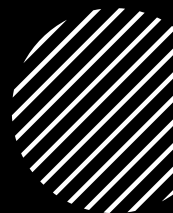




Course:
Mathematics for IT
Professionals



Lecture 1

Number Systems I: Radix

By

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Outline

The topics to be treated in this lecture are:

- Radix
- Signals
- Forms of Number System
 - Decimal
 - Binary
 - Octal
 - Hexadecimal
 - BCD
- Number Conversions
- Application of Number Systems



Lecture Learning Outcomes

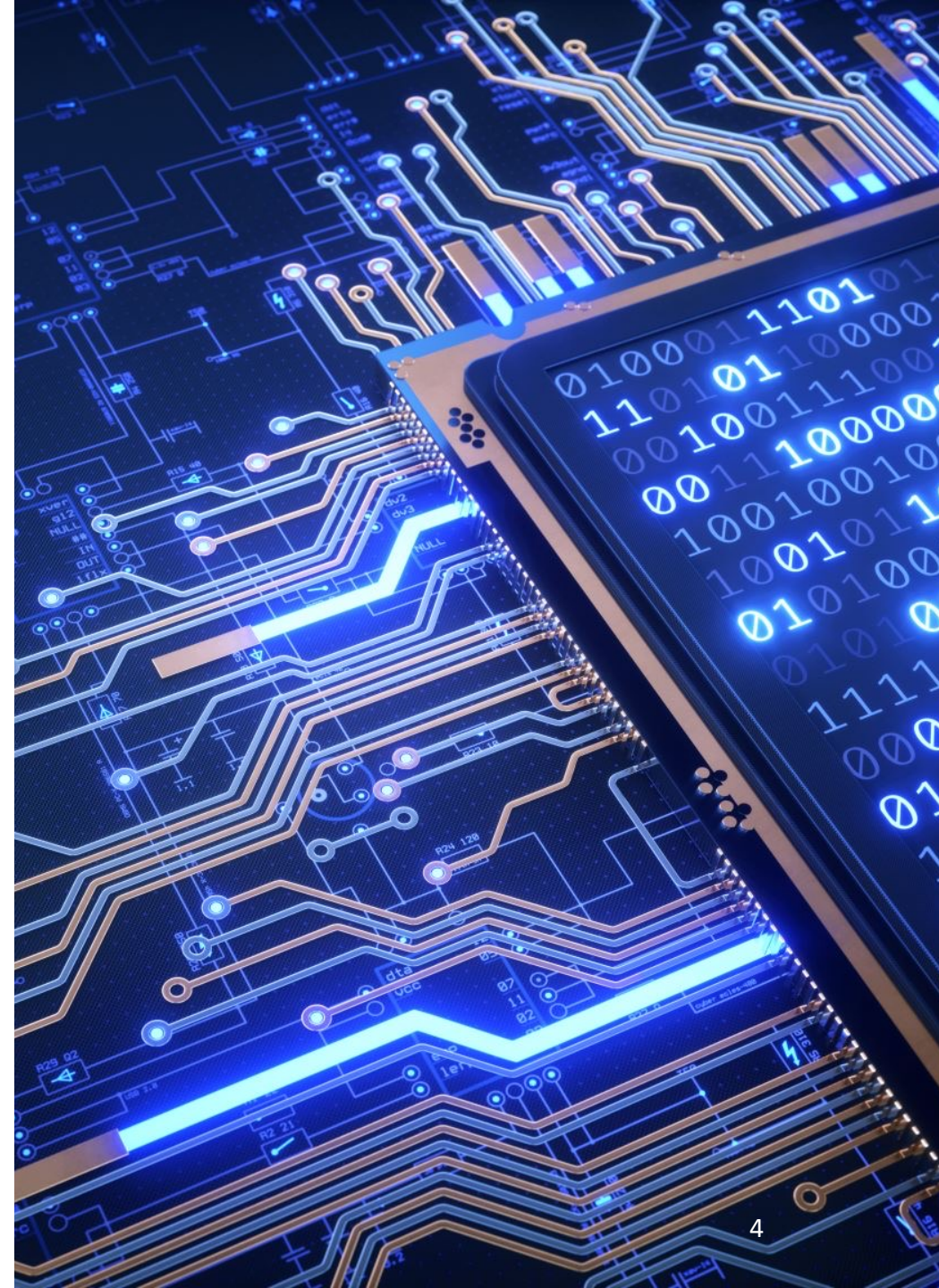
At the end of the session, you will be able to

- define the radix (base) in number systems
- understand the importance of number systems
- know the applications of number systems
- explain the difference between digital and analog signals
- covert numbers from one radix to another form
- represent numbers using ASCII

Introduction

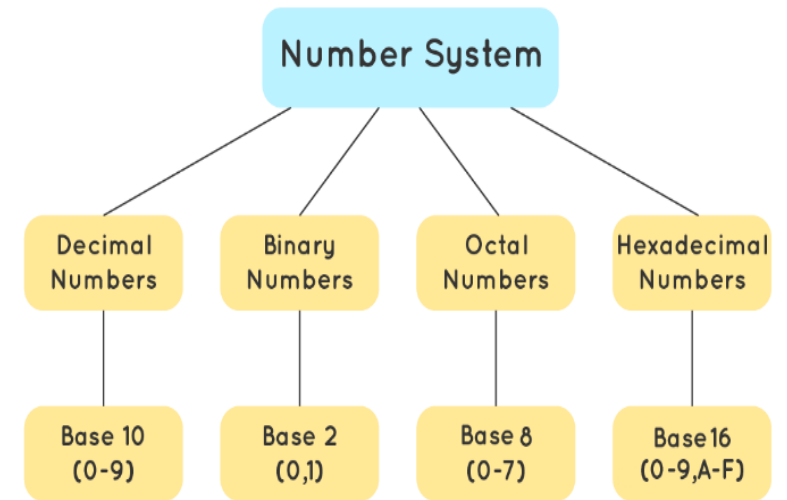
- Decimal or base (radix) ten is used by humans
- Binary or radix two is used by computers
 - Since it only comprehends two states
 - High voltage (+5V) denoted by 1
 - Low voltage (0V) denoted by 0
- Hence, there is the need to convert a number from decimal to binary and vice versa.

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Radix in Number Systems

- Number System:
 - naming, representing or expressing numbers in a particular format using digits or symbols.
 - E.g.: Decimal Number System, Binary Number System, etc.
- Radix or Base:
 - number of digits or symbols in a given number system.
 - E.g.: Radix 10, Radix 2, etc.



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Why study Number Systems?

