

Applied Mechanics

Chapter 6

Analysis of Beam and Frame

Lecture 10 (week 10)

Calculation of Axial Force, Shear Force and Bending Moment for Determinate Beams and Frames

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Learning Objectives:

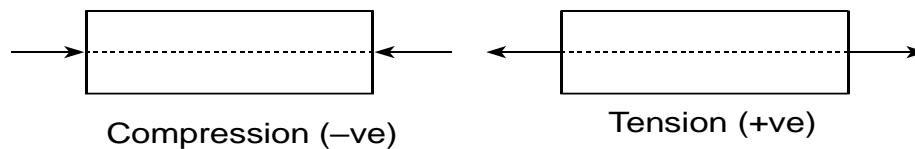
- Demonstrate proficiency in identifying and analyzing determinate beam and frame structures under various loading conditions.
- Apply appropriate mathematical methods and equations to calculate axial forces, shear forces, and bending moments at different sections along the beams and frames.
- Interpret the calculated axial forces, shear forces, and bending moments to assess structural stability and performance.
- Develop critical thinking skills by comparing and contrasting different approaches to solve determinate beam and frame problems.

6.5 Calculation of Axial Force, Shear Force and Bending Moment for Determinate Beams and Frames

6.5.1 Axial forces, shear force and bending moments:

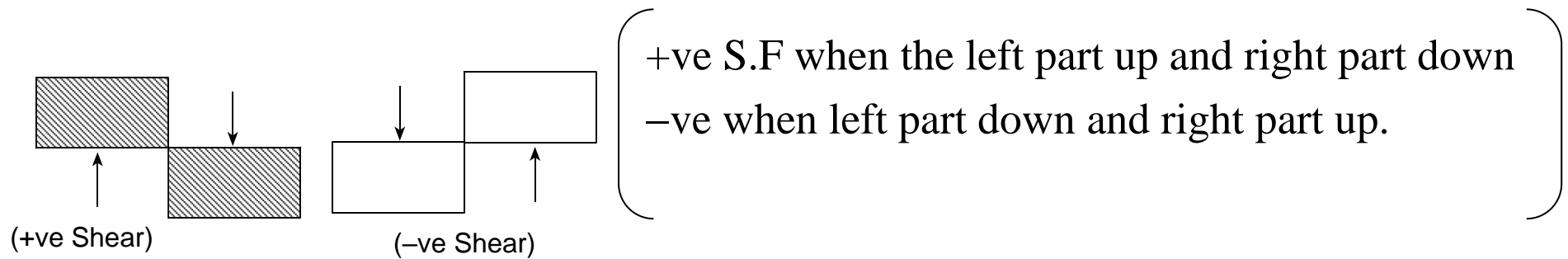
Axial force: Internal forces which are developed along the axis or the member are called axial forces.

Due to Axial force, the member tends to compressed or tensioned.



(a) Axial forces at any cross section of the member is the algebraic sum of all forces acting parallel to the longitudinal axis.

Shear force (S.F): It is the force acting tangential to the cross section of the member. Shear force at a section is defined as the algebraic sum of transverse force acting either side of the section.



***Note:** If we move from left ward to rightward all the upward forces is + ve and downward forces is -ve and vice versa.*

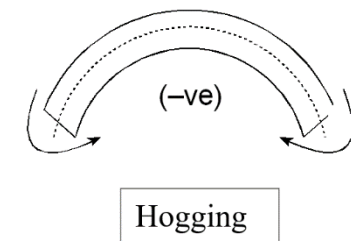
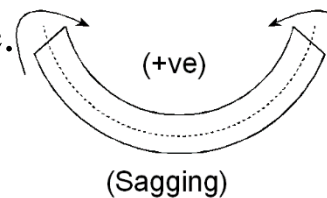
Bending Moment

At any transverse cross section, it is the algebraic sum of the moment of all force as either side of the section.

→ Sagging moment = (+ve) and Hogging moment = (-ve)

It means → From left side clockwise moment positive.

→ From right side clockwise moment negative.



