Applied Mechanics

Chapter 2

Basic Concept In Statics And Static Equilibrium

Lecture 2

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Learning Objectives: By the end of this module, students should be able to:

- Define equilibrium and its significance in structural analysis.
- Know the condition of equilibrium in 2D as well as 3D
- Apply equilibrium principles to analyze forces and moments in stationary systems.
- Formulate and solve equilibrium equations for two-dimensional systems.
- Draw Free Body Diagram and using Equilibrium equation to find unknowns.

2. Basic Concept in Statics and Static Equilibrium

Equilibrium refers to a state in which a body or system experiences no net force or net torque, causing it to remain at rest or to move with constant velocity. In statics, the focus is primarily on systems that are stationary or at rest. This means that the sum of all forces acting on the system is zero, and the sum of all torques (or moments) acting on the system is also zero.

Force Equilibrium: For a body to be in force equilibrium, the vector sum of all external forces acting on it must be zero. This implies that the forces acting in different directions must balance each other out.

Moment Equilibrium: For a body to be in moment equilibrium, the sum of all torques (or moments) about any point must be zero. This ensures that the body is not rotating under the influence of unbalanced torques.

Static Equilibrium of body

When a stationary rigid body which is subjected to many forces in same plane (i.e. concurrent and non-concurrent forces) will be in equilibrium if the algebraic sum of the all the external applied and developed forces is zero and algebraic sum of moment of all these forces about any point in their plane is zero.

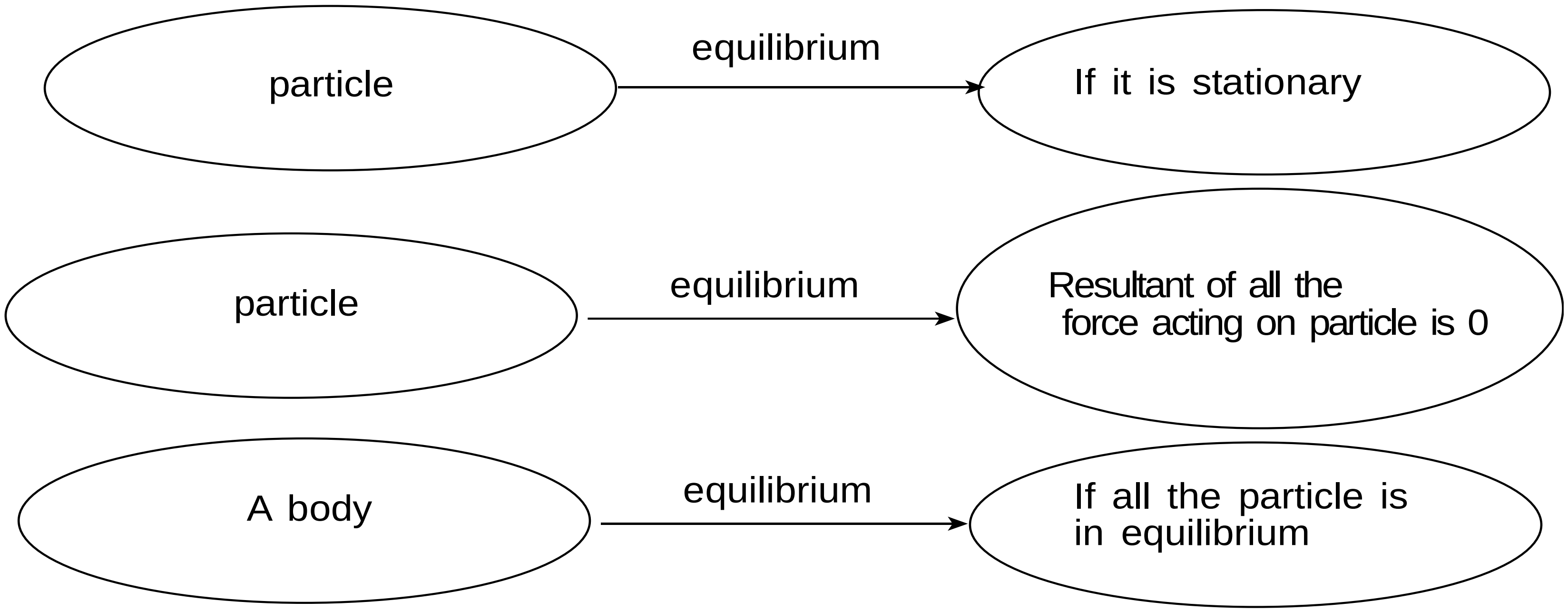
2.1Physical meaning of Equilibrium and its essence in structural application

2.1.1Physical meaning of Equilibrium

If the forces acting on the body are non-concurrent then the equivalent resultant will be single force acting at a common point and a moment about same point. The effect of such a force system will be to translate the body as well as to rotate it. Hence, for equilibrium to exist, both the force and the moment must be null vectors.

