Applied Mechanics

Chapter 8

Kinematics of Particles and Rigid Body

Lecture 15 (week 15)

Uniform Rectilinear Motion of Particles, Uniformly Accelerated Rectilinear Motion of Particles, Curvilinear Motion

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Learning Objectives:

- Define and Understand Basic Concepts of rectilinear and curvilinear motion.
- Understand Curvilinear Motion
- Apply Equations of Motion in Rectangular Coordinates
- Graphical Analysis
- Practical Examples and Problem Solving

8.4 Uniform Rectilinear motion of particles

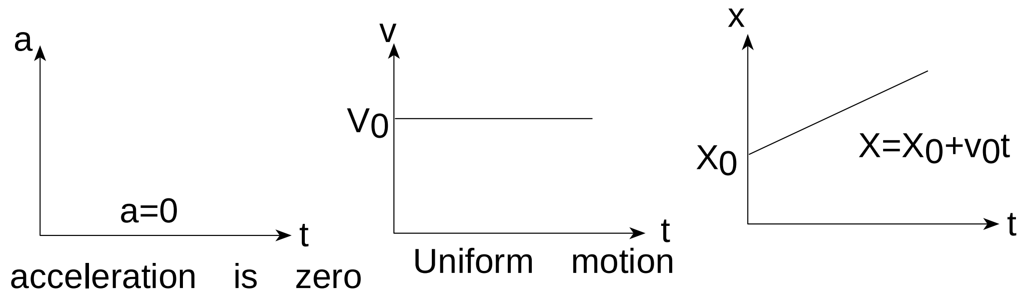
Uniform rectilinear motion is defined as the motion in which the acceleration is zero or in other words, the velocity is uniform or constant. The kinematic equations of motion are.

$[V = v_o]$

$[x = x_o + v_o t]$

(Final velocity is equal to initial velocity)

Since velocity is constant at all the time, the v – t graph is a horizontal straight line.

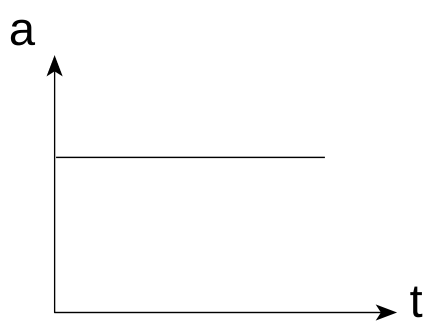


8.5 Uniformly accelerated rectilinear motion of particle

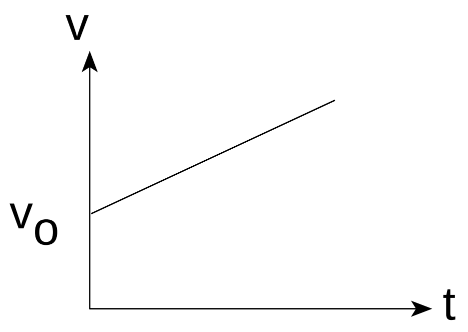
Uniformly accelerated motion is that type of motion in which the acceleration is constant or in other word, the velocity is uniformly varying. The kinematic equations of motion are.

a = constant, $v = v_0 + at$ and $x = x_0 + v_0t + \frac{1}{2}at^2$.

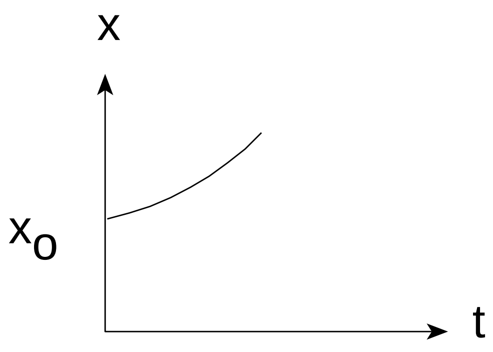
As acceleration is constant, the a– t graph is horizontal straight line.



a=constant



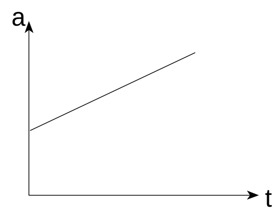
velocity is linear function



position is function of square of time

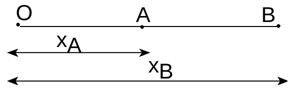
Non uniformly accelerated motion: It is that type of motion in which the acceleration is not constant or in other words,

the acceleration is non-uniformly varying.



