Applied Mechanics

Chapter 3

Forces Acting on Particle and Rigid Body

Lecture 4 (Week 4)

Moments and couples, Resolution of a Force into Forces and a Couple, Resultant of Force and Moment for a System of Force: Examples

Lecturer: Asst. Prof. Sunil Rakhal

Learning Objectives

- Understand the concept of moments and couples in mechanics.
- Identify real-world examples where moments and couples are present.
- Learn methods to resolve forces into component forces and a couple.
- Solve problems involving the resolution of forces and couples in static equilibrium.
- Understand the concept of the resultant of a force and moment in a system.
- Learn techniques to find the resultant force and moment of a system of forces.

3.4 Moments and Couples

3.4.1 Moment of a force

The type of rotational effect of a force is measured by a physical quantity known as the moment of a force. Or in other words, the moment of a force about a point is defined as a measure of the tendency of the force to rotate a body about that point. e.g. Door rotating about its hinge by the application of a force.[2]

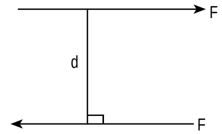
M=F*d

3.4.1.1 Types of moments

- If the tendency of a force is to rotate the body in the clockwise direction, it is said to be a clockwise moment and is generally taken positive.
- If the tendency of a force is to rotate the body in the anticlockwise direction, it is said to be anticlockwise moment and is generally taken negative.

3.4.2 Couple

A couple consists of two parallel forces that are equal in magnitude but opposite in direction separated by a finite distance.



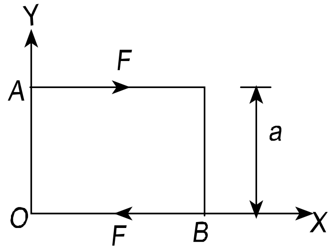
moment of couple = F × d

In vector form

Moment of couple = $\overrightarrow{r_{OA}} \times (F\vec{i})$

$= a\vec{j} \times F\vec{i}$

$= -Fa\vec{k}$ [Clockwise]



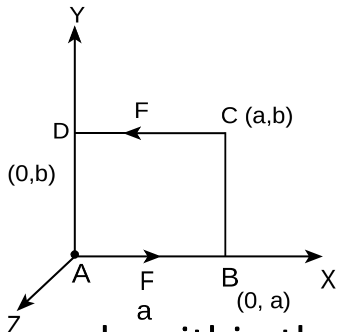
3.4.2.1 Classification of Couples

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts

1.Clockwise couple: A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple.

2.Anticlockwise couple: A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple. [4]

3.4.2.2 Illustration of Couple as a Free Vector



Couple is free vector means the couple moment at D is equal to couple moment at A or any other point on that plate.

Physical meaning

The rotational effect will be the same wherever you apply the couple within the plate.[3]

Proof:

Consider a rectangular plate ABCD having length 'a' and height 'b' lying at plate XY.

Two equal and opposite forces F is separated by a distance b.

Taking moment about point A = $\vec{r} \times \vec{F_0}$

$$= \vec{AD} \times \vec{F} \quad \text{[A be the origin]}$$

$$= b\vec{j} \times F(-\vec{i})$$

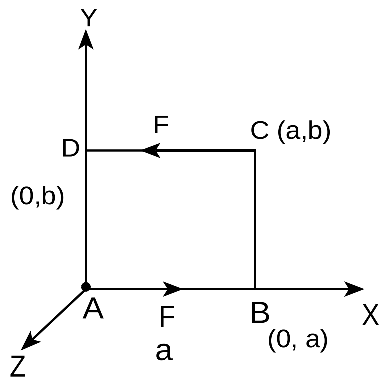
$$= Fb\vec{k}$$

Now,

Taking moment at point D $= \overrightarrow{r_{DA}} \times F(\overrightarrow{i})$

$= -b\overrightarrow{j} \times F(\overrightarrow{i})$

$= Fb\overrightarrow{k}$



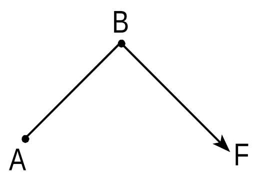
Taking moment at point A $= \overrightarrow{r_{AC}} \times \overrightarrow{F}$

$= (a\overrightarrow{i} + b\overrightarrow{j}) \times F(-\overrightarrow{i})$

$= Fb\overrightarrow{k}$

$\therefore$ $M_A = M_D$ = moment at any other points, hence prove.