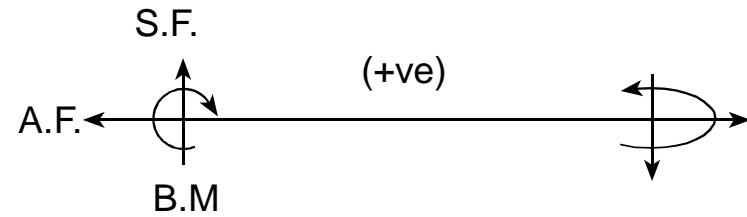
Sign convention for solving problems

While we are going from left side to right side

Tension, upwards and clockwise \rightarrow +ve

While approaching from right to left tension, downward and anticlockwise all (+ve)

AND VICE VERSA



6.5.2 Numerical Examples for A.F,S.F and B.M

Q. Calculate the axial force, shear force and bending moment of the given beam.

Solution:

Calculation of support reaction

There is no horizontal force

so, hinge support B have only one reaction.

$$\curvearrowleft \Sigma M_B = 0$$

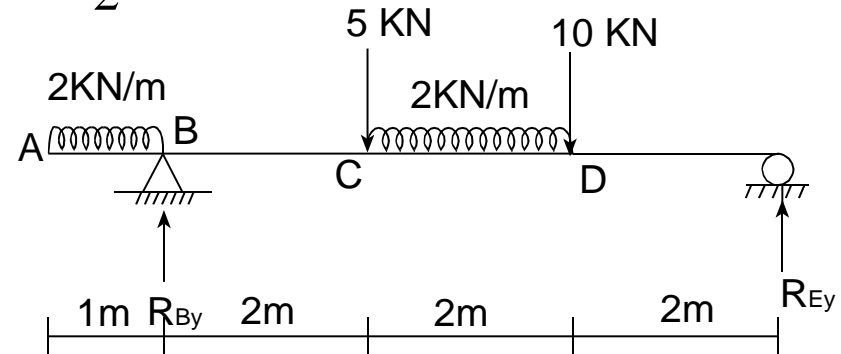
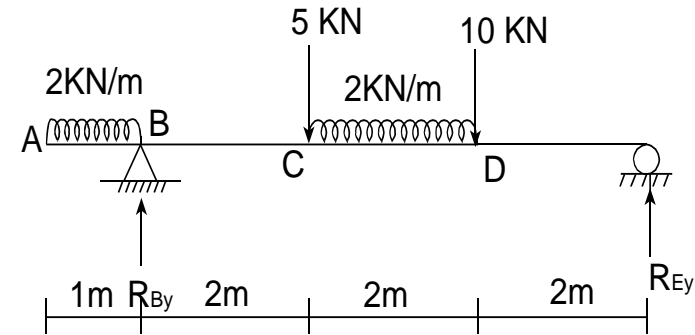
$$R_{EY} \times 6 - 10 \times 4 - 2 \times 2 \times 3 - 5 \times 2 + 2 \times 1 \times \frac{1}{2} = 0 \quad [R_{EY} = 10.16 \text{ KN } (\uparrow)]$$

$$\Sigma F_Y = 0 (\uparrow +)$$

$$R_{BY} + R_{EY} - 5 - 10 - 2 \times 1 - 2 \times 2 = 0$$

$$R_{BY} + 10.16 - 5 - 10 - 2 - 4 = 0$$

$$[R_{BY} = 10.83 \text{ } (\uparrow) \text{ KN}]$$



Shear Force Calculation

$$\text{S.F at A} = 0$$

$$\text{S.F at B}_L = -2 \times 1 = -2 \text{ KN}$$

$$\text{S.F at B}_R = -2 + R_{BY} = 8.83 \text{ KN}$$

$$\text{S.F at C}_L = 8.83 \text{ KN}$$

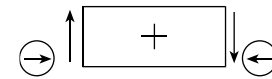
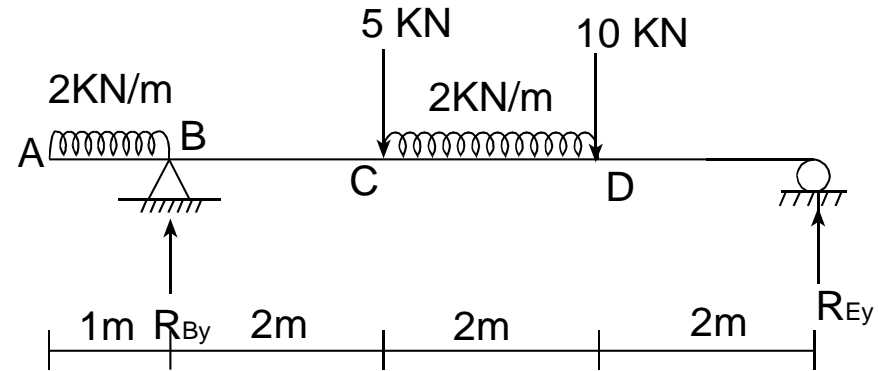
$$\text{S.F at C}_R = 8.83 - 5 = 3.83 \text{ KN}$$

