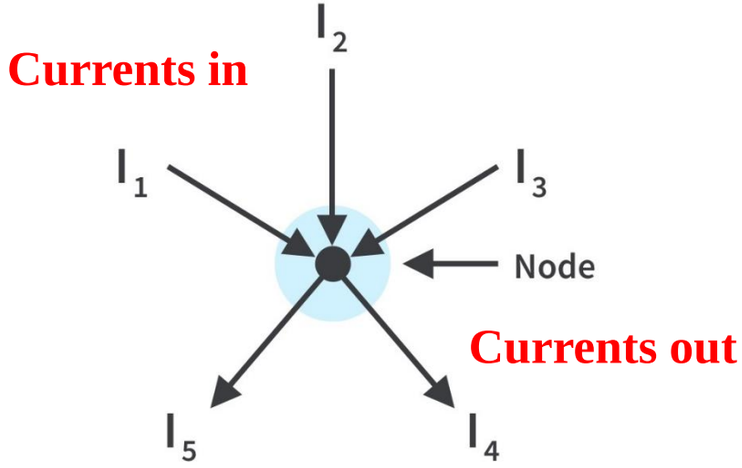


Photo source: [2] - https://www.engineer4free.com/uploads/1/0/2/9/10296972/nodal-analysis-for-circuits-explained_orig.png

Tugunga KCL qo‘llaniladi

$$\sum i_{in} = \sum i_{out}$$



$$I_1 + I_2 + I_3 = I_4 + I_5$$

Photo source: [3] - https://dwma4bz18k1bd.cloudfront.net/tutorials/kcl-site_2021-02-17-013544.jpg

Kuchlanish manbalari bo‘lmagan n ta tugunli zanjir berilgan bo‘lsa, zanjirning tugun tahlili quyidagi uchta bosqichni o‘z ichiga oladi.

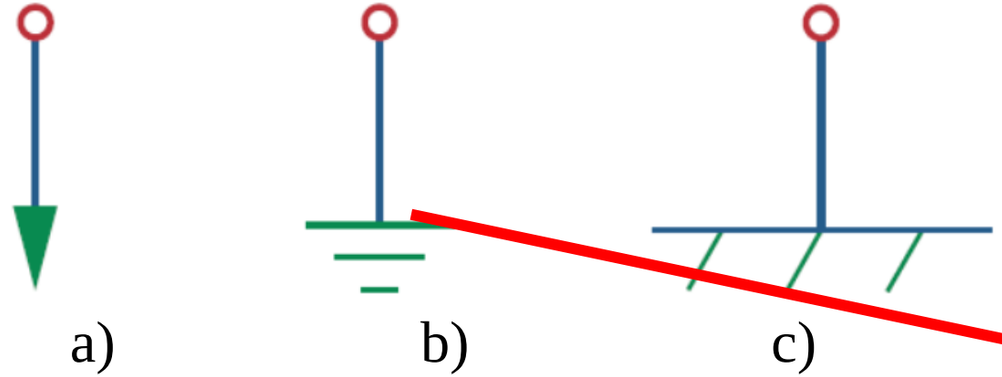
Tugun kuchlanishlarini aniqlash bosqichlari:

1. Yo‘naltiruvchi sifatida bitta tugunni tanlang. Qolgan $n-1$ tugunlarga $u_1, u_2, \dots, u_{n-1}$ kuchlanishlarini belgilang. Kuchlanishlar mos yozuvlar bilan har bir tugunga nisbatan havola qilinadi;

2. $n-1$ noaniq tugunlarining har biriga KCL ni qo‘llang. Shaxobchalardagi tok oqimlarini tugun kuchlanishlari bo‘yicha ifodalash uchun Om qonunidan foydalaning;

3. Noma’lum tugun kuchlanishlarini aniqlash uchun hosil bo‘lgan tenglamalarni yechiladi.

Tugun tahlilining **birinchi bosqichida** tugun shartidan kelib chiqib, zanjirdagi tugun tanlanadi va unga mos belgi qo‘yiladi. Belgilangan tugun odatda yer (*ground – massa*) deb ataladi, chunki u nol potensialga ega deb taxmin qilinadi.



a) b) c)

3.1-rasm. Yo‘naltiruvchi tugunini ko‘rsatish uchun umumiy belgilar:

a) umumiy massa; b) massa; c) tayanch massa.

