# Course: Mathematics for IT Professionals

**Lecture 7**

Vectors and Matrices

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# Outline

The topics to be treated in this lecture are:

- Definition
  - Matrix
  - Vector
- Types of matrices
- Matrix operations
- Transpose of Matrices
- Inverse of Matrices
  - Determinant
  - Minors
  - Co-factors
  - Adjoint
- Linear equations

# Lecture Learning Outcomes

At the end of the session, you will be able to

- understand the difference between a matrix and a vector
- know the types of matrix
- perform various computations using matrices
- find the inverse of a non-singular matrix
- convert a linear system of equations to a matrix and vice versa
- know the applications of matrices

# Introduction

A matrix is a set or group of numbers arranged in a square or rectangular array enclosed by two brackets.

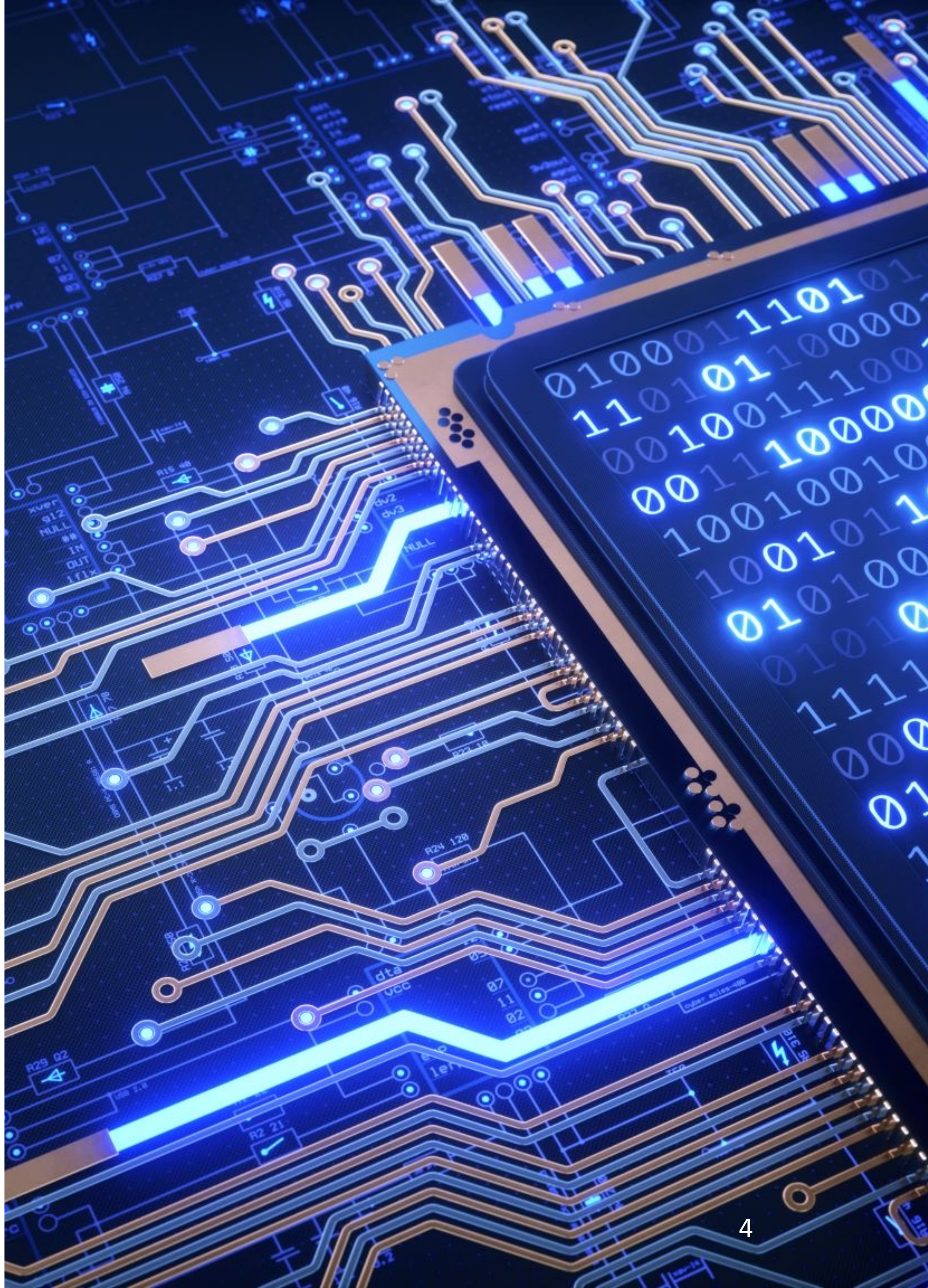
A vector is a matrix with a single row or column.

Matrix algebra has the following two advantages:

- Reduces complicated systems of equations to simple expressions
- Adaptable to systematic method of mathematical treatment and well suited to computers

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 4 & 2 \\ -3 & 0 \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Rosen, K. H. (2012). *Discrete mathematics and its applications (7th Edition)*. McGraw-Hill.



# Introduction

Properties:

- A specified number of rows and a specified number of columns
- Two numbers (rows x columns) describe the dimensions or size of the matrix.

Examples:

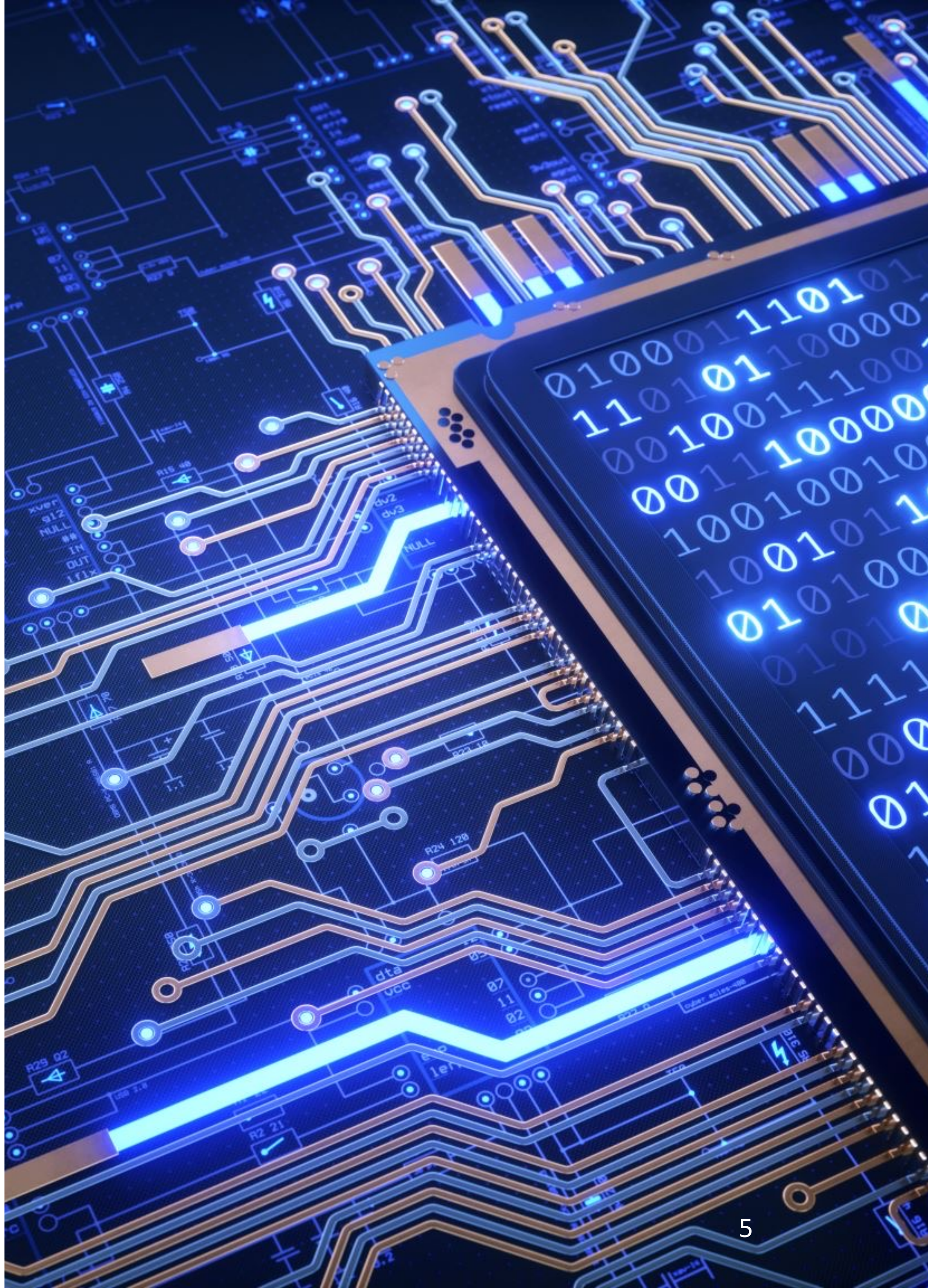
3x3 matrix

2x4 matrix

1x2 matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & -1 & 5 \\ 3 & 3 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 3 & -3 \\ 0 & 0 & 3 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Rosen, K. H. (2012). *Discrete mathematics and its applications (7th Edition)*. McGraw-Hill.



# Introduction

A matrix is denoted by a bold capital letter and the elements within the matrix are denoted by lower case letters

e.g. matrix [**A**] with elements $a_{ij}$

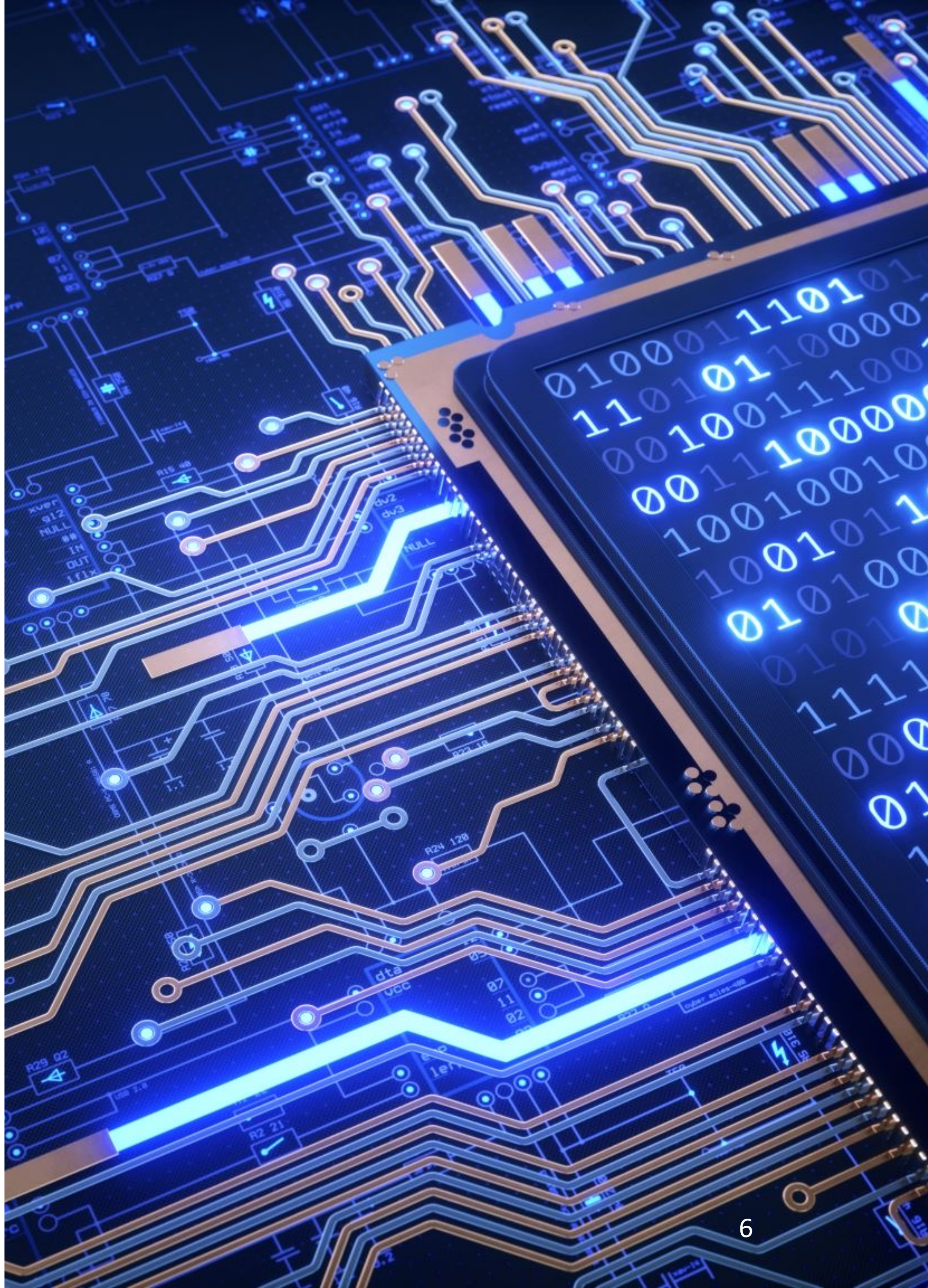
$\mathbf{A}_{mxn}$=

${}_{m}\mathbf{A}^{n}$

$$\begin{bmatrix} a_{11} & a_{12}\ldots & a_{ij} & a_{in} \\ a_{21} & a_{22}\ldots & a_{ij} & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{ij} & a_{mn} \end{bmatrix}$$

