Applied Mechanics

Chapter 6

Analysis of Beam and Frame

Lecture 11 (week 11)

Axial Force, Shear Force and Bending Moment Diagrams and Examples for drawing it.

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Learning Objectives:

- Identify and differentiate between various types of loads and their effects on structural elements.
- Construct shear force diagrams (SFD) and bending moment diagrams (BMD) for simply supported beams, cantilevers, and overhanging beams subjected to various loading conditions.
- Interpret and use S.F and B.M diagrams to understand the internal forces within a structure.
- Relate Diagrams to Structural Behavior.

6.6.1 Axial Force, Shear force and Bending Moment Diagrams

The graphical representation of the axial force, shear force and Bending moment at every section of a structural member is called A.F.D, S.F.D and B.M.D respectively.

REMEMBER

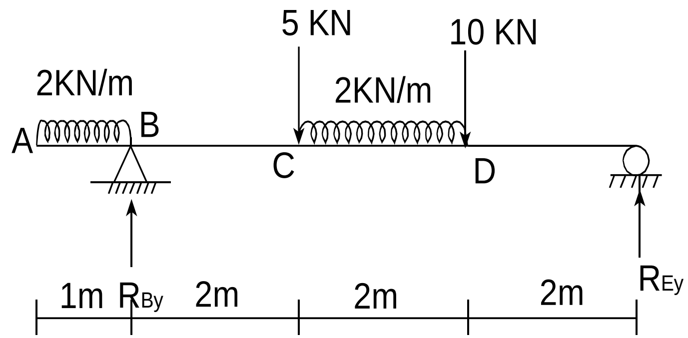
Load	**S.F. D**	**B.M. D**
Between Point load	Horizontal straight line	Inclined straight line
Uniformly Distributed load	Inclined line	Parabolic curve (squares)
Uniformly varying load	Parabolic(square)	Cubic parabolic

Note: *Points to be remember while solving beam frame problems*

1. Maximum B.M occurs at point of zero shear force.
2. B.M is zero at the simple support.
3. The point of Zero B.M is known as point of contraflexure or inflexion point or point of virtual hinge.
4. Under UDL, S.F.D varies linearly but B.M.D varies parabolically.
5. B.M is always zero at the point of internal hinge.
6. Abrupt change in loading causes change in slope of S.F.D.
7. Vertical line is developed by the concentrated load in S.F.D.
8. Take care for zero S.F point, after drawing S.F.D, B.M.D can be drawn by calculated value.
9. If value of S.F. is greater in S.F.D. then the slope of BM curve in B M D is greater and vice versa.[1]

6.6.2 Examples for A.F.D, S.F.D and B.M.D

Q. Draw the axial force, shear force and bending moment diagrams of the given beam.



Solution:

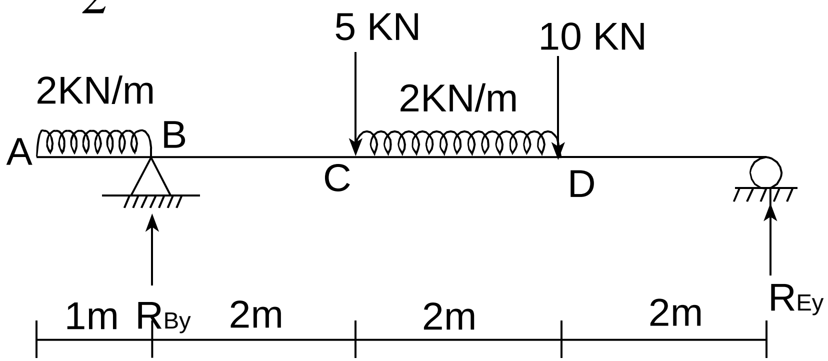
Calculation of support reaction

There is no horizontal force

so, hinge support B have only one reaction.

$\circlearrowleft + \Sigma M_B = 0$

$R_{EY} \times 6 - 10 \times 4 - 2\times2 \times 3 - 5 \times 2 + 2 \times 1 \times \frac{1}{2} = 0$ $[R_{EY} = 10.16 \text{ KN} (\uparrow)]$