When the force acting on a body lie in the same plane (x – y) but are non-concurrent the body will have rotational motion perpendicular to the plane in addition to the translation motion along the plane.

Hence the body will be in equilibrium means there is no rotational motion perpendicular to the plane as well as no translation motion along the plane.



2.1.2Essence in structural application

Equilibrium principles are fundamental in structural analysis as they provide the basis for understanding the behavior of various structural elements and systems. By applying equilibrium equations, engineers can determine the internal forces and reactions within a structure necessary to maintain its stability and structural integrity.

Support Reaction: Equilibrium principles help in calculating the reactions at support points for different types of structural configurations such as beams, trusses, and frames. These reactions provide essential information for designing the structural elements and ensuring overall stability.

Structural Stability: Equilibrium analysis is crucial for assessing the stability of structures under different loading conditions. Engineers use equilibrium equations to determine whether a structure can withstand external loads without collapsing or experiencing excessive deformation.

Analyzing the force balancing and moment balancing: In statics, analyzing the balance of forces and moments in stationary systems involves applying the principles of equilibrium to determine the internal and external forces acting on the system. This analysis aids in understanding how various components of a system interact with each other and with external forces.

Free Body Diagrams: Engineers often use free-body diagrams to represent isolated systems and visualize the forces acting on them. By drawing free-body diagrams, one can accurately analyze the equilibrium conditions and determine the unknown forces or reactions.

Constraint Analysis: In complex systems, where multiple interconnected components are involved, analyzing the balance of forces and moments requires considering the constraints imposed by the connections between the components.

In summary, understanding the physical meaning of equilibrium and its application in structural analysis involves recognizing the balance of forces and moments in stationary systems. By applying equilibrium principles, engineers can determine the reactions, internal forces, and overall stability of structures, ensuring their safe and efficient design and construction.

2.2 Equation of equilibrium (Condition of equilibrium)

Concurrent forces in space

$\Sigma Fx = 0$, $\Sigma Fy = 0$ and $\Sigma Fz = 0$

Coplanar non-concurrent forces(Equilibrium equations for 2D)

$\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_z = 0$(the body may move in any one direction and rotate about itself at the same time.

Where M_z is the moment about the perpendicular z-axis.

Non concurrent forces in space (Equilibrium equations for 3D)

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$$

Where

$\Sigma F_{x=}$ algebraic sum of forces in x direction

$\Sigma F_{y=}$ algebraic sum of forces in y direction

$\Sigma F_{z=}$ algebraic sum of forces in z direction

ΣM_x =algebraic sum of moment about y-z plane

ΣM_y = algebraic sum of moment about x-z plane

$\Sigma M_{z=}$ algebraic sum of moment about x-y plane [2]

2.2.1Mass, Force and Weight

Mass is an indication of the quantity of matter present in a system. If having the more matter means more mass.

Mass is usually measured in kilograms (kg) or grams (g). It's important to note that mass is constant regardless of the object's location in the universe.

In other words, an object will have the same mass whether it's on Earth, the Moon, or in deep space.[1]

Force: force is an external agent which tends to change the speed or direction or momentum of a system. Force can cause an object to accelerate, decelerate, change direction, or deform.

It's often described in terms of Newton's laws of motion. Force is measured in units called Newtons (N).

For example, pushing or pulling an object, gravity acting on an object, or the tension in a rope are all examples of forces.[1]

Effects of Forces:

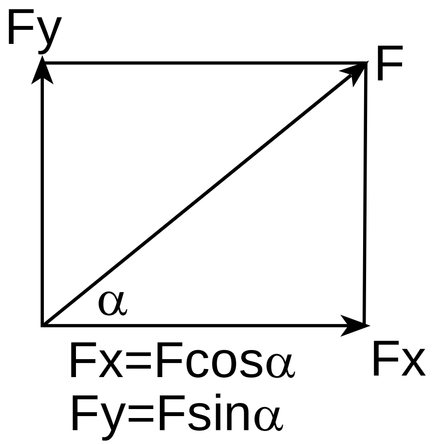
The following effects may be produced in the body:

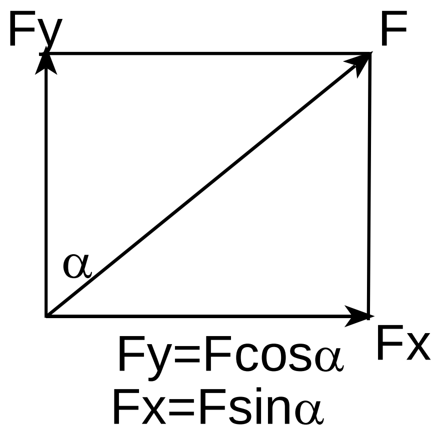
- May change in the motion of the body.
- keep the body in equilibrium
- may change the size or shape of the body
- accelerate or retard its motion
- turn or rotate the body

Characteristics of a force

- have a point of application
- It has magnitude
- Direction or line of action
- nature of forces (Tension, compression) etc.