i goes from 1 to m

j goes from 1 to n



# Types of Matrices

## 1. Column matrix or vector

The number of rows may be any integer, but the number of columns is always 1

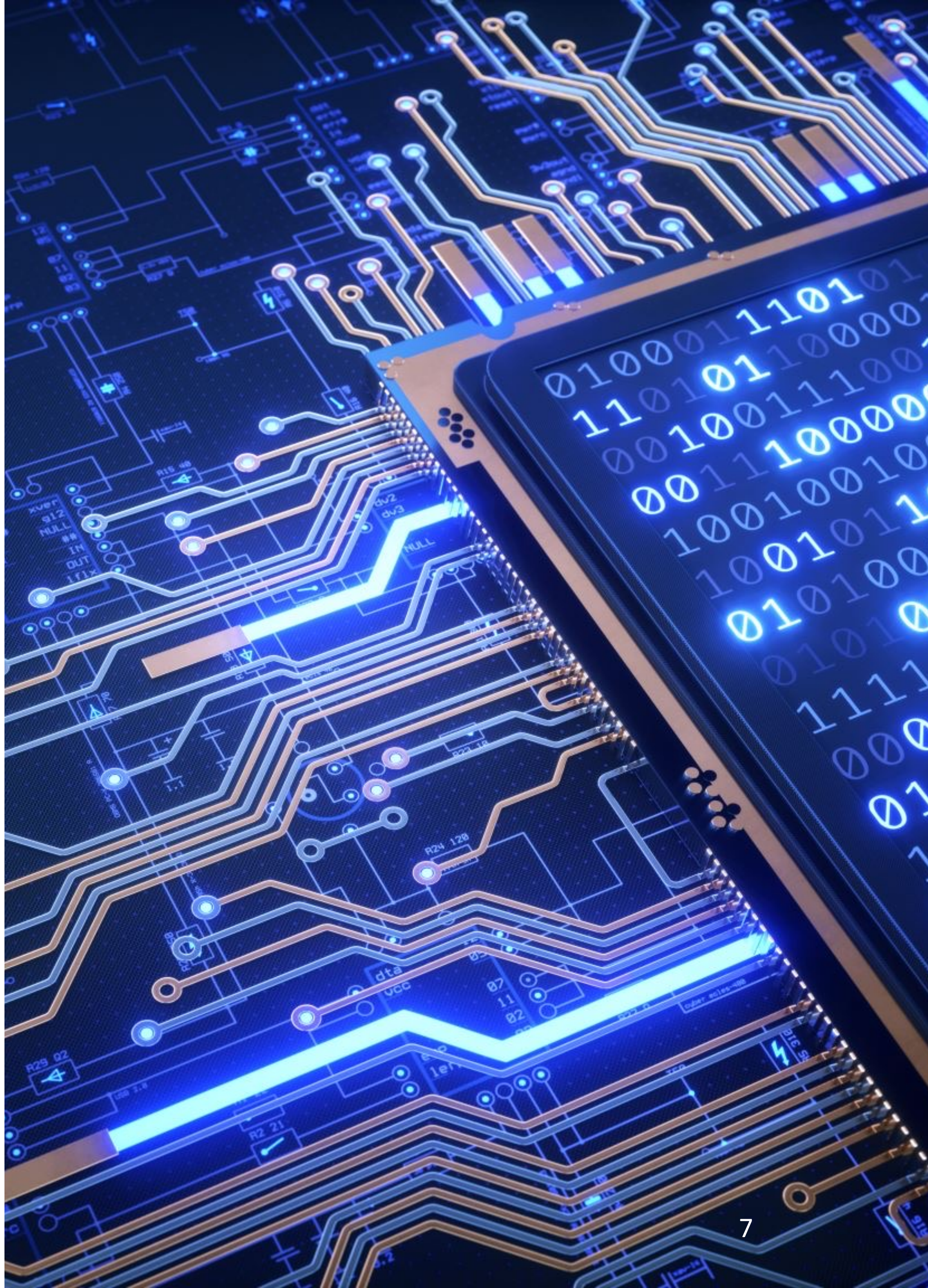
$$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

## 2. Row matrix or vector

Any number of columns but only one row

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 5 & 2 \end{bmatrix}$$



# Types of Matrices

## 3. Rectangular matrix

Contains more than one element and number of rows is not equal to the number of columns

$$\begin{bmatrix} 1 & 1 \\ 3 & 7 \\ 7 & -7 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 3 & 0 \end{bmatrix}$$

$$m \neq n$$

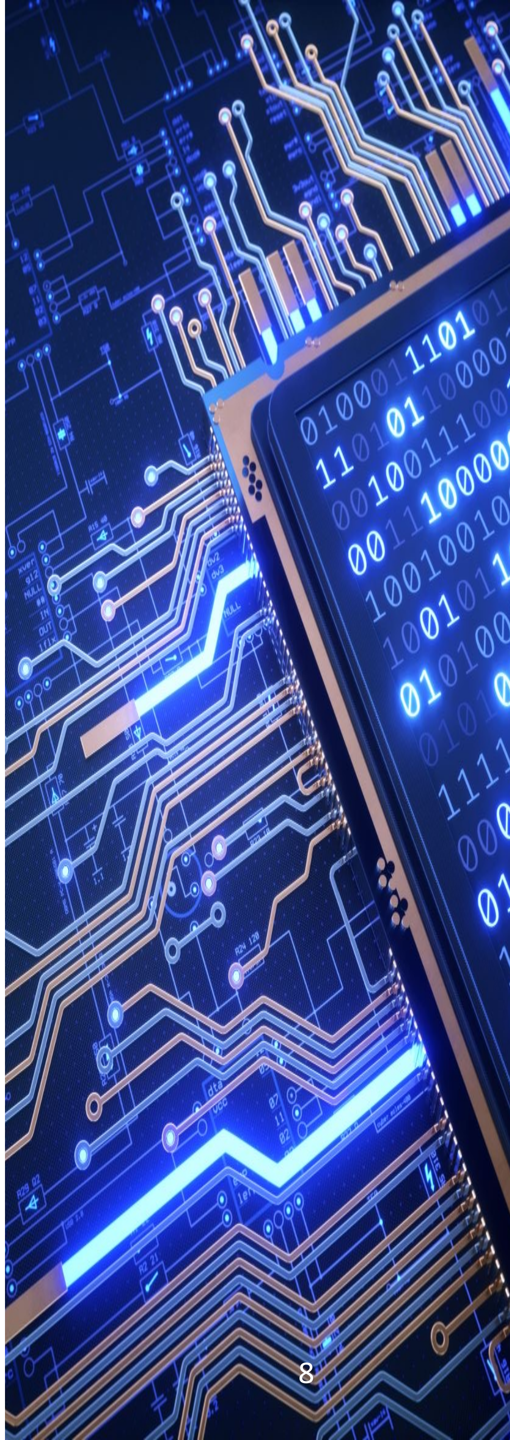
## 4. Square matrix

The number of rows is equal to the number of columns (a square matrix **A** has an order of m)

m x m

$$\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

The principal or main diagonal of a square matrix is composed of all elements $a_{ij}$ for which $i=j$



# Types of Matrices

**5. Diagonal matrix**

A square matrix where all the elements are zero except those on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$

$a_{ij} = 0$ for some or all $i \neq j$

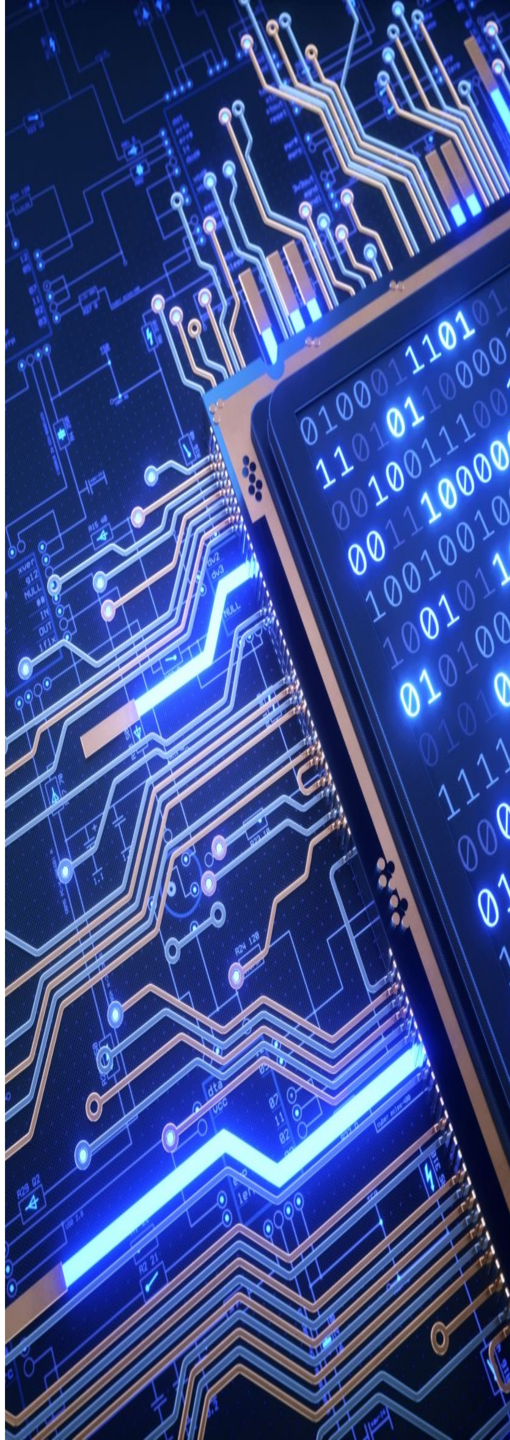
**6. Unit or Identity matrix - I**

A diagonal matrix with ones on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{ij} & 0 \\ 0 & a_{ij} \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$

$a_{ij} = 1$ for some or all $i = j$



# Types of Matrices

## 7. Null (zero) matrix - 0

All elements in the matrix are zero

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$a_{ij} = 0$$

For all $i,j$

## 8. Scalar matrix

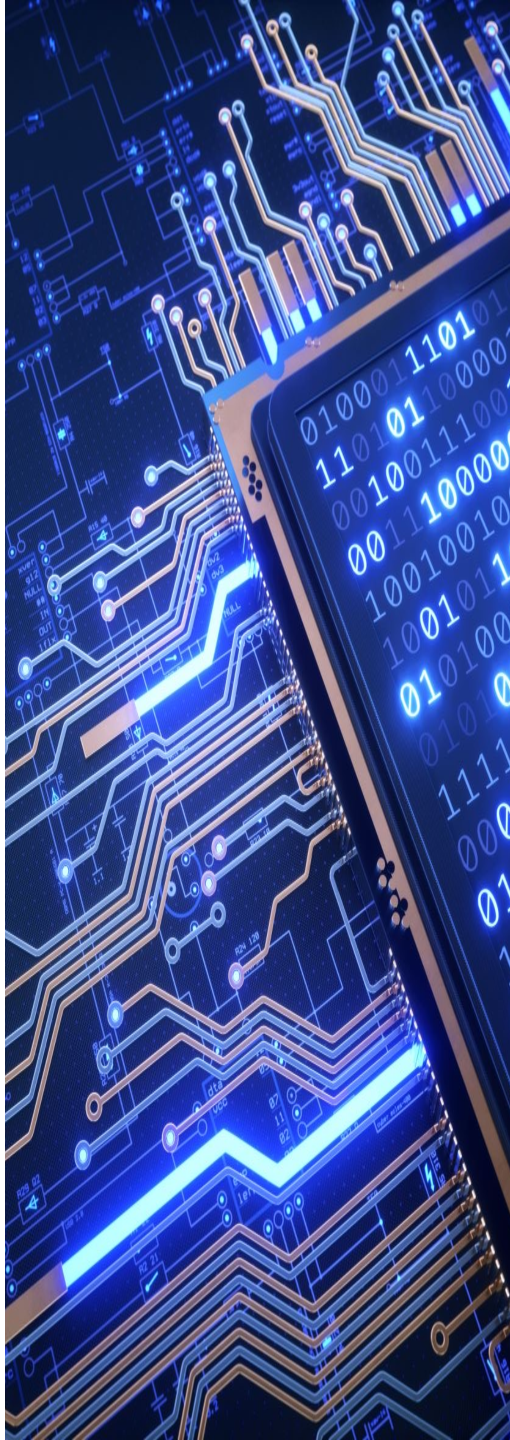
A diagonal matrix whose main diagonal elements are equal to the same scalar

A scalar is defined as a single number or constant

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ 0 & a_{ij} & 0 \\ 0 & 0 & a_{ij} \end{bmatrix} \quad \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i = j$

$a_{ij} = a$ for all $i = j$



# Types of Matrices

### 9. Triangular matrix

A square matrix whose elements above or below the main diagonal are all zero

### 9a. Upper triangular matrix

A square matrix whose elements below the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & a_{ij} & a_{ij} \\ 0 & a_{ij} & a_{ij} \\ 0 & 0 & a_{ij} \end{bmatrix} \begin{bmatrix} 1 & 8 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix}$$