$$\text{S.F at D}_L = 3.83 - 2 \times 2 = -0.17 \text{ KN}$$

$$\text{S.F at D}_R = -0.17 - 10 = -10.17 \text{ KN}$$

$$\text{S.F at E}_L = -10.17 \text{ KN.}$$

$$\text{S.F at E}_R = -10.17 + R_{EY} = 0$$



From left to right upward positive, downward negative

Bending moment Calculation

$$M_A = 0$$

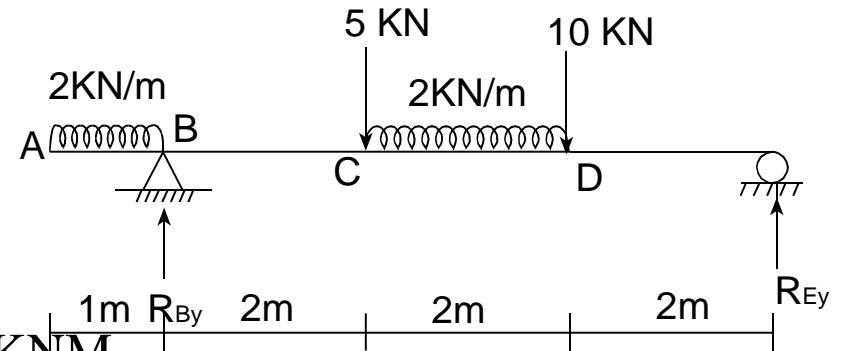
$$M_B = -2 \times 1 \times 1/2 = -1 \text{ KNM}$$

$$M_C = -2 \times 1 \times (2 + \frac{1}{2}) + 10.83 \times 2 = 16.66 \text{ KNM}$$

$$M_D = -2 \times 1 \times (4 + \frac{1}{2}) + 10.83 \times 4 - 5 \times 2 - 2 \times 2 \times \frac{2}{2} = 20.32 \text{ KNM}$$

$$M_E = 0$$

$$M_{AB \text{ (mid)}} = -2 \times 0.5 \times \frac{0.5}{2} = -0.25 \text{ KNM}$$



1. Calculate axial force, shear force and bending moment for the given figure below.

Axial force calculation

AF for member CD = 5 KN (C)

Shear Force Calculation

S.F at $C_L = 0$

S.F at $C_R = 36\text{KN}$

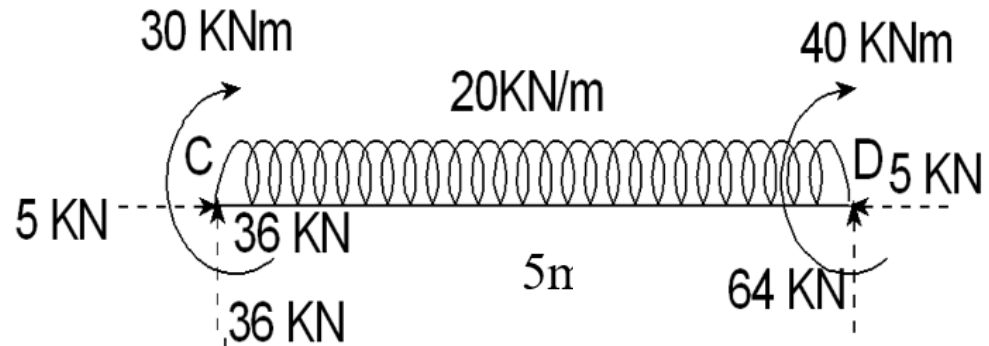
S.F at $\text{mid} = 36 - (20 \times 2.5) = - 14 \text{ KN}$

S.F at $D_L = (36 - 20 \times 5) = - 64 \text{ KN}$

S. F at $D_R = - 64 + 64 = 0 \text{ KN}$

S.F at x section = $36 - 20x$

S.F = 0 at 1.8m from the left end. (by equating S.F equation to 0)