1

Understand how the computer represents data before processing

2

Understand the language of the Digital Computer System

3

Convert numbers from one form to another

4

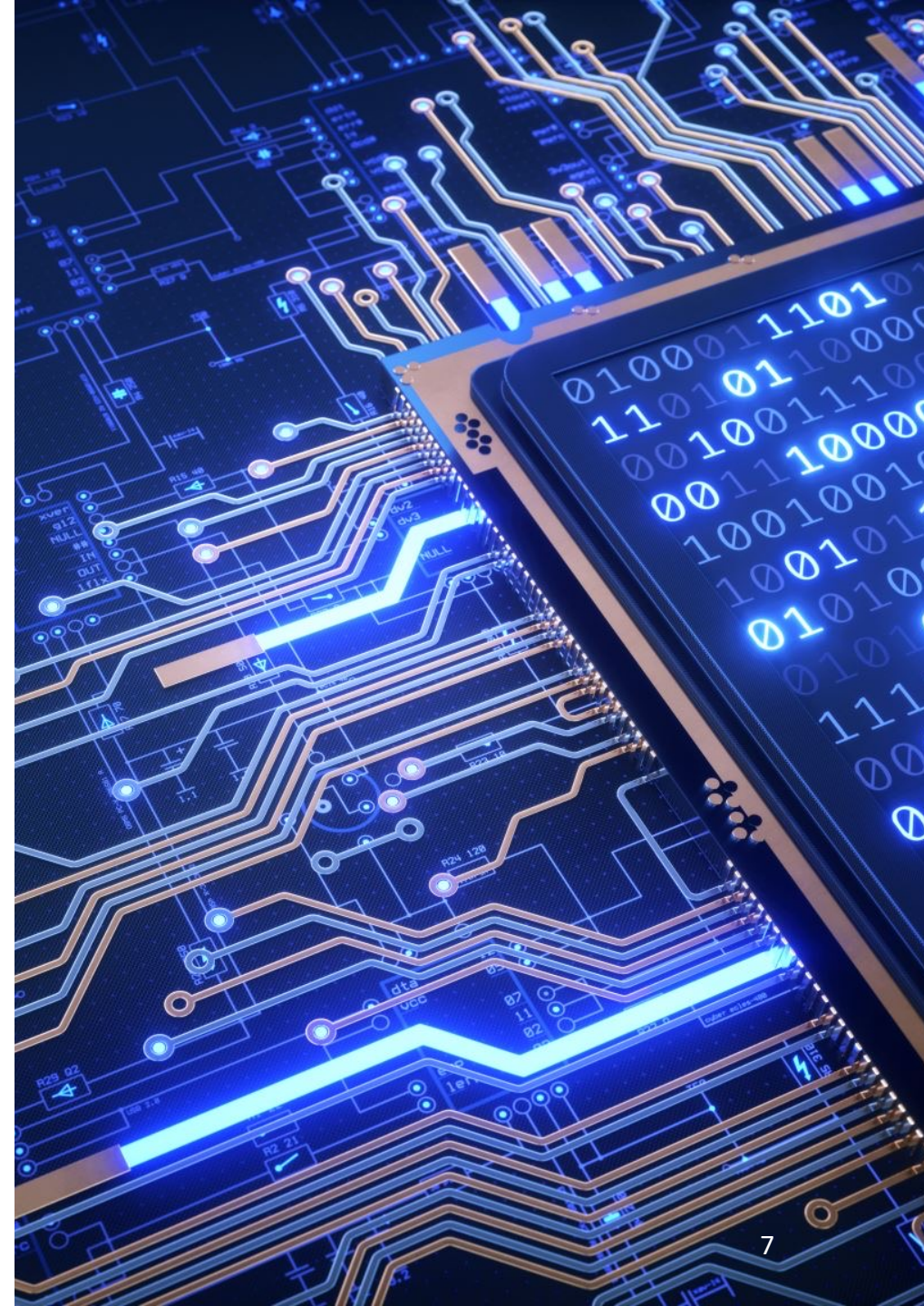
Perform arithmetic computation in any form

5

Analyze and design digital electronics

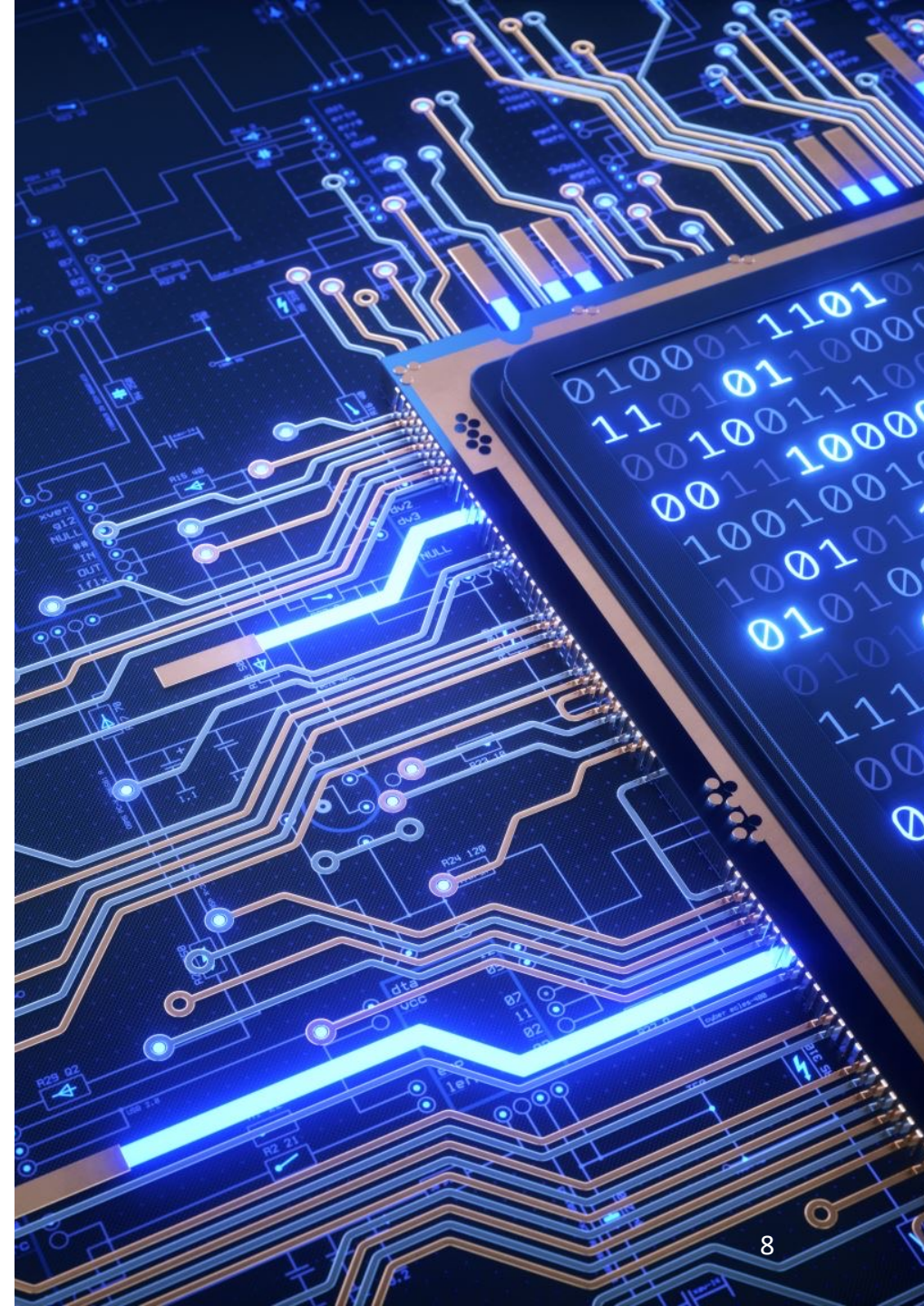
Application of Number Systems (NS)