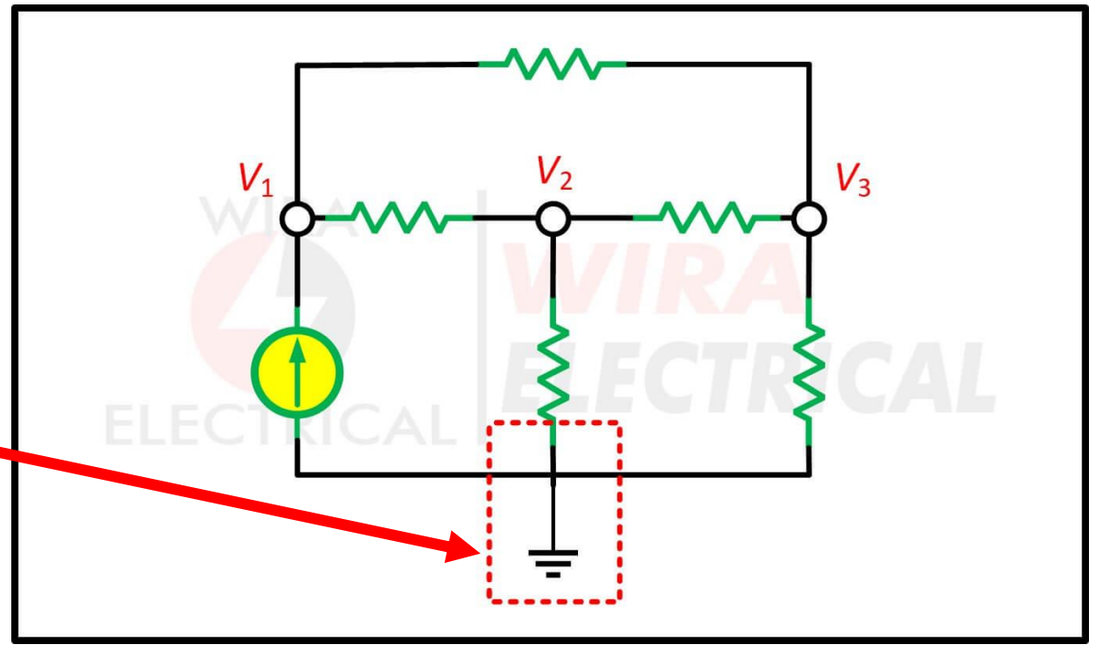
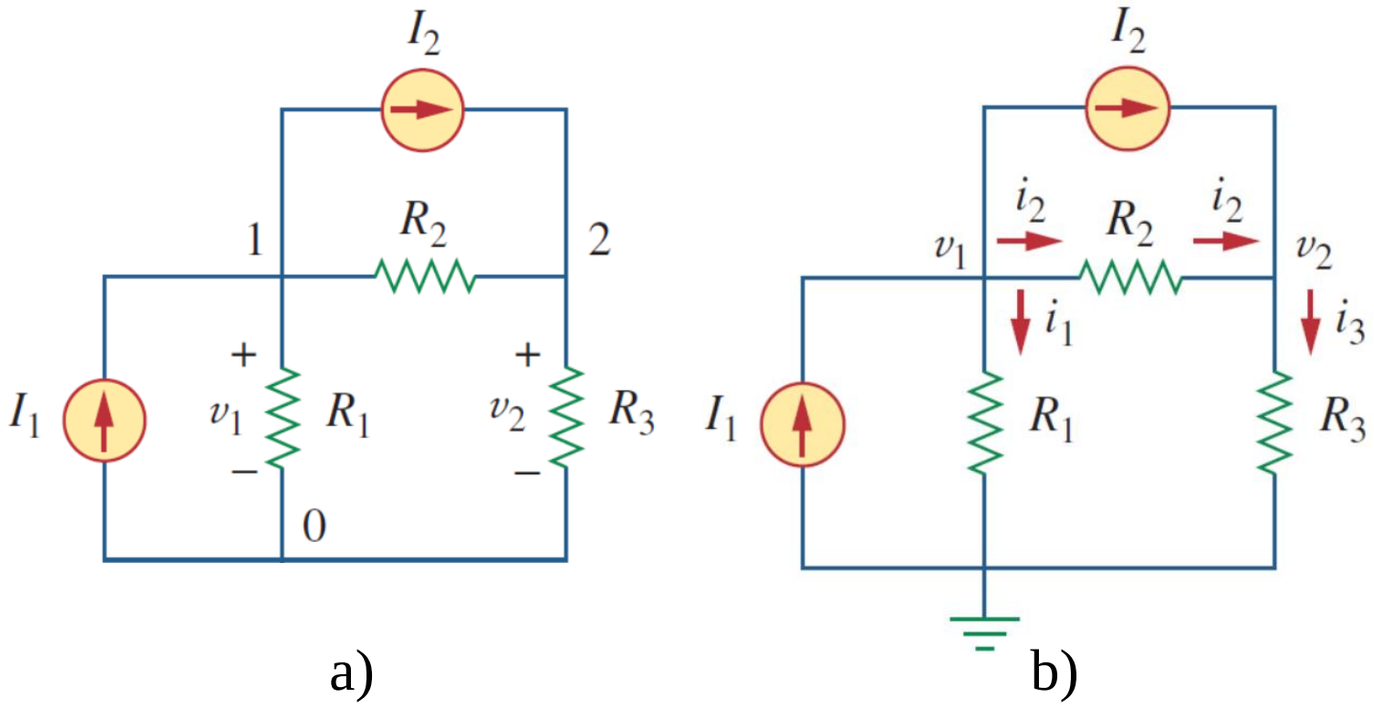


