Solutions to Business Mathematics Examination

Section A

1. C
2. B
3. E
4. D
5. B
6. D
7. A
8. D
9. B
10. C
11. B
12. B
13. A closed system is where everything is consumed internally whereas in an open system some goods or services will be available for the external market
14. Assumptions
    a. The whole economy is divided into two sectors—"inter-industry sectors" and "final-demand sectors," both being capable of sub-sectoral division.
    b. The total output of any inter-industry sector is generally capable of being used as inputs by other inter-industry sectors, by itself and by final demand sectors.
    c. No two products are produced jointly. Each industry produces only one homogeneous product.
    d. Prices, consumer demands and factor supplies are given.
    e. There are constant returns to scale.
    f. There are no external economies and diseconomies of production

Section B

Question one

a) First, construct the profit function $P(x, y)$

$P(x, y) = R(x, y) - C(x, y)$

$= (-0.2x^2 - 0.25y^2 - 0.2xy + 225x + 135y + 1500) - (125x + 45y + 5500)$

$= -0.2x^2 - 0.25y^2 - 0.2xy + 100x + 90y - 4000, \quad \text{for } x, y \geq 0$

So the first partial derivatives of P are

$$P_x = -0.4x - 0.2y + 100, \quad \text{and} \quad P_y = -0.5y - 0.2x + 90.$$

To find the critical points of P, we set $P_x = 0,\ P_y = 0$ and solve the resulting system of equations.

$$-0.4x - 0.2y = -100 \text{ and } -0.2x - 0.5y = -90$$

Subtract twice the second equation from the first equation. This gives

$$-0.2y + y = (-100 + 180) = 80$$

$$\Rightarrow y = 100$$

And so

$$-0.4x - 0.2(100) = -100 \Rightarrow x = 200$$

This gives one critical point, (200, 100). To classify this critical point, we construct the discriminant Δ.

$$\Delta = P_{xx}P_{yy} - P_{xy}^2 = (-0.4)(-0.5) - (-0.2)^2 = 0.16\ \forall\ x, y > 0$$

Therefore, since $\Delta(100,200) > 0$ and $P_{xx}\ (100, 200) < 0, P_{yy}\ (100, 200) < 0$, this point is a relative maximum.

b) Given the following two matrices:

A= $\begin{bmatrix} 0 & 5 & -4 \\ -2 & 3 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} 8 & 4 \\ 3 & -2 \\ 1 & 6 \end{bmatrix}$. Calculate,

(i) $A + BT$ (2 marks)

$$\begin{bmatrix} 0 & 5 & -4 \\ -2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 3 & 1 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 8 & 8 & -3 \\ 2 & 1 & 10 \end{bmatrix}$$

(ii) AB (2 marks)

$$\begin{bmatrix} 0 & 5 & -4 \\ -2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 8 & 4 \\ 3 & -2 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 11 & -26 \\ -3 & 10 \end{bmatrix}$$

c) A two-sector economy of agriculture and mining. Its input - output matrix is given as

$$\begin{matrix} \\ A \\ M \end{matrix} \begin{bmatrix} A & M \\ 0.2 & 0.3 \\ 0.4 & 0.1 \end{bmatrix}$$

(i) If the total production is 300 billion and 500 billion shillings worth of agriculture and mining respectively, calculate the amount of both consumed internally.

$$\begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.1 \end{bmatrix} \begin{bmatrix} 300 \\ 500 \end{bmatrix} = \begin{bmatrix} 210 \\ 170 \end{bmatrix}$$

(1 mark)

(ii) If the external demand is 120 billion and 180 billion shillings worth of agriculture and mining respectively, what would be the total production? $X=(1-A)^{-1}D$

$$1 - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.8 & -0.3 \\ -0.4 & 0.9 \end{bmatrix}$$

$$(1-A)^{-1} = \begin{bmatrix} 1.50 & 0.50 \\ 0.67 & 1.33 \end{bmatrix}$$

$$\begin{bmatrix} 1.50 & 0.50 \\ 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} 120 \\ 180 \end{bmatrix} = \begin{bmatrix} 270 \\ 320 \end{bmatrix}$$

(4 marks)

Question Two (15 marks)

(a) A company has three modes of paying their namely; cash payment, credit card, MPESA.

