

# Atomic and Nuclear Physics

**Week #2**

**Atomic Mass Unit and its relation with MeV**

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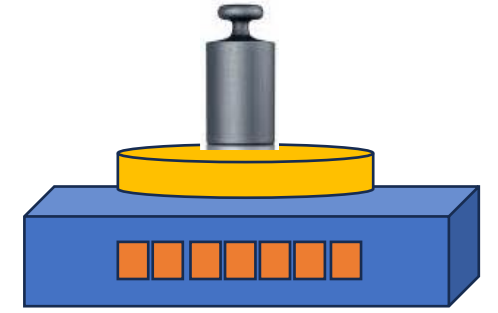
# Topics of discussion

- Atomic Mass Unit (u)
- Experimental methods to determine mass of particles
- Relation between u and MeV
- Numerical Problems

# Mass of a body

## **Fundamental Property:**

- Mass is a fundamental property of substance.
- It is an intrinsic property so that it is same everywhere in the universe.
- For example, a body with one kilogram of matter on Earth will be same mass on the Moon.
- While weights of a body will be different on earth and that on moon, because weight represents the force on it
- Mass plays a fundamental role of the object's inertia by which it opposes the change in the state of rest or uniform motion.
- Acceleration of a body by applying force depends on mass (Newton's law)