Basic concept in Resolution of force





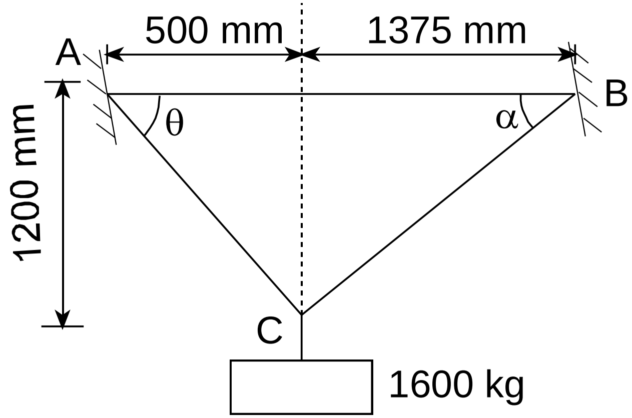
Weight: Weight is the force exerted on an object due to gravity. Weight depends on both the mass of the object and the strength of the gravitational field it's in. Weight is measured in Newtons (N). Unlike mass, weight can vary depending on the gravitational pull of the celestial body the object is on. For instance, an object will weigh less on the Moon compared to Earth due to the Moon's weaker gravitational pull.

Weight of a system equals the product of mass and localize gravitational acceleration.

The value of g at sea level is 9.8066 m/s^2 and generally taken 9.81m/s^2**[1]**

Q. Two cables are tied together at C and loaded as shown determine the tension in AC and BC.

Solution:



$$\theta = \tan^{-1}\left(\frac{1200}{500}\right) = 67.38°$$

$$\propto = \tan^{-1}\left(\frac{1200}{1375}\right) = 41.11°$$

Draw F. B.D of the system

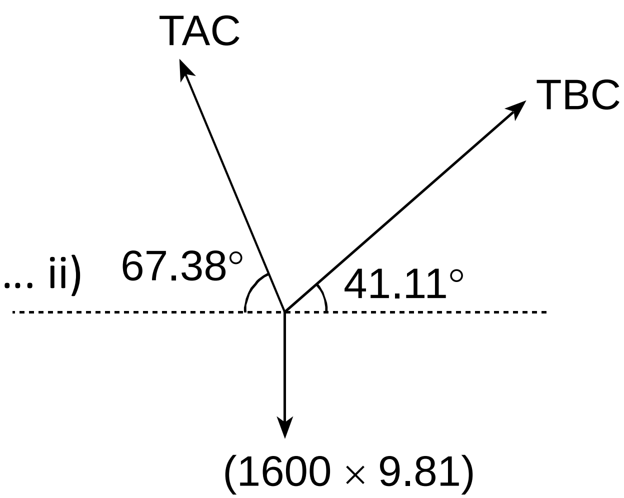
$\overset{+}{\rightarrow}$

$\sum$**Fx = 0**

$-T_{AC}\cos 67.38° + T_{BC}\cos 41.11° = 0$ ...... (i)

(+↑) $\sum$Fy = 0

$T_{AC}\sin 67.38° + T_{BC}\sin 41.11° = (1600 \times 9.81)$ .... ii)



On solving (i) and (ii)

(T_{AC} = 12469.82 N)

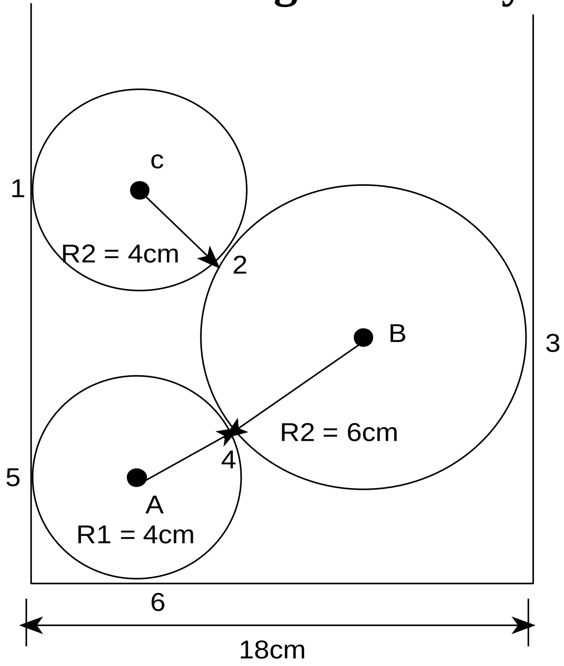
T_{BC} = 6365.55 N)

