

Applied Mechanics

Chapter 6

Analysis of Beams and Frames

Lecture 8 (week 8)

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Learning Objectives:

- Differentiate between discrete and continuum systems in structural engineering.
- Identify examples of discrete and continuum structures in practical applications.
- Identify different types of loads acting on structures and their characteristics.
- Apply standard symbols and conventions for representing loads in engineering drawings and diagrams.
- Understand the concept of rigid joints and their types and significance in structural analysis and design.

6. Analysis of Beams and Frames

Beam

A beam is a horizontal or inclined structural member that primarily carries loads applied perpendicular to its longitudinal axis and are designed to resist bending moments and shear forces, which are induced by the applied loads.[1]

Common examples of beams include floor beams in buildings, bridge girders, and joists in floors or ceilings.

Beams can be made of various materials such as steel, concrete, wood, or composite materials.

Frame

Frame is a network of structure members (i.e. beams and column) which are connected or jointed together to resist load and moment.

6.1 Introduction to Structures: Discrete and Continuum

In structural engineering, the concepts of discrete and continuum systems are fundamental to understanding the behavior of different types of structures.

Discrete System

Definition: Discrete systems are composed of individual, distinct components or elements. These elements are usually separate and identifiable, and they interact with each other through specific connections or joints.

Examples: A truss structure made of individual members connected at their ends by joints, a frame structure composed of columns and beams, and mechanical systems like gear mechanisms or linkage systems are all examples of discrete systems.

Analysis: Analyzed using methods such as equilibrium equations, energy methods, or matrix analysis techniques like the finite element method (FEM). Each component of the system is analyzed individually, and then the overall behavior is determined by considering the interactions between these components.

Continuum System: Continuum systems are characterized by a continuous distribution of material or properties without distinct boundaries between individual components. They are treated as homogeneous and continuous throughout their volume.

Examples: Common examples of continuum systems include beams, plates, shells, and solid bodies like concrete or steel structures. These systems are described by partial differential equations governing their behavior.

Analysis: Continuum systems are analyzed using mathematical models derived from continuum mechanics, which describe how forces, moments, and displacements are distributed within the structure. The behavior of continuum systems is often described by differential equations or variational principles, and solutions are obtained through analytical, numerical, or experimental methods.

Key Differences: The key difference between the two lies in how we perceive the structure. Discrete systems are composed of identifiable parts, while continuum systems are treated as a single, continuous entity. This distinction influences how we analyze and design structures. Discrete systems are often analyzed by considering the individual components and their interactions, while continuum systems are analyzed using mathematical models derived from continuum mechanics.

In practice, many structures exhibit characteristics of both discrete and continuum behavior, depending on the scale of observation and the material properties involved. For example, a concrete beam may be treated as a continuum at the macroscopic level but may exhibit discrete behavior at the microscopic level due to the presence of individual aggregate particles. Understanding and appropriately modeling these behaviors are crucial for accurately predicting the response of structures to various loads and environmental conditions.

6.2 Concept of Load Estimating and Support idealizations: Examples and Standard symbols.

6.2.1 Loads and its types

Load refers to the force applied in the structure and for the design of structure an engineer should have the knowledge of different types of probable load.

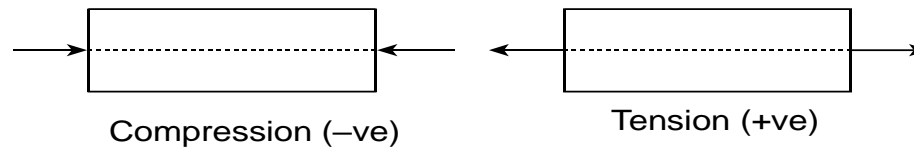
External forces: The loads and forces which are acted on the beam are called external forces.

Internal forces: The shear force, bending moment, axial force, torsional moment etc. Which are introduced in the beam due to external forces are called internal forces.

Axial forces, shear force and bending moments:

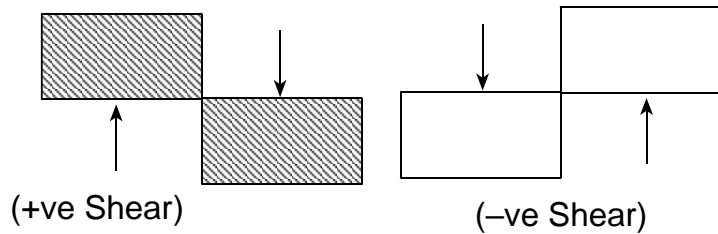
Axial force: Internal forces which are developed along the axis or the member are called axial forces.