# Quantifying mass

- There are several experimental and theoretical methods to quantify mass

**Balance Scales:** work on the principle of equilibrium, where the mass being measured is balanced against a known mass

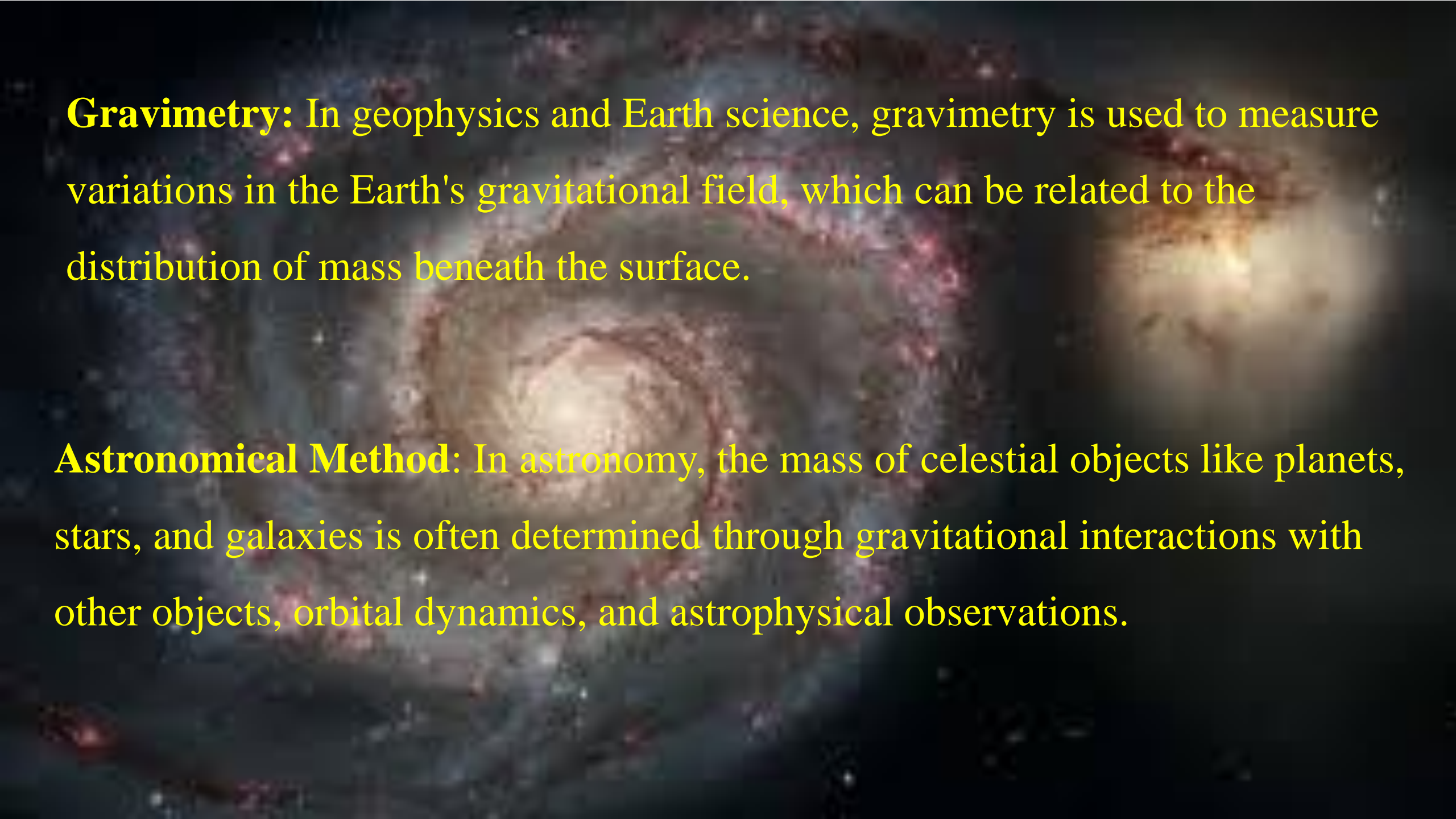


**Weighing Scales:** Weighing scales use load cells, strain gauges, or other sensors to measure the force exerted by the object due to gravity, and then they convert this force into a mass reading.



**Spring Scales:** Spring scales use the deformation of a spring to measure the force exerted by an object due to gravity. The scale is calibrated to display the corresponding mass in units such as kilograms or grams.





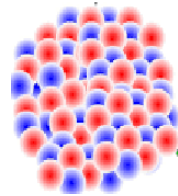
**Gravimetry:** In geophysics and Earth science, gravimetry is used to measure variations in the Earth's gravitational field, which can be related to the distribution of mass beneath the surface.

**Astronomical Method:** In astronomy, the mass of celestial objects like planets, stars, and galaxies is often determined through gravitational interactions with other objects, orbital dynamics, and astrophysical observations.

**Magnetic Resonance Imaging (MRI):** MRI can indirectly measure the mass of tissues in the body based on the interaction of hydrogen nuclei with magnetic fields.



**Nuclear Physics:** In nuclear physics and particle physics, mass spectrometers and particle detectors are used to measure the masses of subatomic particles and nuclei.



**Therefore, determination of mass needs apt method and scale  
to suite the substance**

# How we scale mass of atoms and subatomic particles ???

- Need a suitable reference
- Of comparable quantum
- Should be reliable and stable
- Should be easy to use
- Should be versatile as well

## The atomic mass unit (amu)

- ✓ In modern Physics, the principle of mass energy equivalence says that mass has an equivalent amount of energy and vice – versa or they are interconvertible but this process is difficult to take place.

## The principle argues that mass is concentrated energy.

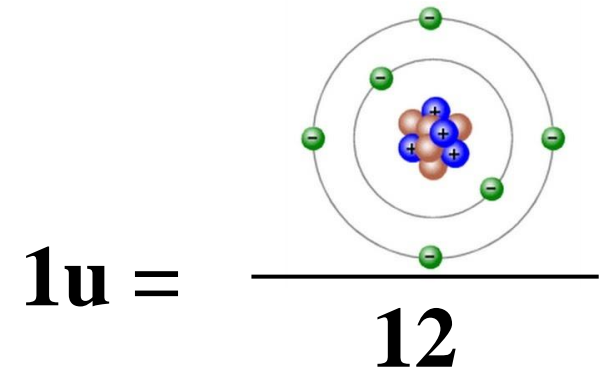
- ✓ Energy can be released through the process matter antimatter annihilation.
- ✓ Since the amount of energy associated with matter is tremendous, subatomic (nuclear) reactions release great amount of energy as compared with chemical reactions.
- ✓ Being the two forms same entity, we can express **mass in energy units and energy in units of mass.**

- To represent the amount or scale mass of atoms we use a simple atom
- The atom is relatively smaller with most of the atoms
- The atom is stable and abundant
- It is easy to use and apply
- The atom we choose as a unit of mass for all atoms is **Carbon-12**
- The unit is somewhat equal to **mass of a nucleon**

## **Atomic mass unit (AMU) (also called Dalton)**

- ❑ An atomic mass unit is equal to 1/12 the mass of a single atom of carbon-12
- ❑ (Carbon has five isotopes –  $C^{10}$ ,  $C^{11}$ ,  $C^{12}$ ,  $C^{13}$  and  $C^{14}$ ).
- ❑ Each isotope is found in a fixed percentage and the most abundant isotope of carbon is  $C^{12}$  followed by  $C^{13}$ .
- ❑ The other three isotopes are negligible for detection.

