Atomic and Nuclear Physics

Week #1

Energy & Mass – energy relation

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We discuss …

- Energy and its units
- The Joule
- Electron-volt (eV)
- Relativistic mass- energy relation.

Energy and its units

- Energy – refers to an entity which helps to perform work
- It appears in various forms and can be transformed from one form to another.
- The SI (International System of Units) unit of energy is the joule (J).

Here are some common forms of energy and their units:

Kinetic energy – by virtue of motion

Mechanical energy (Joules)

Potential energy – by virtue of position in a field

- Gravitational - mgh
- Elastic - ½ kx^2

1. **Thermal Energy**: This is the internal energy of a system due to the motion of its particles. The unit is also the Joule (J) or calories (cal).
2. **Chemical Energy**: This is the energy stored in the bonds between atoms and molecules in a chemical substance. It is typically measured in Joules (J) or calories (cal).
3. **Electrical Energy**: The energy associated with the flow of electric charge. The unit is the Joule (J). In everyday use, we often measure electrical energy in kilowatt-hours (kWh).
4. **Nuclear Energy**: The energy released or absorbed during nuclear reactions. It is often measured in mega electron volts (MeV)

5. Radiant Energy: This is energy carried by electromagnetic waves, such as light, radio waves, and X-rays. The unit is also the Joule (J).

6. **Sound Energy**: The energy associated with the propagation of sound waves. It is typically measured in Joules (J) or decibels (dB).

7. **Wind Energy**: The energy harnessed from the motion of air masses, often measured in Joules (J) or kilowatt-hours (kWh) for electricity generation.

8. **Hydroelectric Energy**: The energy obtained from the flow of water, often measured in Joules (J) or kilowatt-hours (kWh).

9. Solar Energy: Energy from the sun, often measured in Joules (J) or kilowatt-hours (kWh) when used for electricity generation.

10. **Geothermal Energy**: Heat energy stored in the Earth's interior, often measured in Joules (J) or kilocalories (kcal).

11. **Tidal Energy**: Energy generated by the gravitational forces of the moon and sun on Earth's tides, often measured in Joules (J) or kilowatt-hours (kWh).

- ✓ It's important to note that while the joule (J) is the SI unit of energy, other units like the calorie (cal), kilowatt-hour (kWh), and British thermal unit (BTU) are commonly used in specific contexts, especially in everyday life and engineering applications.
- ✓ The conversion between these units can be done using appropriate conversion factors.

- Subatomic or nuclear particles being extremely small and light SI unit of energy – Joule becomes too big for them

- The quantities expressed in Joules may be too small us to understand or comprehend – like 10^{-30} Joules

- Evaluating energy in the units of Joules for such cases will not be convenient.
- For an electron (mass = 9.109×10^{-31} kg) moving a speed of 10 km/s (10^3m/s), kinetic energy of the electron = $\frac{1}{2}mv^2$

$$= \frac{1}{2} \times 9.109\times10^{-31} \times (10^3 m/s)^2 = \mathbf{4.554\times10^{-25}\ J.}$$

This quantity is **too small in human scale** and we cannot mentally apprehend the gravity and scale of the quantity.

- **In subatomic physics, we mostly deal with extremely small objects.**

- Therefore, we have **an alternate unit** while dealing with microparticles in modern physics, called **electron-volt (eV).**

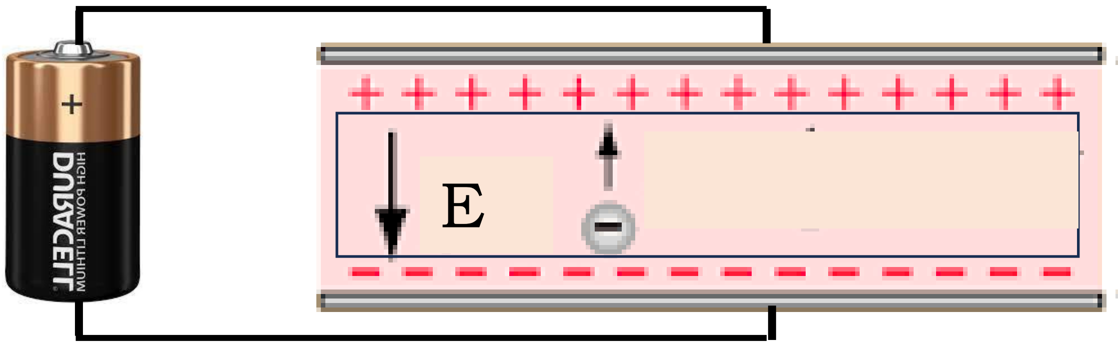
- **One electron volt is the kinetic energy acquired by an electron when it is accelerated through an electrostatic potential of 1 Volt**.

