Applied Mechanics

Chapter 4

Center of Gravity, Centroid and Moment of inertia

Lecture 5 (week 5)

Concepts and Calculation of Centre of Gravity and Centroid: Examples

Lecturer: Asst. Prof. Sunil Rakhal

I.O.E, T.U Nepal

Learning Objectives

- Define center of gravity and centroid.
- Identify center of gravity and centroid for simple geometric shapes.
- Determine the centroid of composite area
- Calculate center of gravity and centroid using appropriate methods.
- Apply concepts to solve real-world engineering problems.

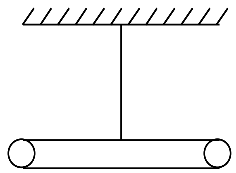
4.1 Concepts and Calculation of Centre of Gravity and Centroid: Examples

4.1.1 Center of gravity

We know, the weights of all parts of a body directed towards the center of the earth as a parallel force, magnitude is equal to their algebraic sum of all these forces.

Thus, centre of gravity of the body may be defined as the point

through which the whole weight (Resultant) of a body may be assumed to act.



It is denoted by c.g(G).

It is related to concentration of whole weight of the body

e.g When the string is in the centre of gravity of the rod, then it is balanced.

4.1.2 Centroid

The centroid or centre of area is defined as the point where the whole area of the figure is assumed to be concentrated. Or it is stable assumed center point of the plane area.[1]

4.1.3 Determination of the centroid of composite area

The location of the centroid of a plane figure can be thought of as the average distance of the area to an axis.

let us consider a whole area is divided into smaller area a_1, a_2 ------ a_n

when x_1, x_2, x_3-----x_n be the horizontal distance from the assume reference y axis to the centroid of the individual figure and Y_1, Y_2, Y_3----- be vertical distance from the assume reference x – axis to the centroid of individual figure.

We know, whole area × centroidal distance of whole figure= Σarea of individual figure × individual centroidal distance from same axis.

so, $(a_1+a_2+a_3+\text{------}a_n) \times \overline{x} = a_1x_1+a_2x_2+a_3x_3+\text{---}a_nx_n$

$$\overline{x} = \frac{a_1x_1+a_2x_2+a_3x_3+\text{------}a_nx_n}{a_1+a_2+a^3+\text{-----}a_n}$$

Where $\overline{x}$ is the centroidal horizontal distance from assumed reference Y axis to the centroid of whole figure

Similarly

$$\bar{y} = \frac{a_1y_1+a_2y_2+a_3y_3+\text{---}a_ny_n}{a_1+a_2+a_3+\text{-----}a_n}\ ]$$

For C.G

Moments are taken of masses, rather than of areas, of individual parts of the solids. The CG will then have the coordinates

$$\bar{x} = \frac{\sum miXi}{\sum mi} \qquad \bar{y} = \frac{\sum miyi}{\sum mi}$$

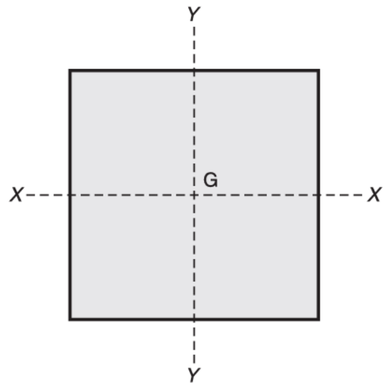
If all the sections are of same material having density ρ then,

$m_1= \rho\ v_1$, $m_2= \rho\ v_2$, $m_3= \rho\ v_3$ and V=V1+V2+V3+……

then

$$\bar{x} = \frac{\sum VXi}{\sum V} \qquad \bar{y} = \frac{\sum Vyi}{\sum V}$$ **[2]**

Centroidal Axis



The axis which passes through the centroid of the given figure is known as centroidal axis. [X-X =centroidal x axis and Y-Y= Centroidal y axis]

Reference Axes

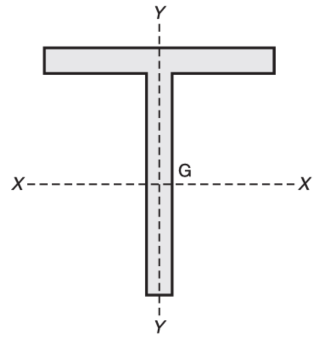
These are the axes with respect to which the centroid of a given figure is determined.

Symmetrical Axis

It is the axis which divides the whole figure into equal parts

- For a figure which is symmetrical about both the axes, $\bar{x} = 0$ and ,$\bar{y} = 0$.