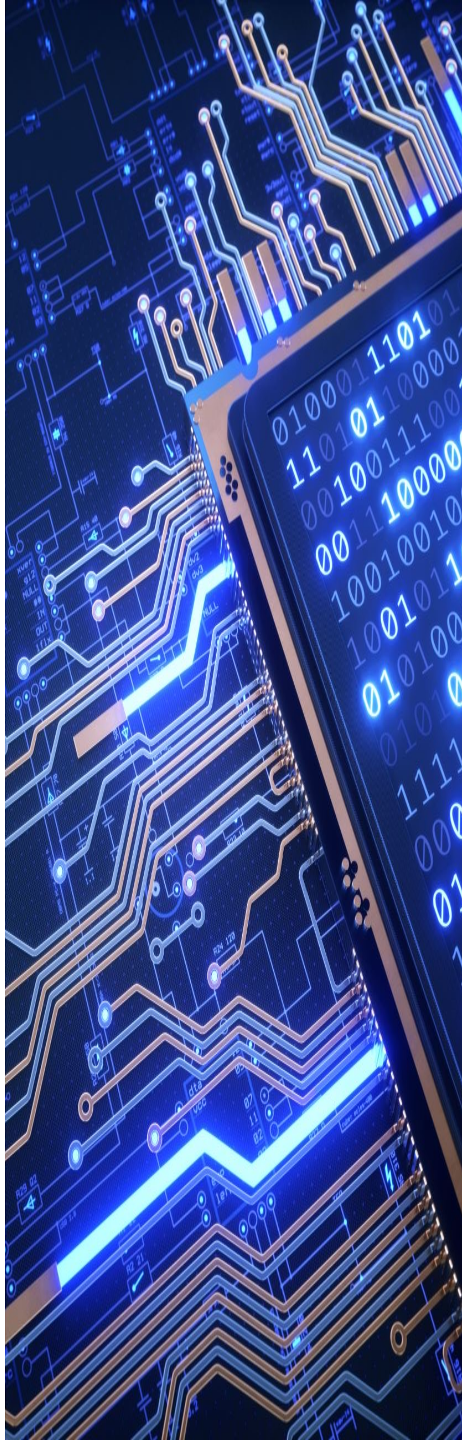
i.e. $a_{ij} = 0$ for all $i > j$

### 9b. Lower triangular matrix

A square matrix whose elements above the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ a_{ij} & a_{ij} & 0 \\ a_{ij} & a_{ij} & a_{ij} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i < j$



# Equality of Matrices

Two matrices are said to be equal only when all corresponding elements are equal

Therefore, their size or dimensions are equal as well

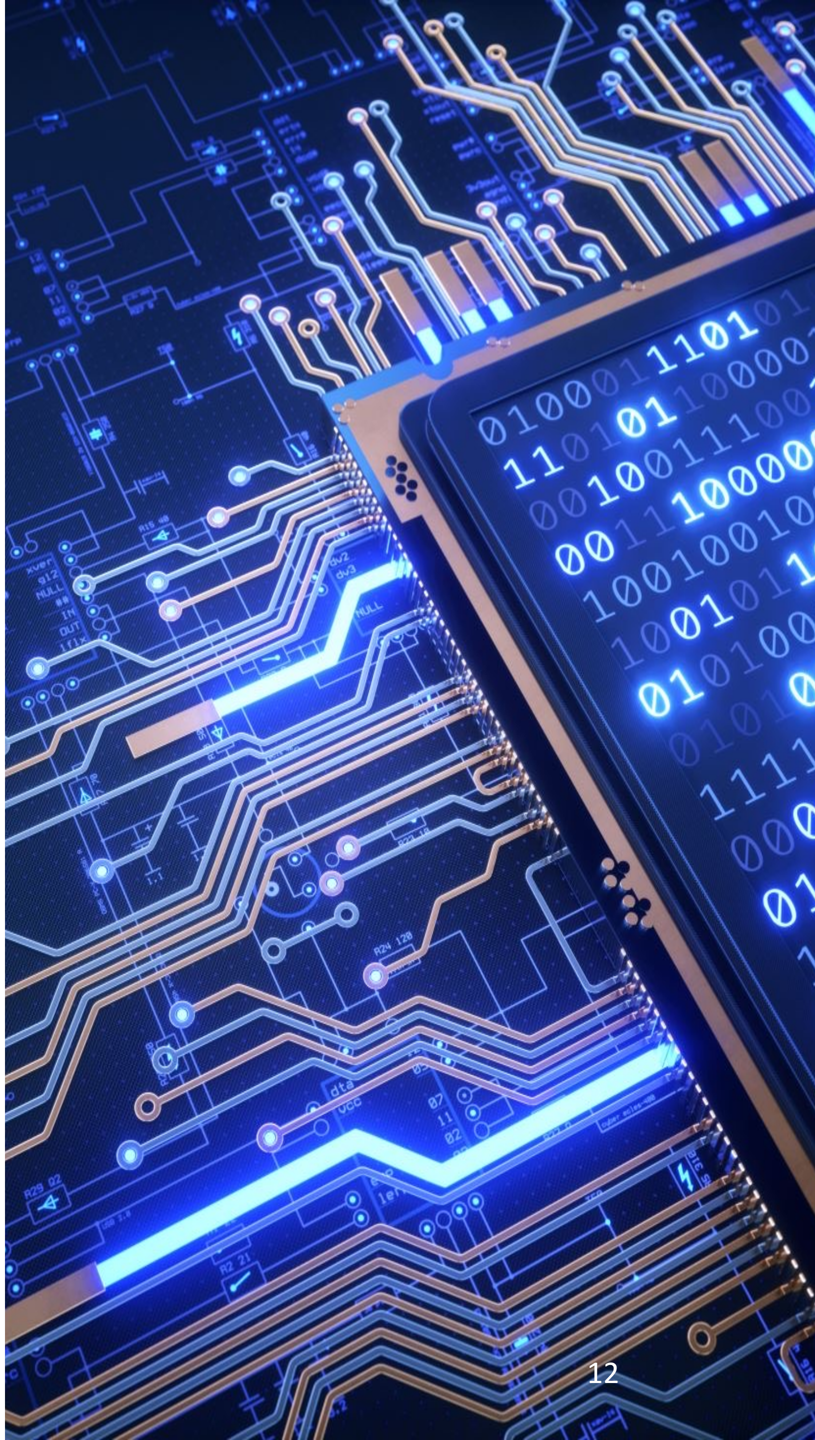
**A** = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$ **B** = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$

**A** = **B**

Some properties of equality:

- If **A** = **B**, then **B** = **A** for all **A** and **B**
- If **A** = **B**, and **B** = **C**, then **A** = **C** for all **A**, **B** and **C**

If **A** = **B**, then $a_{ij} = b_{ij}$



# Addition and Subtraction of Matrices

The sum or difference of two matrices, **A** and **B** of the same size yields a matrix **C** of the same size

$$c_{ij} = a_{ij} + b_{ij}$$

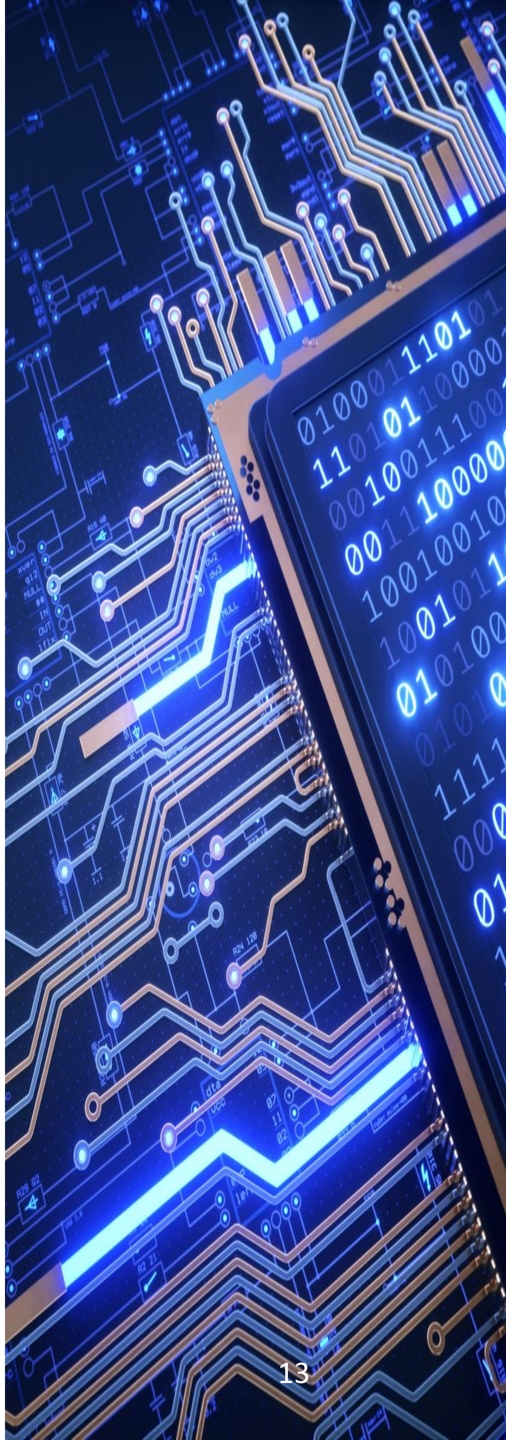
Commutative Law:
**A + B = B + A**

Associative Law:
**A + (B + C) = (A + B) + C = A + B + C**

Matrices of different sizes cannot be added or subtracted

$$\begin{bmatrix} 7 & 3 & -1 \\ 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 6 \\ -4 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 5 \\ -2 & -7 & 9 \end{bmatrix}$$

**A** 2x3 **B** 2x3 **C** 2x3



# Matrix Multiplication

The product of two matrices is another matrix

Two matrices **A** and **B** must be **conformable** for multiplication to be possible

i.e., the number of columns of **A** must equal the number of rows of **B**

Example.

| | | | | |
|---|---|---|---|---|
| **A** | x | **B** | = | **C** |
| (1x3) | | (3x1) | | (1x1) |

**B** x **A** = Not possible!

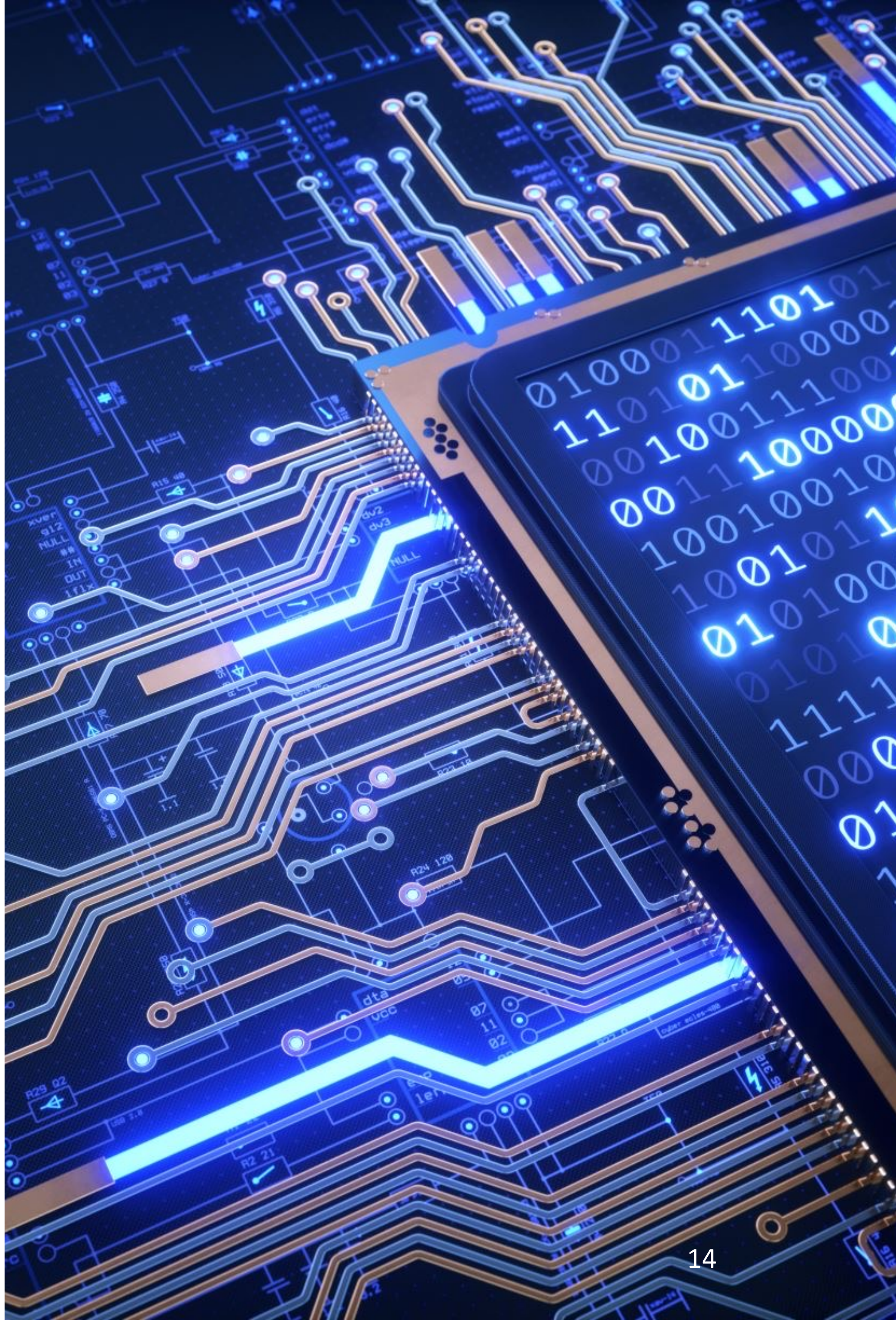
(2x1) (4x2)