$\sum F_Y = 0\ (\uparrow +)$

$R_{BY} + R_{EY} - 5 - 10 - 2 \times 1 - 2\times2 = 0$

$R_{BY} + 10.16 - 5 - 10 - 2 - 4 = 0$

$[R_{BY} = 10.83\ (\uparrow) \text{ KN}]$

Shear Force Calculation

S.F at A = 0

S.F at $B_L = -2 \times 1 = -2$ KN

S.F at $B_R = -2 + R_{BY} = 8.83$ KN

S.F at $C_L = 8.83$ KN

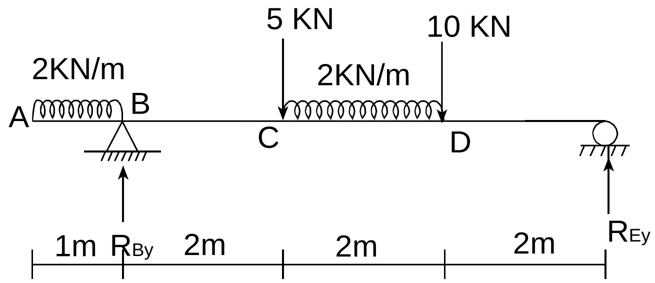
S.F at $C_R = 8.83 - 5 = 3.83$ KN

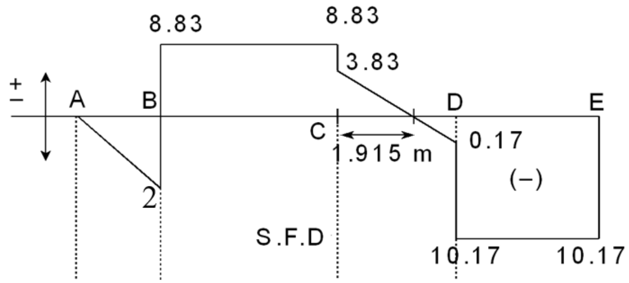
S.F at $D_L = 3.83 - 2 \times 2 = -0.17$ KN

S.F at $D_R = -0.17 - 10 = -10.17$ KN

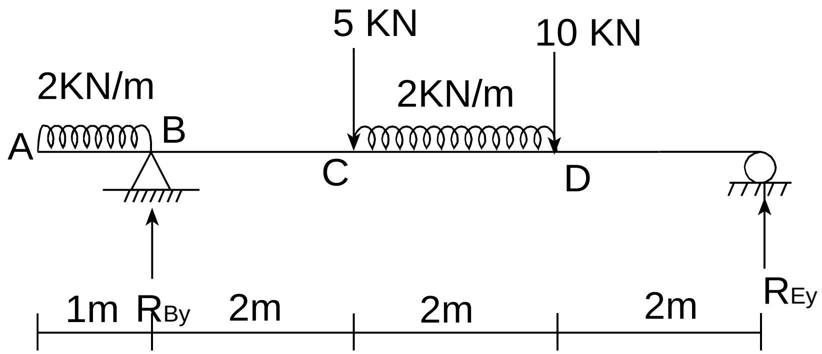
S.F at $E_L = -10.17$ KN.

S.F at $E_R = -10.17 + R_{EY} = 0$





Bending moment Calculation



$M_A = 0$,

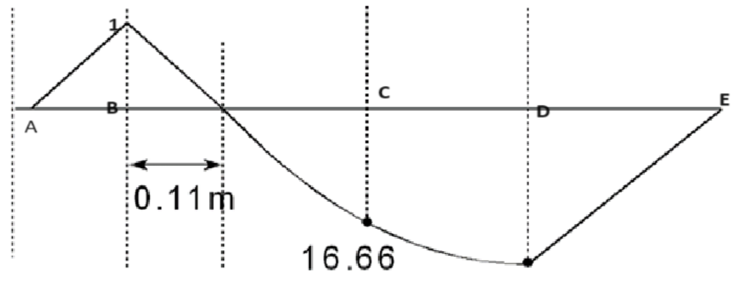
$$M_{AB\,(mid)} = -2 \times 0.5 \times \frac{0.5}{2} = -0.25 \text{ KNM}$$

$$M_B = -2 \times 1 \times 1/2 = -1 \text{ KNM}$$

$$M_C = -2 \times 1 \times (2 + \frac{1}{2}) + 10.83 \times 2 = 16.66 \text{ KNM}$$

$$M_D = -2 \times 1 \times (4 + \frac{1}{2}) + 10.83 \times 4 - 5 \times 2 - 2 \times 2 \times \frac{2}{2} = 20.32 \text{ KNM}$$

$M_E = 0$



B.M.D. (KN m)

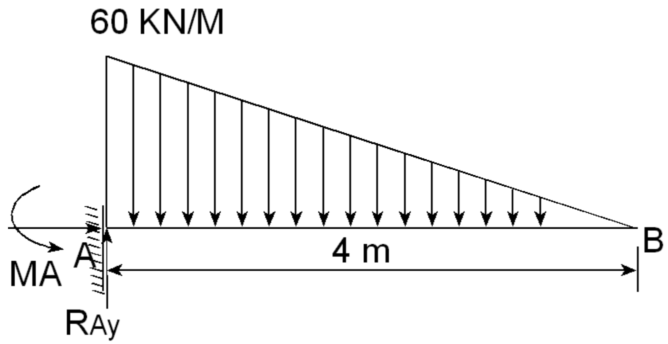
Q. Find the reactions at supports and draw the shear force and bending moment diagrams of the given beam.