Bending moment calculation

For member CD

$$\text{B.m at } C_L = 0$$

$$\text{B.m } C_R = 30\text{KNm}$$

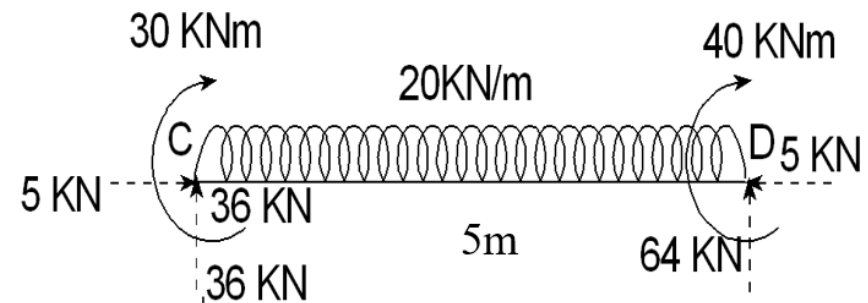
$$\text{B.m } D_L = 30 + 36 \times 5 - 20 \times 5 \times 2.5 = -40$$

$$\text{B.m } D_R = -40 + 40 = 0$$

$$M_{\text{mid}} = 57.5\text{KNm}$$

$$\text{B.mat (x = 1.8)} = 30 + 36 \times 1.8 - 20 \times 1.8 \times \frac{1.8}{2} = 62.4\text{KNm}$$

62.4 kNm is the maximum bending moment at the given section.



Q. Find the reactions at supports and calculate the shear force and bending moment at different section.

Solution: Calculation of support reaction

$$[R_{AX} = 0]$$

(No horizontal force so horizontal reaction is zero)

$$+\uparrow \sum F_y = 0$$

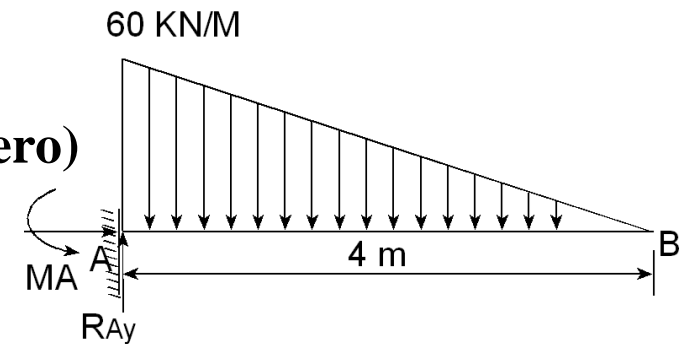
$$R_{AY} - \frac{1}{2} \times 60 \times 4 = 0$$

$$R_{AY} = 120 \text{ KN}(\uparrow)$$

$$\ominus \sum M_A = 0$$

$$-M_A + \frac{1}{2} \times 4 \times 60 \times \frac{4}{3} = 0$$

$$[M_A = 160 \text{ KNM}]$$



Shear force calculation

$$\text{S.F at } A_L = 0$$

$$\text{S.F at } A_R = 120 \text{ KN}$$

$$\text{S.F at } B_R = 120 - \frac{1}{2} \times 4 \times 60 = 0$$

$$\text{S.F at mid} = \frac{1}{2} \times 2 \times 30 = 30 \text{ KN.}$$

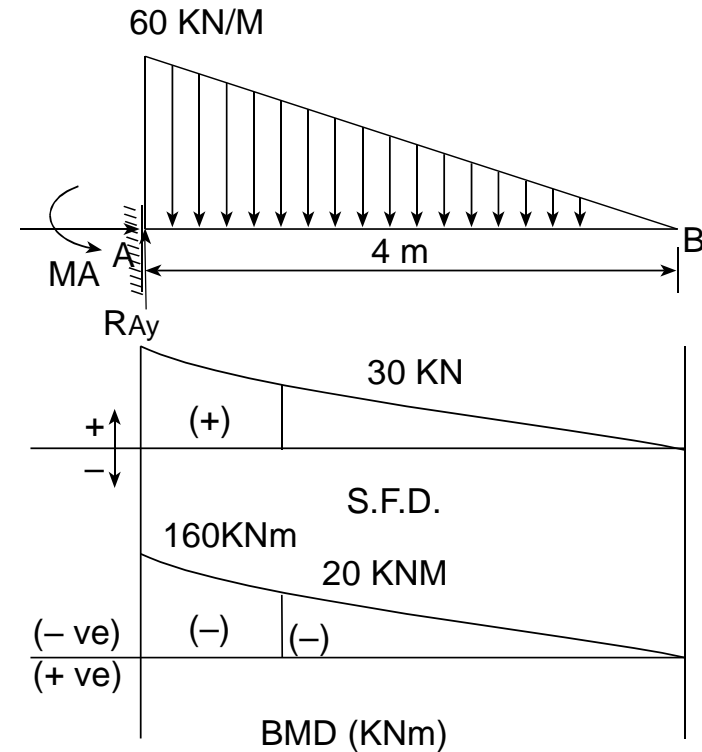
Bending moment calculation

$$M_{A_L} = 0$$

$$M_{A_R} = -160 \text{ KMN.}$$

$$M_B = -160 + 120 \times 4 - \frac{1}{2} \times 4 \times 60 \times \frac{2}{3} \times 4 = 0$$

$$M_{AB_{\text{mid}}} = -\frac{1}{2} \times 2 \times 30 \times \frac{2}{3} = -20 \text{ KNM}$$