- Similarly, 10 eV is the energy acquired by an electron when it is accelerated through an electrostatic potential of 10 Volt.

- The numerical value of 1 eV in joules is equivalent to the numerical value of the charge of an electron in coulombs.

That is 1 eV = 1.6022×10^{-19} J.

Electron volt



Energy acquired by electron:

$$W = qV = 1.6\times10^{-19}\text{C} \times 1\ \text{J/C}$$

$$=1\ \text{eV} = 1.6\times10^{-19}\ \text{J}$$

That is $1\ \text{eV} = 1.6022\times10^{-19}$ J.

Problem 1: Evaluate the **velocity of an electron** accelerated through a potential of 100 Volt?

- ✓ Kinetic energy of electron in 100Volt electric field
- ✓ Energy = 100 eV = 100 × $\mathbf{1.6022\times10^{-19}}$ = 1.6022×10^{-17} Joules.
- ✓ 1.6022×10^{-17} Joules = ½ mv^2 = ½ × $9.109\times10^{-31}\times (v)^2$

Therefore, $v = \sqrt{\frac{2\times1.6022\times10^{-17}}{9.109\times10^{-31}}}$

$= \sqrt{0.3518 \times 10^{14}}$

= $\mathbf{0.5931\times 10^{7}}$ m/s (=5931 km per second)

What happens when a body accelerates at a tremendous rate?

- When objects move with incredible velocities we call it relativistic velocity
- That is, velocity comparable with velocity of light in free space
- Is there any limit for getting accelerated?

Relativistic mass-energy relation.

- In Newtonian mechanics, an object in motion has kinetic energy

$$E = \frac{1}{2} mv^2$$

- Energy can be transformed from one form to another form of energy.

Light to electrical (solar)   Electrical to mechanical (fan) …..

- Can energy be transformed to mass?

How much can we speed up a car?

What happens if speed increases incredibly?

- Einstein proposed that kinetic energy will appear as mass in such cases!

- Some of the kinetic energy will apparently be transformed to Mass

- Therefore, “mass-energy can be inter-changed into one another.

- This happens as $E= mc^2$ and is known as Einstein’s mass-energy equation.

Mass energy equivalence

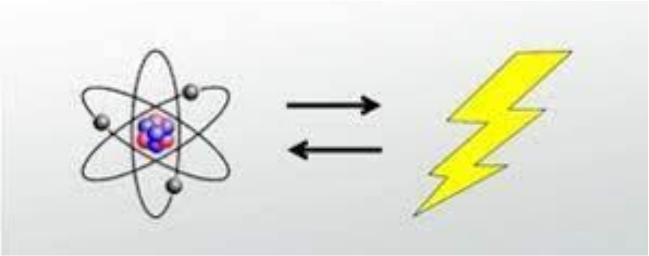
The special theory of relativity (Einstein in 1905)

Two vital ideas in Physics.

(1) variation of mass of a particle with its velocity and

$$\mathbf{m} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

(2) Proportionality between mass and energy



Proportionality between mass and energy

- Helps us to consider **mass as another form of energy**.

- Therefore:

  law of **conservation of energy = law of conservation of mass-energy.**

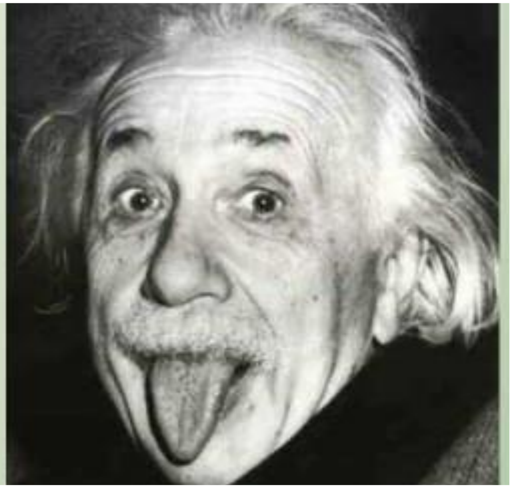
- To be precise, heating a body (adding energy to it) increases it mass;
- Being a tiny fraction of the mass of it is beyond our sensory perceptions and measurement techniques.
- In a nuclear reaction, the energy released is often results in the change in mass that can easily be measured.

