Applied Mechanics

Chapter 6

Analysis of Beam and Frame

Lecture 9 (week 9)

Concept of Statically/Kinematically Determinate and indeterminate Structures

Lecturer: Asst. Prof. Sunil Rakhal

I.O.E, T.U Nepal

Learning Objectives:

- Define statically determinate and statically indeterminate structures in the context of beams and frames.
- Explain the significance of determining the stability and predictability of a structure through the concepts of static and kinematic determinacy.
- Identify the criteria for determining the static determinacy of beams and frames.
- Differentiate between statically determinate and indeterminate structures using examples and diagrams.

6.4 Concept of Statically/Kinematically Determinate and indeterminate Beams and Frames: Relevant Examples

6.4.1 Concept of Statically Determinate structure

The structure is said to be statically determinate, if all the internal member forces and external reactions forces can be evaluated by the three-equilibrium equation of statics.

- Static determinacy implies that the structure can be analyzed using the principles of static equilibrium alone, without the need for additional compatibility equations.
- Such structures have a unique solution for all internal forces and reactions under applied loads.

Criteria for Static Determinacy: Static determinacy in beams and frames is determined by comparing the number of unknown reactions to the number of equilibrium equations available.If the number of unknown reactions equals the number of equilibrium equations, the structure is statically determinate.

Examples of Statically Determinate Beams and Frames

- Simply supported beams, cantilever beams, and other basic beam configurations.
- Rigid frames with no internal redundancies, such as simple portal frames.

Analysis Methods

- Equilibrium equations, including the summation of forces and moments, are used to analyze statically determinate structures.
- For trusses, the method of joints is commonly employed, while the method of sections is used for frames.

Significance in Structural Design and Engineering Practice

- Statically determinate structures are easier to analyze and design compared to indeterminate ones.
- They often result in more efficient designs with lower material and construction costs.

6.4.2 Definition and Characteristics of Static Indeterminacy

- Static indeterminacy occurs when the number of unknown reactions exceeds the number of equilibrium equations available, leading to redundant constraints within the structure.
- Such structures require additional analysis methods beyond statics to determine all internal forces and reactions accurately.[1]

Criteria for Static Indeterminacy

Static indeterminacy in beams and frames arises from the presence of internal redundancies, such as extra supports or redundant members.

Examples of Statically Indeterminate Beams and Frames

Continuous beams, overhanging beams, frames with hinge connections, and other structures with redundant constraints.

Analysis Methods

Compatibility equations, based on kinematic relationships, are used to analyze statically indeterminate structures.

Virtual work principles provide an alternative approach to solving indeterminate systems by considering the work done by virtual displacements.

For beam

External indeterminacy (EI) = R – (3 + C)

Internal indeterminacy (II) = 0

Total degree of static indeterminacy = (EI + II)

= R – (3 + C)

Where, R = no. of supports reaction

C = no of member connected by internal hinge.

For Frame

Total degree of static indeterminacy (T.I) =(3m+r)-(3j+C) for 2D and C=0 if no internal hinge

Total degree of static indeterminacy (T.I) =(6m+r)-(6j+c) for 3D

OR Total internal indeterminacy (TII) = T.I – TEI

Total external indeterminacy (TEI) = R- 3

Where,

j = no. of joint , m = no. of member

C=Σ (m'-1) (if internal hinge is given) and m'= no of member connected by I.H

C= Σ (m'-1) for 2D structure

C=Σ3(m'-1) for 3D structure

Hence **Total degree of static indeterminacy (T.I) =(3m+r)-(3j+**Σ (m'-1) **) for 2D**

Total external indeterminacy (TEI) = R – 3

And total internal indeterminacy (TII) = (TI – TEI)

Or

Total static indeterminacy (T.I) = 3C-R-Σ(m'-1) for 2D

Total static indeterminacy (T.I) = 6C-R-Σ3(m'-1) for 3D

[here C=no. of cut required to make open structure means open tree structure from individual support)

[R=no.of reaction required to make given support to fix support)

6.4.3 kinematic determinacy and indeterminacy.

It is the total no. of movement that are allowed for the joints in a structure.

Kinematic indeterminacy

Kinematic indeterminacy refers to a condition in which a structure has more degrees of freedom (DOFs) than the minimum required to maintain its equilibrium under external loads. In simpler terms, it implies that the structure can move or deform in more ways than necessary to satisfy the equilibrium conditions.

- Kinematic indeterminacy arises when a structure has redundant or excess degrees of freedom beyond what is necessary to maintain its equilibrium.
- These excess degrees of freedom lead to multiple possible configurations or modes of deformation for the structure under load, making it indeterminate from a kinematic perspective.

For beam

Degree of kinematic indeterminacy = total no. of (displacement) movement at joints.