1. Calculate axial force, shear force and bending moment for the frame shown in figure below.

Solution: For calculation of reaction forces we can use 3 equations of equilibrium so,

$$\odot \Sigma M_D = 0$$

Note: Moment can also be taken at support A but try to take moment where maximum unknown forces are acting.

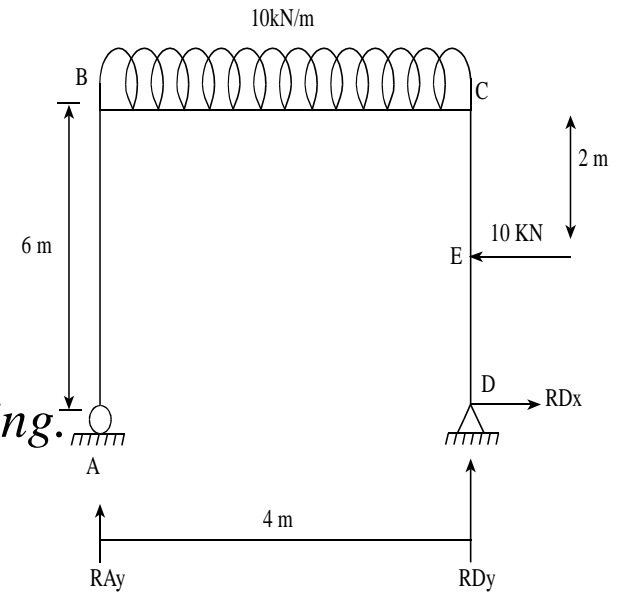
$$R_{AY} \times 4 - 10 \times 4 \left(\frac{4}{2} \right) - 10 \times 4 = 0$$

$[R_{AY} = 30 \text{ KN } (\uparrow)]$ [Assume direction of R_{AY} is ok due to +ve result]

$$(+\uparrow) \Sigma F_Y = 0$$

$$R_{AY} + R_{DY} - 10 \times 4 = 0$$

$$[R_{DY} = 10 \text{ KN } (\uparrow)]$$



(\rightarrow)

$$\sum F_X = 0, R_{DX} - 10 = 0$$

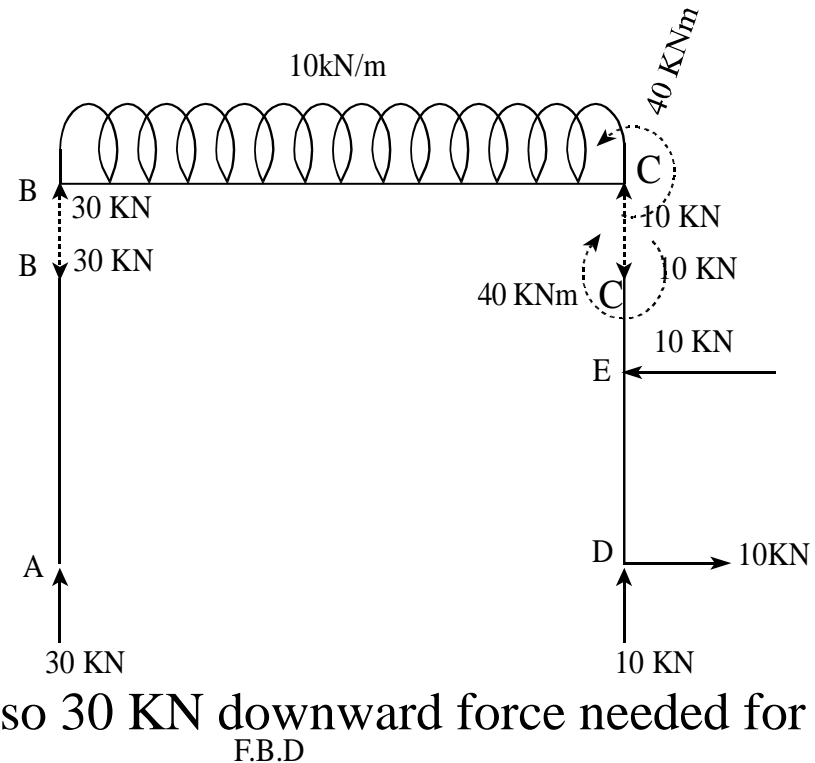
$$[R_{DX} = 10 \text{ KN } (\rightarrow)]$$

