Course title: Atomic and Nuclear Physis

Week # 2

Main Topics: Atomic mass unit and its relation with MeV

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Lecture Learning Outcomes:

At the end of the lecture, you will be able to:

(i) Explain Atomic Mass Unit (u)
(ii) Determine mass of elements in u
(iii) Estimate the energy equivalent to amu

Introduction: Expressing mass of atoms and subatomic particles in the SI unit is difficult. It was found suitable to define a unit convenient in the scale of mass of atoms based on the mass of a simple atom. This unit explains the use of carbon as a scale to express the mass of atoms and subatomic entities and its energy equivalence.

1. The atomic mass unit (amu)

The perception of the atomic mass unit aroused from the necessity to define and compare the masses of different molecules, atoms and subatomic particles in modern physics. Atoms are extremely small and their masses are often expressed in very small numbers. When we use SI units to express masses of these particles, it becomes extremely small so that one may not get clear idea about the quantum of the entity. As we have adopted electron volt to express energy in place of Joules, a new idea was mooted to define the mass using an atom. The new scale would act as a common reference point.

Atomic mass unit (AMU), also known as Unified mass units (u) or called Dalton, is a unit for expressing masses of atoms, molecules, or subatomic particles. The atomic mass unit is defined as one-twelfth the mass of an atom of carbon-12 ($_6C^{12}$). In other words, the mass of one $_6C^{12}$ atom is exactly 12 atomic mass units. This definition was adopted in 1961 by the International Union of Pure and Applied Chemistry (IUPAC). Carbon has five isotopes – C^{10}, C^{11}, C^{12}, C^{13} and C^{14}. Each isotope is found in a fixed percentage and the most abundant isotopes of carbon are C^{12} followed by C^{13}. The other three isotopes are negligible for detection. Carbon-12 was chosen as the reference because it is abundant, stable, and its isotopic composition is relatively uniformand simple. This warrants that the definition of the atomic mass unit remains consistent and accurate over time.

The *atomic mass unit* (u) is defined as $\left(\frac{1}{12}\right)^{\text{th}}$ the mass of $_6C^{12}$. Since 12 grams of $_6C^{12}$ forms a mole of carbon atoms, it will contain one Avogadro's number of atoms. Therefore, the atomic mass unit can be defined and estimated as:

$$1\,u = \frac{1}{12}\left(\frac{0.012kg/mole}{6.0221x10^{23}/mole}\right) = 1.6606x10^{-27}\,kg$$

For example, we have fixed that the atomic mass of $_6C^{12}$ as 12 AMU which is numerically equal to $1.660538921\times10^{-27}$ kg in SI units. But, when we say atomic mass of carbon in general, it is the weighted average of the isotopes of carbon in its natural form.

Expressing atomic masses in atomic mass units allows for more convenient calculations in physical science. It simplifies stoichiometry, chemical reactions, and other computations involving atomic and molecular masses.

To express atomic masse of particular element, we take the weighted average of all the isotopes of the element. The atomic mass unit is used to express the atomic masses of elements in the periodic table. The atomic mass recorded for each element is an average of the masses of all its naturally occurring isotopes, weighted by their abundance. To determine the mass of atoms and particles we use mass spectrometry and other sophisticated measurement methods are used to determine the atomic masses of various elements and isotopes with high accuracy.

The mass of $_6C^{12}$ is 12 amu. Mass of $_6C^{13}$ is 13.003355 amu. The natural abundance of $_6C^{13}$ is 1.11%, and the natural abundance of $_6C^{12}$ is 98.89%. The average atomic mass of Carbon is the weighted average of the masses of the two most abundant isotopes.

Therefore, average mass C = (0.9889 x 12 amu) + (0.0111 x 13.00335) amu

= 11.8668 + 0.144337

= 12.011137185

= **12.01 amu**

2. Experimental methods to determine the mass of particles

Experimental determination of mass of atoms and subatomic particles is a vital part of experimental physics. Several methods are there to experimentally measure the masses of subatomic particles with varying degrees of precision. Needless to mention that all the methods are designed to determine the mass indirectly as in all other experiments for microparticles. Enlisted here are some of the most important experimental methods for estimating the mass of atoms and subatomic particles. Among several methods, mass spectrometry is the most general experimental tool. Various properties of the particles are utilised to determine the mass experimentally.

1. Mass Spectrometry:

   a) Magnetic Sector Mass Spectrometry: Fast moving charged particles or ions are subjected a suitable magnetic field and are separated by the Lorentz force acting on the charges due to magnetic field. When the magnetic field applied is transverse to the motion of the particles, Lorentz force (= q (v×B)) provides a centripetal force (= m $\frac{v^2}{r}$). The radius of curvature of the charge path in the magnetic field depends on the mass-to-charge ratio. This method helps to determine the mass precisely.
   b) Time-of-Flight Mass Spectrometry: Charged particles or ions accelerated using a known electrostatic field. The electrostatic field applied helps us to calculate the energy imparted to the particle. Measuring the time of flight of the particles between electrodes, mass-to-charge ratio of the charges can be determined.
   c) Quadrupole Mass Spectrometry: This method also works with the principle of selecting ions based on their mass to charge ratio. Each of the two opposing rods are applied with equal potential (DC) superimposed with a high-frequency oscillating voltage. The voltages applied induce transverse oscillations to the ions traversing between the rods. The amplitudes of almost all oscillations intensify so that ultimately the ions will make contact with the rods. Therefore, only ions with a certain ratio of mass to charge (m/q) is in the resonance condition which allows passage through the system.
   d) Penning Trap Mass Spectrometry: Penning trap method is a precise experimental technique used to measure the mass of charged particles, namely atomic ions or subatomic particles. It operates based on the principles of electromagnetic trapping and the measurement of the oscillations of these trapped particles. The applied electric field provides axial confinement (along the trap's central axis), while the simultaneously applied magnetic field confines the charged particles radially (perpendicular to the central axis). When a charged particle enters the trap, it orbits around the central axis of the trap, with a frequency that depends on its charge-to-mass ratio. This frequency is called the cyclotron frequency. The trapped particle's cyclotron frequency is measured by applying an oscillating electric field to the trap. This oscillating field imparts energy to the trapped particle, causing it to move in response. By carefully tuning the frequency of the applied electric field, we can drive the particle's