**A** x **B** = Not possible!

(6x2) (6x3)

**A** x **B** = **C**

(2x3) (3x2) (2x2)



# Matrix Multiplication

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

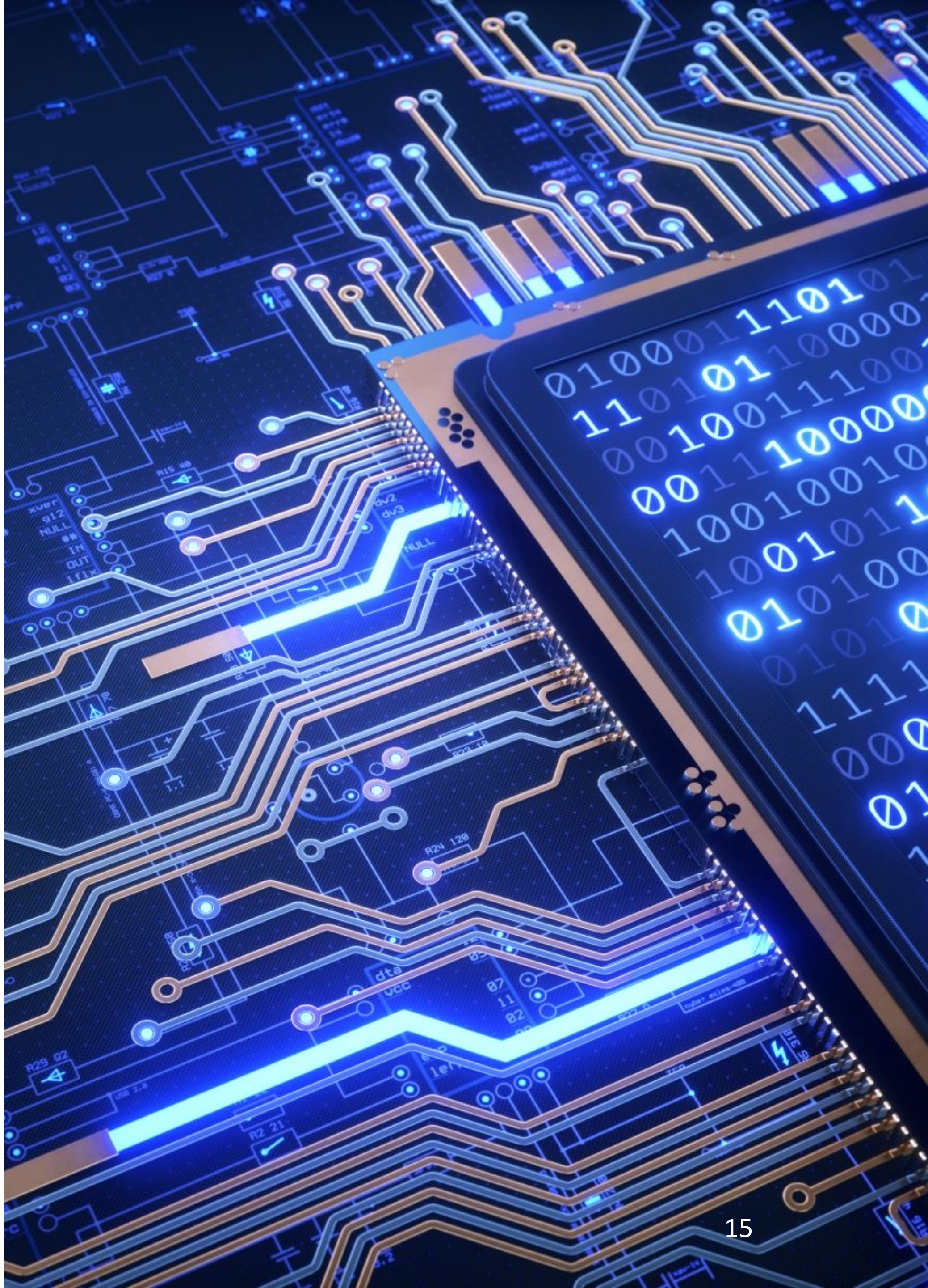
$$(a_{11} \times b_{11}) + (a_{12} \times b_{21}) + (a_{13} \times b_{31}) = c_{11}$$

$$(a_{11} \times b_{12}) + (a_{12} \times b_{22}) + (a_{13} \times b_{32}) = c_{12}$$

$$(a_{21} \times b_{11}) + (a_{22} \times b_{21}) + (a_{23} \times b_{31}) = c_{21}$$

$$(a_{21} \times b_{12}) + (a_{22} \times b_{22}) + (a_{23} \times b_{32}) = c_{22}$$

Successive multiplication of row *i* of **A** with column *j* of **B** – row by column multiplication



# Matrix Multiplication: Worked Example

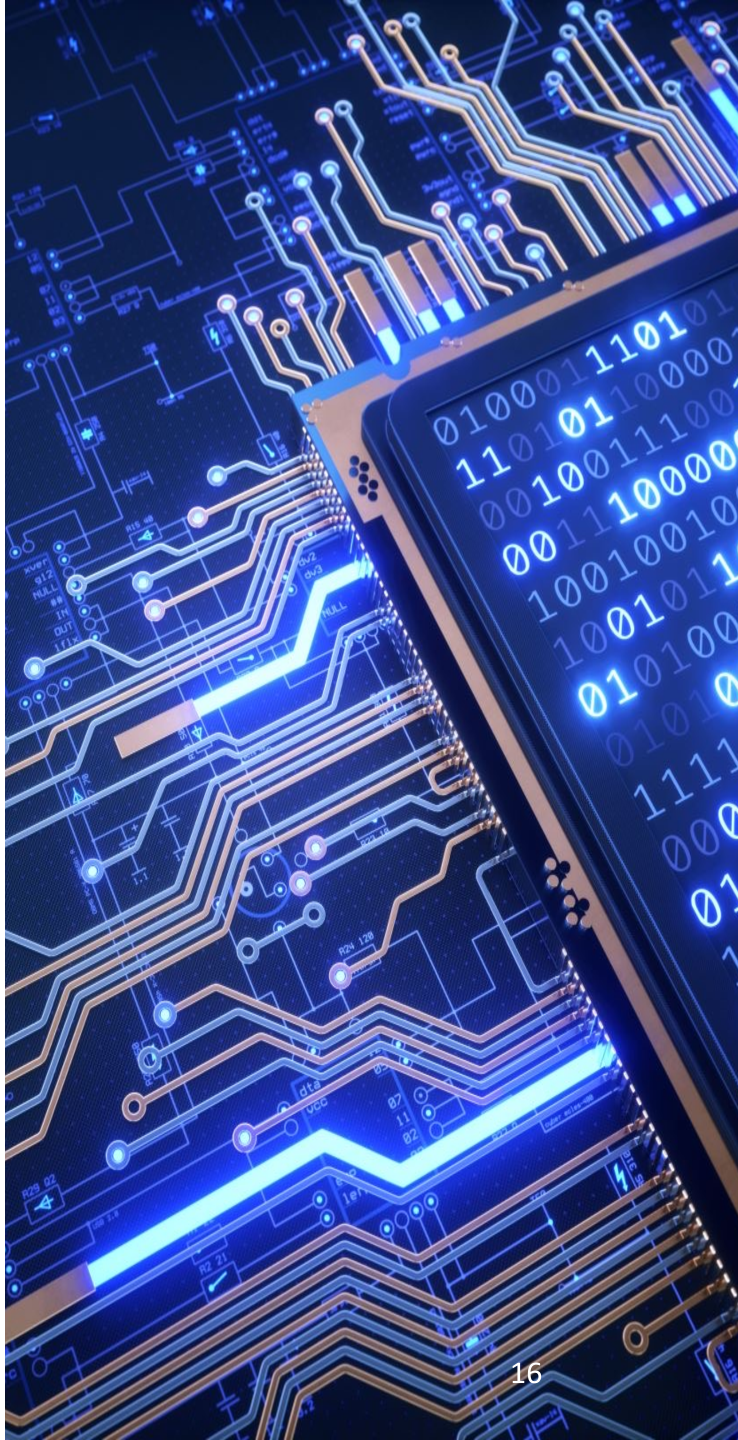
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 7 \end{bmatrix}\begin{bmatrix} 4 & 8 \\ 6 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} (1\times4)+(2\times6)+(3\times5) & (1\times8)+(2\times2)+(3\times3) \\ (4\times4)+(2\times6)+(7\times5) & (4\times8)+(2\times2)+(7\times3) \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$

Remember also:

**IA = A**

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix} = \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$



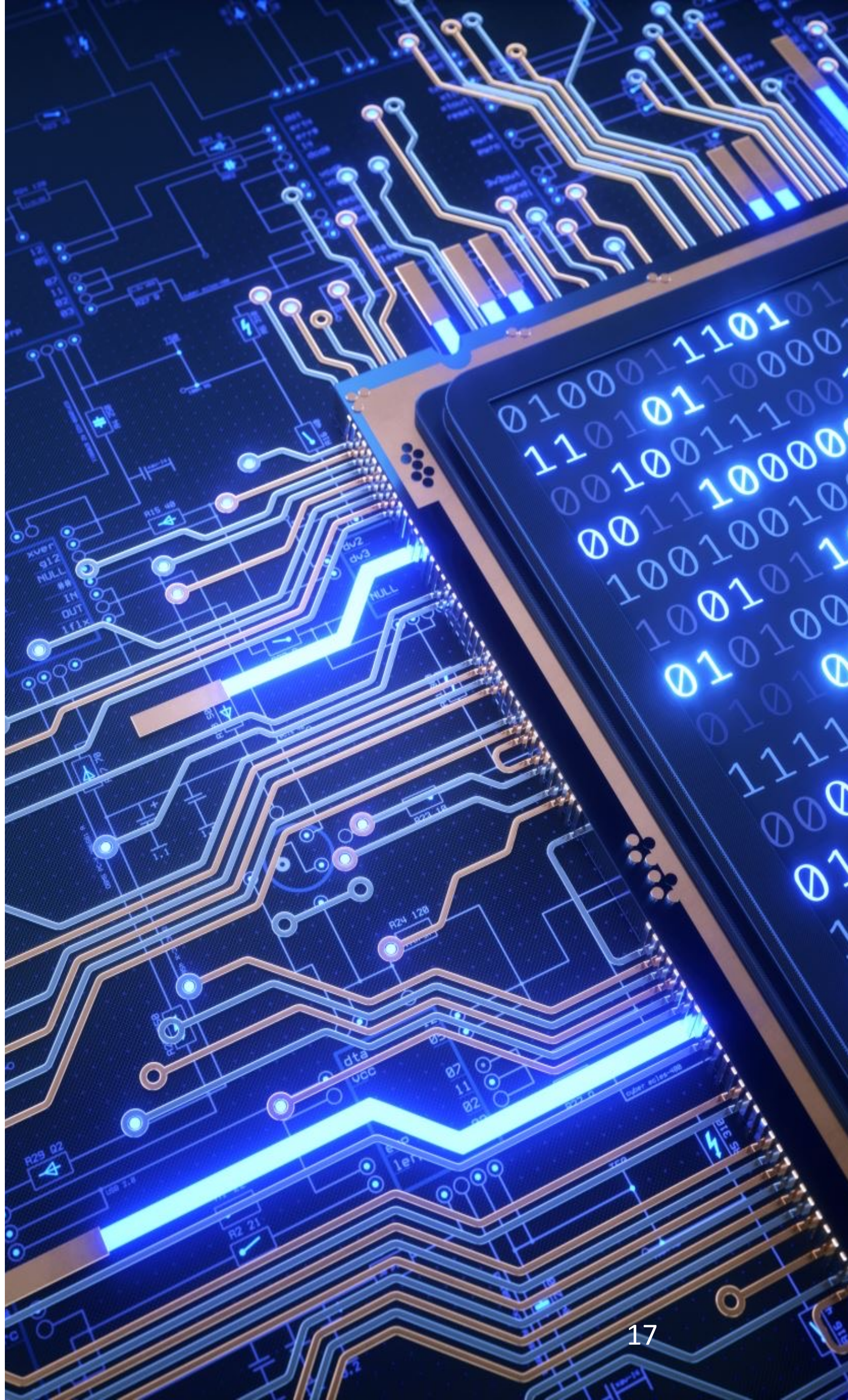
# Matrix Operations

Assuming that matrices **A**, **B** and **C** are conformable for the operations indicated, the following are true:

1. $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$
2. $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C} = \mathbf{ABC}$ - (associative law)
3. $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{AB} + \mathbf{AC}$ - (first distributive law)
4. $(\mathbf{A}+\mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$ - (second distributive law)

**Caution!**

1. **AB** not generally equal to **BA**, **BA** may not be conformable
2. If $\mathbf{AB} = \mathbf{0}$, neither **A** nor **B** necessarily $= \mathbf{0}$
3. If $\mathbf{AB} = \mathbf{AC}$, **B** not necessarily $= \mathbf{C}$



# Transpose of Matrix

If :

$$\underset{2x3}{A} = {}_2A^3 = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix}$$

Then transpose of A, denoted $A^T$ is:

$$A^T = {}_2A^{3^T} = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix}$$

$$a_{ij} = a^T_{ji}$$ For all *i* and *j*

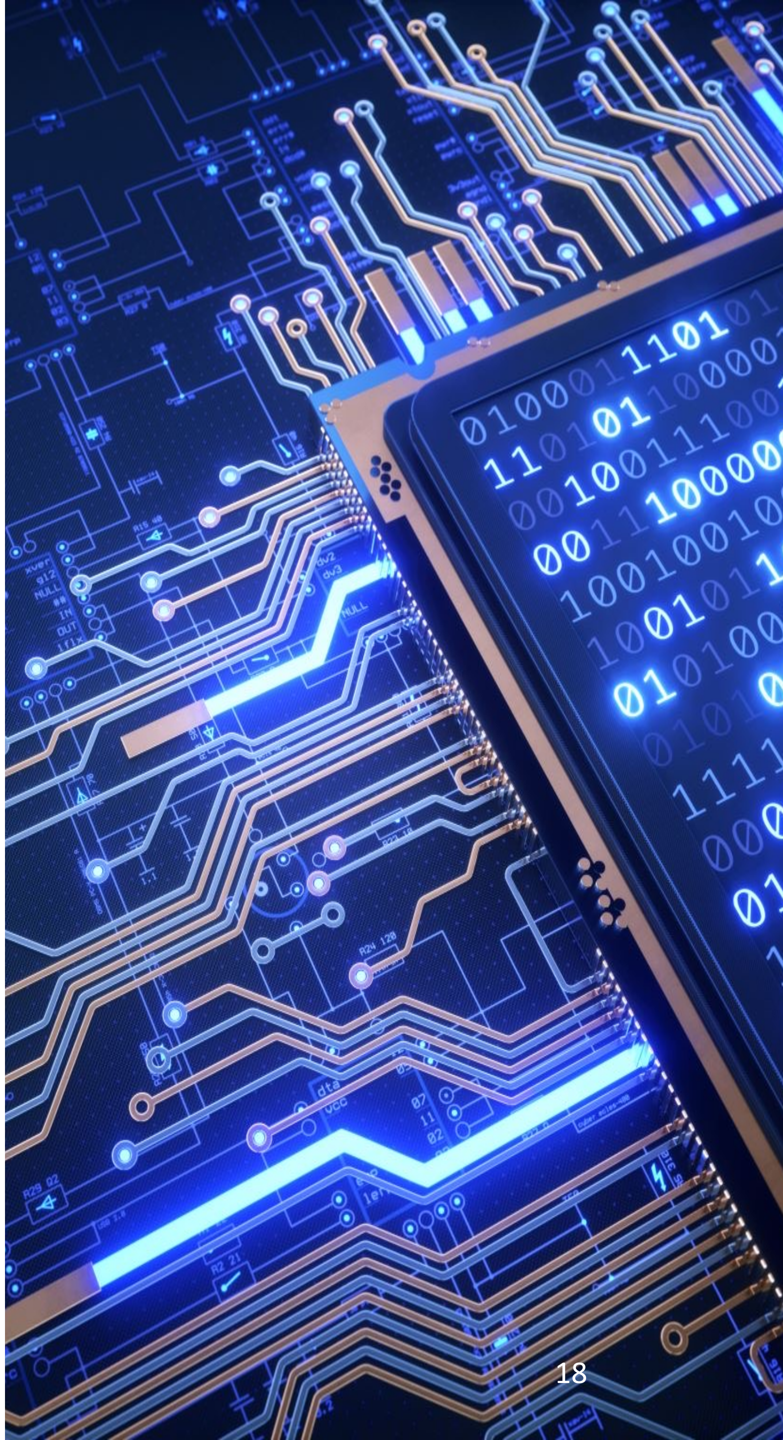
To transpose:

Interchange rows and columns

The dimensions of $\mathbf{A}^T$ are the reverse of the dimensions of **A**

Properties of transposed matrices:

1. $(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
2. $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$
3. $(k\mathbf{A})^T = k\mathbf{A}^T$
4. $(\mathbf{A}^T)^T = \mathbf{A}$



# In-class Exercise

Prove the following:

1. $(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
2. $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$

Solution:

1. $(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$

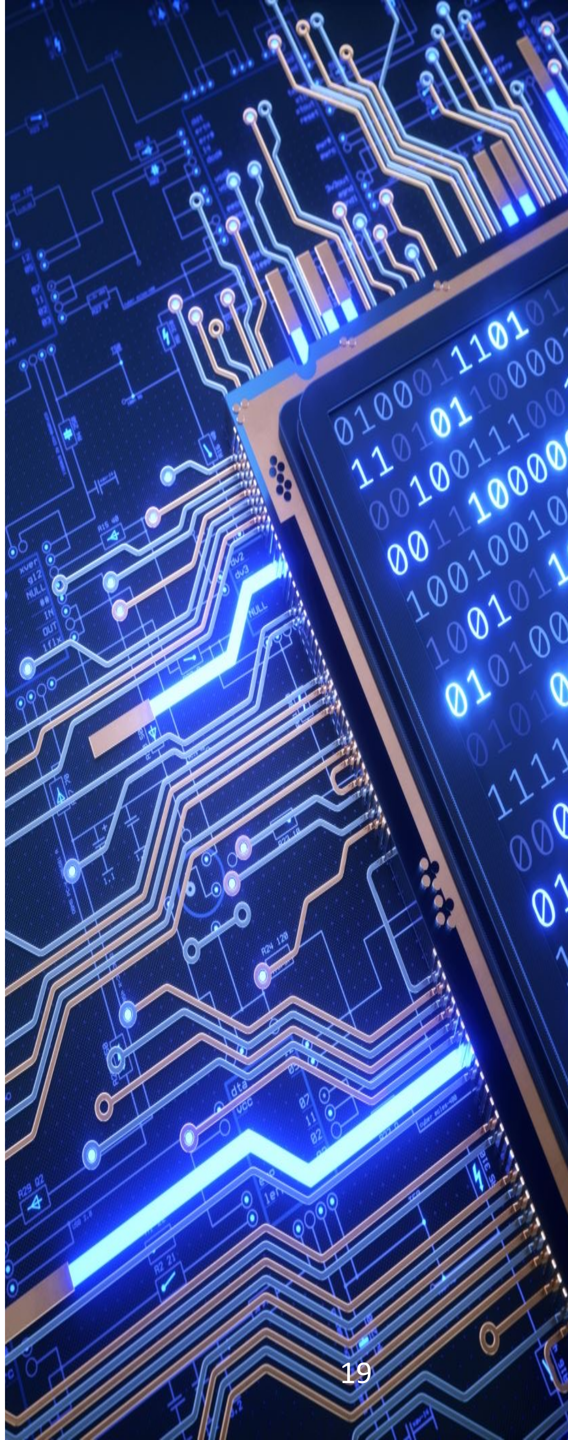
$$\begin{bmatrix} 7 & 3 & -1 \\ 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 6 \\ -4 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 5 \\ -2 & -7 & 9 \end{bmatrix} \longrightarrow \begin{bmatrix} 8 & -2 \\ 8 & -7 \\ 5 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 \\ 3 & -5 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 5 & -2 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 8 & -2 \\ 8 & -7 \\ 5 & 9 \end{bmatrix}$$

2. $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 8 \end{bmatrix}$$



# Inverse of Matrices

## DETERMINANT OF A MATRIX

To compute the inverse of a matrix, the determinant is required

Each square matrix **A** has a unit scalar value called the determinant of **A**, denoted by Det(**A**) or |**A**|

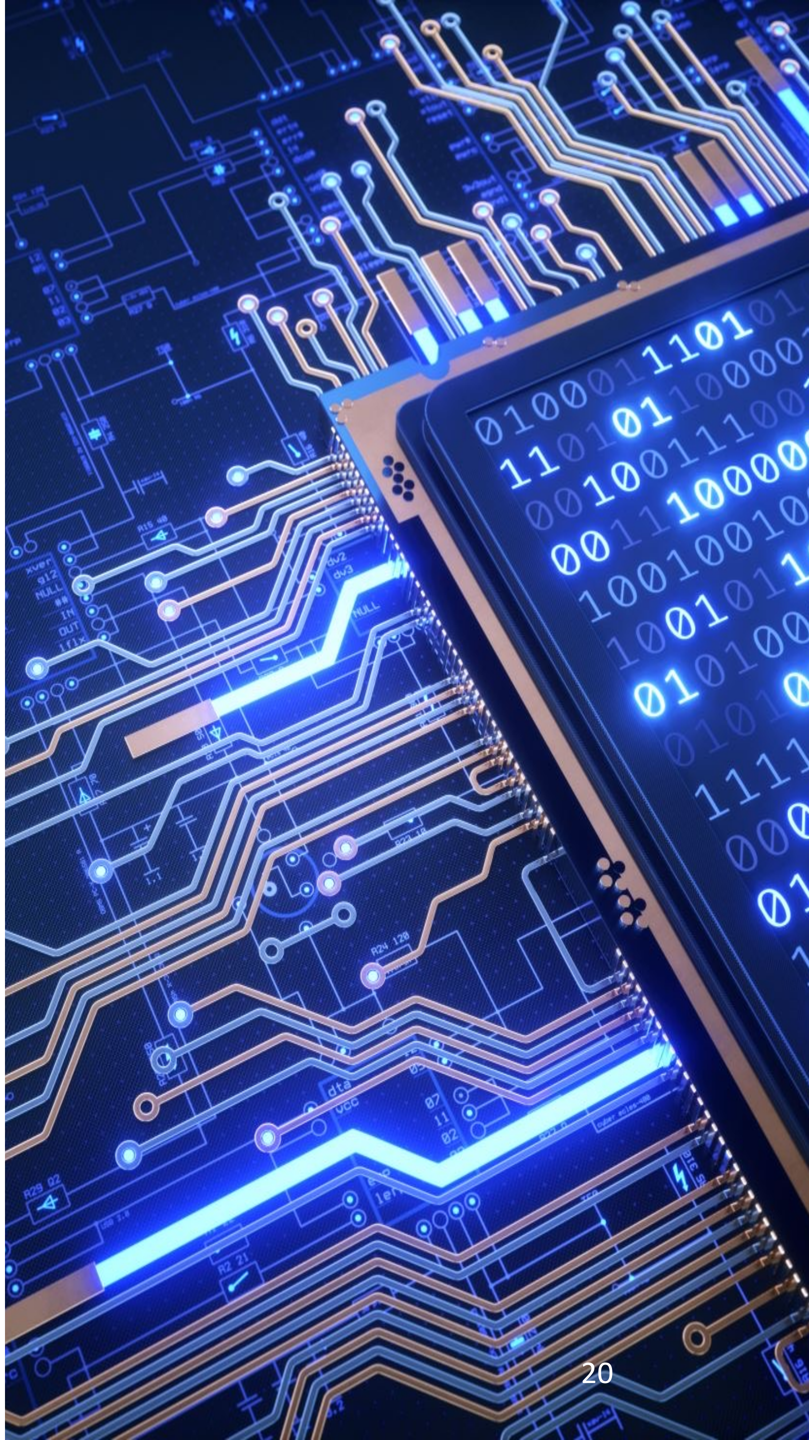
If $$A = \begin{bmatrix} 1 & 2 \\ 6 & 5 \end{bmatrix}$$

then $$|A| = \begin{vmatrix} 1 & 2 \\ 6 & 5 \end{vmatrix}$$

If **A** = [**A**] is a single element (1x1), then the determinant is defined as the value of the element

Then |**A**| = det (**A**) = $a_{11}$

If **A** is (n x n), its determinant may be defined in terms of order (n-1) or less.