Note: The individual member should be balance in three way longitudinally, transversely (i.e shear) and rotation (moment at joint)

For balancing member:

30 KN load acted at upward direction so 30 KN downward force needed for balance.

Here the added 30 KN force should be in equilibrium so; at the same joint of another member upwards 30 KN force is applied. Member AB is balance in axial force and added force is also balance.



Here is no need of horizontal (transverse) force because no any forces acted in this member horizontally.

Moment is also balance no rotation produced at the joint. i.e. $\Sigma m = 0$

And for member BC, no need of Axial force (i.e. no horizontal force) and for transverse (shears balance) balance, we should adding upward 10 KN force at the joint so, we achieve vertical force balance and at the same joint of another member 10 KN. downward force is required for balancing added force and member CD is vertically balanced.

There is also no need for horizontal forces for member CD.

Note: Our target is to make members

$$\Sigma F_x = 0)$$

$$\Sigma F_y = 0$$

$$\Sigma M_{joint} = 0 \text{ after balancing}$$

For balancing moment

First of all check for rotation that is in which direction the member tends to rotate

For member BC

$$\text{Moment at C} = 30 \times 4 - 10 \times 4 \times \left(\frac{4}{2}\right)$$

$$= 40 \text{ KNM (+ve) so clockwise.}$$

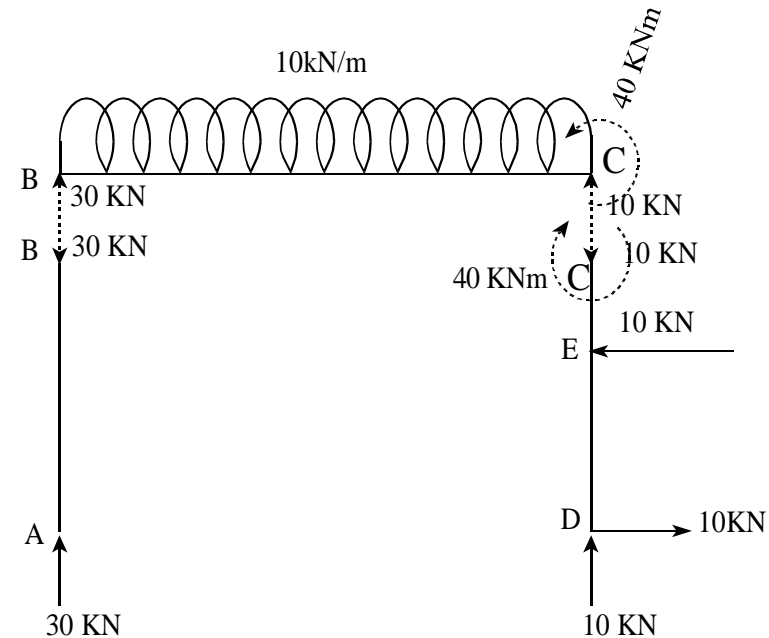
It means the member tends to rotate in clockwise direction about point C.

So, for balancing of rotation we should apply 40 KNM moment in anticlockwise direction.

But at the same point of another member 40 KNM

F.B.D

in clockwise direction should applied for balancing these anticlockwise 40 KNm and after this last member CD should be balanced itself. [i.e. check $M_D = 40 - 10 \times 4 = 0$]



By this way all the members are balanced in Axial, shear and moment

For calculation of Axial force

A.F for member AB = - 30 KN or, 30 KN (c)

A.F for member BC = 0 (i.e. no axial force through members) 10kN/m

A.F for member CD = - 10 KN or, 10 KN (c)

For calculation of S.F

S.F for member AB.