motion at the cyclotron frequency. The mass of the trapped particle can be determined from its cyclotron frequency and the known values of the charge of the particle and the strength of the magnetic field.

2. Cyclotron and Linear Accelerators: Particle accelerators like cyclotron and linear accelerators can determine the mass-to-charge ratio of ions by accelerating them to known velocities and measuring their trajectories in magnetic fields. This information can be used to calculate the mass.

3. Beta Spectroscopy: This method is exclusively for the determination of the mass of electrons. It involves measuring the energy spectrum of electrons emitted during beta decay processes. The shape of the spectrum is related to the electron mass.

4. Electron Mass Spectrometry: Here, the mass of electrons is determined by measuring their cyclotron frequency in a magnetic field. The mass-to-charge ratio of electrons is calculated from this frequency.

5. Neutrino Mass Measurements: Neutrino is an extremely light particle. Therefore, detection of neutrino directly is very difficult. Tritium beta decay experiments is used to measure the electron's energy spectrum to infer the neutrino mass through the conservation of energy and momentum.

6. Nuclear Reactions: Certain nuclear reactions can be used to deduce the masses of specific nuclei. The energy released or absorbed in these reactions is related to the mass difference between reactants and products, as described by Einstein's mass-energy equivalence ($E=mc^2$).

7. Muon and Electron Scattering: By scattering electrons or muons off atomic nuclei, researchers can obtain information about nuclear masses and charge distributions.

These experimental methods vary in their precision and applicability to different particles, but collectively, they allow scientists to measure the masses of atoms and subatomic particles with remarkable accuracy. Advances in technology and experimental techniques continue to improve our understanding of particle masses and the fundamental constants of nature.

3. Relation between amu and MeV.

In modern Physics, the principle of mass energy equivalence says that mass has an equivalent amount of energy and vice – versa or they are interconvertible but this process is difficult to take place. The principle argues that mass is concentrated energy. Energy can be released through the process matter antimatter annihilation. Since the amount of energy associated with matter is tremendous, subatomic (nuclear) reactions release great amount of energy as compared with chemical reactions. Being the two forms same entity, we can express mass in energy units and energy in units of mass.We know that 1 amu is numerically equals to $1.660538921\times10^{-31}$ kg. Using Einstein's relation:

$$E = mc^2 = 1.660538921\times10^{-27}\ \text{kg} \times (3 \times 10^8\ \text{m/s})^2 = 14.94485 \times 10^{-11}\ \text{Joules}$$

$$= \frac{[14.94485\times10^{-11}]}{1.6\times10^{-19}} = 9.31\times 10^8 = 931 \times 10^6\ \text{eV}$$

Therefore, **1 amu = 931 MeV**

Problem 1 Determine the average atomic mass of oxygen using the following data:

Isotope	Mass (amu)	Abundance (%)
$_8O^{16}$	15.99491	99.759
$_8O^{17}$	16.99913	0.037
$_8O^{18}$	17.99916	0.204

The average mass of O = (15.99491×0.99759) + (16.99913×0.00037) + (17.99916×0.00204)

= 15.956 + 0.0063 + 0.0367 = **15.999 amu**

Problem 2 Copper has two isotopes $_{29}Cu^{63}$ and $_{29}Cu^{65}$. The atomic mass of copper is 63.54amu. If the atomic masses of $_{29}Cu^{63}$ and $_{29}Cu^{65}$ are 62.9296 and 64.9278 amu respectively, what is the natural abundance of each isotope?

Isotope	Mass (amu)	Abundance (%)
$_{29}Cu^{63}$	62.9296	x
$_{29}Cu^{65}$	64.9278	y

Here x and y are the abundance of isotopes. Evidently, $x + y = 100\% = 1$

Further, mass of Cu = 63.54 = $(62.9296\times x) + (64.9278\times y)$

$$= (62.9296\times x) + (64.9278\times(1-x)) = 64.9278 + x\,(62.9296-64.9278)$$

$$= 64.9278 - x\ 1.9982$$

Solving for x , we get x = 0.6945 and hence y = *0.3055*

Therefore, the natural abundance of $_{29}Cu^{63}$ and $_{29}Cu^{65}$ are **69.45% and 30.55%** respectively.

References:

1. Nuclear Physics: Experimental and Theoretical, 2nd Revised edition by H S Hans. New Academic Science, 2011.
2. Littlefield, T.A. & Thorley, N., Atomic and Nuclear Physics, 3rd edition, (ELBS and van Nostrand Reinhold Co., 1979).
3. **Noz, M.E., & McGuire, G.O., Radiation Protection in the Radiologic and Health Sciences, Lea & Fibiger (2005).**