Solution:Calculation of support reaction

$[R_{AX} = 0]$

(No horizontal force so horizontal reaction is zero)

$+\uparrow \sum F_y = 0$



$$R_{AY} - \frac{1}{2} \times 60 \times 4 = 0$$

$$R_{AY} = 120 \text{ KN}(\uparrow)$$

$$\circlearrowright \Sigma M_A = 0$$

$$-M_A + \frac{1}{2} \times 4 \times 60 \times \frac{4}{3} = 0$$

$$[M_A = 160 \text{ KNM}]$$

Shear force calculation

S.F at $A_L = 0$

S.F at $A_R = 120$ KN

S.F at mid $= \frac{1}{2} \times 2 \times 30 = 30$ KN.

S.F at $B_L = 120 - \frac{1}{2} \times 4 \times 60 = 0$

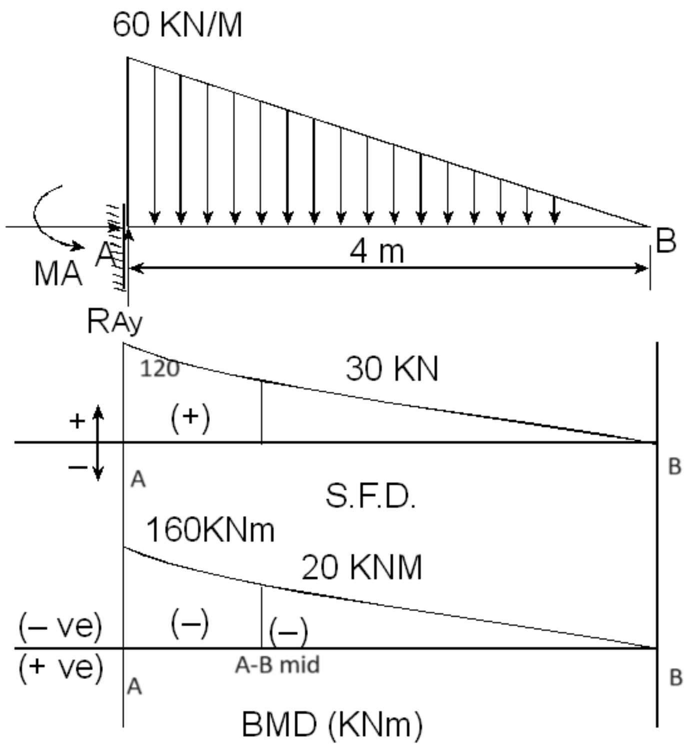
Bending moment calculation

$\mathbf{MA_L = 0}$

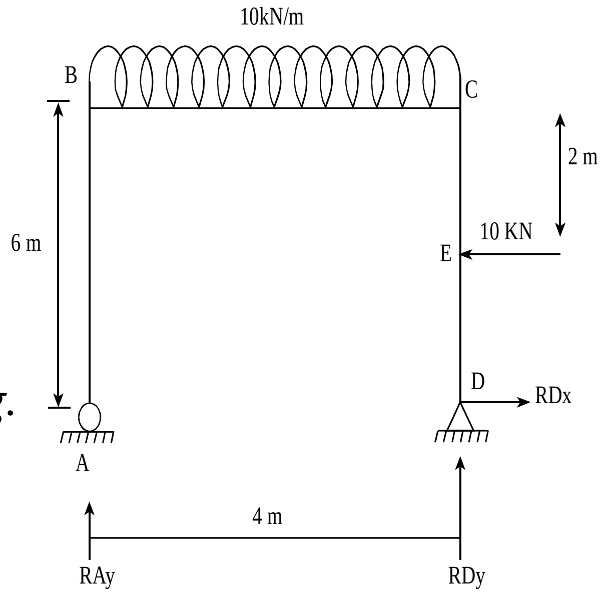
$\mathbf{MAB_{mid} = -\frac{1}{2} \times 2 \times 30 \times \frac{2}{3} = -20}$ **KNM**

$\mathbf{MA_R = -160}$ **KMN.**

$\mathbf{MB = -160 + 120 \times 4 - \frac{1}{2} \times 4 \times 60 \times \frac{2}{3} \times 4 = 0}$



Q.Draw axial force, shear force and bending moment diagrams for the frame shown in figure below.



