Applied Mechanics

Chapter 8

Kinematics of Particles and Rigid Body

Lecture 14 (week 14)

Kinematics of Particles and Rigid Body, Rectilinear Kinematics, Position, Velocity and Acceleration of a Particle and Rigid Body, Determination of Motion of Particle and Rigid Body

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Learning Objectives:

- Understand kinematics concepts.
- Describe rectilinear motion.
- Analyze position, velocity, and acceleration.
- Determine motion using kinematic principles.

8. Kinematics of Particles and Rigid Body

Dynamics: The study of bodies/ particles under motion is known as dynamics.

Kinematics: The study of motion of bodies /particles without considering the force causing the motion is known as kinematics.

Kinematics of Particles

Particle:

An idealized object with mass but no size or shape. It can be considered as a point mass.

Used to simplify the analysis of motion.

Position:

The location of a particle in space, usually described by coordinates (x, y, z) in a given reference frame.

Displacement:

The change in position of a particle.

A vector quantity with both magnitude and direction.

Velocity:

The rate of change of position with respect to time.

Acceleration:

The rate of change of velocity with respect to time.

Equations of Motion for Particles (For constant acceleration)

$v=u+at$

$s=ut+0.5at^2$

$v^2=u^2+2as$ Where u is the initial velocity, v is the final velocity, a is the acceleration, s is the displacement, and t is the time.

Kinematics of Rigid Bodies

Rigid Body:

An object with a definite shape and size that does not deform under motion.

All points in a rigid body maintain the same relative positions.

Rotational Motion:

The motion of a rigid body around a fixed axis.

Described using angular quantities.

Angular Position (θ):

The orientation of a line with another line or plane.

Measured in radians or degrees.

Angular Displacement (Δθ):

The change in angular position. $\Delta\theta = \theta final - \theta initial$.

Angular Velocity (ω):

The rate of change of angular position with respect to time.

Angular Acceleration (α):

The rate of change of angular velocity with respect to time.

Equations of Motion for Rigid Bodies

For constant angular acceleration:

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + 0.5\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

Where ω_0 is the initial angular velocity, ω is the final angular velocity, α is the angular acceleration, θ is the angular displacement, and t is the time.

8.1 Rectilinear Kinematics

Rectilinear motion: If the path followed by the particle is straight line, then the resulting motion is known as rectilinear motion. Also called one dimensional motion

e.g A car travelling a straight road

For rectilinear motion the change of position of particle at any time may be x the velocity is given by

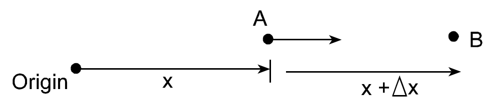
$$v = \frac{dx}{dt} \text{ and acceleration (a)} = \frac{dv}{dt}$$

8.2 Position, Velocity and Acceleration of a Particle and Rigid Body

Consider a particle which is moving along a straight line as shown in figure.

At any time, t, it is at the position A and position vector be

$\vec{r} = (x - 0)\,\vec{i} = x\,\vec{i}$ ....(i)



At (time $(t + \Delta t)$ it will be at position B so position vector $(\vec{r}) = (x + \Delta x)\,\vec{i}$......(2)

Then average velocity $(\vec{v}_{av}) = \dfrac{\text{change in position vector}}{\text{time interval}}$

$$= \frac{(x + \Delta x) - x}{(t + \Delta t) - t} = \frac{\Delta x}{\Delta t}$$

$$\therefore \vec{v}_{av} = \frac{\Delta x}{\Delta t}$$

As $\Delta t \rightarrow 0$, $B \rightarrow A$ then instantaneous velocity at A i.e. $\vec{v} = \lim_{\Delta t \rightarrow = 0} \left(\frac{\Delta x}{\Delta t}\right)$

$\left[\therefore\ v = \frac{dx}{dt}\right]$ The rate of change of displacement is known as velocity.[1]

Average acceleration of particle

$$\vec{a}_{av} = \frac{\text{change in velocity}}{\text{time interval}} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \left[\ \vec{a} = \frac{d^2x}{dt^2}\vec{i}\ \right]$$

In scalar form $\left[\ a = \frac{dv}{dt}\right]$ Hence, the rate change of velocity is called acceleration. Again $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$

$$\therefore\ a = v.\ \frac{dv}{dx}$$

8.3 Determination of Motion of Particle and Rigid Body

We know, Acceleration may be a function of position (x), velocity and time (t). The condition of motion will be satisfied by the types of acceleration that the particle passes. And if the position of particle is known at any instant then the motion of particle is known.[2]

Here are the 3 cases for acceleration

Case 1. When the acceleration is given function of time i.e. a = f(t)

we know, $a = \frac{dv}{dt}$ or, $f(t) = \frac{dv}{dt}$

$$dv = f(t)\, dt$$

on integration

$$\int_{v_o}^{v} dv = \int_{o}^{t} f(t)\, dt$$

$$v - v_o = \int_o^t f(t)\, dt \quad \text{or, } [v = v_o + \int_o^t f(t)\, dt - 1]$$