# Inverse of Matrices

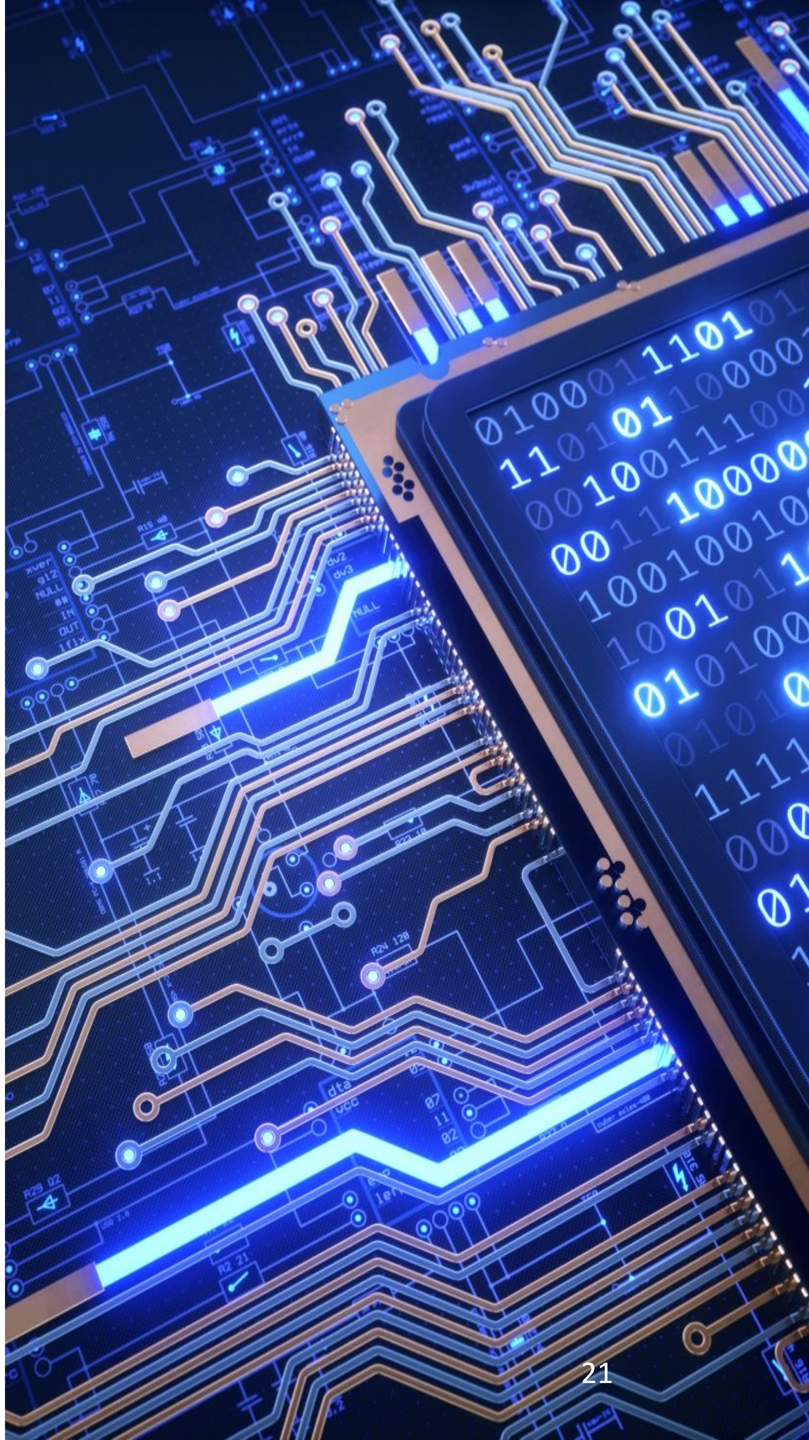
## DETERMINANT OF A MATRIX

Worked Example: Find the determinant of matrix A

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$$

$$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$|A| = (1)(2-0) - (0)(0+3) + (1)(0+2) = 4$$



# Inverse of Matrices

**MINORS**

If **A** is an n x n matrix and one row and one column are deleted, the resulting matrix is an (n-1) x (n-1) submatrix of **A**.

The determinant of such a submatrix is called a minor of **A** and is designated by $m_{ij}$ , where *i* and *j* correspond to the deleted

row and column, respectively.

$m_{ij}$ is the minor of the element $a_{ij}$ in **A**.

Example:

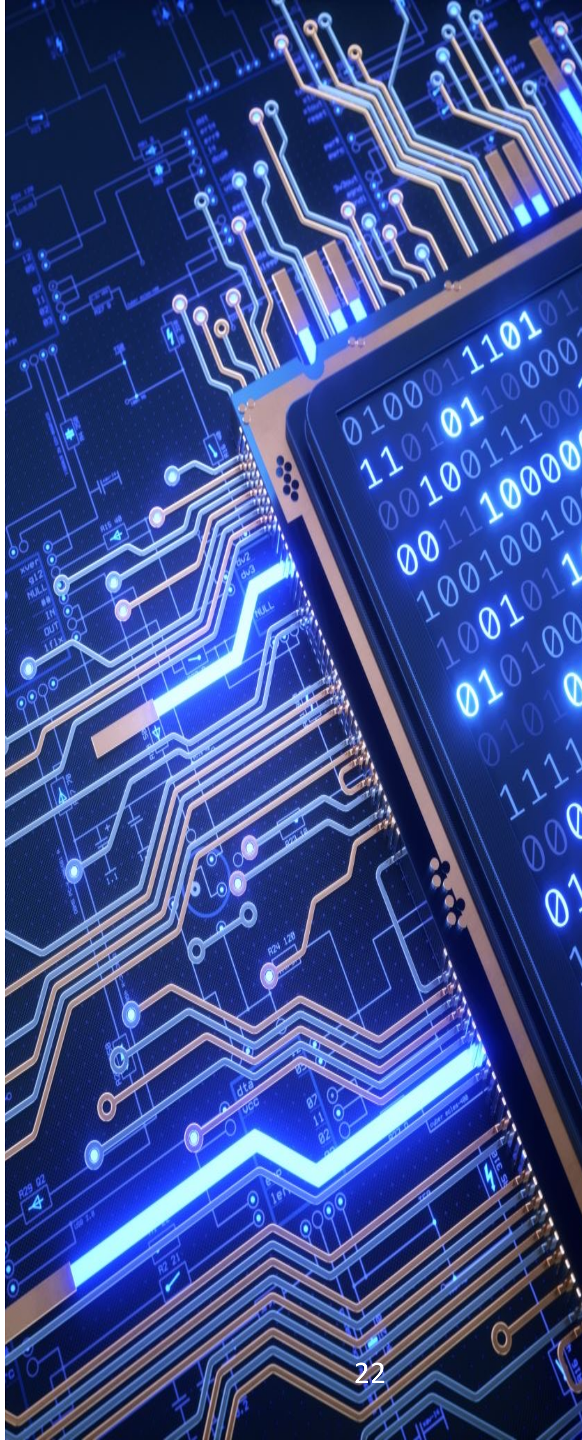
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Each element in **A** has a minor

Delete first row and column from **A** .

The determinant of the remaining 2 x 2 submatrix is the minor ($m_{11}$) of $a_{11}$.

$$m_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad m_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \qquad m_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$



# Inverse of Matrices

**CO-FACTORS**

The co-factor $C_{ij}$ of an element $a_{ij}$ is defined as:

$$C_{ij} = (-1)^{i+j} m_{ij}$$

When the sum of a row number *i* and column *j* is even, $c_{ij} = m_{ij}$ and when *i+j* is odd, $c_{ij} = -m_{ij}$

$$c_{11}(i=1, j=1) = (-1)^{1+1} m_{11} = +m_{11}$$

$$c_{12}(i=1, j=2) = (-1)^{1+2} m_{12} = -m_{12}$$

$$c_{13}(i=1, j=3) = (-1)^{1+3} m_{13} = +m_{13}$$

For the 2 x 2 matrix :

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Has co-factors:

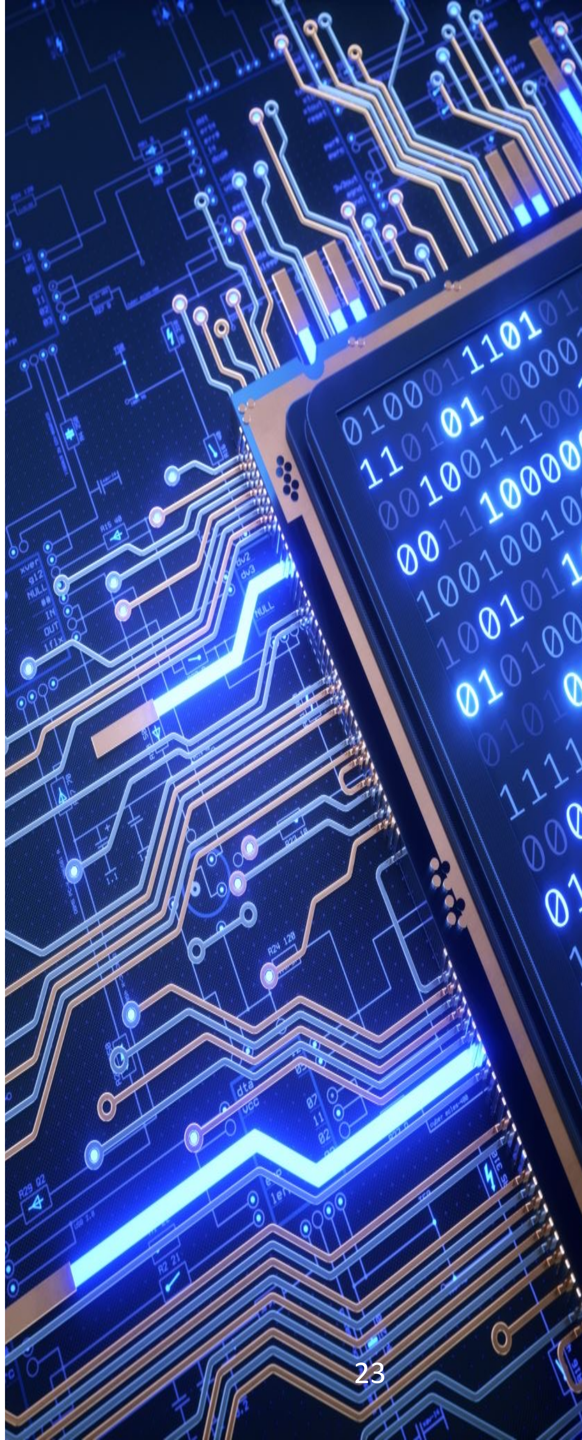
$$c_{11} = m_{11} = |a_{22}| = a_{22}$$

And:

$$c_{12} = -m_{12} = -|a_{21}| = -a_{21}$$

And the determinant of **A** is:

$$|A| = a_{11}c_{11} + a_{12}c_{12} = a_{11}a_{22} - a_{12}a_{21}$$



# Inverse of Matrices

**ADJOINT MATRICES**

The adjoint matrix of **A**, denoted by adj **A**, is the transpose of its cofactor matrix

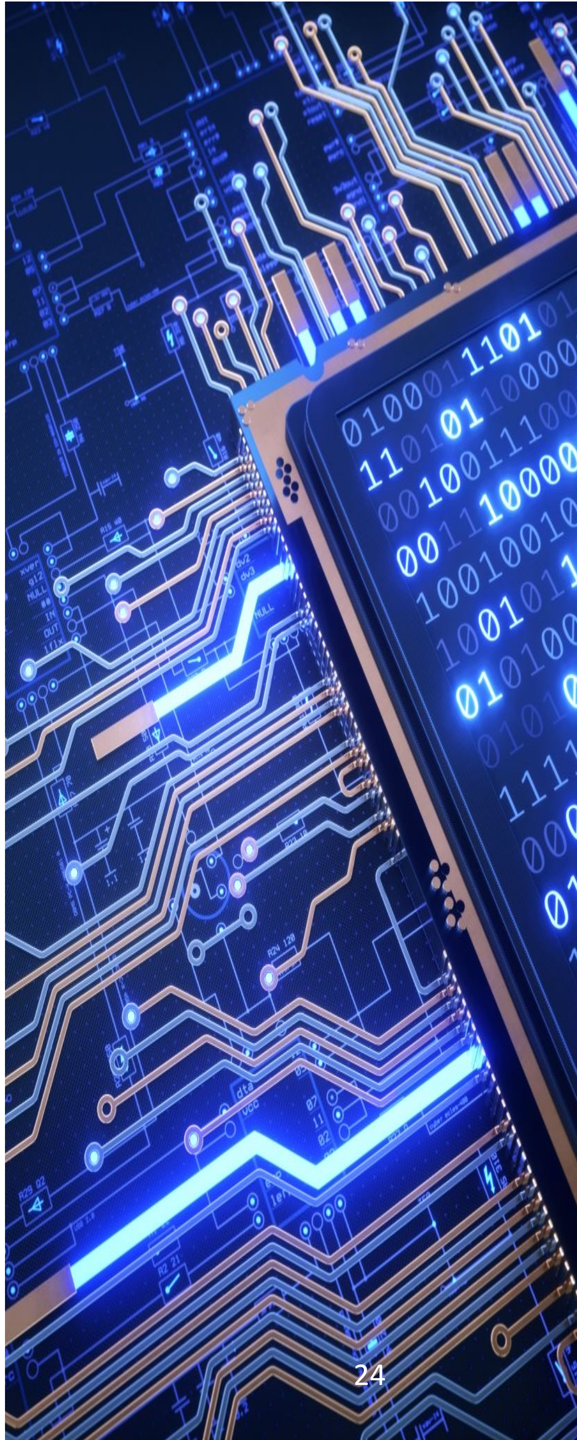
$$adjA = C^T$$

The inverse of matrix A is defined as follows:

$$A^{-1} = \frac{adjA}{|A|}$$

Worked Example: Find the inverse of matrix A

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$



# Inverse of Matrices

Solution

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

The determinant of **A** is

|**A**| = (3)(-1-0)-(-1)(-2-0)+(1)(4-1) = -2

The elements of the cofactor matrix are

$$c_{11} = +(-1), \quad c_{12} = -(-2), \quad c_{13} = +(3),$$
$$c_{21} = -(-1), \quad c_{22} = +(-4), \quad c_{23} = -(7),$$
$$c_{31} = +(-1), \quad c_{32} = -(-2), \quad c_{33} = +(5),$$