Photo source: [1] - https://wiraelectrical.com/wp-content/uploads/2021/06/Nodal-voltage-analysis-1.jpg

Ikkinchi bosqichda, zanjirdagi har bir tugunlar uchun KCLni qo‘llaymiz.



a) b)

3.2-rasm. Oddiy zanjir uchun tugun tahlili.

1 tugunga nisbatan KCL quyidagi ifodani beradi:

$$\boldsymbol{I_1 = I_2 + i_1 + i_2} \qquad (3.1)$$

2 tugunga nisbatan esa,

$$\boldsymbol{I_2 + i_2 = i_3} \qquad (3.2)$$

Demak, tok kuchi rezistordagi yuqori potensialdan quyi potensialga qarab harakatlanadi.

$$\boldsymbol{i = \frac{u_{yuqori} - u_{quyi}}{R}} \qquad (3.3)$$

Buni hisobga olib, 3.2-rasm, *b* dan quyidagini olamiz,

$$i_1 = \frac{u_1 - 0}{R_1} \quad \text{yoki} \quad i_1 = G_1 u_1$$

$$i_2 = \frac{u_1 - u_2}{R_2} \quad \text{yoki} \quad i_2 = G_2(u_1 - u_2) \qquad (3.4)$$

$$i_3 = \frac{u_2 - 0}{R_3} \quad \text{yoki} \quad i_3 = G_3 u_3$$

(3.4) tenglamani mos ravishda (3.1) va (3.2) tenglamalar natijalariga almashtiramiz.

$$I_1 = I_2 + \frac{u_1 - 0}{R_1} + \frac{u_1 - u_2}{R_2} \qquad (3.5)$$

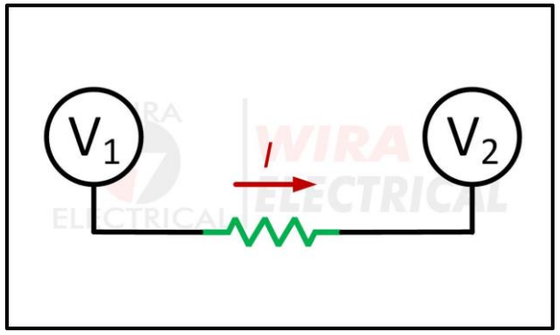
$$I_2 + \frac{u_1 - u_2}{R_2} = \frac{u_2 - 0}{R_3} \qquad (3.6)$$

O‘tkazuvchanlik nuqtai nazaridan (3.5) va (3.6) tenglamalar quyidagicha bo‘ladi.

$$I_1 = I_2 + G_1 u_1 + G_2(u_1 - u_2) \qquad (3.5)$$

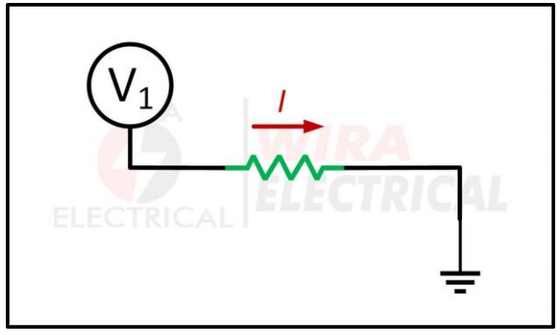
$$I_2 + G_2(u_1 - u_2) = G_3 u_3 \qquad (3.6)$$