Albert Einstein “**the mass of an object increases with its speed**”

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

where $\sqrt{1 - \left(\frac{v}{c}\right)^2} < 1$ and m_0 = rest mass

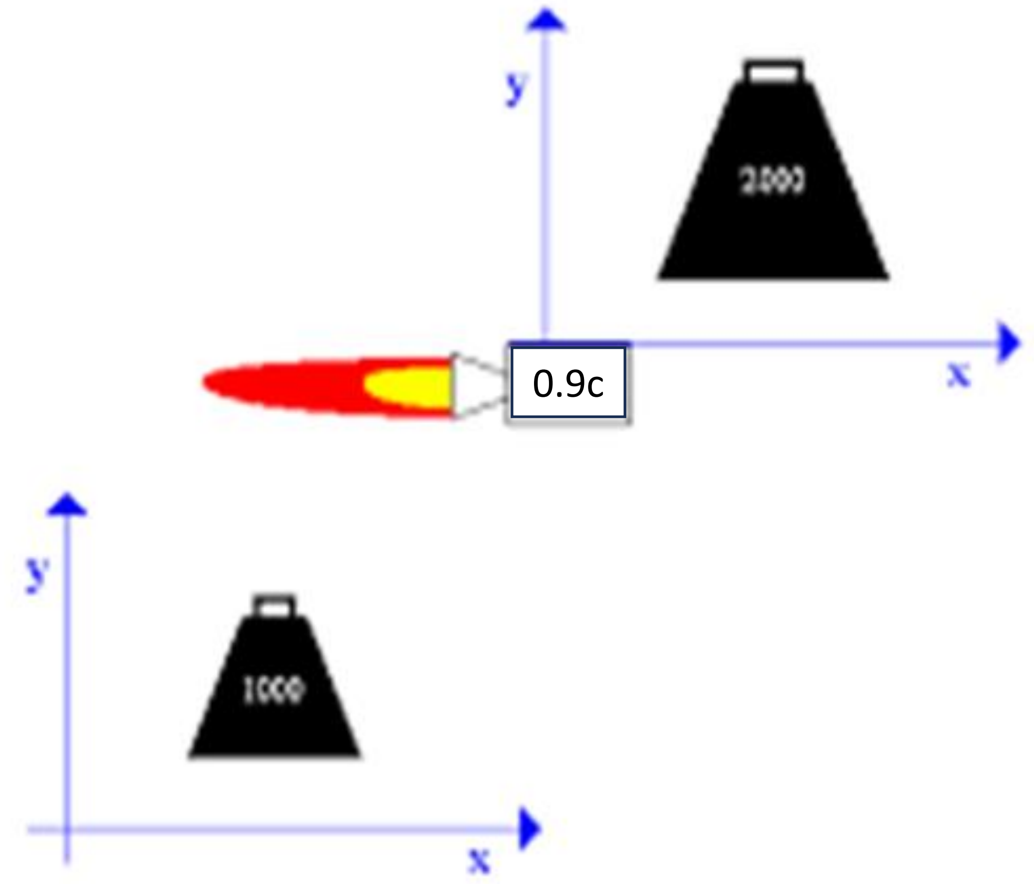
v = speed of object



Albert Einstein “performing work on mass increases its energy”

This implies that:

mass and energy are equivalent, in the sense that a *gain or loss of mass* can be regarded equally well as a *gain or loss of energy*



$$\mathbf{m} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\text{m} = \frac{1000}{\sqrt{1-\left(\frac{0.9c^2}{c^2}\right)^2}}$$