- The *atomic mass unit* (u) is defined as  $\left(\frac{1}{12}\right)^{\text{th}}$  the mass of  ${}_6\text{C}^{12}$ .
- Since 12 grams of  ${}_6\text{C}^{12}$  forms a mole of carbon atoms, it will contain one Avogadro's number of atoms.
- Therefore, the atomic mass unit can be defined and estimated as:
- $1\text{u} = \frac{1}{12} \left[ \frac{0.012 \text{ kg/mole}}{6.0221 \times 10^{23} / \text{mole}} \right] = 1.6606 \times 10^{-27} \text{ kg}$
- **$1\text{u} = 1.6606 \times 10^{-27} \text{ kg}$**



- We have fixed that the atomic mass of  $^{12}\text{C}$  as  $12 \text{ AMU} = 1.660538921 \times 10^{-27} \text{ kg}$ .
- But, when we say atomic mass of carbon in general, it contains 2 isotopes of carbon.
- Therefore, we take the weighted average of the isotopes of carbon in its natural form.

*Let us calculate it:*

- The mass of  $^{12}\text{C}$  is 12 amu. Mass of  $^{13}\text{C}$  is 13.003355 amu. The natural abundance of  $^{13}\text{C}$  is 1.11%, and the natural abundance of  $^{12}\text{C}$  is 98.89%.

- The mass of  $^{12}\text{C}$  is 12 amu. Mass of  $^{13}\text{C}$  is 13.003355 amu. The natural abundance of  $^{13}\text{C}$  is 1.11%, and the natural abundance of  $^{12}\text{C}$  is 98.89%.
- The average atomic mass of Carbon is the weighted average of the masses of the two most abundant isotopes.
- Therefore, average mass C =  $(0.9889 \times 12 \text{ u}) + (0.0111 \times 13.00335) \text{ u}$   
 $= 11.8668 + 0.144337$   
 $= 12.011137185$   
 $= \mathbf{12.01 \text{ u}}$

**Therefore, average atomic mass of carbon is 12.01 amu**

Example:

Determine the average atomic mass of oxygen using the following data:

Isotope	Mass (amu)	Abundance (%)
$^{16}\text{O}$	15.99491	99.759
$^{17}\text{O}$	16.99913	0.037
$^{18}\text{O}$	17.99916	0.204

$$\begin{aligned}\text{The average mass of O} &= (15.99491 \times 0.99759) + (16.99913 \times 0.00037) + (17.99916 \times 0.00204) \\ &= 15.956 + 0.0063 + 0.0367 \\ &= \mathbf{15.999 \text{ amu}}\end{aligned}$$

- ✓ Using this principle, we can also evaluate the natural abundance of nuclei
- ✓ Once we have the precise masses of two isotopes of a specific species, we can determine the nuclear abundance of the isotopes!!

Copper has two isotopes  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$ . The atomic mass of copper is 63.54amu. If the atomic masses of  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  are 62.9296 and 64.9278 amu respectively, what is the natural abundance of each isotope?

Isotope	Mass (amu)	Abundance (%)
$^{63}\text{Cu}$	62.9296	$x$
$^{65}\text{Cu}$	64.9278	$y$

Here  $x$  and  $y$  are the abundance of isotopes. Evidently,  $x + y = 100\% = 1$

$$\begin{aligned}\text{Further, mass of Cu} &= 63.54 = (62.9296 \times x) + (64.9278 \times y) \\ &= (62.9296 \times x) + (64.9278 \times (1-x)) = 64.9278 + x(62.9296 - 64.9278) \\ &= 64.9278 - x \cdot 1.9982\end{aligned}$$

Solving for  $x$ , we get  $x = 0.6945$  and hence  $y = 0.3055$

Therefore, the natural abundance of  $^{63}\text{Cu}$  and  $^{65}\text{Cu}$  are **69.45%** and **30.55%** respectively.