Tugun tahlilini resistor bilan qo‘llanilishi



$$I = \frac{U_1 - U_2}{R}$$

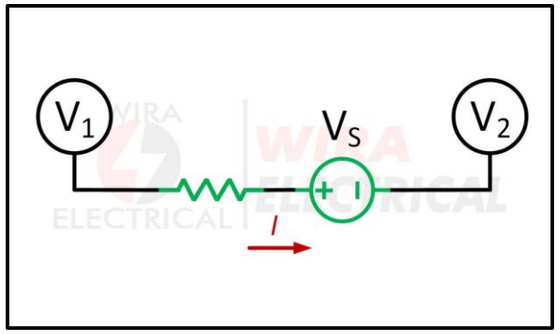
Photo source: [4] - https://wiraelectrical.com/wp-content/uploads/2021/06/Nodal-voltage-analysis-2.jpg



$$I = \frac{U_1 - 0}{R} = \frac{U_1}{R}$$

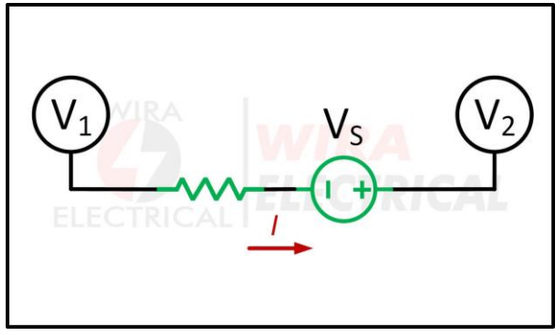
Photo source: [5] - https://wiraelectrical.com/wp-content/uploads/2021/06/Nodal-voltage-analysis-3.jpg

Tugun tahlilini kuchlanish manbai bilan qo‘llanilishi



$$I = \frac{U_1 - U_s - U_2}{R}$$

Photo source: [6] - https://wiraelectrical.com/wp-content/uploads/2021/06/Nodal-voltage-analysis-4.jpg



$$I = \frac{U_1 + U_s - U_2}{R}$$

Photo source: [7] - https://wiraelectrical.com/wp-content/uploads/2021/06/Nodal-voltage-analysis-5.jpg

Tugun tahlilini tok kuchi manbai bilan qo‘llanilishi

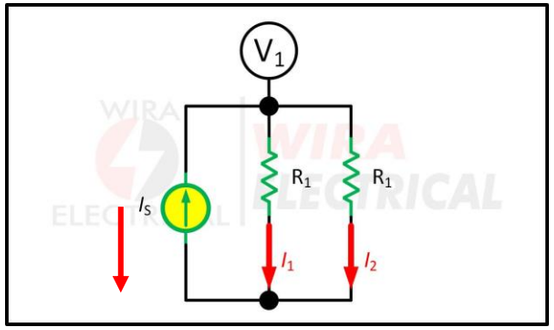


Photo source: [8] - https://wiraelectrical.com/wp-content/uploads/2021/06/Nodal-voltage-analysis-6.jpg

$$I_s = I_1 + I_2$$

$$I_s = \frac{U_1}{R_1} + \frac{U_1}{R_2}$$

$$-I_s + I_1 + I_2 = 0$$

$$-I_s + \frac{U_1}{R_1} + \frac{U_1}{R_2} = 0$$

Tugun tahlilining **uchinchi bosqichi** tugun kuchlanishlarini hal qilishdir.

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \qquad (3.9)$$

u_1 va u_2 tugun kuchlanishlarini har qanday standart usul yordamida aniqlashimiz mumkin.

- **Inversion matritsa usuli**
- **Kramer qoidasi**
- **Soddalashtirish usuli**
- **Almashtirish usuli**

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad\quad \vdots \qquad\quad \vdots & \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_2 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$\mathbf{AX} = \mathbf{B}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta}, \quad \dots, \quad x_n = \frac{\Delta_n}{\Delta}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