For frame

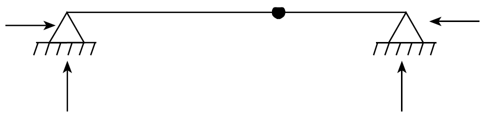
Degree of kinematic indeterminacy

(DKI) = 3j – (m + r)

6.4.4 Important Examples

Beam

Q. Calculate external indeterminacy of given beam



Solution:

Here, no. of reaction (R) = 4 (2 hinge support so 4 reaction)

No. of conditional equation(c) = (m’-1)=2-1= 1

We know, external indeterminacy = R – (3 + C)

= 4 – (3 + 1)

= 0

0 means it can be solved by using 3 equilibrium equations.

Q. Calculate external and kinematic indeterminacy of given beam

Solution

(R) = 4

(C) = 0 (no internal hinge so]

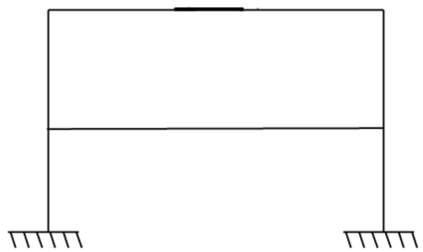
External indeterminacy = R – (3 + C)

= 4 – (3 + 0) = 1

So the given beam is indeterminate.

Kinematic indeterminacy=2

Q. Calculate external, internal and total statical indeterminacy of given frame.



Here, no. of member (m) = 6

No. of unknown reaction (R) = 6

(fixed support have 3 unknown reaction ,so 2 fixed support =6 reaction)

No. of joint (j) = 6

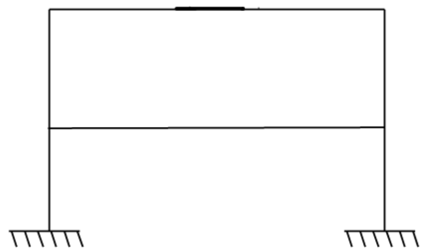
No. of condition (C) = $\Sigma(m'-1)=0$ (there is no internal hinge)

Total static indeterminacy as

(TI) = (3m + r) – (3j + C)

= (3 × 6 + 6) – (3 × 6 + 0)

= 24 – 18 = 6



Total external indeterminacy (TEI) = R – (3)

= 6 – (3)

= 3

And total internal indeterminacy (TII) = (TI – TEI)

=6-3=3

Or Total static indeterminacy (T.I) = 3C-R-Σ(m'-1)=3×2-0-0=6

[here C=no. of cut required to make open structure means open tree structure from individual support)

[R=no.of reaction required to make given support to fix support)

Total static indeterminacy (T.I) = TEI + TII

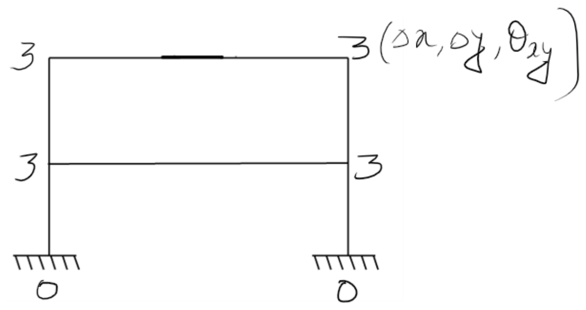
= 3 + 3 = 6

And total internal indeterminacy (TII) = (TI – TEI)