Again we know, $v = \frac{dx}{dt}$, $dx = v.dt$

On integration

$$\int_{x_o}^{x} dx = \int_o^t v.dt$$

$$x - x_o = \int_o^t [v_o + \int_o^t f(t)\, dt]\, .dt$$

$$x = x_o + \int_o^t [v_o + \int_o^t f(t)\, dt]\, dt \ldots\ldots\ldots..(2)$$

Case 2: When acceleration is the function of velocity i.e. a = f (v) then

$$a = \frac{dv}{dt} = f(v) \text{ or, } \frac{dv}{f(v)} = dt$$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v.\frac{dv}{dx}$$

$$\text{or, } v.\frac{dv}{dx} = f(v)$$

$$\left[v.\frac{dv}{f(v)} = dx\right]$$

Case 3: When the acceleration is the function of position i.e. a = f(x). then we can write. $a = f(x)$

$$v.\frac{dv}{dx} = f(x)$$

$$v.dv = F(x)\, dx$$

Worked out Examples

1. The position of a particle in rectilinear motion is defined by the relation $x = t^3 - 2t^2 + 10t - 6$, where x is in meter and t is in seconds. Determine

(i) position velocity and acceleration of the particle at time t = 3 s

(ii) the average velocity during t = 2 s and t = 3 sec and

(iii) the average acceleration during the third second.

Solution:

Given, expression for position

$(x) = t^3 - 2t^2 + 10t - 6$……………..(1)

we know, velocity $(v) = \frac{dx}{dt} = 3t^2 - 4t + 10$

Acceleration $(a) = \frac{dv}{dt} = 6t - 4$

(i) position velocity and acceleration at time t = 3sec

Position at (3sec) $X = 3^3 - 2 \times 3^2 + 10 \times 3 - 6 = 33m$

Velocity at t = 3 sec $V(3) = 3 \times 3^2 - 4 \times 3 + 10 = 25m/sec$

Acceleration at t = 3 sec $a(3) = 6 \times 3 - 4 = 14m/s^2$

(ii) Average velocity during t = 2 s and t = 3s

we know, $v_{av} = \dfrac{\text{net displacement}}{\text{time interval}}$

So, we 1st calculate net displacement at t = 2sec and t = 3 sec

Net displacement = x (3) – x(2)

$= (33 - (2^3 - 2 \times 2^2 + 10 \times 2 - 6)]m$

[take t = 2 in equation (1)]

= 19m

$\therefore V_{avg} = \dfrac{19}{3-2} = 19m/s$

(iii) Average acceleration during t = 2 (s) and t = 3s

we know, $a_{vg} = \dfrac{\text{change in velocity}}{\text{time interval}}$

change in velocity at these two time instant = v(3) – v (2)

$= [25 - (3 \times 2^2 - 4 \times 2 + 10)]$ m/sec

$= 11$m/sec

$\therefore\ a_{avg} = \dfrac{11}{3-2} = 11$ m/sec^2

2. The acceleration of a particle is defined by the relation, $a = kt^2$ knowing that the velocity is – 33 m/sec when time is zero second and again velocity is + 33 when time is 5 sec.

(i) Determine the value of constant k

(ii) Also develop equation of motion of particle, knowing that the position of particle is zero at the instant of 5 sec.

Solution:

Given that, $a = kt^2$ [function of time]

Initial velocity (v_o) = – 33m/s

Velocity at 5 sec (v_5) = + 33m/s

Now, $a = kt^2$

$$a = \frac{dv}{dt} = kt^2$$

$$dv = kt^2 dt$$

Integrating on both sides we get

$\int dv = k\int t^2 dt$

or, $\left[v = k \times \frac{t^3}{3} + C_1\right]$..............(1)

To find C_1 using given condition on equation (1)

At t = 0, v = – 33m/s

$-33 = k \times \frac{0^3}{3} + C_1$

$[C_1 = -33]$

The equation (1) becomes

$\left[v = \frac{kt^3}{3} - 33\right]$..............(2)

At t = 5 sec, v = + 33m/sec, $V = Kt^3/3 + C1$

$33 = k \times \frac{5^3}{3} - 33$

[k = 1.584]

The equation (2) becomes

$$v = \frac{1.584t^3}{3} - 33$$

$[v = 0.528t^3 - 33]$………….(3)

<u>For the equation of position</u>

equation (3) becomes

$$v = 0.528t^3 - 33$$

$$\frac{dx}{dt} = 0.528t^3 - 33$$

$$dx = (0.528t^3 - 33)\ dt$$

On integration on both sides we get

$\left[x = \frac{0.529t^4}{4} - 33t + c_2\right]$……………………..(4)

To find C_2, using given condition, ($x = 0$ when $t = 5$sec) so

$$0 = \frac{0.528 \times 5^4}{4} - 33 \times 5 + C_2$$

$$C_2 = 82.5$$

Then equation (4) becomes

$$x = \frac{0.528t^4}{4} - 33t + 82.5$$

$$\therefore x = 0.132t^4 - 33t + 82.5$$

Thus, the equations of motion of particle are

$$a = kt^2 = 1.584t^2$$

$$v = 0.528t^3 - 33$$

$$x = 0.132t^4 - 33t + 82.5$$

References

[1] Kumar, D. (2019). *Engineering Mechanics.* New delhi: S.K Kataria and Sons.

[2] Neupane, P. a. (2024). *A Text book of Engineering Mechanics.* Bhotahity Kathmandu: Heritage Publisher and Distributors PVT .LTD.

Thank You!!!