Solution: For calculation of reaction forces we can use 3 equations of equilibrium so,

$\circlearrowright^{+}$ $\mathbf{\Sigma M_D = 0}$

Note: *Moment can also be taken at support A but try to take moment where maximum unknown forces are acting.*

$$R_{Ay} \times 4 - 10 \times 4 \left(\frac{4}{2}\right) - 10 \times 4 = 0$$

$[R_{AY} = 30 \text{ KN } (\uparrow)]$ [Assume direction of R_{Ay} is ok due to +ve result]

$\mathbf{(+\uparrow)\ \Sigma FY = 0}$

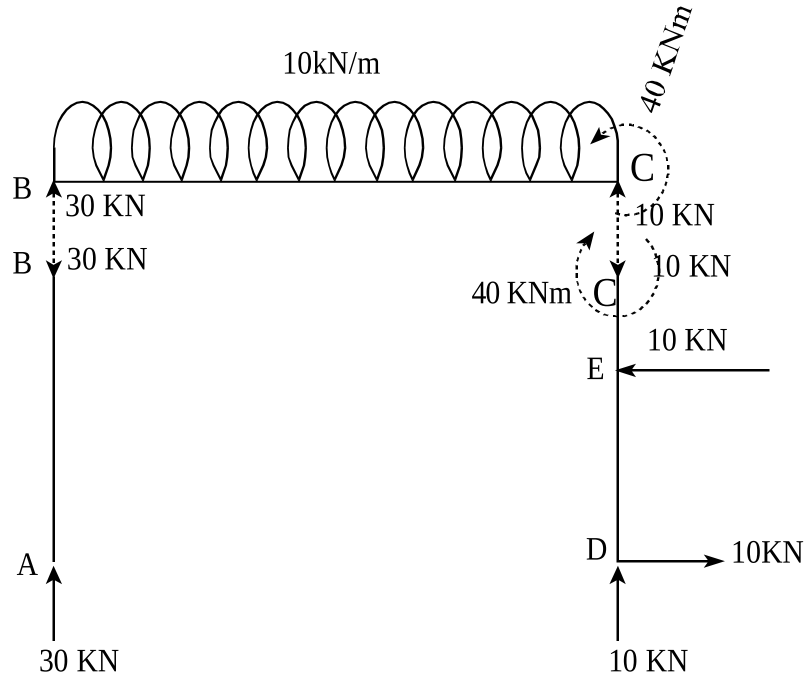
$$R_{AY} + R_{DY} - 10 \times 4 = 0$$

$[R_{DY} = 10 \text{ KN } (\uparrow)]$

$(\overset{+}{\rightarrow})$

$\sum FX = 0$, $R_{DX} - 10 = 0$

$[R_{DX} = 10 \text{ KN} (\rightarrow)]$



F.B.D

Note: *The individual member should be balance in three way longitudinally, transversely (i.e shear) and rotation (moment at joint)*

For balancing member:

30 KN load acted at upward direction so 30 KN downward force needed for balance.

Here the added 30 KN force should be in equilibrium so; at the same joint of another member upwards 30 KN force is applied. Member AB is balance in axial force and added force is also balance.

Here is no need of horizontal (transverse) force because no any forces acted in this member horizontally.

Moment is also balancing no rotation produced at the joint. i.e. $\Sigma m = 0$

And for member BC, no need of Axial force (i.e. no horizontal force) and for transverse (shears balance) balance, we should add upward 10 KN force at the joint so, we achieve vertical force balance and at the same joint of another member 10 KN downward force is required for balancing so added force and member CD is vertically balanced.

There is also no need for horizontal forces for member CD.