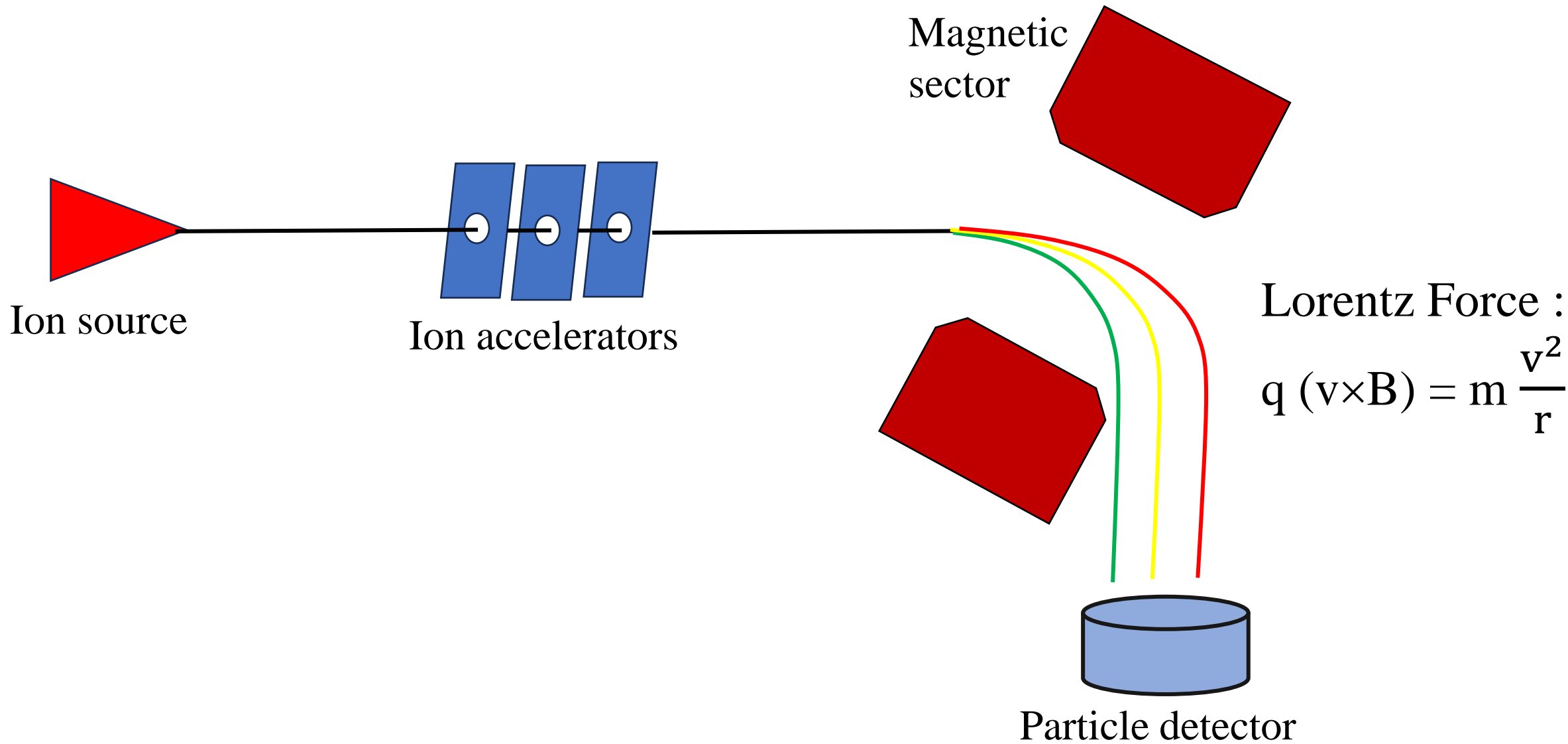
## **Experimental methods to determine the mass of subatomic particles**

- We have defined a unit to represent the mass of atoms and subatomic particles
- How to determine the mass experimentally?
- Experimental determination of mass of atoms and subatomic particles is a vital part of experimental physics.
- Several methods are there to experimentally measure the masses of subatomic particles with varying degrees of precision.
- All methods are designed to determine the mass indirectly as in all other properties of microparticles.

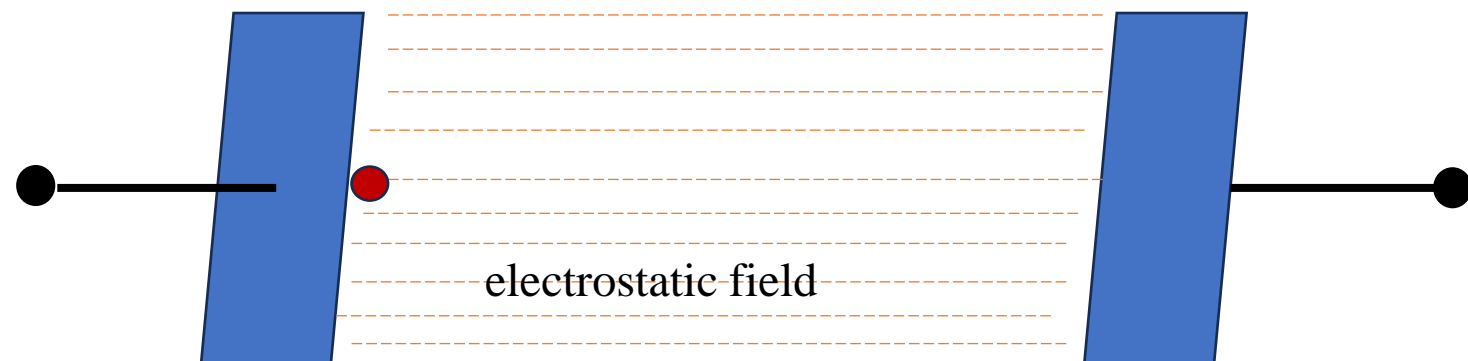
- ✓ Following are some of the most important experimental methods for estimating the mass of atoms and subatomic particles.
- ✓ Among several methods, mass spectrometry is the most general experimental tool.
- ✓ Various properties of the particles are utilised to determine their mass experimentally.