$$\text{m} = \frac{1000}{\sqrt{1-(0.9)^2}}$$

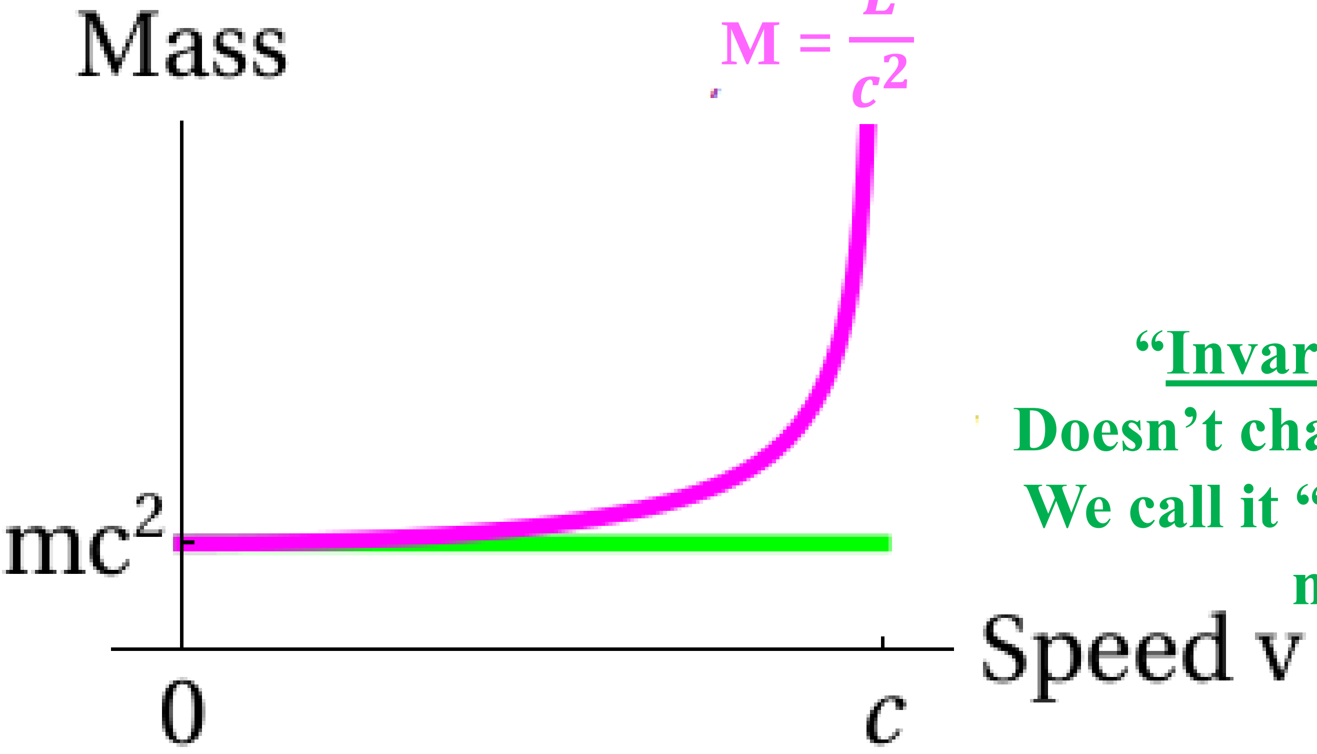
$$\text{m} = \frac{1000}{\sqrt{1-0.81}} = \frac{1000}{\sqrt{0.19}}$$

$$= \frac{1000}{0.4359} = 2294 \text{ g}$$

"Relativistic" Mass

Just energy in disguise

$$M = \frac{E}{c^2}$$



"Invariant" Mass

Doesn't change with speed

We call it "Mass" or "rest mass"

The **variation of mass with velocity** leads to modifications of our ideas about kinetic energy, T = $\int F.ds.$ F = m a = $m\frac{d(v)}{dt} = \frac{d(mv)}{dt}$

The relation for **kinetic energy** can be written as :

$$T = \int_o^s \frac{d(mv)}{dt}.ds \quad = \int_o^s \frac{ds}{dt}.d(mv) \quad = \int_o^{mv} v\, d(mv)$$

$$\mathbf{m} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$T = \int_o^v v d\left(\frac{m_o v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

Upon integration we get: $\mathbf{T = mc^2 - m_o c^2}$

$$\mathbf{T = mc^2 - m_o c^2}$$

Therefore, total energy of a body, $\mathbf{mc^2 = T + m_o c^2}$

Apparent mass   Rest mass

If a quantity of energy is transferred with a velocity υ, we can write for the magnitude of the associated momentum P = mv.

Energy, $E = mc^2$ gives: $m = \frac{E}{c^2}$

Therefore, we can have $P = \frac{Ev}{c^2}$

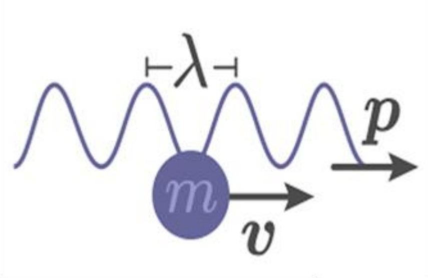
The relation for total energy:

$\mathbf{E = T + m_o c^2}$ is the first order relation for total energy

$\mathbf{E^2 = p^2 c^2 + E_o^2}$ is the second order relation.