- Decimal NS:
 - Counting, measuring and labeling items
 - Performing arithmetic calculations
 - Used in communication systems such as phone numbers, zip codes, etc
- Binary NS:
 - Used to represent and process data in computers
 - Performing binary arithmetic computations
 - Image manipulation and computer graphics
 - Binary operations extract specific information from images



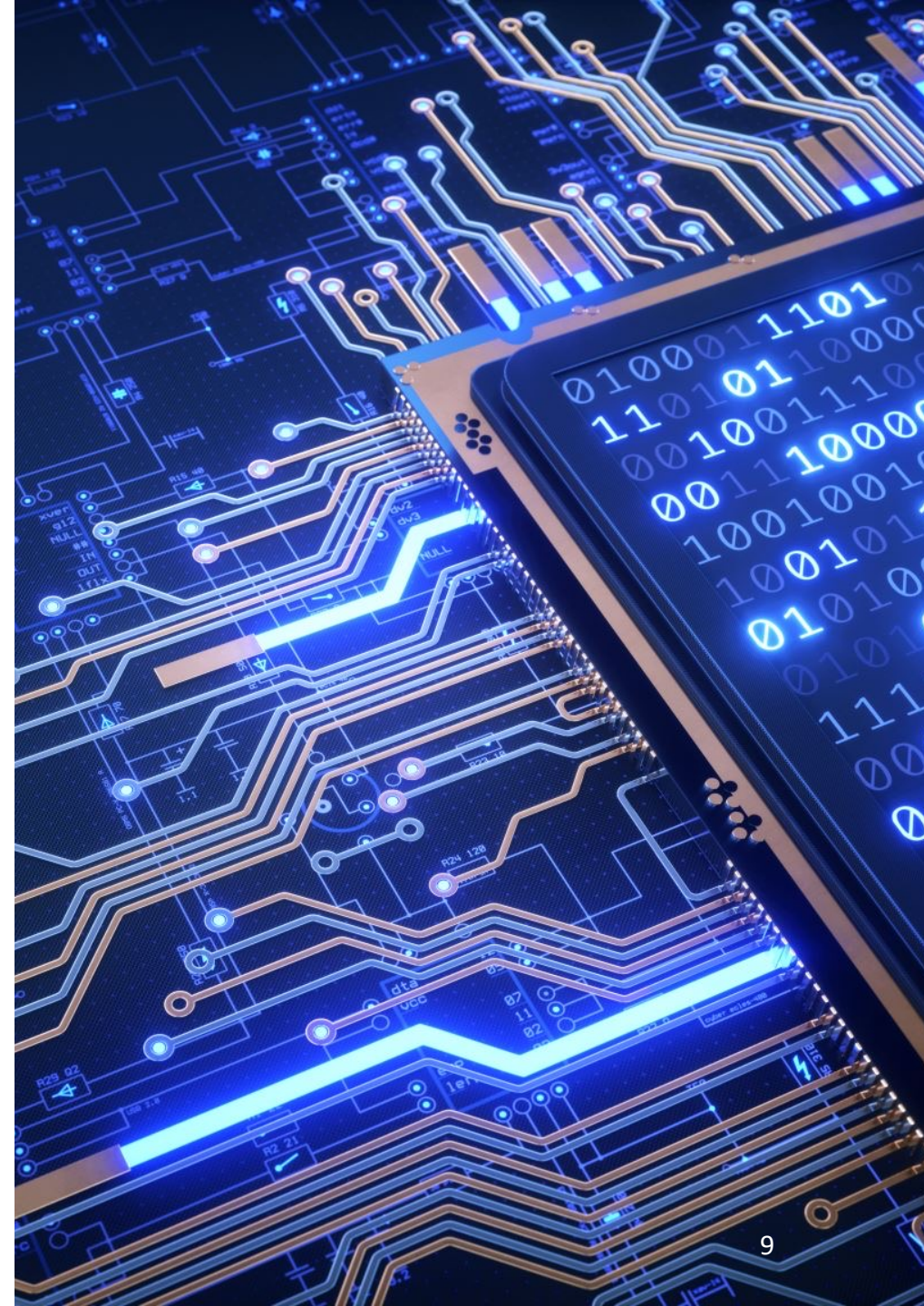
Application of Number Systems (NS)

- Octal NS:
 - Represent file permissions in UNIX OS (Linux and Mac OS)
 - Used for error detection caused by noise or other transmission issues
 - Used to distinguish different aircraft on a radar based on "squawk" codes
- Hexadecimal NS:
 - Large numbers can be represented using fewer digits
 - MAC addresses are represented using Hex
 - Colours are represented in Hex for graphic design



Application of Number Systems (NS)

- Binary Coded Decimal NS:
 - For encoding product codes using barcodes
 - Used in digital clocks and timekeeping systems
 - Employed in error detection and corrections techniques
 - E.g.: checksums and parity bits
 - For analog-to-digital conversions
 - Used in accounting systems for accurate manipulation of monetary values



Analog vs Digital Signals

- Signal refers to data values converted into electrical pulse readily available for transmission and processing.
 - Signals are used for
 - arithmetic computation
 - communication
 - networking
- Forms of numerical value representation
 - Analog signal
 - Digital signal