Due to Axial force, the member tends to compressed or tensioned.



(a) Axial forces at any cross section of the member is the algebraic sum of all forces acting parallel to the longitudinal axis.

Shear force (S.F): It is the force acting tangential to the cross section of the member. Shear force at a section is defined as the algebraic sum of transverse force acting either side of the section.[2]



+ve S.F when the left part up and right part down
 -ve when left part down and right part up.

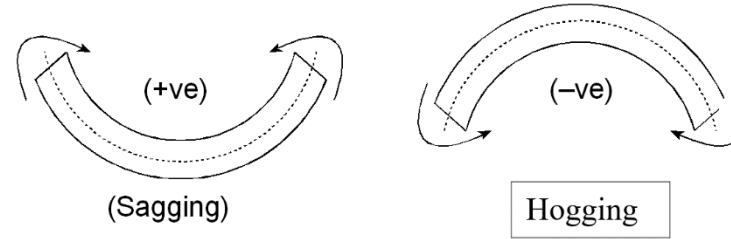
Note: If we move from left ward to rightward all the upward forces is + ve and downward forces is -ve and vice versa.

Bending Moment

At any transverse cross section, it is the algebraic sum of the moment of all force as either side of the section.

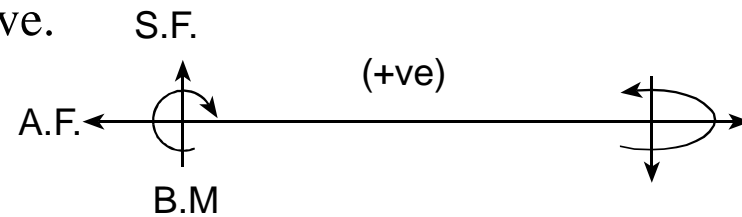
→ Sagging moment = (+ve)

→ Hogging moment = (-ve)



It means → From left side clockwise moment positive.

→ From right side clockwise moment negative.



sign convention for solving problems

While we are going from left side to right side

Tension, upwards and clockwise → +ve

While approaching from right to left tension, downward and anticlockwise all (+ve)

AND VICE VERSA

Types of loads

a. According to the nature

1. **Dead load:** Fixed load having fixed magnitude e.g. self-weight of the structure, wall load etc.
2. **Live/imposed load:** The loads vary in both magnitude and direction with respect to time. It is also called live loads e.g. loads of vehicle over bridge, all movable load.
3. **Dynamic load:** The loads which produce vibrations. E.g. earthquake load.
4. **Natural loads:**→ All the loads caused by the nature.[3]
 - wind load
 - Earth pressure
 - temperature effects
 - settlement
 - hydrostatic load etc.

b. According to the intensity / distribution

1. Point load: loads which acts at a single point.



2. Distributed load: The loads which are distributed over some length.

(i) Uniformly distributed load: load which distributed along structure uniformly. (i.e. e.g. 5 KN/m (5 KN per meter)

Total load (W) = (w × distance)

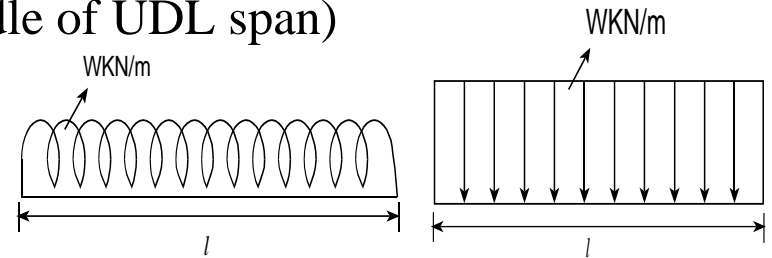
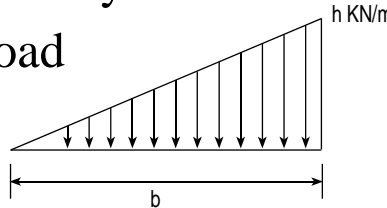
Total load is assumed to concentrated at cg (middle of UDL span)

(ii) Uniformly varying load (UVL)

Load varies with uniform intensity.