The cofactor matrix is therefore

$$C = \begin{bmatrix} -1 & 2 & 3 \\ 1 & -4 & -7 \\ -1 & 2 & 5 \end{bmatrix}$$

so

$$adjA = C^T = \begin{bmatrix} -1 & 1 & -1 \\ 2 & -4 & 2 \\ 3 & -7 & 5 \end{bmatrix}$$

and

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{-2}\begin{bmatrix} -1 & 1 & -1 \\ 2 & -4 & 2 \\ 3 & -7 & 5 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 & 0.5 \\ -1.0 & 2.0 & -1.0 \\ -1.5 & 3.5 & -2.5 \end{bmatrix}$$

# Linear Equations

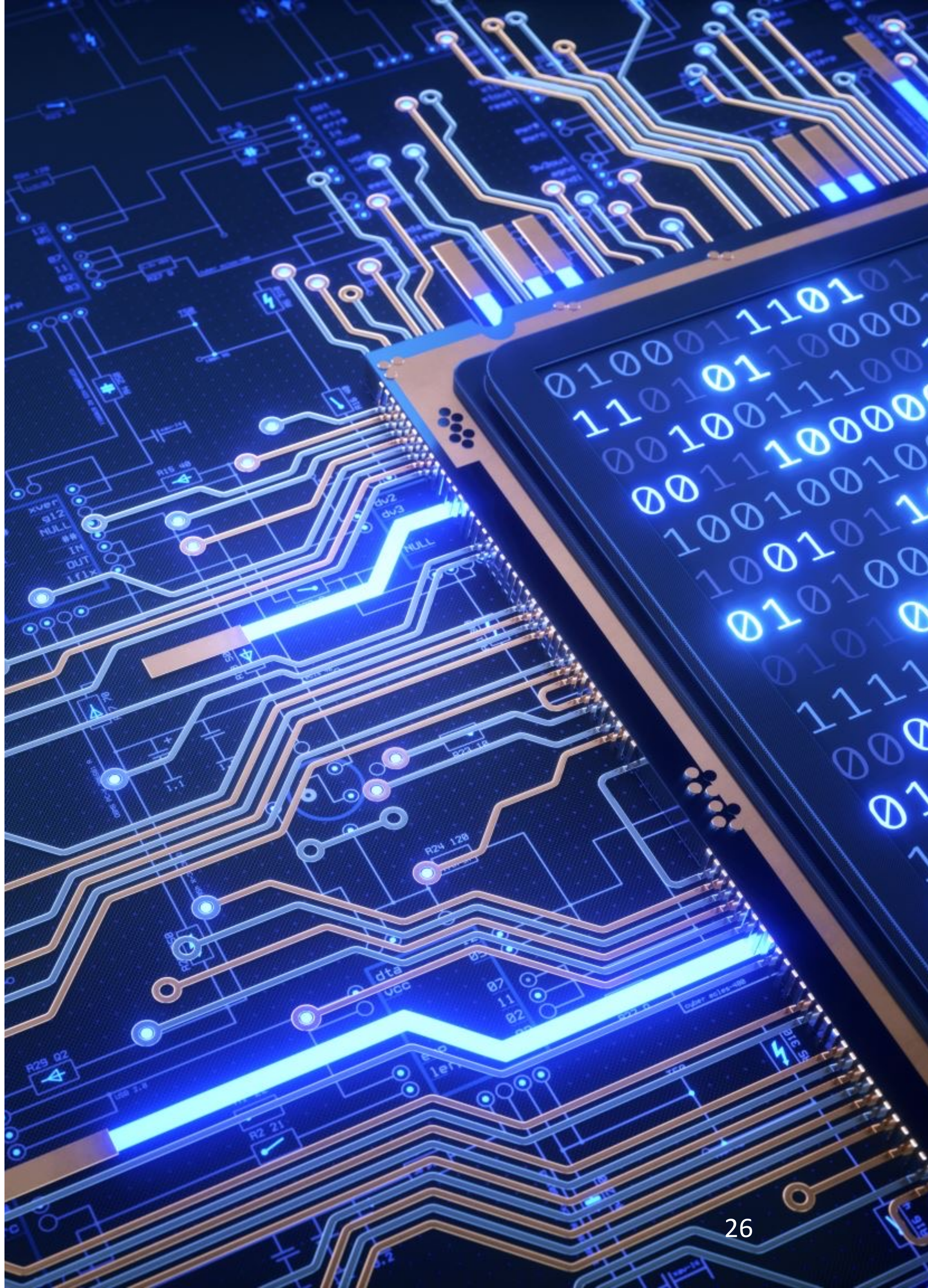
Linear equations are common and important for survey problems

Matrices can be used to express these linear equations and aid in the computation of unknown values

Example

*n* equations in *n* unknowns, the $a_{ij}$ are numerical coefficients, the $b_i$ are constants and the $x_j$ are unknowns

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n
\end{aligned}
$$



# Linear Equations

The equations may be expressed in the form

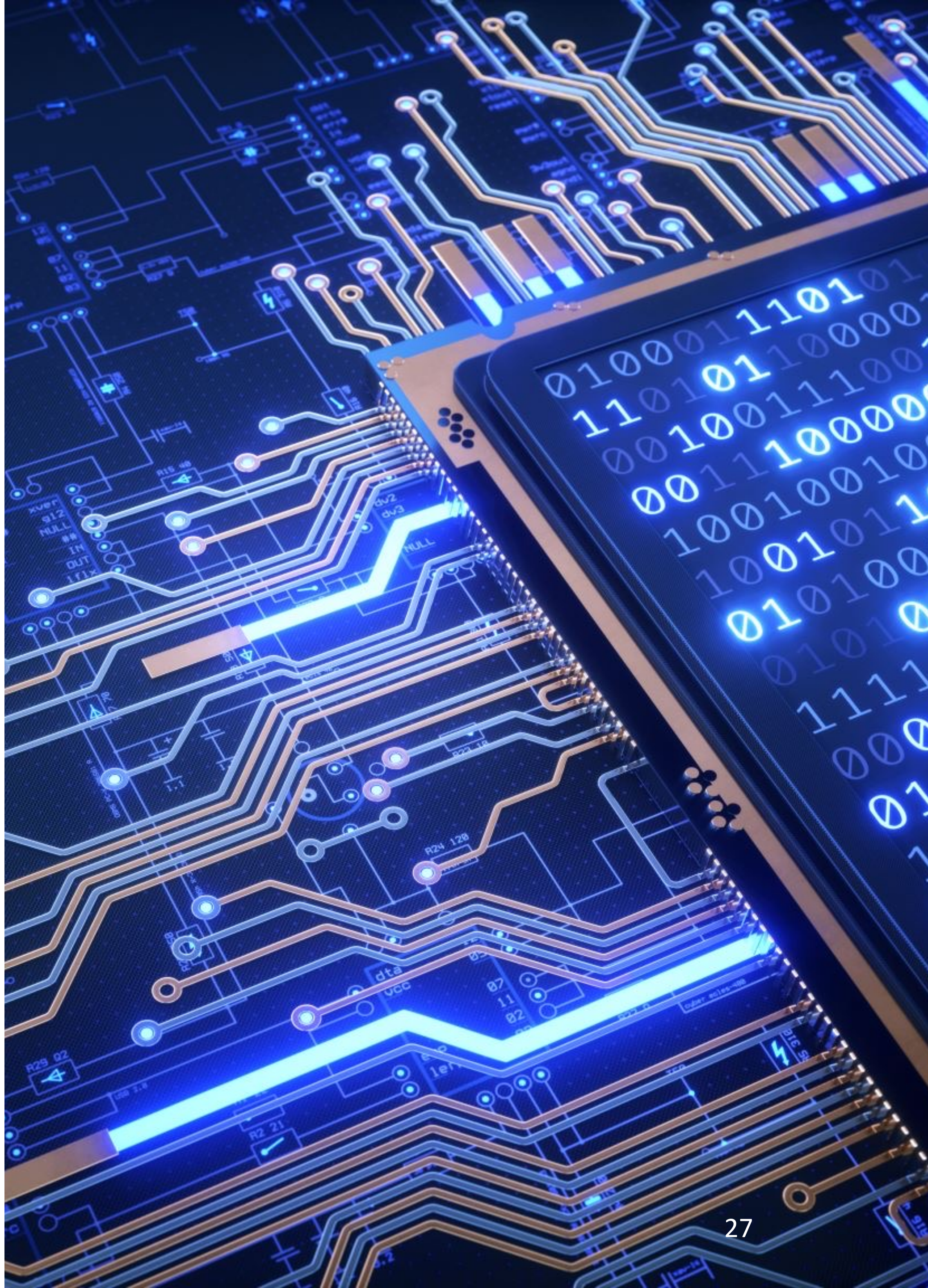
**AX = B**

where

$$A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n1} \cdots & a_{nn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \text{ and } \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

n x n      n x 1      n x 1

Number of unknowns = number of equations = n



# Linear Equations

If the determinant is nonzero, the equation can be solved to produce n numerical values for x that satisfy all the simultaneous equations

To solve, pre-multiply both sides of the equation by $\mathbf{A}^{-1}$ which exists because $|\mathbf{A}| \neq \mathbf{0}$

$$\mathbf{A}^{-1}\,\mathbf{A}\mathbf{X} = \mathbf{A}^{-1}\,\mathbf{B}$$

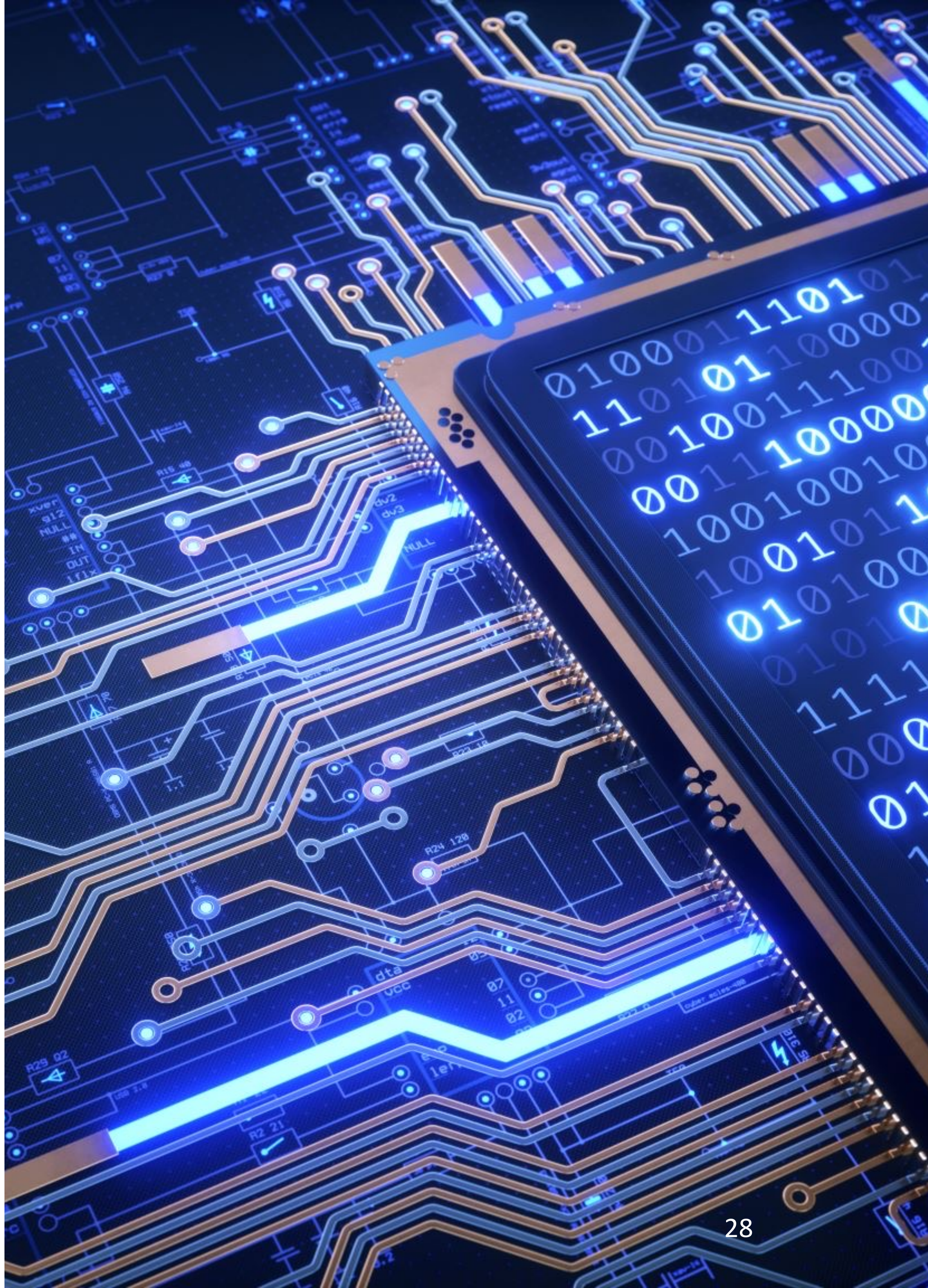
Now since

$$\mathbf{A}^{-1}\,\mathbf{A} = \mathbf{I}$$

We get

$$\mathbf{X} = \mathbf{A}^{-1}\,\mathbf{B}$$

So, if the inverse of the coefficient matrix is found, the unknowns, **X** would be determined