(i) two payments; and (4 marks)

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 0.85 & 0.10 & 0.05 \\ 0.04 & 0.90 & 0.06 \\ 0.05 & 0.25 & 0.70 \end{bmatrix} = \begin{bmatrix} 0.608 & 0.275 & 0.117 \end{bmatrix}$$

$$[0.608 \quad 0.275 \quad 0.117]\begin{bmatrix}0.85 & 0.10 & 0.05\\ 0.04 & 0.90 & 0.06\\ 0.05 & 0.25 & 0.70\end{bmatrix} = [0.517 \quad 0.338 \quad 0.145]$$

(ii) in the long run. (6 marks)

$$[x \quad y \quad z]\begin{bmatrix}0.85 & 0.10 & 0.05\\ 0.04 & 0.90 & 0.06\\ 0.05 & 0.25 & 0.70\end{bmatrix} = [x \quad y \quad z]$$

$$0.85x + 0.04y + 0.05z = x$$

$$0.1x + 0.9y + 0.25z = y$$

$$0.05x + 0.06y + 0.70z = z$$

Solve any of the above two equations together with $x + y + z = 1$ simultaneously.

$X = 0.22, y = 0.62$ and $z = 016$

b) Formulate a linear program for this problem (5 marks)

Since the exercise is asking for the number of ounces of each food required for the optimal daily blend, my variables will stand for the number of ounces of each:

x: number of ounces of Food X
y: number of ounces of Food Y

Min C = 0.2x + 0.3y,

Subject to:

fat: $8x + 12y > 24$
carbs: $12x + 12y > 36$
protein: $2x + 1y > 4$
Non negativity: x > 0 and y > 0.

Question Three (15 marks)

(a) If the decision maker knows nothing about the probabilities of the four states of nature, what would be the best decision using;

(i) Optimistic approach (1 mark)

	S1	S2	S3	S4	Max
D1	140	90	120	50	140
D2	110	100	80	90	110
D3	90	150	120	110	150
D4	80	60	110	130	130

Maximax D_3=150

(ii) Pessimistic approach (1 mark)

	S1	S2	S3	S4	Min
D1	140	90	120	50	50
D2	110	100	80	90	80
D3	90	150	120	110	90

D4	80	60	110	130	60

Maximin=D_3=90

(iii) Laplace approach (2 marks)

Calculate the average

$A = 100$

$B = 95$

$C = 117.5$

$D = 95$

The best decision would C

(iv) Opportunity loss approach (3 marks)

Opportunity loss table

	S_1	S_2	S_3	S_4	max
D_1	0	60	0	80	80
D_2	30	50	40	40	50
D_3	50	0	0	20	50
D_4	60	90	10	0	90

Minimax regret D_2 or D_3=50

(b) Suppose the decision maker obtain information that enables the computation of the following probabilities $P(S_1)=0.4$, $P(S_2)=0.2$, $P(S_3)=0.1$, $P(S_4)=0.3$

(i) Which would be the best decision using expected value approach? (3 marks)

$A = 101$

$B = 99$

$C = 111$

$D = 94$

The best decision is C

(ii) Calculate the value of perfect information (3 marks)

$$(0.4 \times 140) + (0.2 \times 150) + (0.1 \times 120) + (0.3 \times 130) - 111 = 26$$

d) Explain two main differences between payback period and net present value methods of investment appraisal.

(i) Unlike NPV, Payback period does not consider the time value of money

(ii) Payback period does not consider the cash inflow after payback period where as NPV considers all the cash inflow. (2 marks)