# Magnetic Sector Mass Spectrometry

- Fast moving charged particles or ions are subjected a suitable magnetic field and are separated by the Lorentz force acting on the charges due to magnetic field.
- When the magnetic field applied is transverse to the motion of the particles, Lorentz force ( $= q (\mathbf{v} \times \mathbf{B})$ ) provides a centripetal force ( $= m \frac{v^2}{r}$ ).
- The radius of curvature of the charge path in the magnetic field depends on the mass-to-charge ratio.
- This method helps to determine the mass precisely.



- **Time-of-Flight Mass Spectrometry:**
- Charged particles or ions accelerated using a known electrostatic field.
- The electrostatic field applied helps us to calculate the energy imparted to the particle.
- Measuring the time of flight of the particles between electrodes, mass-to-charge ratio of the charges can be determined.

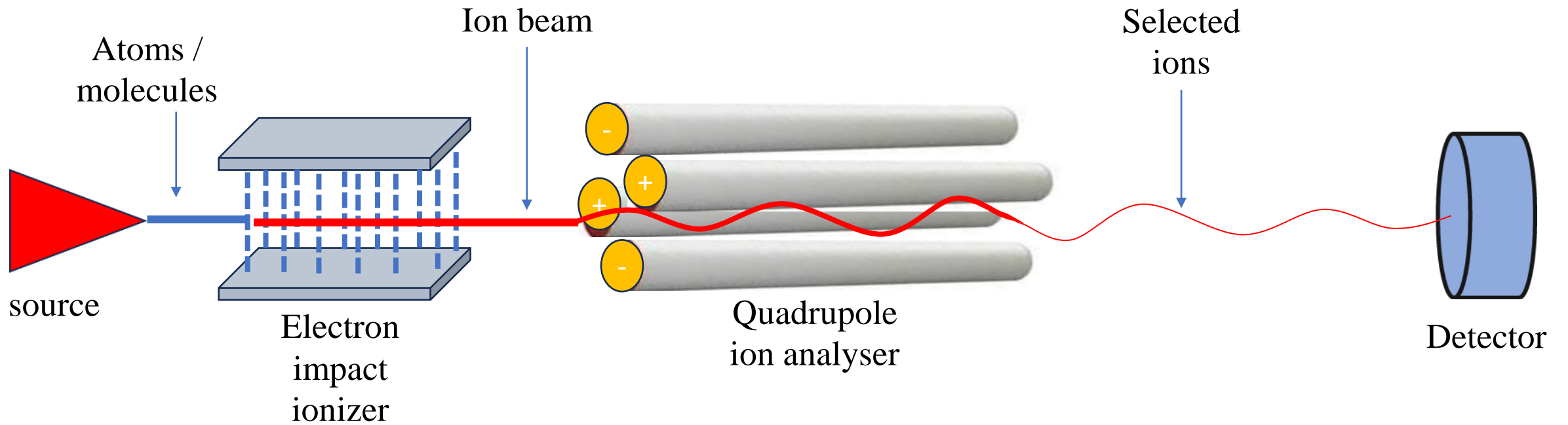


## Quadrupole Mass Spectrometry:

- The method also works with the principle of selecting ions based on their mass to charge ratio ( $m/q$ ).
- Each of the two opposing rods are applied with equal potential (DC) superimposed with a high-frequency oscillating voltage.
- The voltages applied induce transverse oscillations to the ions traversing between the rods.
- The amplitudes of almost all oscillations intensify so that ultimately the ions will make contact with the rods.