Note: For easy remembrance



$M_A = \overrightarrow{r_{AB}} \times \overrightarrow{F}$ (position from A to the force acted point)

3.4.2.3 Characteristic of Couple

a) Having same magnitude but opposite direction

b) Couple cannot be balanced by a single force.

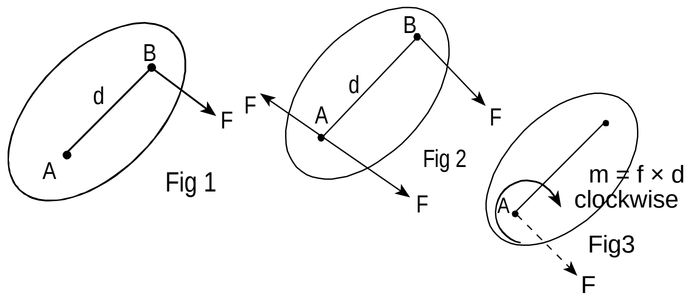
c) The resultant forces constituting the couple is zero.

d) The translation effect of couple on the body is zero.

e) Couple must rotate the body about fixed axis either clockwise or anticlockwise.

f) Two forces must be involved

3.5 Resolution of forces into force and a couple



F force is acting at point B in figure 1. Now the same magnitude of two equal forces F but in opposite direction is added to the system at point A shown in figure 2. These means the system is same as figure 1 because force is opposite in direction and cancel each other. And the couple is formed by the force F which is acted at point A and B shown by figure 3. The moment is clockwise moment and one single force is remaining in this system.

Hence by this way we can convert the single force F into force and couple moment (F × d) at another point.

3.6 Resultant of Force and Moment for a System of Force: Examples

3.6.1 Magnitude, Direction and Position of Resultant Force

In a coplanar non-concurrent force system, the magnitude, direction and position of resultant can be determined by two approaches:

1. Scalar method

- Calculate the algebraic sum of all the forces acting in the x-direction (i.e. Fx) and also in the y-direction (i.e. Fy).
- Determine the magnitude of the resultant using the formula, $R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$
- Determine the direction of the resultant using the formula

$$\propto = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

- The position of resultant can be determined by using the Varignon's theorem as

$$R \times d = F_1 \times d_1 + F_2 \times d_2 \ldots\ldots.$$

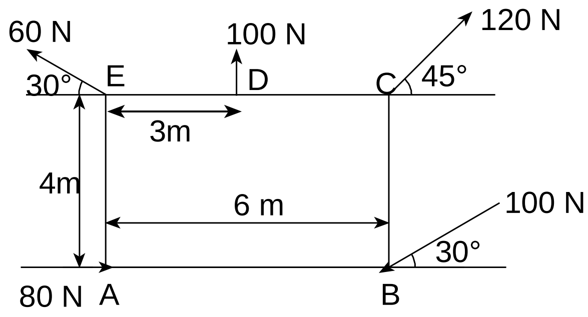
2. Vector approach (alternative method)

Resolving forces into components in vector form, find Resultant and calculate direction (i.e inclination) and apply Varignon's theorem in vector form for location i.e.