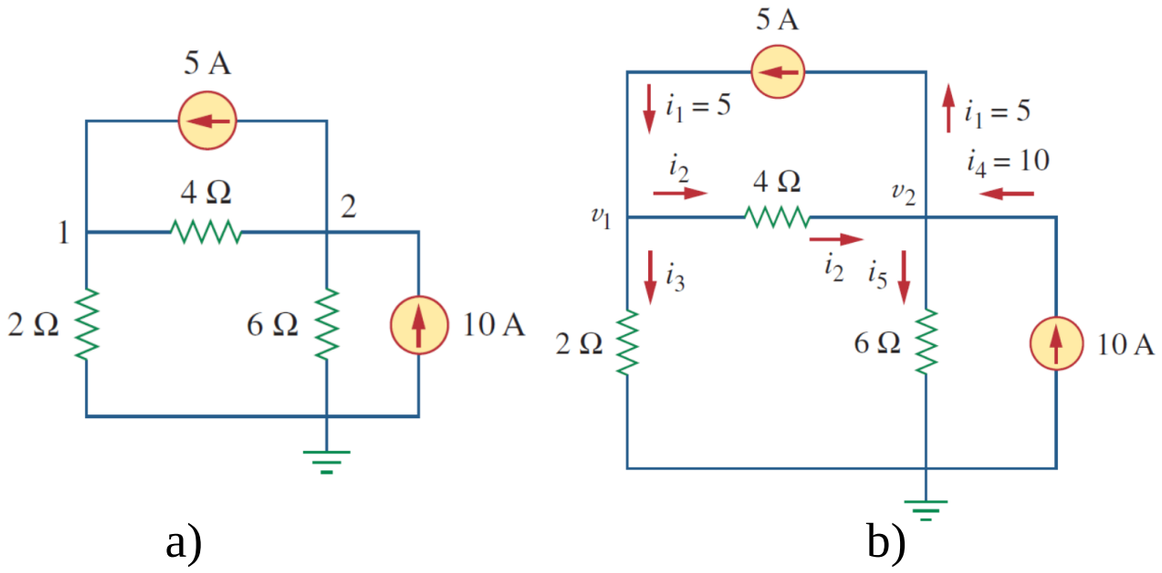
$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 & \cdots & a_{1n} \\ a_{21} & b_2 & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & b_n & \cdots & a_{nn} \end{vmatrix}, \dots, \Delta_n = \begin{vmatrix} a_{11} & a_{12} & \cdots & b_1 \\ a_{21} & a_{22} & \cdots & b_2 \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & b_n \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} \quad \begin{matrix} - \\ - \\ - \end{matrix} \quad \begin{matrix} + \\ + \\ + \end{matrix}$$

$$= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{33}a_{12}a_{21}$$

Equation source: [9] - Fundamentals of Electric Circuits, Charles K. Alexander and Matthew N. O. Sadiku / 5th edition, the McGraw-Hill Companies, Inc., -2013. – p A-A-1.

3.1.1-masala: 3.3-rasmda ko‘rsatilgan zanjirdagi tugun kuchlanishlarini hisoblang.



a) b)

3.3-rasm.
a) asl zanjir; b) tahlil qilingan zanjir.

Yechish: 1 tugunga KCL va Om qonunini qo‘llaymiz

$$i_1 = i_2 + i_3 \rightarrow 5 = \frac{u_1 - u_2}{4} + \frac{u_1 - 0}{2}$$

$$20 = u_1 - u_2 + 2u_1 \text{ yoki } 3u_1 - u_2 = 20 \quad (3.1.1)$$

2 tugunga KCL va Om qonunini qo‘llaymiz.

$$i_2 + i_4 = i_1 + i_5 \rightarrow \frac{u_1 - u_2}{4} + 10 = 5 + \frac{u_2 - 0}{6}$$

$$3u_1 - 3u_2 + 120 = 60 + 2u_2 \text{ yoki } -3u_1 + 5u_2 = 60 \quad (3.1.2)$$

1-usul:

$$3u_1 - u_2 = 20$$
$$-3u_1 + 5u_2 = 60$$
$$4u_2 = 80 \rightarrow u_2 = 20$$

$$3u_1 - 20 = 20$$
$$u_1 = \frac{40}{3} = 13{,}33\ V$$

2-usul: $\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

$$\Delta_1 = \begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix} = 100 + 60 = 160$$

$$\Delta_2 = \begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix} = 180 + 60 = 240$$