# Quadrupole Mass Spectrometry:

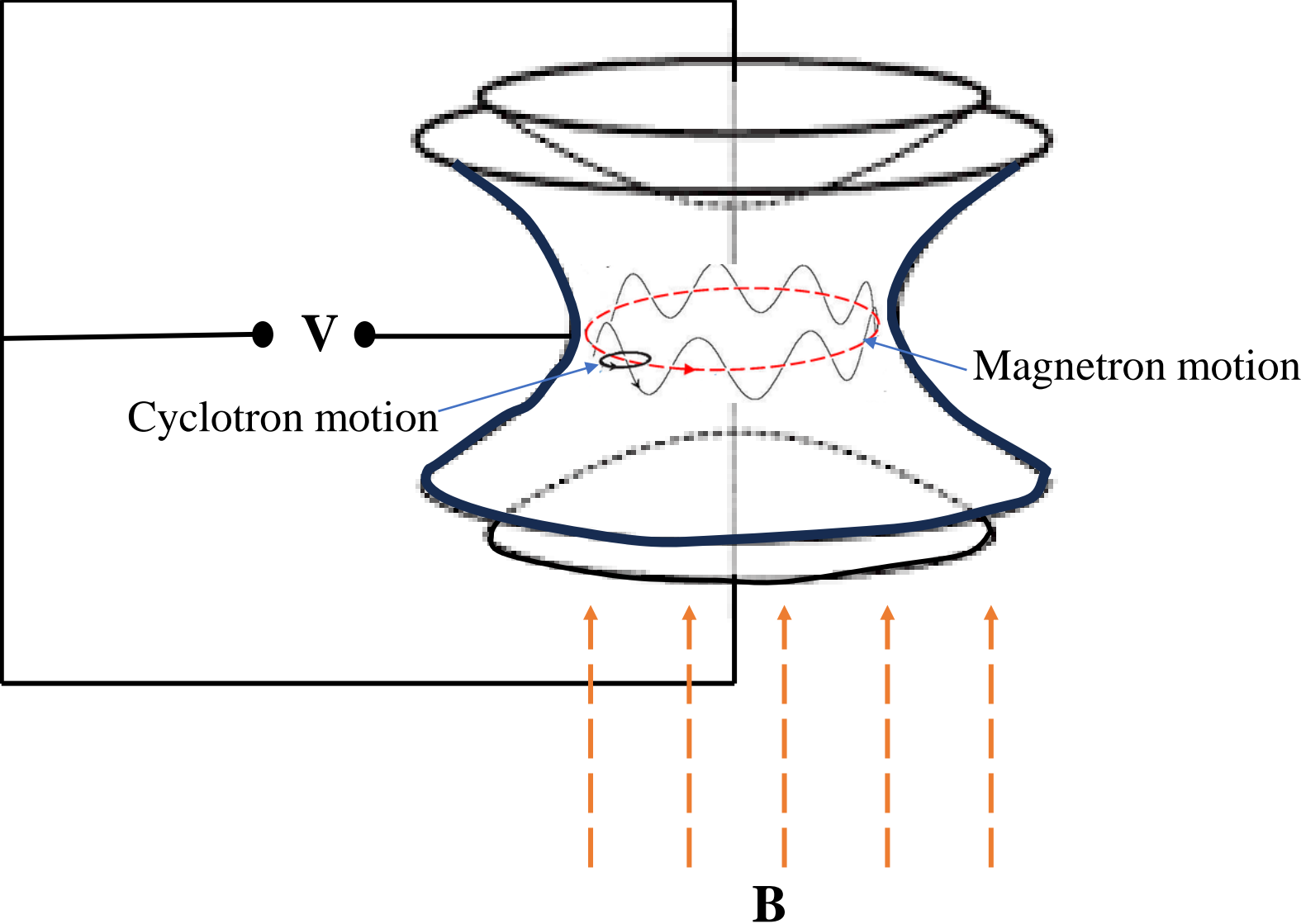
Ions with a specific ratio of mass to charge ( $m/q$ ) is in the resonance condition which allows passage through the system.



## Penning Trap Mass Spectrometry:

- Penning trap method is a precise experimental technique used to measure the mass of charged particles, namely atomic ions or subatomic particles.
- It operates based on the principles of electromagnetic trapping and the measurement of the oscillations of these trapped particles.
- The applied electric field provides axial confinement (along the trap's central axis), while the simultaneously applied magnetic field confines the charged particles radially (perpendicular to the central axis).
- When a charged particle enters the trap, it orbits around the central axis of the trap, with a **frequency that depends on its charge-to-mass ratio**.