While talking about momentum and De Broglie wave lengths,

we can have two limiting cases.



$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

(1) **Non-relativistic** realm (**E_o >> T**, Rest mass energy is much greater than the KE)

Momentum, P = $m_o v = \sqrt{2m_oT}$ and De Broglie, $\lambda = \frac{h}{m_o v} = \frac{h}{\sqrt{2m_oT}}$

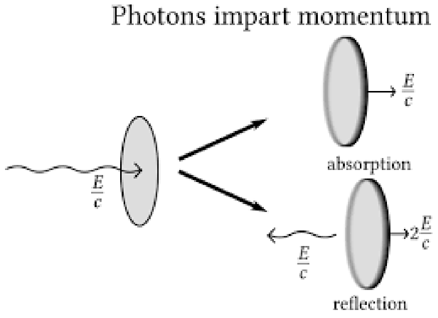
(1) **Relativistic case**, (**T >> E_o**, KE is much greater than rest mass energy)

Momentum, P = $\frac{E}{c}$ and De Broglie, $\lambda = \frac{hc}{E}$

- The **rest mass of the photon must be zero**

Otherwise, the mass of a photon traveling with the velocity of light would be infinite.

$$\mathbf{m} = \frac{\boldsymbol{m_o}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_o v}{\sqrt{1-\frac{c^2}{c^2}}} = \frac{m_o v}{0} = \alpha$$



Although the rest mass of a photon is zero, the photon has a mass (momentum) associated with its kinetic energy.

Mass of photon is = $\frac{h}{c^2}$ and corresponding momentum, p = $\frac{h}{\nu c}$

1. Einstein's mass-energy relation (derivation):

- Consider an object travelling with the speed of light.
- Energy and momentum are **induced** in it due to the applied force.
- The increase in **momentum** of the object

  = mass x change in velocity of the body because of the force.

- **Energy acquired**= Force x Distance (**E = F x d**)
- The **momentum gained** = the force x the time (P = F x t)
- The momentum gained by the object, P = m x c Hence, $\mathbf{F} = \boldsymbol{\frac{mc}{t}}$
- Combining, E = F x d and F = $\frac{mc}{t}$, we have E = $\frac{mc}{t}$ x d = $\frac{mcd}{t}$ = mc^2.
- Therefore, $\mathbf{E = mc^2}$.

Physical Significance of Einstein's Equation: $E = mc^2$

Imagine, a ***brick being heated*** or a ***battery being charged*** in the inertial reference frame in which they are at rest.

- As the brick is heated, or the battery charged, it absorbs an amount of energy E_o as measured in its rest frame.
- Einstein's equation tells us that the mass of the brick, or battery, after it absorbs an amount of energy E_o is increased exactly by an amount $\frac{E_0}{C^2}$.
- The value of the **mass** of the hot brick or charged battery **is greater than it was before** it absorbed energy, though this **quantity is so negligible**.

Imagine, a brick being heated through 100^{o}C. Its heat capacity is 80 kJ/kg^{o}C. Calculate the mass gained by the brick? Assume that mass of brick is 2kg

Energy gained while heating through 100^{o}C = $80000 \times 2 \times 100 = 16 \times 10^6$ J

Energy gained is equivalent = Δmc^2

Therefore, $\Delta m = (16 \times 10^6 \text{ J}) / (3 \times 10^8)^2 = 1.8 \times 10^{-10}$ kg is unimaginably small !!

Similar way, assume a mobile phone cell charged with 1000mA at 3V for an hour.

Energy gained while charging = $1000 \times 10^{-3} \times 3 \times 3600 = 108 \times 10^{2}$ J

Energy gained is equivalent = Δmc^2

Therefore, $\Delta m = (108 \times 10^{2} J) / (3 \times 10^{8})^{2} = 12 \times 10^{-14}$ kg is unimaginably small !!

Recapitulation

- There are numerous forms of energy which are interchangeable
- The SI (International System of Units) unit of energy is the joule (J)
- Electron Volt (eV) is a convenient unit of energy for subatomic particles
- For relativistic particles, total energy is the sum of dynamic energy and rest mass energy
- Just like two forms of energy, mass and energy are also inter convertible

References:

1. Littlefield, T.A. & Thorley, N., Atomic and Nuclear Physics, 3rd edition, (ELBS and van Nostrand Reinhold Co., 1979).

2. Noz, M.E., & McGuire, G.O., Radiation Protection in the Radiologic and Health Sciences, Lea & Fibiger (2005).

THANK YOU