$[\vec{r_1} \times \vec{F_1} + \vec{r_2} + \vec{F_2} = \vec{r} \times \vec{F_R}]$

Q. A plate of size 6m × 4m is acted upon by a set of forces in its planer as shown in figure below. Determine magnitude, direction and position of resultant force.[1]

Solution:



Resolving force into x – components and take sum

$\xrightarrow{+}$ **(Rightward direction positive)**

$\sum$**Fx =** 80 – 100 cos30° + 120 cos 45° – 60 cos 30°

= 80 – 86.6 + 84.85 – 51.96, = 26.29 N

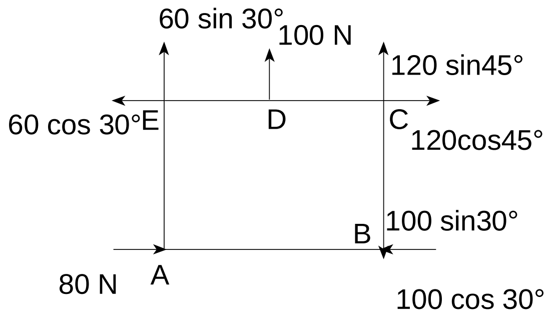
$(+\uparrow)\ \Sigma Fy = -100 \sin 30^0 + 120 \cos 45^0 + 100 + 60 \sin 30^0$

= 164.85N

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2} = \sqrt{(26.29)^2 + (164.85)^2}$

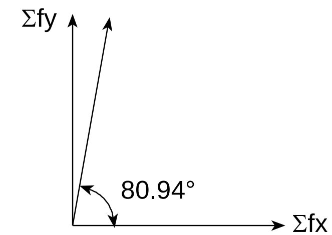
= 166.93 N

$\tan\propto = \dfrac{\sum Fy}{\sum Fx} = \dfrac{164.85}{26.2} = 6.27$ and $\propto = 80.94$



Since $\sum Fx$ and $\sum Fy$ both positive,

so resultant lies in first quadrant as shown in figure



Applying Varignon's theorem,for location as:

Moment of resultant about any point = sum of moment of all the forces about same point.

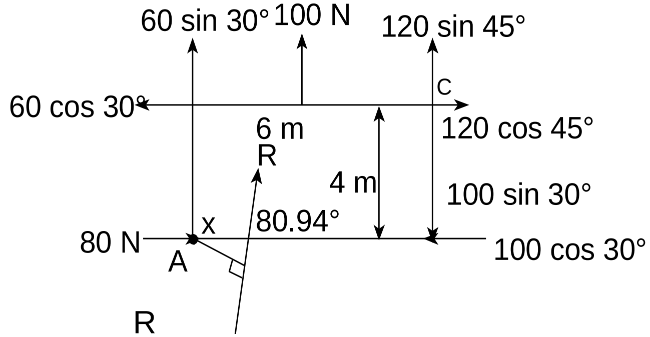
$R \times d = F_1 \times d_1 + F_2 \times d_2$ ....

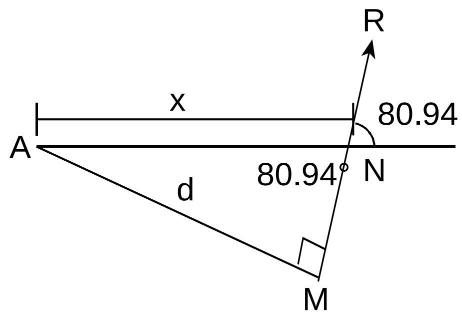
(Take anticlockwise moment positive)