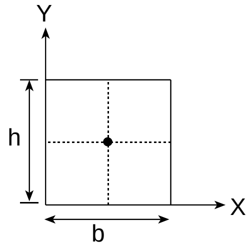
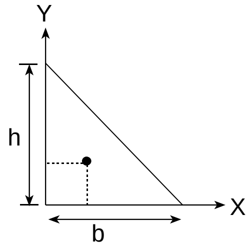
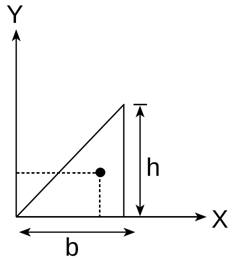
- For a figure which is symmetrical about the Y–Y axis, $\bar{x} = 0$.

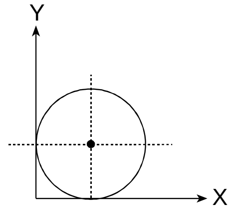
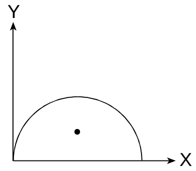
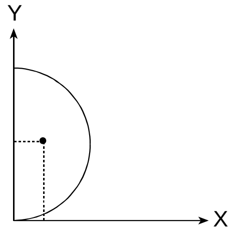
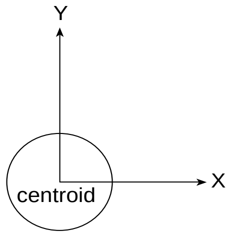


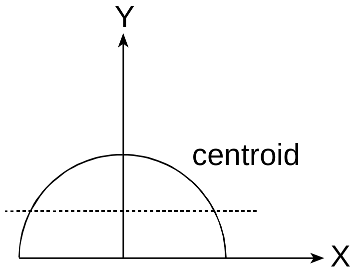
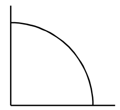
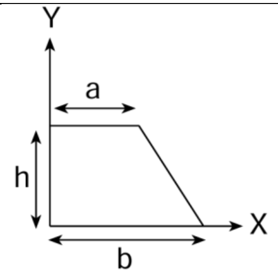
Note: The area on the left-side of the Y–Y axis is equal to the area on the right side of the Y–Y axis.

- For a figure which is symmetrical about the X–X axis, $\bar{y} = 0$.
- For a figure which does not have any axis of symmetry, we calculate both centroidal x and y. [3]

4.1.4 Position of centroid of plane geometrical figures

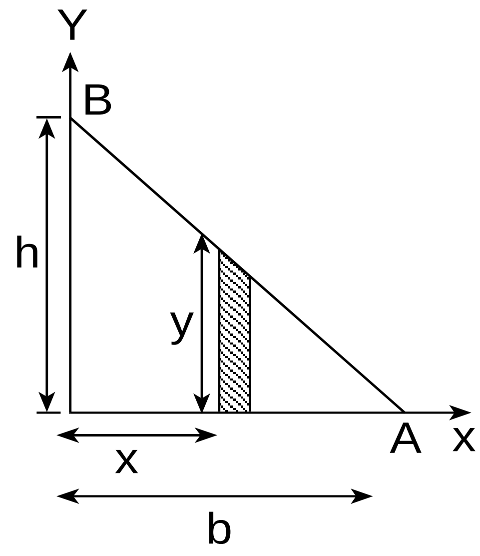
Name of geometrical	Figure	Area	$\bar{x}$	$\bar{y}$
Rectangle		$b\times h$	$\frac{b}{2}$	$\frac{h}{2}$
Triangle		$\frac{1}{2}\times b\times h$	$\frac{b}{3}$	$\frac{h}{3}$
Triangle		$\frac{1}{2}bh$	$\frac{2b}{3}$	$\frac{h}{3}$

Circle		πr^2	r	r
Semicircle		$\frac{\pi r^2}{2}$	r	$\frac{4r}{3\pi}$
Semicircle		$\frac{\pi r^2}{2}$	$\frac{4r}{3\pi}$	r
Circle		πr^2	0	0

Semicircle		$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$
Quarter circle		$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Trapezium		$\frac{1}{2}\ (a + b) \times h$	$\frac{a^2 + b^2 + ab}{3(a+ b)}$	$\frac{(2a + b)\ h}{3\ (a + b)}$

Q. Centroid of right-angle triangle by methods of integration.

Solution: 1st of all determine the equations of inclined line of triangle AB by similar triangles properties



i.e. $\frac{h}{b} = \frac{y}{(b-x)}$

or, $y = \frac{h}{b}(b-x)$

<u>Taking vertical strip parallel to Y. axis having width dx</u>

$$A = \int dA = \int y \times dx = \int_0^b h/b \times (b-x) \times dx = h/b \times [bx - \frac{x^2}{2}]$$