Area of triangle gives total load

Total load = $\frac{1}{2} \times b \times h$



(Note: for taking moment of total load take a distance from centroid of triangle.)

6.2.2 Types of supports, symbols and their no. of unknown reaction

Name of supports	Symbol and reaction	Degrees of freedom	NO. of unknown reaction
Roller supports		2	1
Hinge/pin supports		1	2
Fixed support		0	3

Note: *D.o.f* means the no. of possible way in which a joint can move in 2D structure.

e.g In roller supports, the vertical direction is constrains and the horizontal and rotation is possible so, D.O.F of roller support is 2.

Q. Find the reactions at support.

Solution:

(i) Calculation of support reaction

[$R_{AX} = 0$] (No horizontal force so horizontal reaction is zero)

$$+\uparrow \sum F_y = 0$$

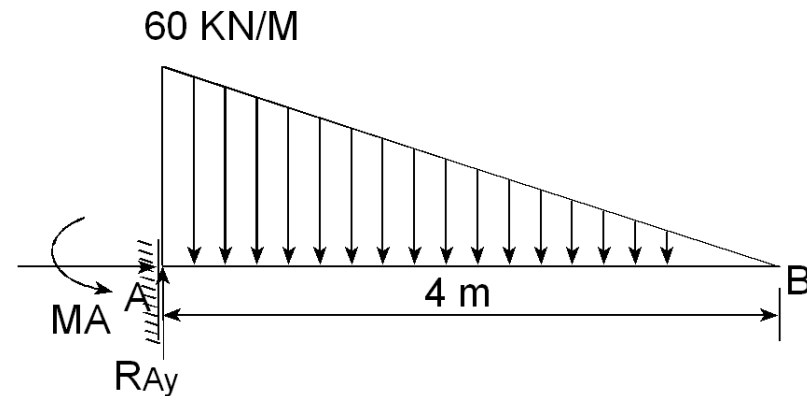
$$R_{AY} - \frac{1}{2} \times 60 \times 4 = 0$$

$$R_{AY} = 120 \text{ KN}(\uparrow)$$

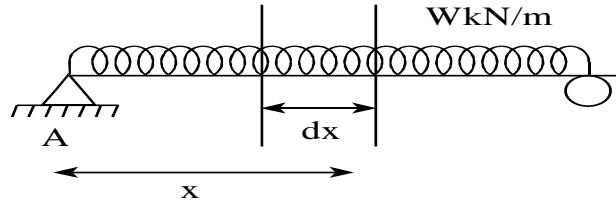
$$\ominus \sum M_A = 0$$

$$- M_A + \frac{1}{2} \times 4 \times 60 \times \frac{4}{3} = 0$$

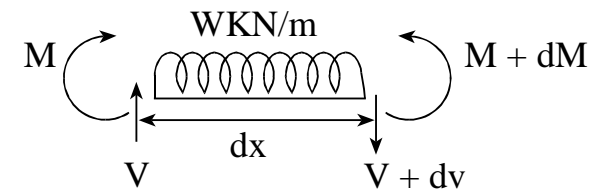
$$[M_A = 160 \text{ KNM}]$$



6.2.3 Relationship between rate of loading, shear force and bending moment



Consider a simply supported beam and take a small strip of beam having very small width dx at x distance from left support A. $W \text{ kN/m}$ be the intensity of UDL. V is the shear force and M be the B.M at a section.[3]



Take a direction as shown in figure. (i.e. take positive sign)

Here, Beam is in equilibrium so, dx portion is also in equilibrium.

So, $(+\uparrow) \sum F_y = 0$

$$V - Wdx - (v + dv) = 0$$

$$\left(\therefore \frac{dv}{dx} = -w \right)$$

The rate of change of S.F with respect to x is equals to intensity of loading.

$$\therefore \frac{dv}{dx} = -w$$

$$: [v = -w dx + c]$$

Area under the curve with (-ve) sign gives change in shear force.