= 6 – 3=3

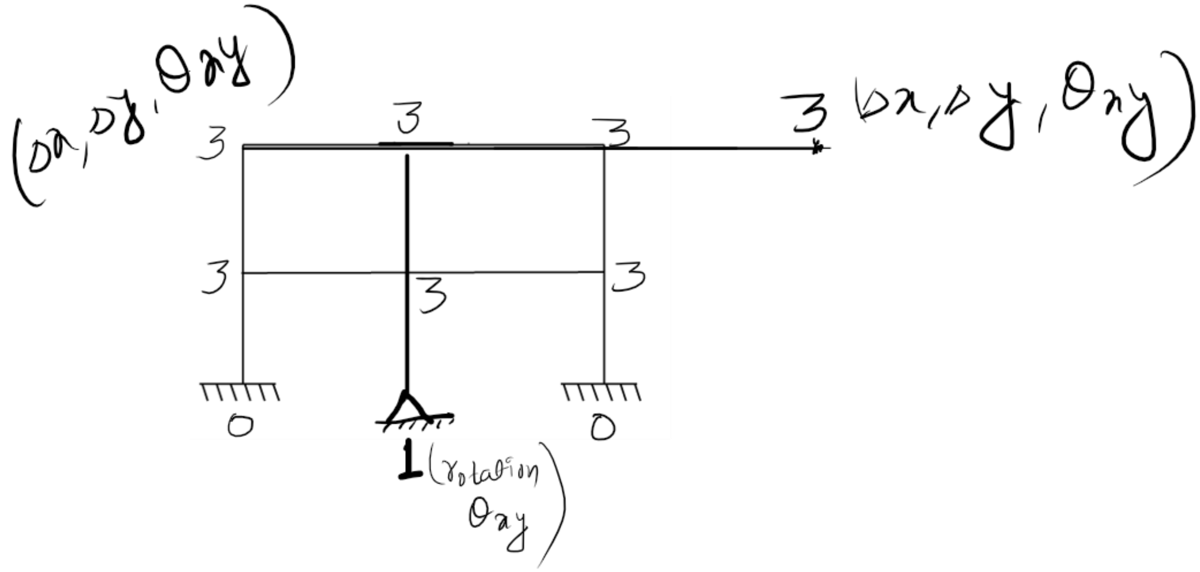
Dk=no of unknown joint displacement

=3+3+3+3+0+0=12

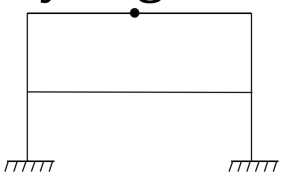


Q. Find the no of kinematic indeterminacy

D_k=7*3+1=22



Q. Calculate external, internal and total statical indeterminacy of given frame.



Solution:

Here, no. of member (m) = 6

No. of unknown reaction (R) = 6 (fixed support have 3 unknown reaction ,so 2 fixed support =6 reaction)

No. of joint (j) = 6

No. of condition (C) = 2-1=1

Total external indeterminacy (TEI) = R – (3)

= 6 – (3)= 3

you can calculate total static indeterminacy as

(TI) = (3m + r) – (3j +Σ(m'-1))

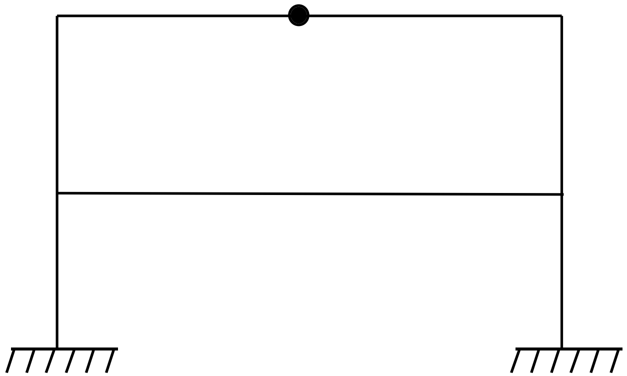
= (3 × 6 + 6) – (3 × 6 + 1)

= 24 – 19 = 5

And total internal indeterminacy (TII) = (TI – TEI)

= 5 – 3

= 2

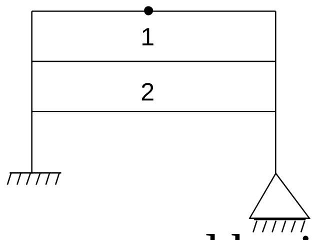


Or Total static indeterminacy (T.I) = 3C-R-Σ(m'-1)

=3×2-0-(2-1)=5

Q. Calculate external, internal and total statical indeterminacy of given frame.

Solution:



no. of member (m) = 9

no. of unknown reaction (R) = 5

no.of condition (c) =Σ(m'-1)= (2-1)=1 (member connected by internal hinge)

no. of joint (J) = 8

TEI = R – (3 + C)

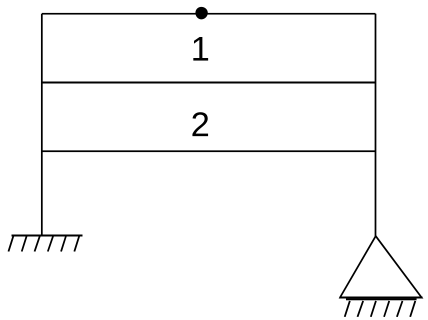
= 5 – (3 + 1)= 1

OR TI = (3m + r) – (3j + Σ(m'-1))

= (3 × 9 + 5) – (3 × 8 + 1) = 7

Or Total static indeterminacy (T.I) = 3C-R-Σ(m'-1)

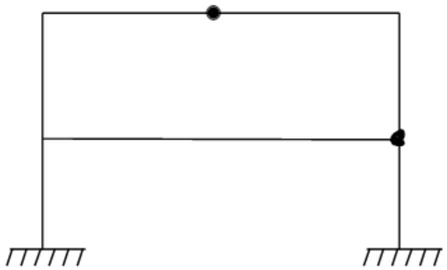
=3×3-1-(2-1)=7



T.II = TI – TEI

= 7 – 1 = 6

Q. Calculate external, internal and total statical indeterminacy of given frame.



Solution: no. of member (m) = 6

no. of unknown reaction (R) = 6

no.of condition (c) =Σ(m'-1)= (2-1)+(3-1)=3

(m'=member connected by internal hinge)

no. of joint (J) = 6

TEI = R – (3)

= 6– 3 = 3

OR TI = (3m + r) – (3j + Σ(m'-1))

= (3 × 6 + 6) – (3 × 6 + 3) = 3

Or Total static indeterminacy (T.I) = 3C-R-Σ(m'-1) [support is fix support already)

=3×2-0-3=3

T.II = TI – TEI = 3 – 3 = 0

References

[1] Neupane, P. a. (2024). *A Text book of Engineering Mechanics.* BhotahityKathmandu: Heritage Publisher and Distributors PVT .LTD.

Thank You!!!