$A = h/b \times [b^2 - \frac{b^2}{2}]$ or, $A = h/b \times \frac{b^2}{2}$

$[A = ½ \times b \times h]$

Calculation of centroid

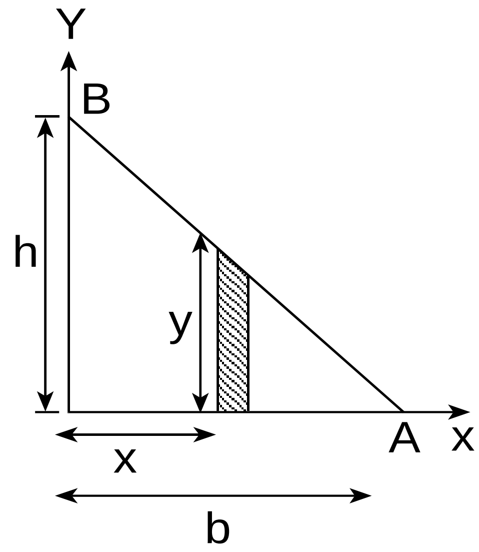
$\overline{x} = 1/A \int x_e . dA$ $\overline{x} = \frac{2}{bh} \times \int_0^b x \times y \times dx$

$\overline{x} = \frac{2}{bh} \int_0^b x \times h/b \times (b-x)dx$ $\overline{x} = \frac{2}{bh} \times \frac{h}{b} \left[\frac{bx^2}{2} - \frac{x^3}{3} \right]_0^b$

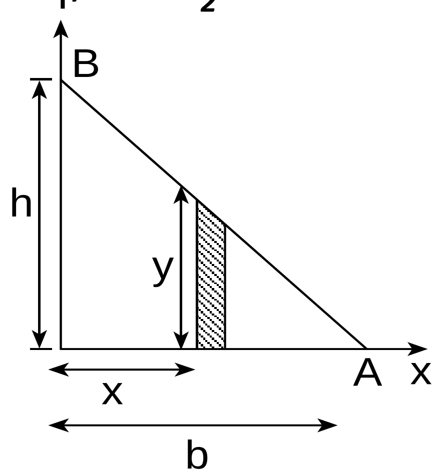
$\overline{x} = \frac{2}{b^2} \times \left[b^{3/2} - \frac{b^3}{3} \right]$

$\overline{x} = \frac{2}{b^2} \times \frac{b^3}{6}$

$\overline{x} = b/3$



Where Ye = distance from reference x axis to the centroid of the strip i.e. Ye= $\frac{y}{2}$



And again

$$\bar{y} = 1/A \int_0^b Y_e dA \quad \text{or,} \quad \bar{y} = \frac{2}{bh} \times \int_0^b \frac{Y}{2} \times Y dx$$

$$\bar{y} = \frac{2}{bh} \times \frac{1}{2} \times \int_0^b y^2 \, dx \quad \bar{y} = \frac{1}{bh} \times \int_0^b \frac{h^2}{b^2} (b-x)^2 dx$$

$$\bar{y} = \frac{h}{b^3} \times \int_0^b (b^2 - 2bx + x^2) dx$$

$$\bar{y} = h/b^3 \left[b^2x - \frac{2bx^2}{2} - \frac{x3}{3} \right]_0^b \quad \text{or,} \quad \bar{y} = \frac{h}{b^3} \left[b^3 - \frac{2b^3}{2} + \frac{b^3}{3} \right] \quad \text{or,} \quad \bar{y} = \frac{h}{b^3} \times \frac{2b^3}{6}$$

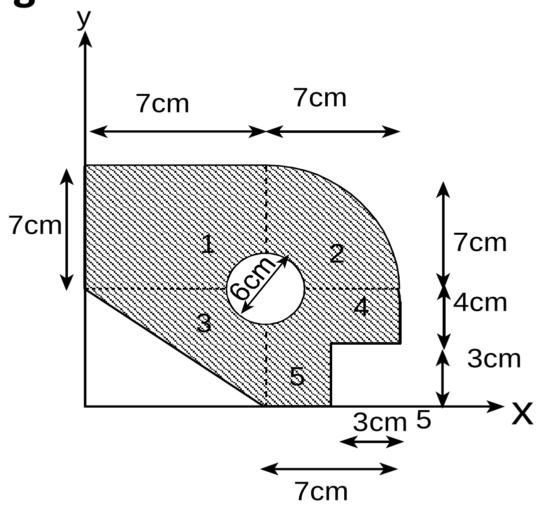
$[\bar{y} = h/3]$

Hence, the centroid of right angled triangle is $(\bar{x}, \bar{y}) = (b/3, h/3)$