Analog



Digital



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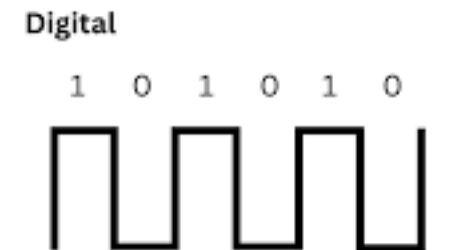
Analog vs Digital Signals

- Analog signal

- represents numerical values or figures of a quantity between two expected extreme values.
- provides an output which is continuous
- E.g.: Temp from 0°C – 100°C

- Digital signal

- express numerical values or figures of a quantity into steps of values.
- provides an output which is discrete
- E.g.: Temp of 9°C as 00001001



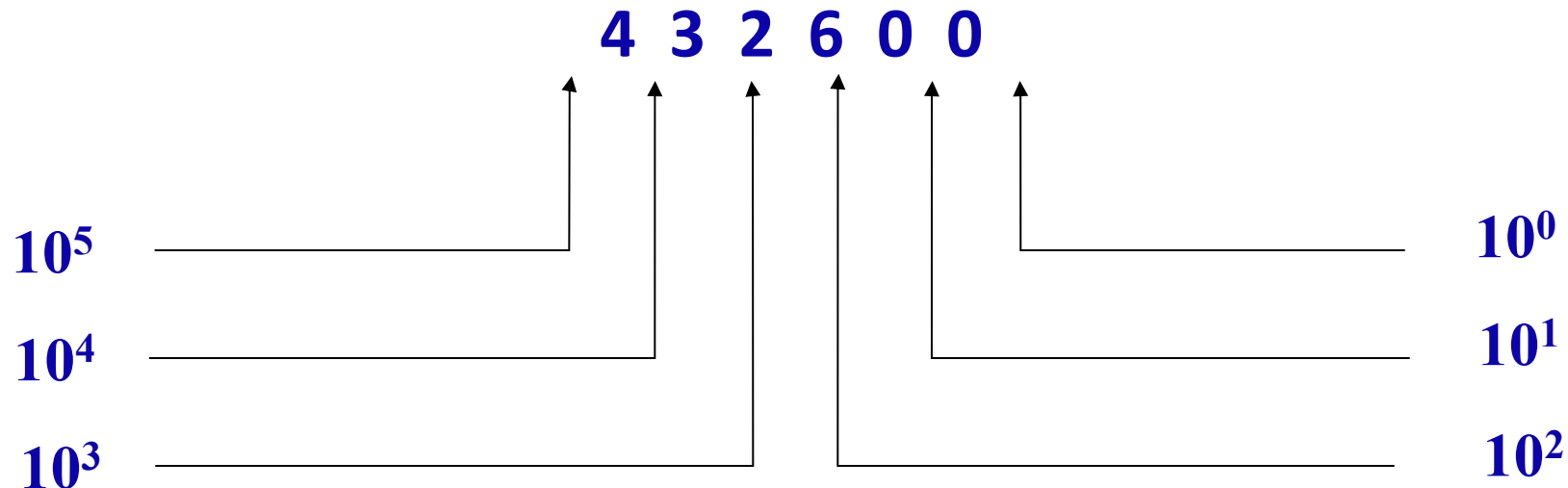
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Decimal Number System

- Radix ten or Decimal System

- {0 1 2 3 4 5 6 7 8 9}

E.g.: 432,600



Powers of ten:

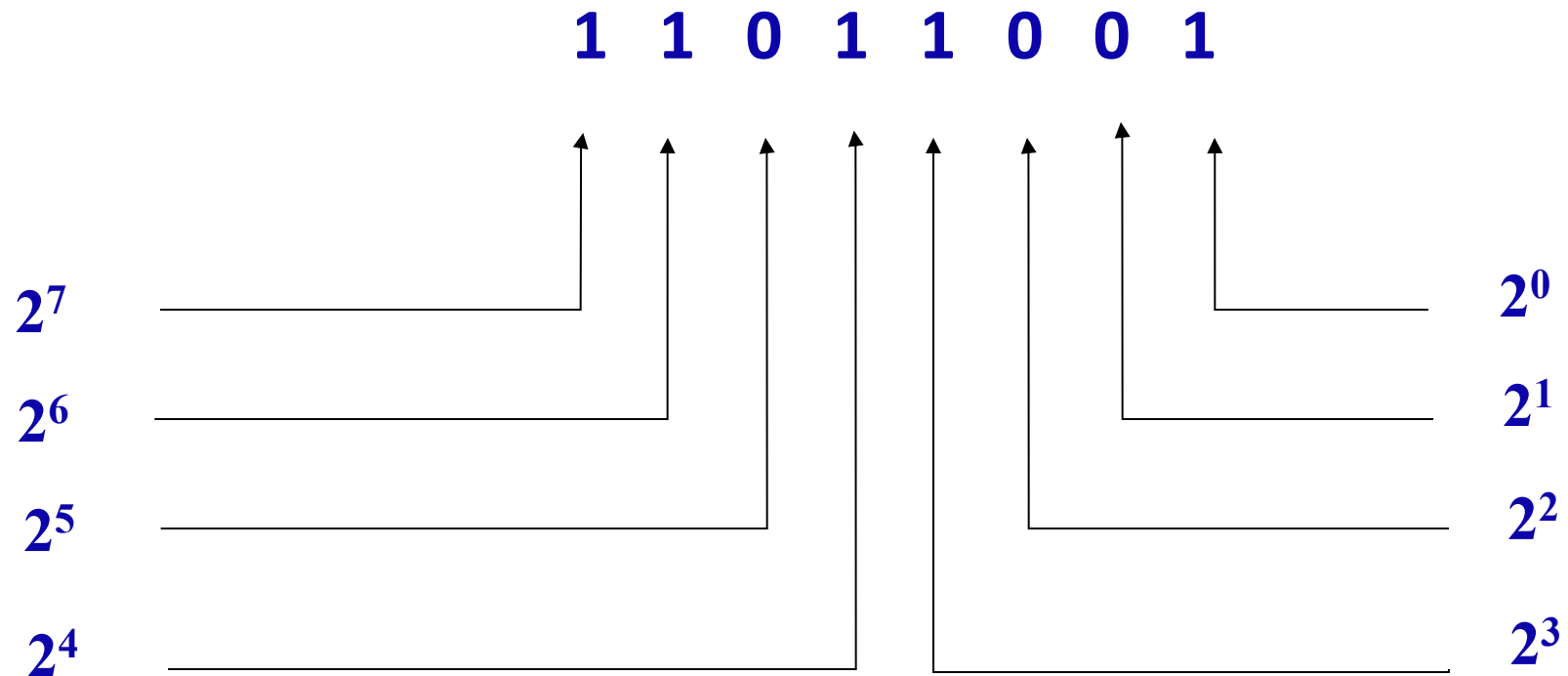
$$10^0 = 1 \quad 10^4 = 10000$$

$$10^1 = 10 \quad 10^3 = 1000 \quad 10^5 = 100000$$

Binary Number System (Radix 2) {0, 1}

Binar	Decimal
0	0
1	1
10	2
11	3
100	4
101	5
110	6
111	7
1000	8
1001	9
1010	10

Example



Binary to Decimal

Convert 1101 1001 to decimal

$$\begin{array}{r} 1 \times 2^7 = 128 \\ + 1 \times 2^6 = 64 \\ + 1 \times 2^4 = 16 \\ + 1 \times 2^3 = 8 \\ + 1 \times 2^0 = 1 \\ \hline 217 \end{array}$$

Try this!

1. What is 10011010 in decimal?
2. What is 00101001 in decimal?

Solution:

1. 154
2. 41

Decimal to Binary

Steps:

- 1. Determine the greatest power of two that is less than the number
- 2. Deduct the number from that power of two.
- 3. Repeat steps 1 and 2 for the updated outcome until you get to zero.
- 4. Write the binary number

E.g.: Convert 574 to radix 2

- $2^9 = 512$
- $574 - 512 = 62$
- $2^5 = 32$ $62 - 32 = 30$
- $2^4 = 16$ $30 - 16 = 14$
- $2^3 = 8$ $14 - 8 = 6$
- $2^2 = 4$ $6 - 4 = 2$
- $2^1 = 2$ $2 - 2 = 0$
- 1000111110

Decimal to Binary Conversion

- Example: 144
 - $2^7 = 128$ $144 - 128 = 16$
 - $2^4 = 16$ $16 - 16 = 0$
- Result: 10010000
- 53
 - = $32 + 16 + 4 + 1$
 - = $2^5 + 2^4 + 2^2 + 2^0$
 - = 110101 in binary
 - = 0011 0101 as a full byte in binary
- Exx. Convert 211 to binary
 - 1101 0011

Fraction(Decimal) to Binary

- ◆ Use Radix to multiply the number
- ◆ Pick the 0 or 1 integer as a partial result
- ◆ Pick the fraction part and repeat the process

Example: $(0.625)_{10}$

	Integer	Fraction	Result
$0.625 * 2 =$	1	. 25	$x_{-1} = 1$
$0.25 * 2 =$	0	. 5	$x_{-2} = 0$
$0.5 * 2 =$	1	. 0	$x_{-3} = 1$

Result: $(0.x_{-1} x_{-2} x_{-3})_2 = (0.101)_2$

In-Class Exercise

1. For a given binary number system, list the first sixteen digits.
2. Let 0, 1 and X be the independent digits in any arbitrary number system.
 - (a) What is this number system's radix?
 - (b) In this number system, list the first ten numbers.

Please try it!

Solution:

1. 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110 and 1111
- 2(a). The radix of the proposed number system is 3
- 2(b). The first 10 numbers in this number system would be 0, 1, X, 10, 11, 1X, X0, X1, XX and 100

Octal (Radix 8)

- Has eight different digits because its radix is 8.
- The 8 digits are as follows:

$\{0, 1, 2, 3, 4, 5, 6, 7\}$

- The place values in this number system are
 - $8^0, 8^1, \dots, 8^7$ (integer)
 - $8^{-1}, 8^{-2}, \dots, 8^{-7}$ (fraction)
- Exercise:
 - Write the next 10 numbers that follow '7'

Solution: 10, 11, 12, 13, 14, 15, 16, 17, 20 and 21

Decimal to Octal

- Convert 175.3125_{ten} to radix of 8

Converting the integer part: 175_{ten}

	Partial Quotient	Partial Remainder	Result
$175 / 8 =$	21	7	$x_0 = 7$
$21 / 8 =$	2	5	$x_1 = 5$
$2 / 8 =$	0	2	$x_2 = 2$

$175_{\text{ten}} = (x_2 x_1 x_0)_8 = 257_8$

Converting the fraction part: 0.3125_{ten}

	Partial Integer	Partial Fraction	Result
$0.3125 \times 8 =$	2	. 5	$x_{-1} = 2$
$0.5 \times 8 =$	4	. 0	$x_{-2} = 4$

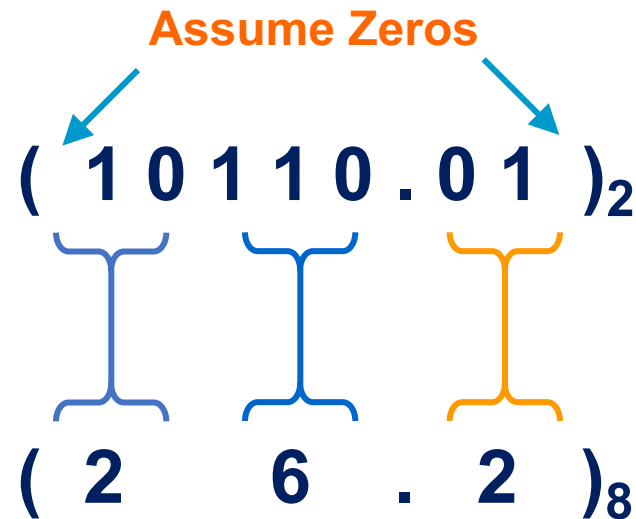
$0.3125_{\text{ten}} = (0.x_{-1} x_{-2})_8 = (0.24)_8$

$$175.3125_{\text{ten}} = 257.24_{\text{eight}}$$

Binary to Octal

- ◆ $8 = 2^3$
- ◆ An octal digit is represented by group of 3 bits

Example:



Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Works **both** ways (Binary to Octal & Octal to Binary)