# Worked Example

Example

$$3x_1 - x_2 + x_3 = 2$$
$$2x_1 + x_2 = 1$$
$$x_1 + 2x_2 - x_3 = 3$$

The equations can be expressed as

$$\begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

When $\mathbf{A}^{-1}$ is computed the equation becomes

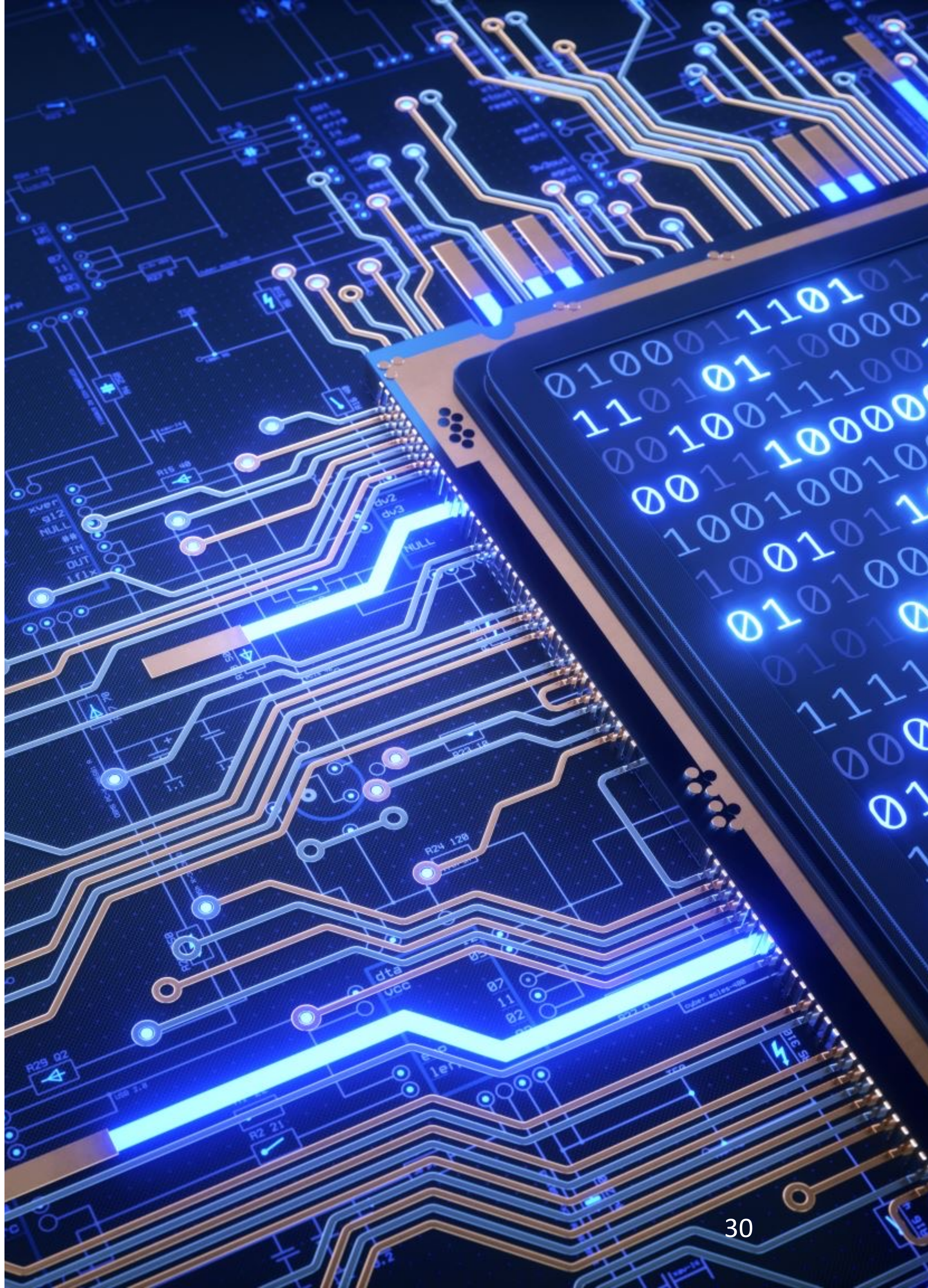
$$X = A^{-1}B = \begin{bmatrix} 0.5 & -0.5 & 0.5 \\ -1.0 & 2.0 & -1.0 \\ -1.5 & 3.5 & -2.5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -7 \end{bmatrix}$$

Therefore

$$x_1 = 2,$$
$$x_2 = -3,$$
$$x_3 = -7$$

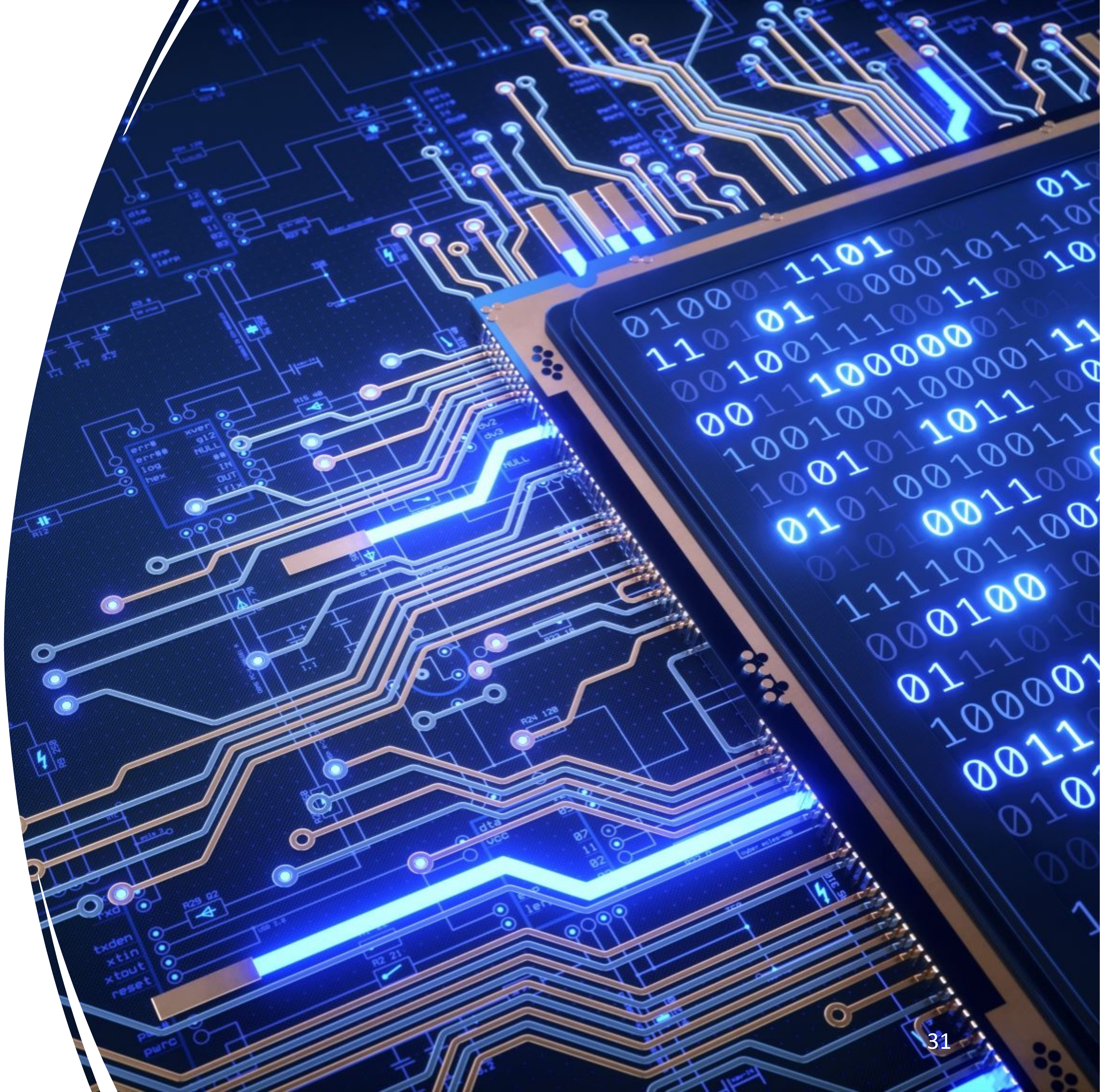
# Applications of Matrices

- Linear Algebra
- Computer Graphics
- Data Analysis and Statistics
- Machine Learning and Data Mining
- Signal Processing
- Finance and Economics
- Physics and Engineering
- Network Analysis:
- Optimization



# Summary

- A matrix is a square or rectangular array of numbers arranged in rows and columns.
    - Each element of a matrix is identified by its row and column index.
- A vector is a single row or column matrix.
- Forms of matrices.
- Inverse of matrices.
- Matrices are used to express linear equations.



# Reference

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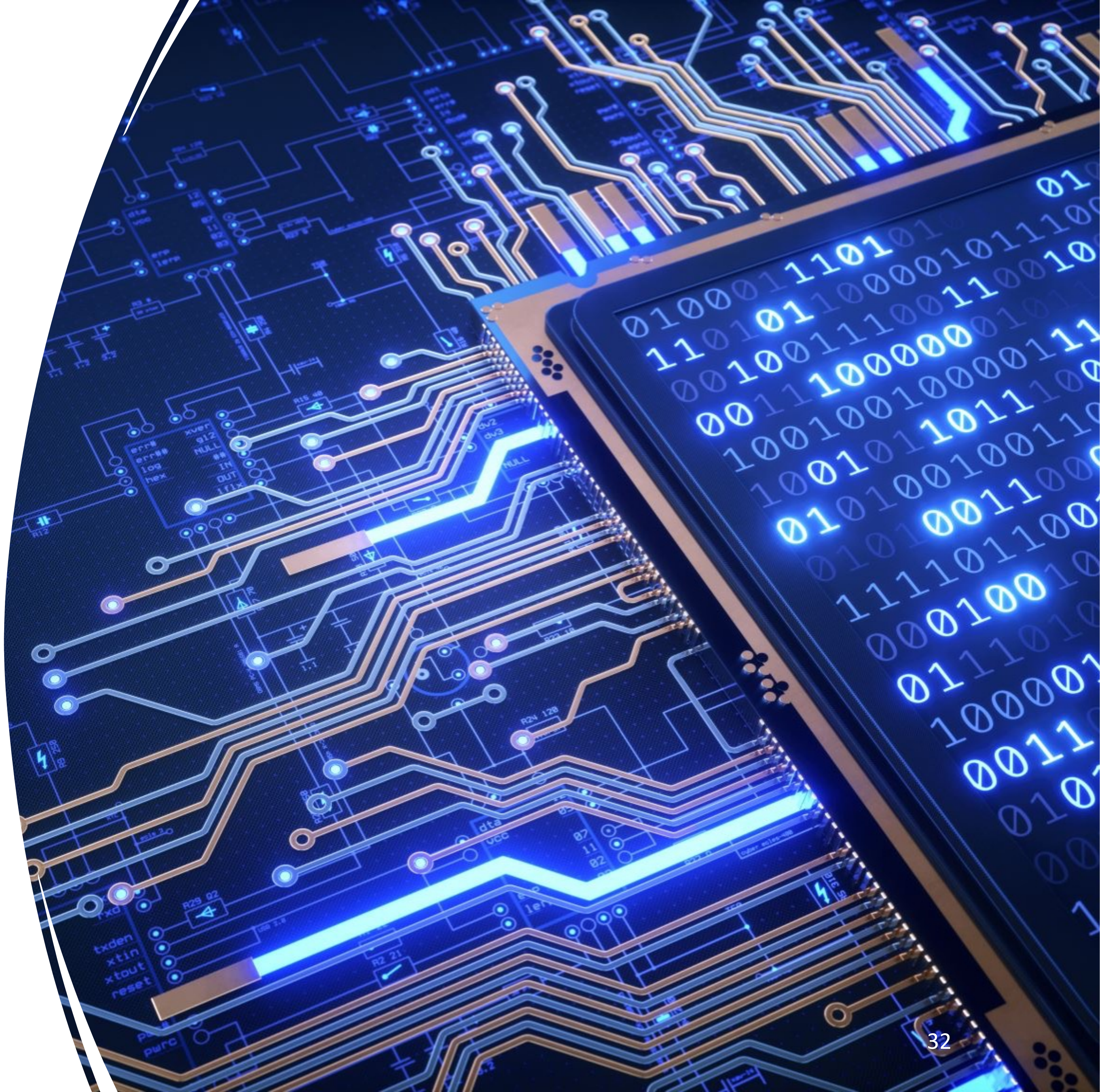
Chapter 2



See you next time!

Thank you!