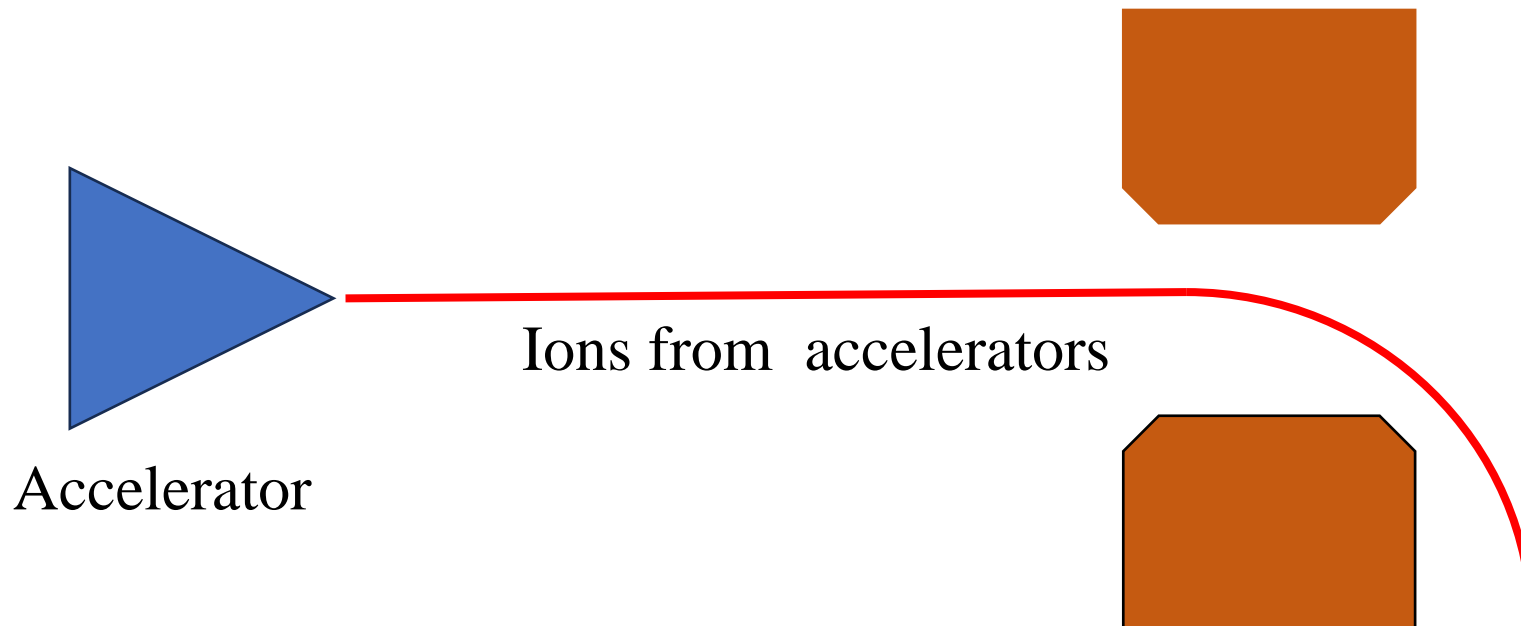
# Penning Trap



- This frequency is called the cyclotron frequency.
- The trapped particle's cyclotron frequency is measured by applying an oscillating electric field to the trap.
- This oscillating field imparts energy to the trapped particle, causing it to move in response.
- By carefully tuning the frequency of the applied electric field, we can drive the particle's motion at the cyclotron frequency.
- The mass of the trapped particle can be determined from its cyclotron frequency and the known values of the charge of the particle and the strength of the magnetic field.

## Cyclotron and Linear Accelerators:

- ❑ Particle accelerators like cyclotron and linear accelerators can determine the mass-to-charge ratio of ions by accelerating them to known velocities and measuring their trajectories in magnetic fields.
- ❑ This information can be used to calculate the mass.



Lorentz Force :

$$q (\mathbf{v} \times \mathbf{B}) = m \frac{v^2}{r}$$

$$\frac{m}{q} = \frac{r (\mathbf{v} \times \mathbf{B})}{v^2}$$

## **Beta Spectroscopy:**

- ✓ This method is exclusively for the determination of the mass of **electrons**.
- ✓ It involves measuring the energy spectrum of electrons emitted during beta decay processes.
- ✓ The shape of the spectrum is related to the electron mass.

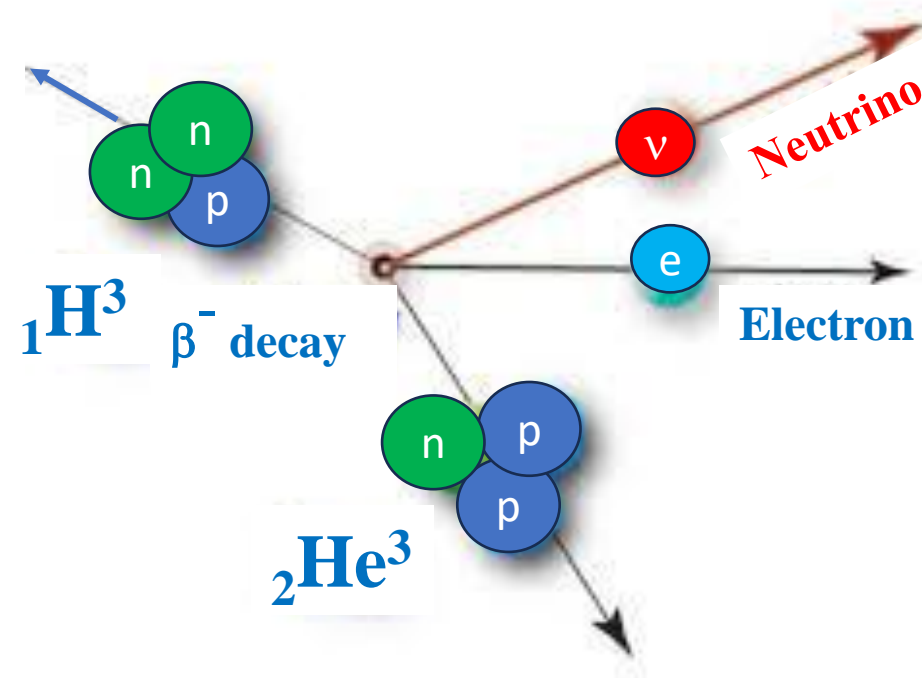
## Electron Mass Spectrometry:

- Here, the **mass of electrons** is determined by measuring their cyclotron frequency in a magnetic field.
- The mass-to-charge ratio of electrons is calculated from this frequency.

### Cyclotron frequency

$$f = \frac{qB}{2\pi m}$$

$$\text{Mass to charge ratio} = \frac{m}{q} = \frac{B}{2\pi f} = \frac{B}{\omega}$$



# Neutrino Mass Measurements

- Neutrino is an extremely light particle.
- Therefore, detection of neutrino directly is very difficult.
- Tritium beta decay experiments is used to measure the electron's energy spectrum to infer the neutrino mass through the conservation of energy and momentum.

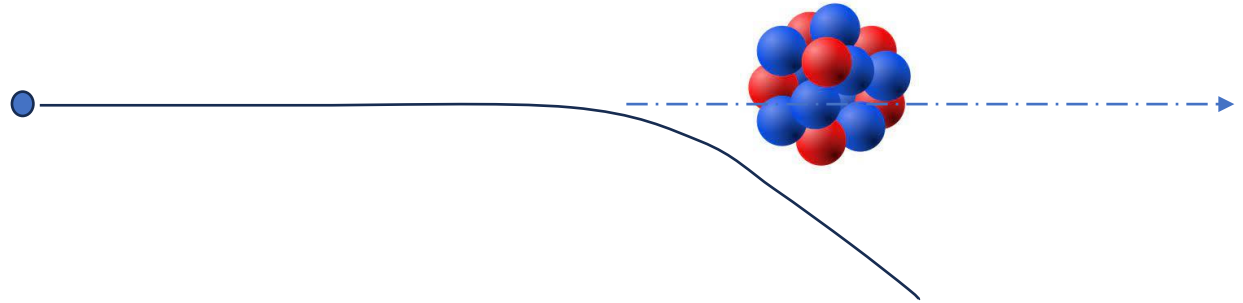
## Nuclear Reactions

- ✓ Certain nuclear reactions can be used to deduce the masses of specific nuclei.
- ✓ The energy released or absorbed in these reactions is related to the mass difference between reactants and products, as described by Einstein's mass-energy equivalence ( $E=mc^2$ ).

$$\mathbf{m} = \frac{\mathbf{E}}{c^2}$$

# Muon and Electron Scattering

- By scattering electrons or muons off atomic nuclei, researchers can obtain information about nuclear masses



- ❑ The experimental methods describe vary in their precision and applicability to different particles
- ❑ At the same time they allow scientists to determine the masses of atoms and subatomic particles with remarkable accuracy.

## Relation between amu and MeV.

- In modern Physics, the principle of mass energy equivalence says that mass has an equivalent amount of energy and vice – versa or they are interconvertible but this process is difficult to take place.
- The principle reasons that mass is concentrated energy.
- Energy can be released through the process matter antimatter annihilation.
- Since the amount of energy associated with matter is tremendous, subatomic (nuclear) reactions release great amount of energy as compared with chemical reactions.

- ✓ Being the two forms same entity, we can express mass in energy units and energy in units of mass.

We know that 1 amu is numerically equals to  $1.660538921 \times 10^{-31}$  kg.

Using Einstein's relation:

$$E = mc^2 = 1.660538921 \times 10^{-27} \text{ kg} \times (3 \times 10^8 \text{ m/s})^2 = 14.94485 \times 10^{-11}$$

Joules

$$= \frac{[14.94485 \times 10^{-11}]}{1.6 \times 10^{-19}} = 9.31 \times 10^8 = 931 \times 10^6 \text{ eV}$$

Therefore, **1 amu = 931 MeV**

# *Numerical Problems*

**Problem 1** Determine the average atomic mass of oxygen using the following data:

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*Thank You*