$166.93 \times d = -100 \sin 30° \times 6 - 120 \cos 45 \times 4 +$

$120 \sin 45° \times 6 + 100 \times 3 + 60 \cos 30° \times 4$

$$d = \frac{377.55}{166.93} = 2.261 \text{ m}$$

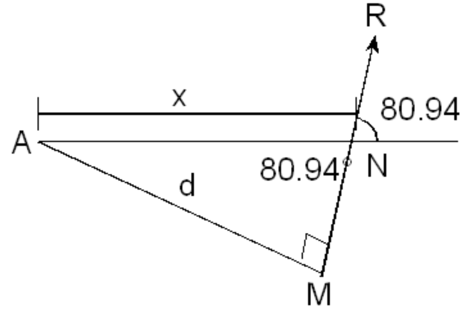




∴ In small triangle AMN

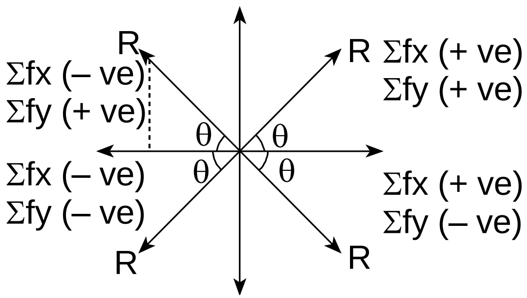
$$\text{Sin } 80.94° = \frac{d}{x}$$

$$\therefore x = \frac{2.261}{\sin 80.94} \quad (x = 2.29\text{m})$$



Hence the resultant lies 2.29m from point A.

Note: *for direction*



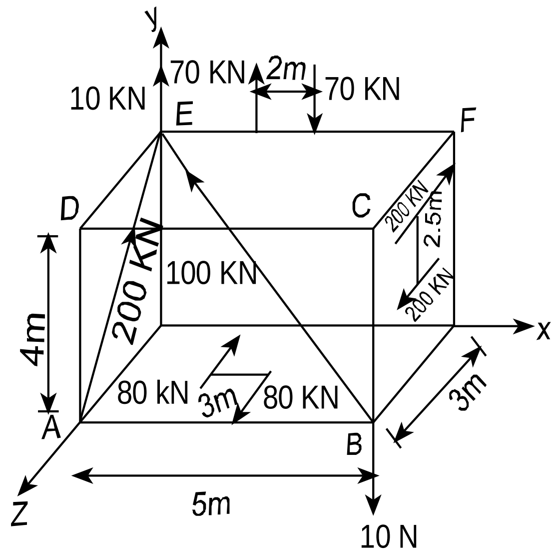
Q. Determine the resultant force and moment of the following system about 'O' as shown in figure below.

Solution:

Given force 10 KN, 200 KN and 100KN are in the magnitude form so,

We calculate magnitude along with direction in vector form.

So, $\vec{F_{AE}}$ =magnitude of force × unit vector along force line)



$= (F_{AE}) \times \hat{F_{AE}}$

$$= 200 \times \frac{\vec{AE}}{|\vec{AE}|}$$

The coordinates of different point are

A = (0, 0, 3)

B = (5, 0, 3)

E = (0, 4, 0)

so, $\vec{F}_{AE} = 200 \times \dfrac{(4\vec{j} - 3\vec{k})}{\sqrt{(4)^2 + (-3)^2}} = (160\,\vec{j} - 120\,\vec{k})$

$\vec{F}_{BE} = F_{BE} \times \hat{F}_{BE}\ \vec{BE}$

$= 100 \times \dfrac{\vec{BE}}{|\vec{BE}|} = 100 \times \dfrac{(-5\vec{i} + 4\vec{j} - 3\vec{k})}{\sqrt{(-5)^2 + (4)^2 + (-3)^2}}$

$= (-70.7\vec{i} + 56.56\vec{j} - 42.42\vec{k})$

Now, the resultant force ($\vec{F}$) in vector form is

$= (160\vec{j} - 120\vec{k}) + (-70.7\vec{i} + 56.56\vec{j} - 42.42\vec{k}) + 10\vec{j} - 10\vec{j}$