Vat $A_{L,R} = 0$ and Vat $B_{L,R} = 0$ [v = shear force)

For member B.C

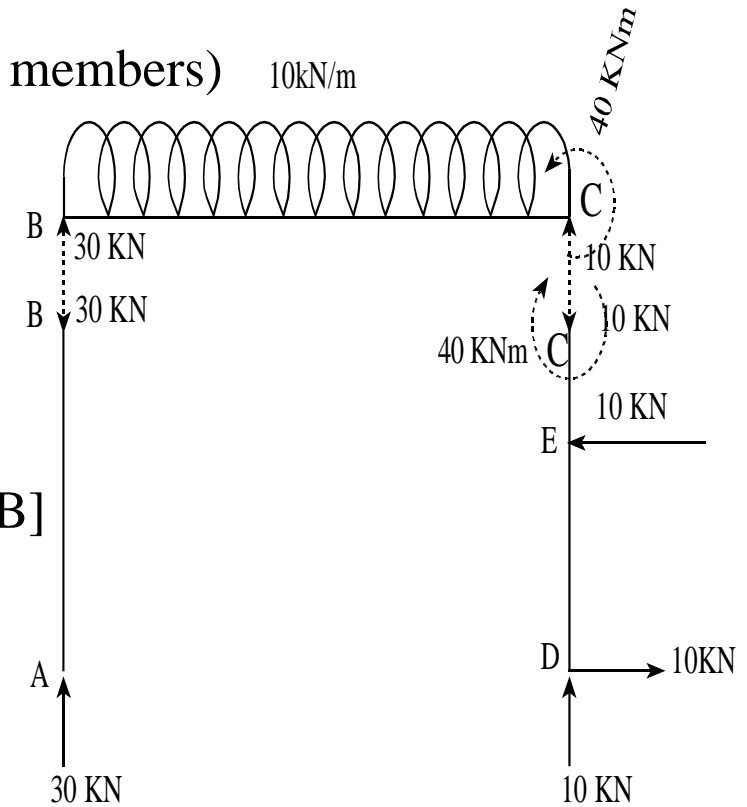
V at $B_L = 0$ KN (B_L means S.F. at just left of point B)