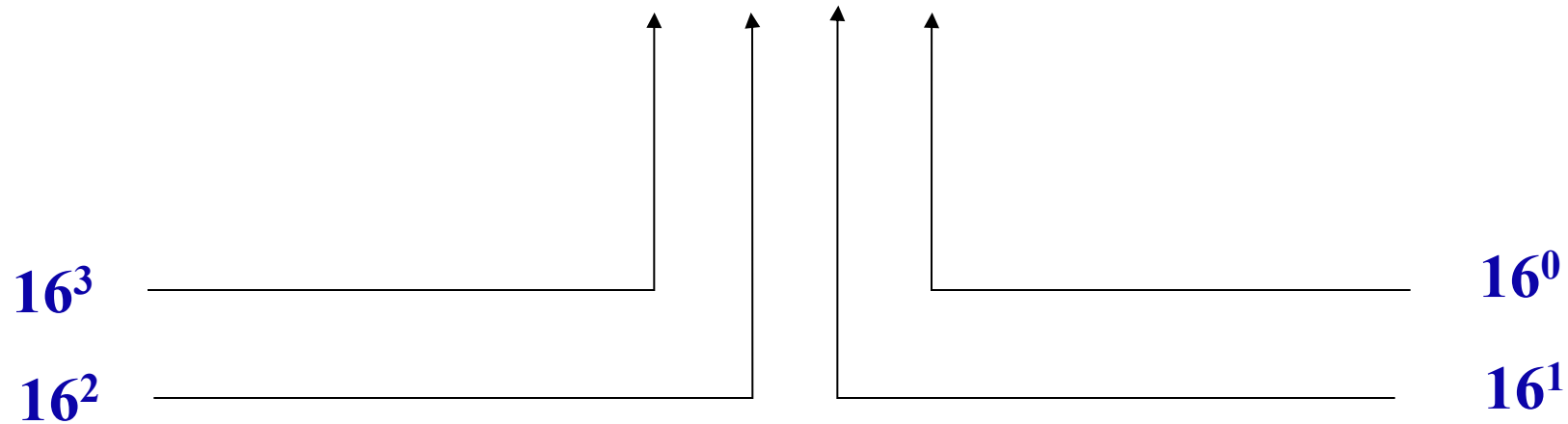
Hexadecimal (Radix 16)

- ◆ A hex digit is represented by group 4 bits
{0 1 2 3 4 5 6 7 8 9 A B C D E F}
- ◆ Decimal to Hexadecimal equivalents shown in Table

Dec	Hex	Dec	Hex
0	0	8	8
1	1	9	9
2	2	10	A
3	3	11	B
4	4	12	C
5	5	13	D
6	6	14	E
7	7	15	F

Example: Hexadecimal to Decimal

3 B 6 E



$$3 * 16^3 = 12,288$$

$$11 * 16^2 = 2,816$$

$$6 * 16^1 = 96$$

$$14 * 16^0 = 14$$

$$= 15214$$

Hexadecimal to Binary Conversion

$$16 = 2^4$$

Each hexadecimal digit will be represented by 4 bits (nibble)

Binary	Hex	Binary	Hex
0000	0	1001	9
0001	1	1010	A
0010	2	1011	B
0011	3	1100	C
0100	4	1101	D
0101	5	1110	E
0110	6	1111	F
0111	7		
1000	8		

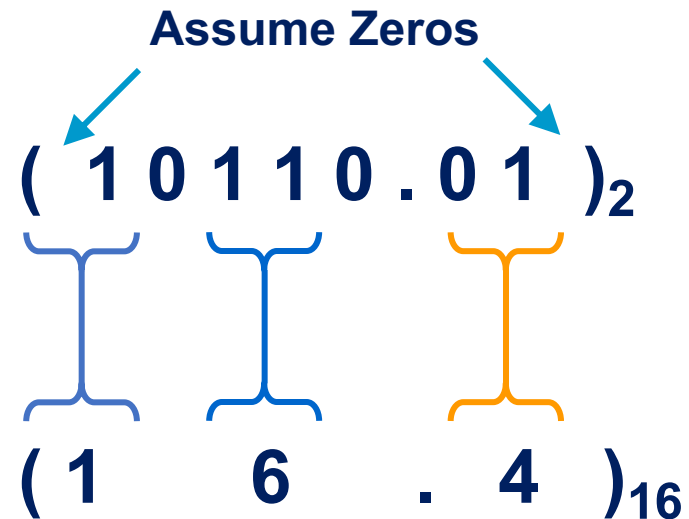
Binary to Hexadecimal

- Put binary numbers in groups of four (nibbles)
 - 1101 1001 0110
- Translate each nibble's representation into hex
- 1101 1001 0110
 - D 9 6

Binary to Hexadecimal

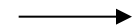
◆ $16 = 2^4$

Example:



Can be done in both ways

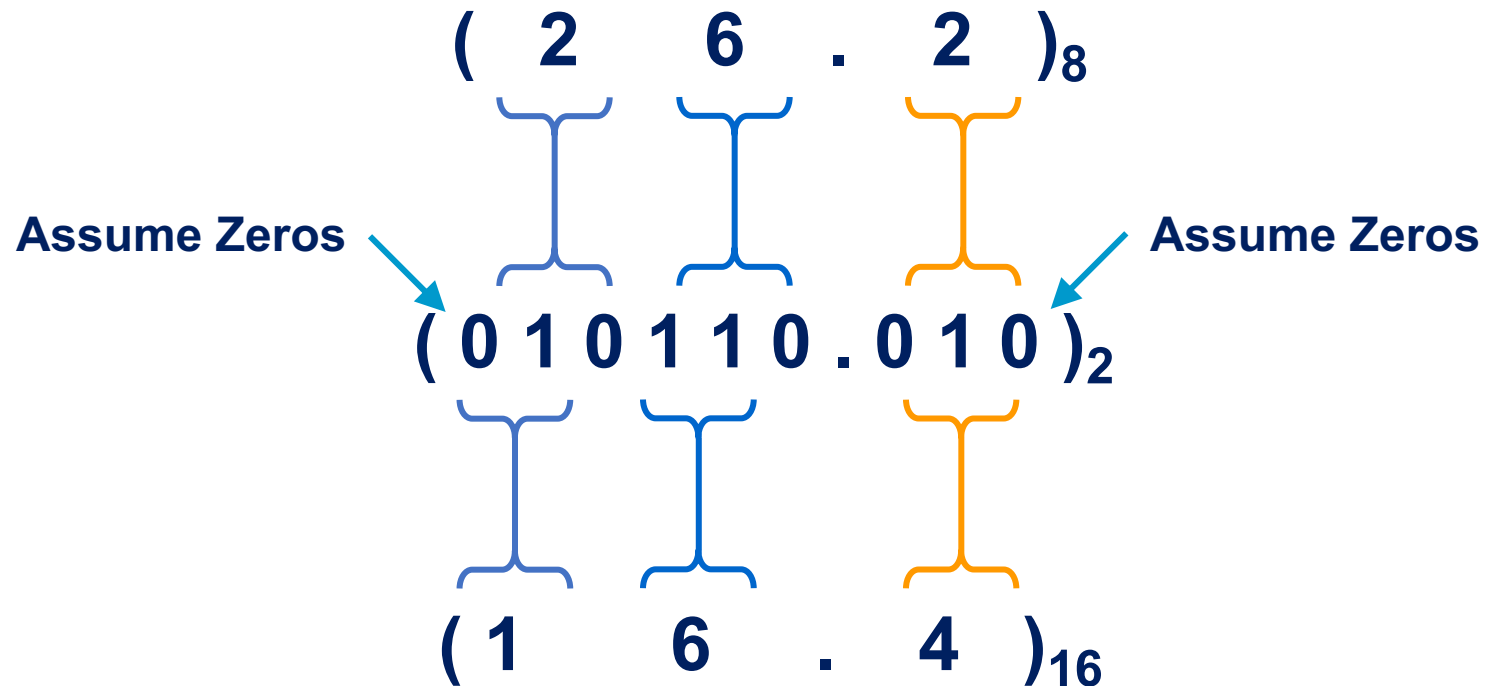
Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1