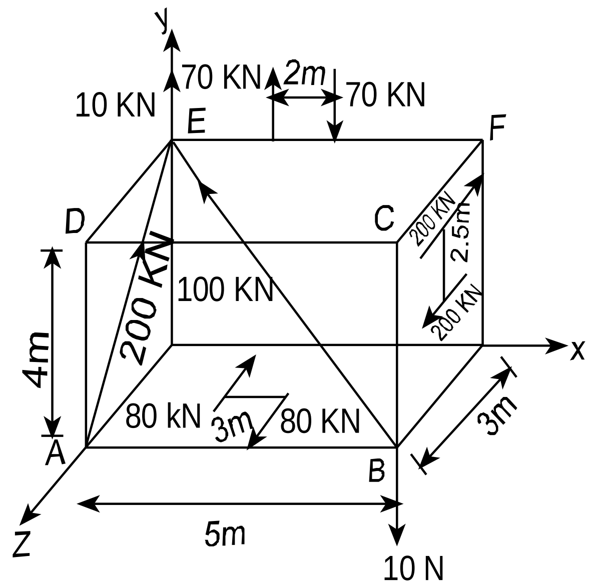
$\vec{F} = (-70.7\,\vec{i} + 216.56\vec{j} - 162.42\vec{k})$

Here are 3 couples so,

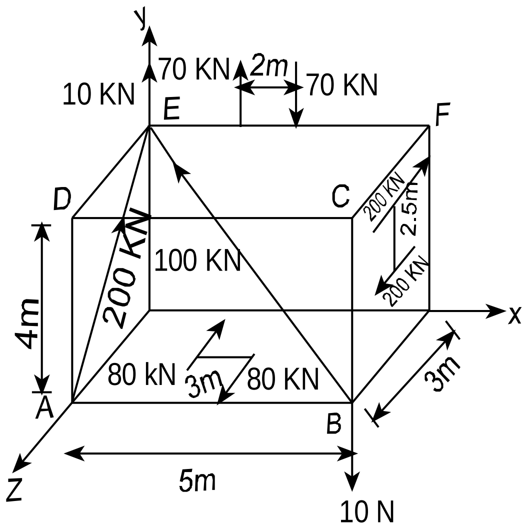
$C_1 = 2\vec{i} \times (-70\vec{j}) = -140\vec{k}$

$C_2 = 3i \times (80\,\vec{k}) = -240\vec{j}$

$C_3 = 2.5\vec{j} \times (-200\vec{k}) = -500\,\vec{i}$



Resultant moment about point '0'



$\vec{M}_O = \overrightarrow{OA} \times \overrightarrow{F_{AE}} + \overrightarrow{OB} \times \overrightarrow{F_{BE}} + C_1 + C_2 + C_3 + \overrightarrow{OB} \times (-10\vec{j})$

$= 3\vec{k} \times (160\vec{j} - 120\vec{k}) + (5\vec{i} + 3\vec{k}) \times (-70.7\vec{i} + 56.56\vec{j} - 42.42\vec{k})$

$-140\vec{k} - 240\vec{j} - 500\vec{i} + (5\vec{i} + 3\vec{k}) \times (-10\vec{j})$

$= -480\vec{i} + 282.8\vec{k} + 212.1\vec{j} - 212.1\vec{j} - 169.68\vec{i} - 140\vec{k} - 500\vec{i} - 240\vec{j} - 50\vec{k} + 30\vec{i}$

$\vec{M}_O = [-1119.68\vec{i} - 240\vec{j} + 92.8\vec{k}]$

References

[1] Kumar, D. (2019). *Engineering Mechanics.* New delhi: S.K Kataria and Sons.

[2] M.N. SHESHA PRAKASH, G. B. (August, 2014). *ELEMENTS OF CIVIL ENGINEERING AND ENGINEERING MECHANICS.* Rimjhim House, 111, Patparganj, Delhi: PHI Learning Private Limited.

[3] Neupane, P. a. (2024). *A Text book of Engineering Mechanics.* Bhotahity Kathmandu: Heritage Publisher and Distributors PVT .LTD.

[4] Khurmi, R. (1967). *A Tect Of Engineering Mechanics.* RAM NAGAR, NEW DELHI - 110 055: S. CHAND & COMPANY LTD