$$u_1 = \frac{\Delta_1}{\Delta} = \frac{160}{12} = 13{,}33\ V$$

$$u_2 = \frac{\Delta_2}{\Delta} = \frac{240}{12} = 20\ V$$

$i_1 = 5\ A,$

$i_2 = \frac{u_1 - u_2}{4} = -1{,}66\ A,$

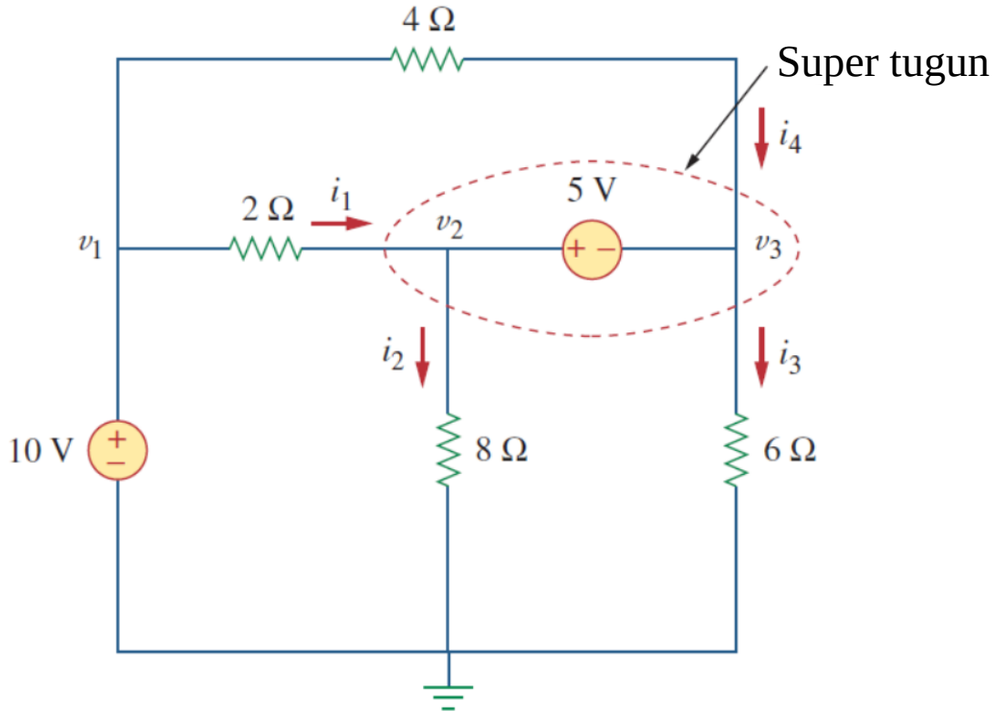
$i_3 = \frac{u_1}{2} = 6{,}66\ A,$

$i_4 = 10\ A,$

$i_5 = \frac{u_2}{6} = 3{,}33\ A$

3.2. Kuchlanish manbalari bilan tugunlarni tahlil qilish.

Kuchlanish manbalarining tugun tahlilida quyidagi ikkita imkoniyatlarni ko‘rib chiqaylik.



3.4-rasm. Super tugunli zanjir.

1-holat: Agar kuchlanish manbai yo‘naltiruvchi tugun va yo‘naltiruvchi bo‘lmagan tugunlar bilan o‘zaro ulangan bo‘lsa, biz shunchaki kuchlanish manbasining kuchlanishiga teng bo‘lmagan noaniq tugundagi kuchlanishni o‘rnatamiz.

$$u_1 = 10\ V \qquad (3.10)$$

2-holat: Agar kuchlanish manbai (*bog‘liq yoki mustaqil*) ikkita yo‘naltiruvchi bo‘lmagan tugunlar orasiga ulangan bo‘lsa, ushbu tugunlar umumlashtirilgan tugun (*generalized node*) yoki super tugun (*supernode*) ni hosil qiladi.

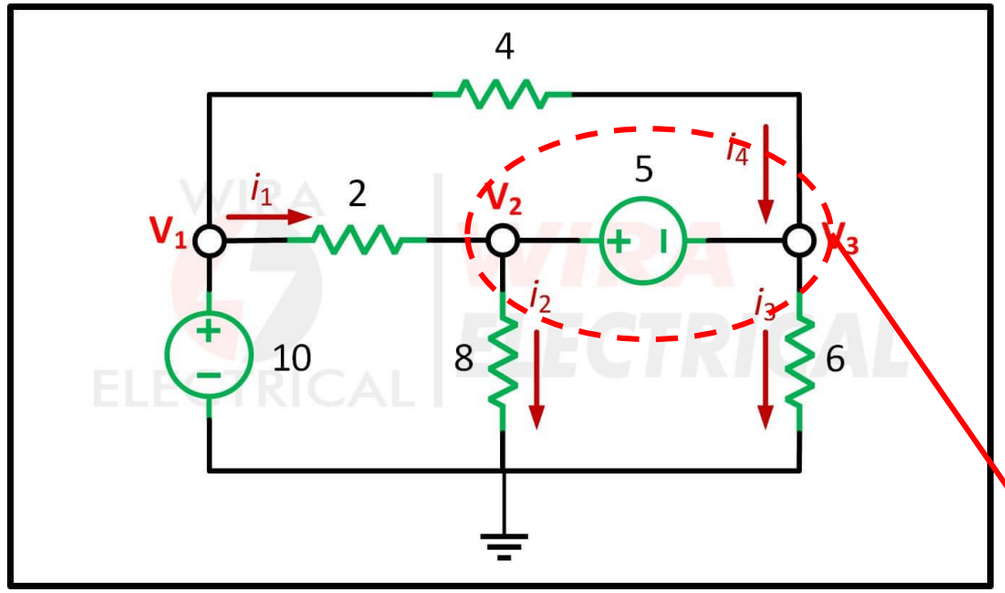


Photo source: [10] - https://wiraelectrical.com/wp-content/uploads/2021/07/supernode-analysis-2.jpg

Super tugunni kuchlanish manbasini va uning ikkita tugunini o‘rab turgan yopiq sirt deb hisoblash mumkin.