Octal to Hexadecimal

- ◆ Convert to Binary as a baseline step

Example:



Can be done in both ways

Decimal to Hexadecimal

- Eg. 284
 - $16^2 = 256$ $284 - 256 = 28$
 - $16^1 = 16$ $28 - 16 = 12$ (Hex C)
 - Result: 1 1 C
 - **Repeated Division Approach:**

	Quotient	Remainder	Coefficient
284/ 16 =	17	12	$a_0 = C$
17/ 16 =	1	1	$a_1 = 1$
1/ 16 =	0	1	$a_2 = 1$

Answer: **$(284)_{10} = (a_2 a_1 a_0)_{16} = (11C)_{16}$**

- Exercise: Try 1054
- Result: 4 1 E

Hexadecimal to Decimal

- Convert **2DB** in hex to radix of ten

Place Value	16^2	16^1	16^0				
Hexadecimal	2	D	B				
	$(16^2 \times 2)$	$(16^1 \times 13)$	$(16^0 \times 11)$				
Decimal	512	+	208	+	11	=	731

Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Application Example

- **ASCII** (American Standard Code for Information Interchange)
 - assigns standard binary codes to letters, numerals, punctuation marks, and other characters used in computers.
 - is a 7-bit character set containing 128 characters
 - the 8th bit is often used for error detection (parity bit)
- **E.g.:** Use the ASCII codes to represent **Digital**

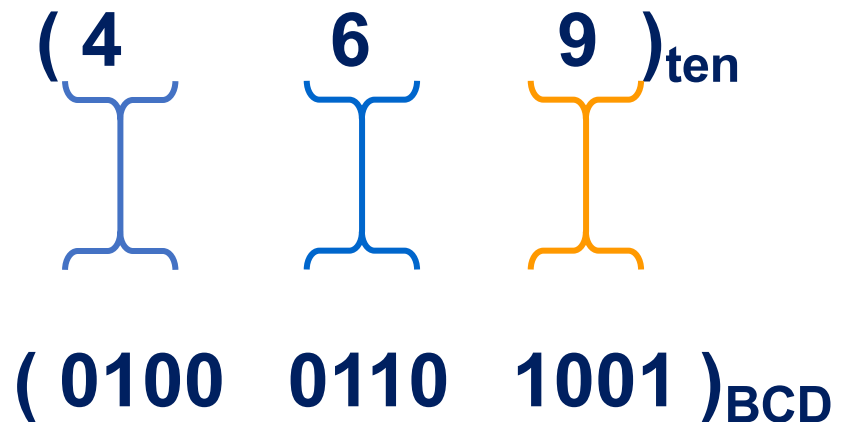
<u>Character</u>	<u>Binary Code</u>	<u>Hexadecimal Code</u>
D	1000100	44
i	1101001	69
g	1100111	67
i	1101001	69
t	1110100	74
a	1100001	61
l	1101100	6C

BCD Number System

- The Binary Coded Decimal (BCD) replaces each decimal digit with a four-bit binary code.

0 1 2 3 4 5 6 7 8 9

- E.g.: Convert 469_{ten} to BCD
- Answer:



BCD Number System

- Exx.: Find the decimal equivalent of $0110\ 0100\ 1011_{BCD}$

Answer:

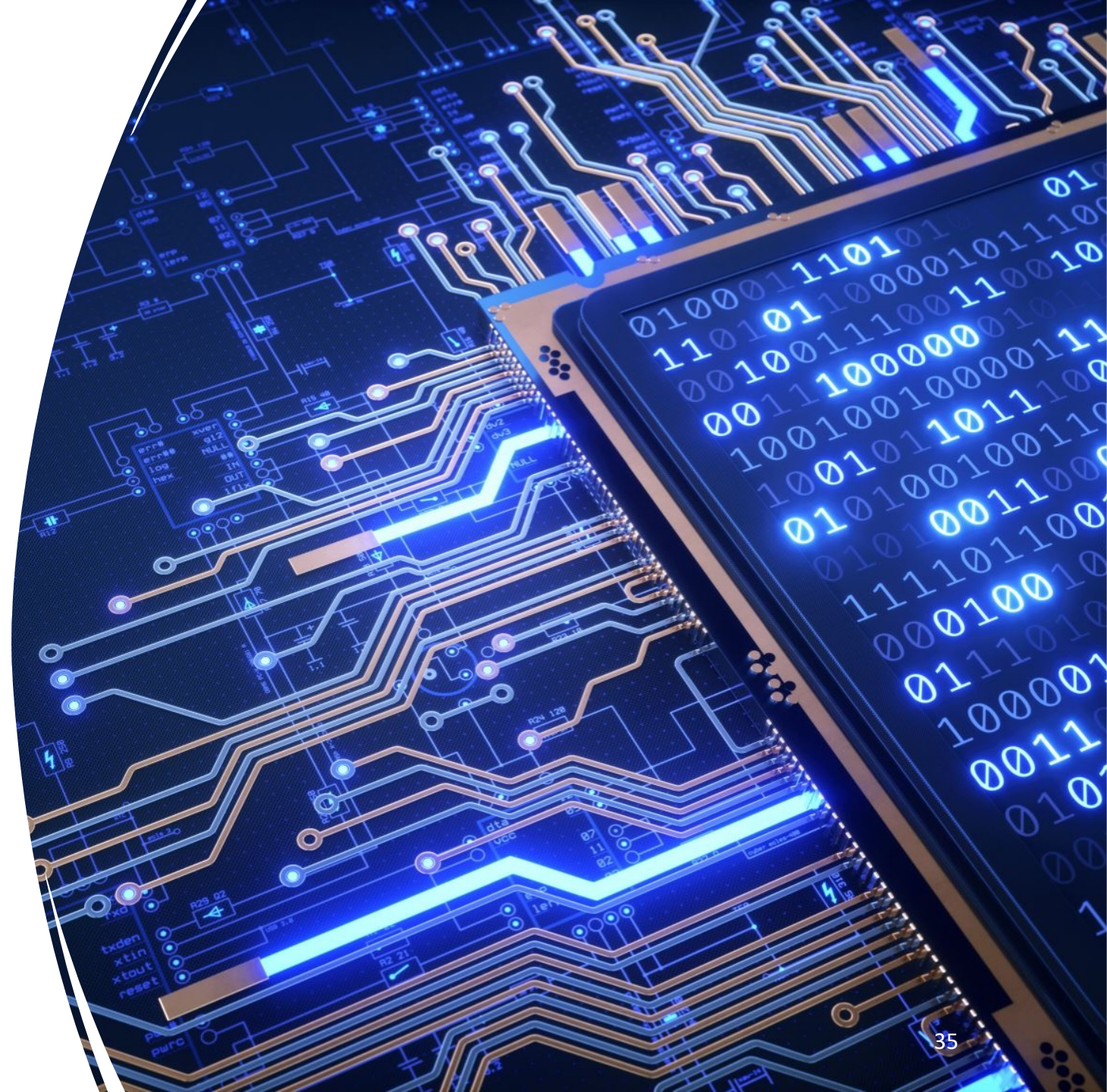
$$\begin{array}{ccc} (0110 & 0100 & 1011)_{BCD} \\ \left. \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right\} & \left. \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right\} & \left. \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right\} \\ (6 & 4 & \#)_{ten} \end{array}$$