Q. Locate the centroid if the shaded area as shown in the figure.

Divide the whole figure into Known geometrical shapes

1. Rectangle (7cm × 7cm) (+ve)
2. quater circle with 7cm radius (+ve)
3. Triangle (+ve)
4. Rectangle (7cm × 4cm) (+ve)
5. Rectangle (4cm × 3cm) (+ve)
6. circle with 6cm diameter (-ve)



$A_1 = 7 \times 7 = 49cm^2$ $x_1 = 3.5cm$ $y_1 = 7 + 3.5 = 10.5cm$	$A_2 = \dfrac{\pi \times 7^2}{4} = 38.5cm^2$ $x_2 = 7 + \dfrac{4 \times 7}{3\pi} = 9.97cm$ $y_2 = \left(7 + \dfrac{4r}{3\pi}\right) = 9.97cm$

$A_3 = ½ \times 7 \times 7 = 24.5cm^2$ $x_3 = 2 \times 7/3 = 4/3cm = 4.67cm$ $y_3 = (7 - 7/3) = 4.67cm$	$A_4 = 7 \times 4 = 28cm^2$ $x_4 = 7 + 7/2 = 10.5cm$ $y_4 = 3 + 4/2 = 5cm$
$A_5 = 4 \times 3 = 12\ cm^2$ $x_5 = 7 + 4/2 = 9cm$ $y_5 = 3/2 = 1.5cm$	$A_6 = \pi \times 3^2 = 28.27cm^2$ $x_6 = 7cm$ $y_6 = 7cm$

we known

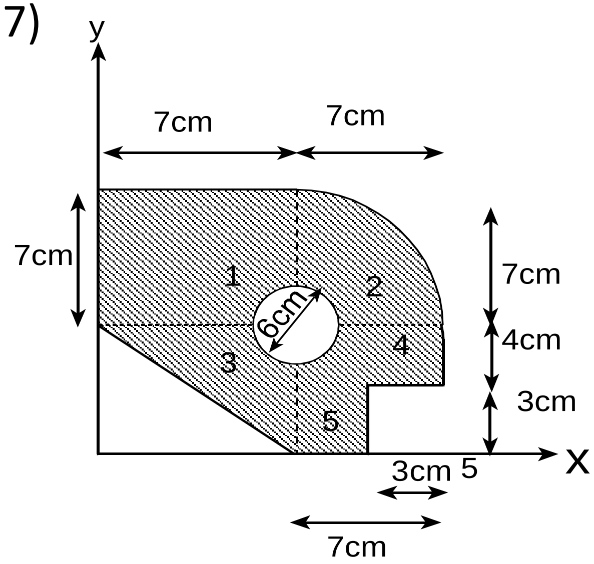
$$\bar{x} = \frac{(49\times3.5+(38.5\times9.97)+(24.5\times4.67)+(28\times10.5)+(12\times9)-28.27\times7)}{(49+38.5+24.5+28+12-28.27)}$$

$\bar{x} = 7.06cm$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + \ldots\ldots\ldots\ldots A_5y_5 - A6y_6}{(A_1 + A_2 + \ldots\ldots\ldots\ldots + A_5 - A_6)}$$

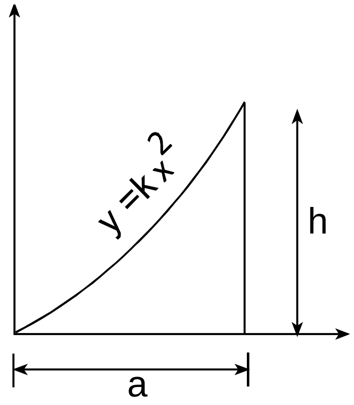
$$\bar{y} = \frac{514.5 + 383.9 + 114.3 + 140 + 18 - 197.82}{123.7}$$

$\bar{y} = 7.86cm$



Q. Determine centroid of the given plane figure.

Solution:



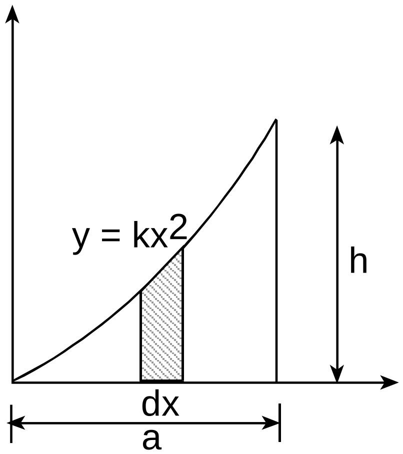
(For the value of constant (k))

$h = Ka^2$

$K = \frac{h}{a^2}$

Equation becomes

$y = \frac{h}{a^2} X^2$



$$\mathbf{A = \int dA = \int_0^a y.dx}$$

$$= \int_0^a \frac{h}{a^2} x^2\, dx = \frac{h}{a^2}\left[\frac{x3}{3}\right]_0^a = \frac{h}{a^2} \cdot \frac{a^3}{3}$$

$[\therefore A = ah/3]$

Now, $\bar{x}$ =1/A ∫xe.dA

$$=\frac{1}{\left(\frac{ah}{3}\right)}\int_0^a Xe.y\,dx = \frac{3}{ah}\times\int_0^a X\times\frac{h}{a^2}X^2dx = \frac{3}{ah}\times\frac{h}{a^2}\left[\frac{x^4}{4}\right]_0^a$$

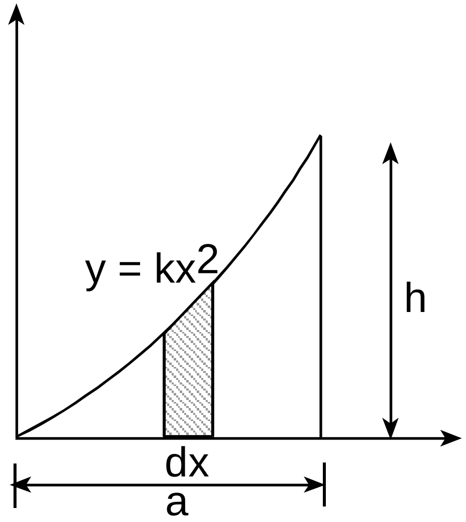
[∴ $\bar{x}$ =3a/4]

And $\bar{y}$ =1/A ∫ye. dA $=\frac{3}{ah}.\int_0^a \frac{y}{2}\times y\,dx = \frac{3}{2ah}\int_0^a Y^2.dx$

$$=\frac{3}{2ah}\int_0^a \frac{h^2}{a^4}X^4dx = \frac{3}{2ah}.\frac{h^2}{a^4}\times\left[\frac{x^5}{5}\right]_0^a = \frac{3h}{2a^5}\times\frac{a^5}{5}$$

$$\bar{y} = \frac{3h}{10}$$

Hence, centroid of the whole given figure from the given axis ($\bar{x}$,$\bar{y}$) = ($\frac{3a}{4}$,$\frac{3h}{10}$)

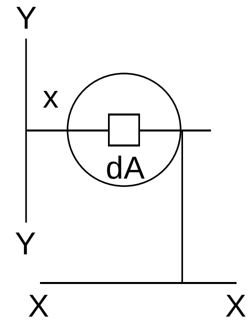


4.1.5 Moment of Inertia

The cumulative product of area and square of its distance from an axis is called the moment of inertia of a section about that axis.

i.e. $I_X = \int y^2 dA$ and

$I_Y = \int x^2 dA$



Or in other words

First moment of elemental area about y-y axis = X×dA.

(Moment of first moment) second moment of elemental area = (X ×dA) ×X = $X^2 dA$

∴ 2^{nd} moment of entire area A w. r. to y-y axis is obtained by integrating above expression,

$I_Y = \int x^2 dA$ Similarly, $I_{xx} = \int y^2 dA$ ∴ And the second moment of an area is termed as M.O.I

4.1.6 Physical meaning of moment of Inertia

It gives the measure of resistance to bending of beams under the action of loads. The greater the second moment of area (M.O.I), the greater the resistance to bending So M.O.I of cross-section is essential for designing.

References

[1] Khurmi, R. (1967). *A Tect Of Engineering Mechanics.* RAM NAGAR, NEW DELHI - 110 055: S. CHAND & COMPANY LTD.

[2] Kumar, D. (2019). *Engineering Mechanics.* New delhi: S.K Kataria and Sons.

[3] M.N. SHESHA PRAKASH, G. B. (August, 2014). *ELEMENTS OF CIVIL ENGINEERING AND ENGINEERING MECHANICS.* Rimjhim House, 111, Patparganj, Delhi: PHI Learning Private Limited.

Thank You!!!