Tugun kuchlanishlarini aniqlash uchun ikkala KCL va KVL ni qo‘llaymiz.

Super tugun ikkita noaniq tugun va unga parallel ulangan har qanday elementlar o‘rtasida bog‘langan (*bog‘liq yoki mustaqil*) kuchlanish manbasini o‘rab olish orqali hosil bo‘ladi.

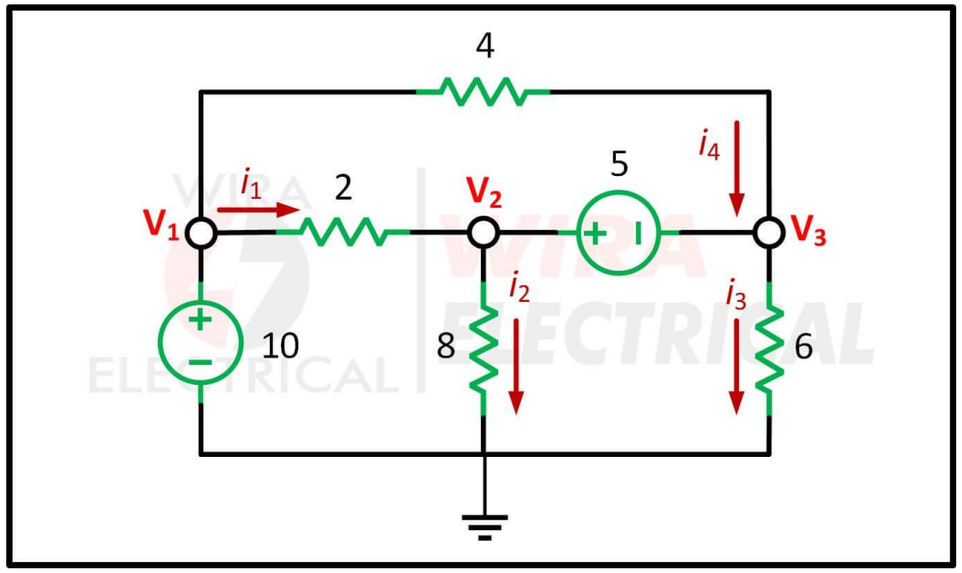


Photo source: [10] - https://wiraelectrical.com/wp-content/uploads/2021/07/supernode-analysis-2.jpg

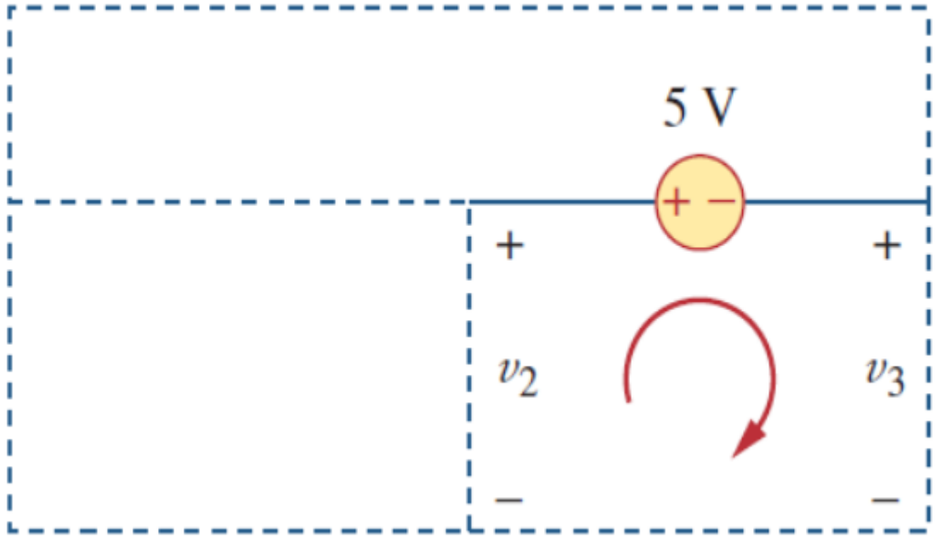
Tugunni tahlil qilishning muhim komponenti KCL ni qo‘llashdir. Bu har bir element orqali tok kuchini bilishni talab qiladi.