Vat $B_R = 30$ KN

Vat $C_L = 30$ KN - $10 \times 4 = - 10$ KN

Vat $C_R = -10$ KN + 10 KN = 0 KN

Vat mid = $30 - 10 \times 2 = 10$ KN



F.B.D

For member CD

$V_{at C_L} = 0$

$V_{at C_R} = 0$

$V_{at E_L} = 0$

$V_{at E_R} = -10 \text{ KN}$

$V_{at D_L} = -10 \text{ KN}$

$V_{at D_R} = -10 + 10 = 0 \text{ KN}$

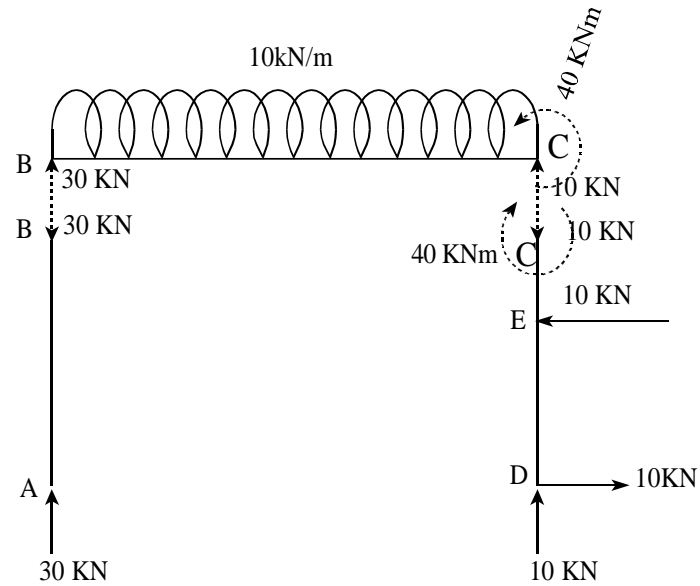
Bending moment calculation

For member AB

B Mat A = 0

B Mat B_L = 0

B Mat B_R = 0



F.B.D

For member BC

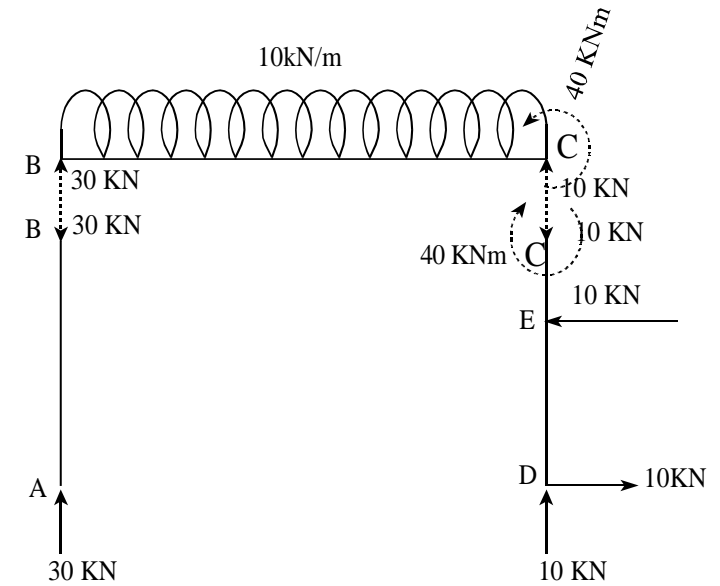
$B M_{at B_L} = 0$, $B M_{at B_R} = 0$

$B M_{at C_L} = 30 \times 4 - 10 \times 4 \left(\frac{4}{2}\right) = 40 \text{ KNM}$

$B M_{at C_R} = 40 - 40 = 0 \text{ KNM}$

$B M_{at mid} = 30 \times 2 - 10 \times 2 \times \left(\frac{2}{2}\right) = 40 \text{ KNM}$

$B.M \text{ max (at 3m from B)} = 30 \times 3 - 10 \times 3 \times \left(\frac{3}{2}\right) = 45 \text{ KNM.}$



F.B.D

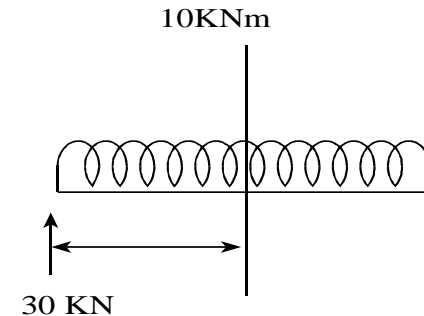
B.M can also be calculated by equations so, $B.M \text{ at } X = 30x - 10 \times x \times \frac{x}{2} = 30x - 5x^2$

$B.M \text{ at } B(x = 0) = 0$

$B.M \text{ at mid } (x = 2m) = 30 \times 2 - 5 \times 2^2 = 40 \text{ KNm}$

$B.M \text{ at } (x = 4) = 30 \times 4 - 5 \times 4^2 = 40 \text{ KNm}$

$B. M \text{ at } C_R = 40 - 40 = 0 \text{ KNm}$



For member CD

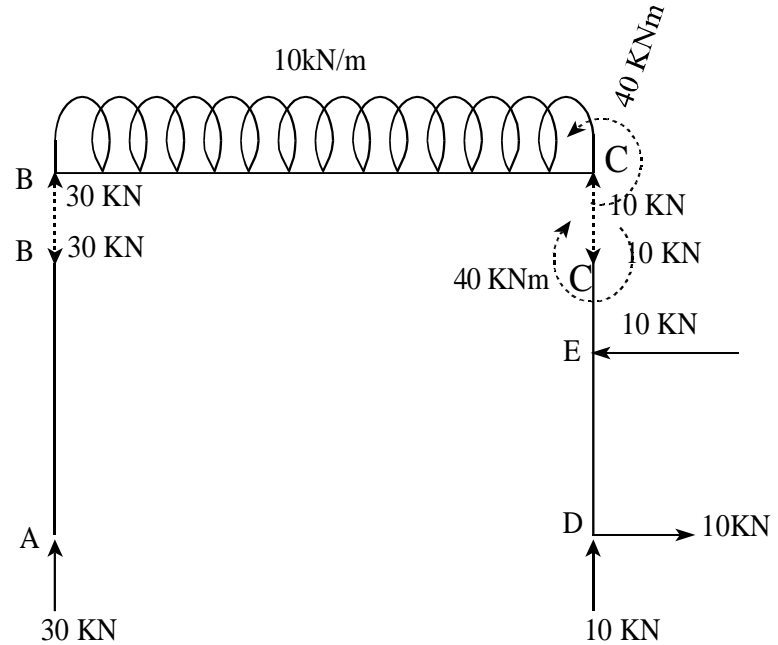
B. M at $C_L = 0$

B. M at $C_R = 40 \text{ KNm}$

B. M at E = 40 KNm

B. M at $D_L = 40 - 10 \times 4 = 0 \text{ KNm}$

B. M at $D_R = 0 \text{ KNm}$



F.B.D

Thank You!!!