Note: Our target is to make members

$\Sigma Fx = 0$)

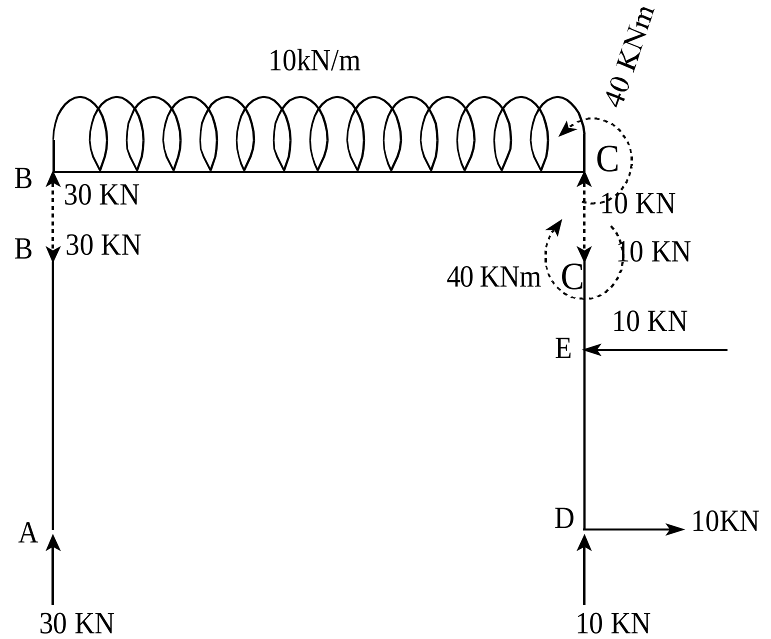
$\Sigma Fy = 0$

$\Sigma M_{joint} = 0$ after balancing

For balancing moment

First of all, check for rotation that is in which direction the member tends to rotate

For member BC



F.B.D

Moment at C $= 30 \times 4 - 10 \times 4 \times \left(\frac{4}{2}\right)$

$= 40$ KNM (+ve) so clockwise.

It means the member tends to rotate in clockwise direction about point C.

So, for balancing of rotation we should apply 40 KNM moment in anticlockwise direction.

But at the same joint of another member, 40 KNM in clockwise direction should applied for balancing these anticlockwise 40 KNm moment and after that last member CD should be balanced itself. [i.e. check $M_D = 40 - 10 \times 4 = 0$]

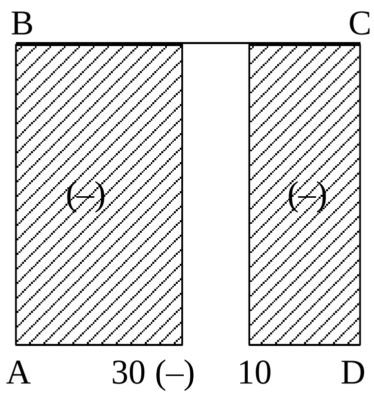
By this way all the members are balanced in Axial, shear and moment

For calculation of Axial force

A.F for member AB = – 30 KN or, 30 KN (c)

A.F for member BC = 0 (i.e. no axial force through members)

A.F for member CD = – 10 KN or, 10 KN (c)



A.F.D.

For calculation of S.F

S.F for member AB.

Vat $A_{L,R}$ = 0 and Vat $B_{L,R}$ = 0 [v = shear force)

For member B.C

V at B_L = 0 KN (B_L means S.F. at just left of point B]