Kuchlanish manbai orqali tok kuchini oldindan bilishning hech qanday usuli yo‘q. Biroq, KCL har qanday boshqa tugun kabi super tugunda qanoatlantirilishi kerak.

$$\boldsymbol{i_1 + i_4 = i_2 + i_3} \qquad (3.11a)$$

$$\boldsymbol{\frac{u_1 - u_2}{2} + \frac{u_1 - u_3}{4} = \frac{u_2 - 0}{8} + \frac{u_3 - 0}{6}} \qquad (3.11b)$$

Kontur (*loop*) bo‘ylab soat yo‘nalishi bo‘yicha tok kuchining harakati belgilanadi.



3.5-rasm. KVLni super tugunga qo‘llash.

$$-u_2 + 5 + u_3 = 0$$

$$u_2 - u_3 = 5 \qquad (3.12)$$

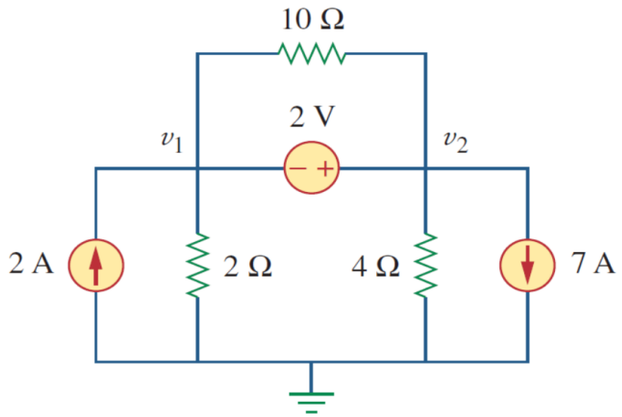
Super tugunning quyidagi xususiyatlariga e’tibor bering:

1. Super tugun ichidagi kuchlanish manbai tugun kuchlanishlarini hal qilish uchun zarur bo‘lgan cheklash tenglamasini beradi;

2. Super tugunni o‘ziga xos kuchlanishi yo‘q;

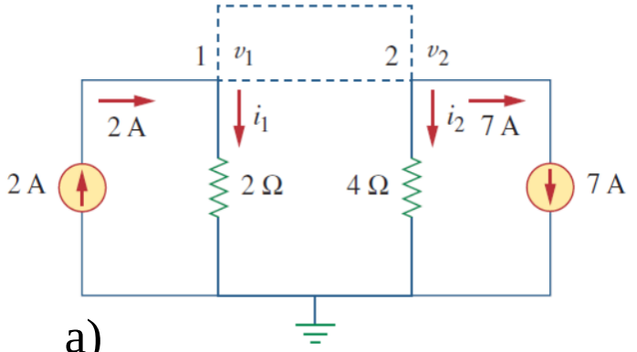
3. Super tugun ham KCL, ham KVL qo‘llanilishini talab qiladi.

3.2.1-masala: 3.6-rasmda ko‘rsatilgan zanjirdagi tugun kuchlanishlarini hisoblang.

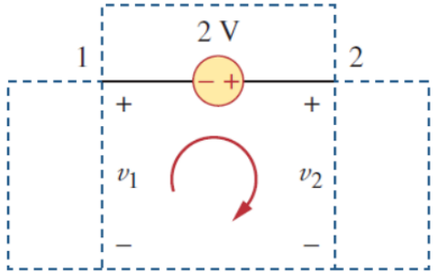


3.6-rasm.

$$\sum i_k = \sum i_{ch}$$



a)



b)

$$\sum U = 0$$

3.7-rasm.

a) super tugunga KCL qo‘llanilgan; b) konturga KVL qo‘llanilgan.

Yechish: KCL ni super tugunga qo‘llash quyidagi ifodani beradi.

$$2 = i_1 + i_2 + 7$$

$$2 = \frac{u_1 - 0}{2} + \frac{u_2 - 0}{4} + 7$$

$$8 = 2u_1 + u_2 + 28$$

$$u_2 = -20 - 2u_1 \quad (3.2.1)$$

KVL ni qo‘llaymiz.

$$-u_1 - 2 + u_2 = 0$$

$$u_2 = u_1 + 2 \quad (3.2.2)$$

(3.2.1) va (3.2.2) tenglamalardan quyidagi hosil bo‘ladi.

$$u_2 = u_2$$

$$-20 - 2u_1 = u_1 + 2$$

$$3u_1 = -22$$

$$u_1 = -7{,}33\ V$$

$$u_2 = u_1 + 2 = -7{,}33 + 2 = -5{,}33\ V$$

FOYDALANILGAN MANBALAR:

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