Also, taking a moment at a point(right end of section) and clockwise moment is positive.

$$\curvearrowright \Sigma M_{\text{right end}} = 0$$

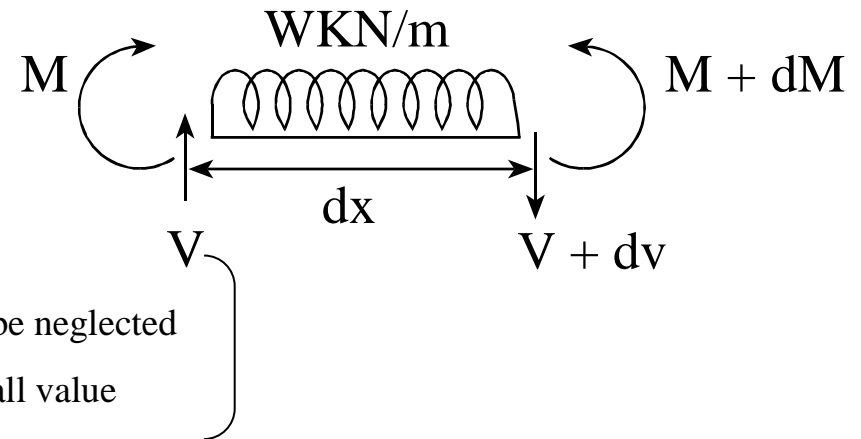
$$v \cdot dx + M - w \times dx \times \frac{dx}{2} - (M + dM) = 0$$

$$\text{or, } v dx + M - M - dM = 0$$

$$\left[\frac{dm}{dx} = v \right]$$

$$\therefore m = \int v dx + c_1$$

Area under the S.F gives change in Bending.



6.3 Use of Beam/Frame and concept of Rigid joint

6.3.1 Use of Beam and Frame

Beams: Beams are structural elements that primarily experience bending. They are commonly used to support loads perpendicular to their longitudinal axis. Examples include floor beams in buildings or bridge girders.

Frames: Frames are assemblies of beams and columns arranged in a geometric pattern to provide structural support and stability. They are commonly used in building and bridge structures to resist both gravity loads (such as the weight of floors or vehicles) and lateral loads (such as wind or earthquakes).

Beams and frames are fundamental structural elements used in construction to support loads and resist forces. Typical uses of beam and frame are as follows:

Beam

Horizontal Support: Beams primarily carry loads perpendicular to their length. They're commonly used to support the weight of floors, ceilings, roofs, and bridges.

Spanning Openings: Beams can span openings such as doors, windows, and archways, distributing the weight of the structure above.

Used For Load Distribution: They distribute loads from above to vertical supports, such as columns or walls, transferring these loads to the foundation.

For flexural Resistance: Beams are designed to resist bending moments caused by applied loads, distributing these moments as tension and compression forces along their length.

Frame:

Structural Stability: Frames provide structural stability to buildings and other structures by resisting lateral forces such as wind, seismic activity, or accidental impact.

Load Transfer: Frames transfer vertical and horizontal loads to the foundation, ensuring the stability and safety of the entire structure.

Rigid Connection: Unlike beams, frames consist of interconnected beams and columns, forming a rigid structure that resists deformation under load.

Space Enclosure: Frames define the shape and form of a structure, providing support for walls, floors, roofs, and other building elements.

6.3.2 Concept of Rigid joint

Rigid joint: A rigid joint is a connection between structural members that restricts relative movement between them. In other words, it behaves as if it were a single, continuous element rather than two separate ones.

Importance: Rigid joints are essential in frames because they allow for the transfer of moments (bending forces) between connected members. Without rigid joints, frames would behave differently under loading, leading to inaccurate structural analysis and potentially unsafe designs.

Examples: Welded connections, bolted connections with sufficient stiffness, and moment-resisting connections in steel or reinforced concrete structures are examples of rigid joints.

References

- [1] Kumar, D. (2019). *Engineering Mechanics*. New delhi: S.K Kataria and Sons.
- [2] M.N. SHESHA PRAKASH, G. B. (August, 2014). *ELEMENTS OF CIVIL ENGINEERING AND ENGINEERING MECHANICS*. Rimjhim House, 111, Patparganj, Delhi: PHI Learning Private Limited.
- [3] Neupane, P. a. (2024). *A Text book of Engineering Mechanics*. Bhotahity Kathmandu: Heritage Publisher and Distributors PVT .LTD.

Thank You!!!