- not possible because 1011 is not a valid BCD

- Assignment:
 - Convert $(9750)_{10}$ to BCD
 - Answer: $(9750)_{10} = (1001011101010000)_{BCD}$

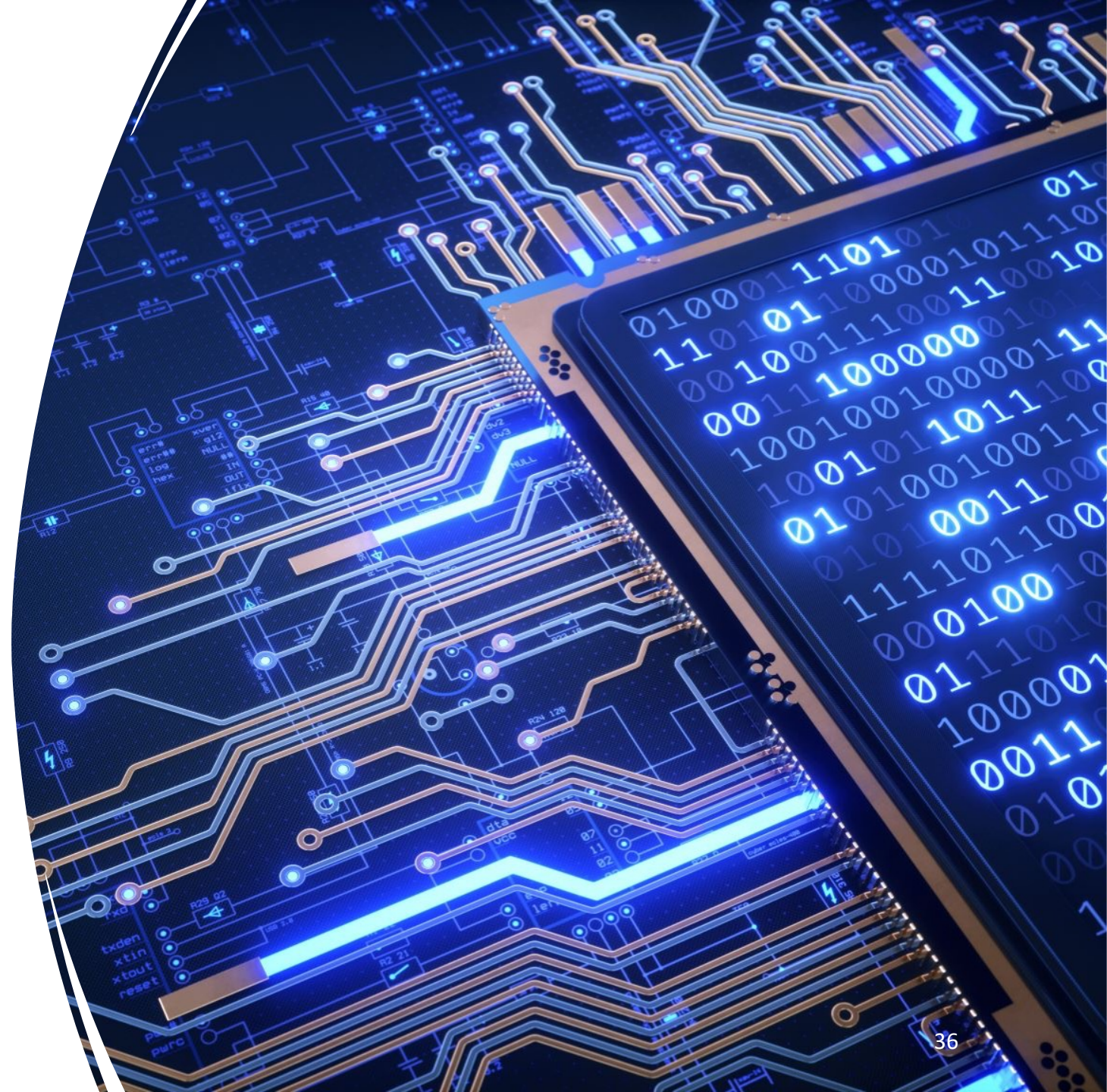
Summary

- Digital electronics use base-two (binary)
 - +5 voltage (1) and 0V voltage (0)
 - High and Low
- Analog electronics operate on continuously varying electrical/physical magnitudes – temperature, pressure, velocity, etc.
- Number conversions
 - Binary, Octal, Hexadecimal, BCD



Reference

Maini, A. K. (2007). "Digital Electronics: Principles, Devices and Applications". *John Wiley & Sons, Ltd.* ISBN: 978-0-470-03214-5
Chapter 1



See you next
time!

*Thank
you!*