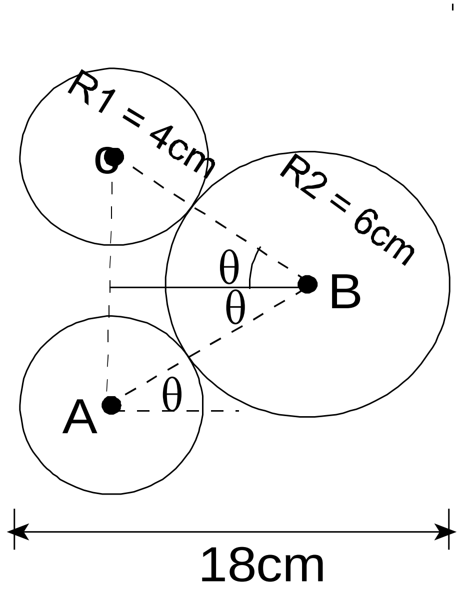
Q. Determine the reaction at the contact points, if three cylinders are piled in a rectangular ditch as shown in figure. Given that weight of the cylinders are:

W_A = 2 KN

W_B = 5 KN

W_C = 3 KN





Solution:

we have b = 18 – 4 – 6 = 8cm

$$\therefore \cos\theta = \left(\frac{8}{10}\right)$$

$$\theta = 36.87°$$

For cylinder C [have less no. of unknown reaction so taking 1st C for fast calculation]

($\overset{+}{\rightarrow}$)

$\sum$Fx = 0

$R_1 - R_2 \cos 36.87 = 0$

$[R_1 = 0.8 R_2]$

(+$\uparrow$) $\sum$Fy = 0

$R_2 \sin 36.87° - W_C = 0$

$$R_2 = \frac{3 \text{ KN}}{\sin 36.87}$$

$[R_2 = 5 \text{ KN}]$

$\therefore$ and $[R_1 = 4 \text{ KN}]$

Considering free body diagrams of cylinder A.

(+$\uparrow$) $\sum$Fy = 0

$- R_4 \sin 36.87 - W_A + R_6 = 0$

$- 0.6 R_4 + R_6 = 2 \text{ KN}$ .... (1)

($\overset{+}{\rightarrow}$)

$\sum$Fx = 0

$R_5 - R_4 \cos 36.87° = 0$

$[R_4 = 1.25 R_5]$ .... (2)

For F.B.D of cylinder B

(+$\uparrow$) $\sum$Fy = 0

$R_4 \sin 36.87 - R_2 \sin 36.87 - W_B = 0$

$0.6 R_4 - 5 \times 0.6 - 5 = 0$

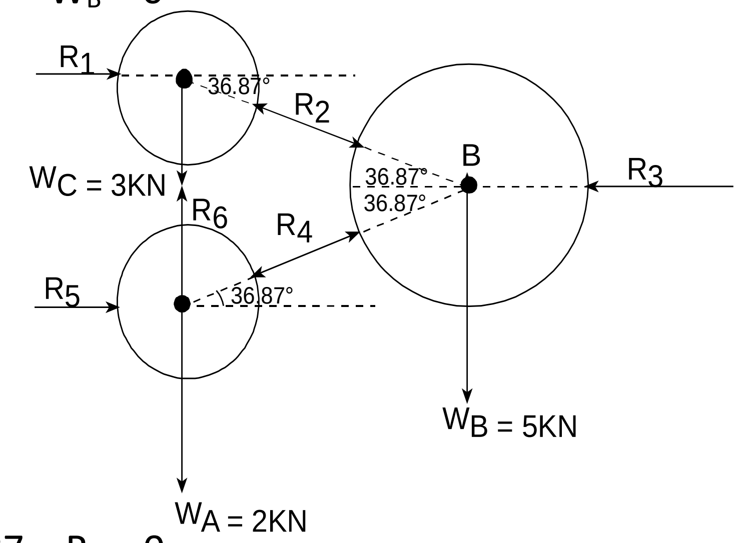
$[R_4 = 13.33 \text{KN}]$

$\therefore [R_5 = 10.67 \text{ KN}]$

From equation (i)

$-0.6 \times 13.33 + R_6 = 2$

$[R_6 = 10 \text{ KN}]$



($\overset{+}{\rightarrow}$)

$\sum$Fx = 0

$R_4 \cos 36.87 + R_2 \cos 36.87 - R_3 = 0$

$13.33 \times \cos 36.87 + 5 \cos 36.87 - R_3 = 0$

$[R_3 = 14.67 \text{ KN}]$

References

[1] Kumar, D. (2019). *Engineering Mechanics.* New delhi: S.K Kataria and Sons.

[2] Parajuli, N. a. (2024). *A Text book of Engineering Mechanics.* Bhotahity Kathmandu: Heritage Publisher and Distributors PVT .LTD.

Thank You!!!