Vat B_R = 30 KN

Vat C_L = 30 KN – 10 × 4 = – 10 KN

Vat C_R = –10 KN + 10 KN = 0 KN

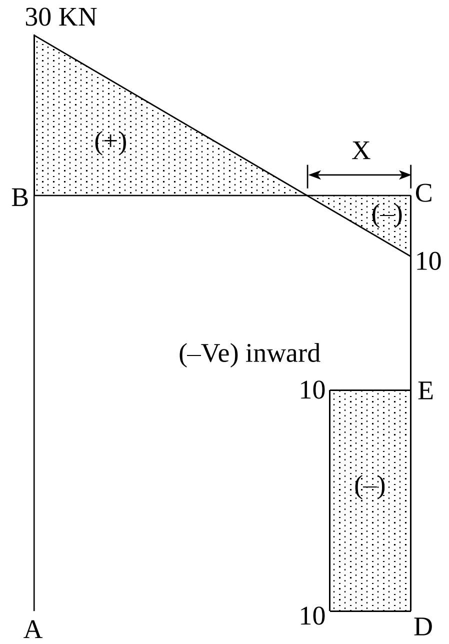
Vat mid = 30 – 10 × 2 = 10 KN

For member CD

Vat C_L = 0, Vat C_R = 0

Vat E_L = 0, Vat E_R = – 10 KN

Vat D_L = – 10 KN, Vat D_R = –10 + 10 = 0 KN



Bending moment calculation

For member AB

B Mat A = 0

B Mat B_L= 0

B Mat B_R = 0

For member BC

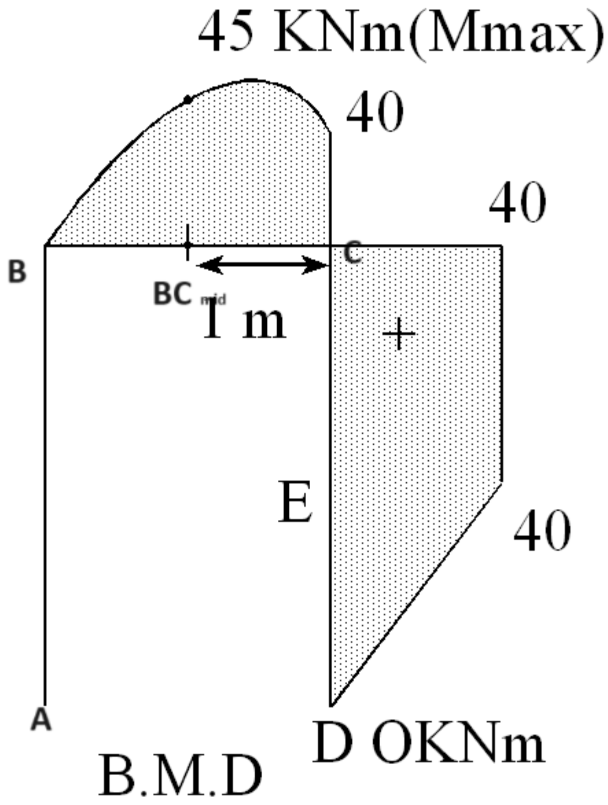
B M_{at} B_L = 0 , B M_{at} B_R = 0

B M at $C_L = 30 \times 4 - 10 \times 4 \left(\frac{4}{2}\right) = 40$ KNM

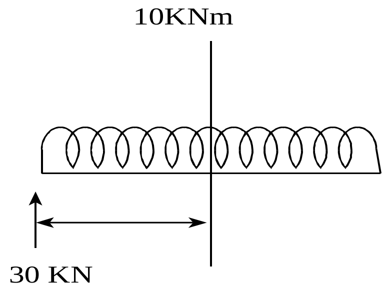
B M at $C_R = 40 - 40 = 0$ KMN

B M at mid $= 30 \times 2 - 10 \times 2 \times \left(\frac{2}{2}\right) = 40$ KNM

B.M max (at 3m form B) $= 30 \times 3 - 10 \times 3 \times \left(\frac{3}{2}\right) = 45$ KNM.



B.M can also be calculated by equations so, B.M at X = $30\,x - 10 \times x \times \frac{x}{2} = 30x - 5x^2$



B.M at B(x = 0) = 0

B.M at mid (x = 2m) = $30 \times 2 - 5 \times 2^2 = 40$ KNm

B.M at (x = 4) = $30 \times 4 - 5 \times 4^2 = 40$ KNm

B. M at $C_R = 40 - 40 = 0$ KNm

For member CD

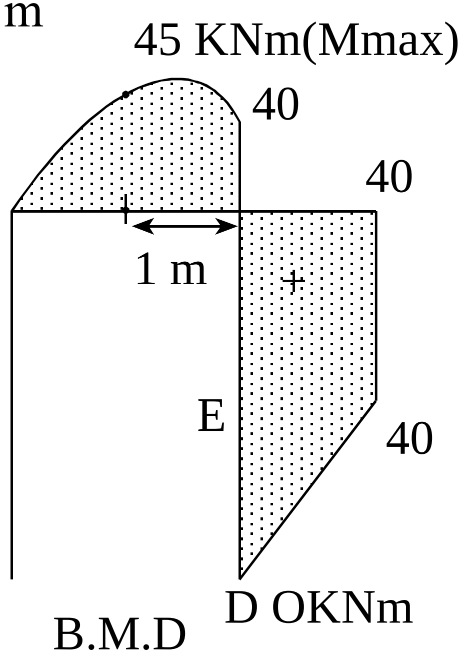
B. M at $C_L = 0$

B. M at $C_R = 40$ KNm

B M at E = 40 KNm

B M at $D_L = 40 - 10 \times 4 = 0$ KNm

B M at $D_R = 0$ KNm



References

[1] Kumar, D. (2019). *Engineering Mechanics.* New delhi: S.K Kataria and Sons.

Thank You!!!