<u>Relative motion of two particles</u>

Let us consider o is the origin and the coordinates of A and B is x_A and x_B respectively.



Relative position of B with respect to A can be written as $x_{B/A}$ or x_{AB}

$\therefore\ x_{B/A} = x_B - x_A$ $\qquad$ $\therefore\ x_B = x_A + x_{B/A}$

on derivatives

$V_B = V_A + V_{B/A}$

Again, differentiating

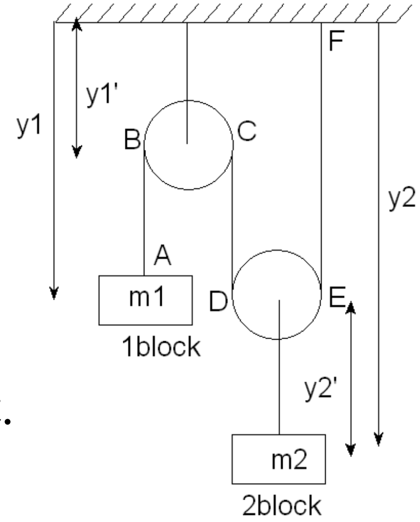
$a_B = a_A + a_{B/A}$

Motion of connected Bodies /Dependent motion

If the motion of one particle depends upon motion of another or several particles such motion is said to be dependent motion. eg. block and pulley system. In this system, we must draw separate free body diagram for each body and applying the equation of motion to each body and solve for the unknowns.

Kinetic equations of motion for mass and pulley system

Q. Prove that the acceleration of block 2 is equal to half of block 1 in opposite direction. And also write the equation of motion.



Let us consider a system and the string is inextensible its total length is constant.

i.e. AB + arc BC + CD + arc DE + EF = constant

we see from figure that the lengths of the string embracing the pulley i.e. arc BC and DE always constant.

$\therefore$ AB + CD + EF = constant

since the length $y_{1'}$ and $y_{2'}$ are constants, we can also write the above expression as

or, $(y_1 - y_1') + (y_2 - y_2' - y_1') + (y_2 - y_2') =$ constant

$(AB + y_1') = y_1$

$(EF + y_2') = y_2$

$\boxed{y_1 + 2y_2 = \text{constant}}$ …………(i)

As y_1 and y_2 vary with time, so differentiating then w.r.t time we get

$v_1 + 2v_2 = 0$………..(ii)

$a_1 + 2a_2 = 0$…………(iii)

$\therefore\ [a_2 = -\frac{a_1}{2}\ ]$

[Hence, the acceleration of the block 2 is equal to the half of acceleration of the block 1 and -ve sign indicates opposite direction.

Rectilinear motion along vertical Y – axis

When the Bodies thrown upwards

$V = V_o - gt$

$V^2 = V_o^2 - 2g\,(y - y_o)$ or, $v^2 = v_o^2 - 2gs$

$y = y_o + v_o t - \frac{1}{2} gt^2$ or, $S = v_0 t - \frac{1}{2} gt^2$

Where y_o is the initial displacement from the ground level

Note: *If the body is thrown from the ground level then $y_o = 0$, if the body is thrown from the top of the building then y_o = height of the building.*

At the top, velocity becomes 0, and the body changes direction.

For maximum height

We know

$$v^2 = v_o^2 - 2gs$$

$0 = v_o^2 - 2g \times (hmax)$

$$h_{max} = \frac{v_0^2}{2g}$$

Time taken to reach maximum height

We know, $V = v_o - gt$

$0 = v_o - gt$

$[t = v_o/g]$

Total time of ascent and Descent for object thrown upward to reach the ground $T = 2 \times t = \frac{2v_o}{g}$

Bodies thrown Downward or dropped

$v = v_o + gt, \; v^2 = v_o^2 + 2gs$

$$S = y = V_o t + \frac{1}{2} gt^2$$

8.6 Curvilinear motion

The motion of a particle along a curved path other than a straight line is known as curvilinear motion e.g. motion of satellite, projectile motion etc.

Or, when a particle undergoes translational motion along a curved path other than straight line, the motion is said to be curvilinear, e.g. bike travelling on a curved path.

i.e. $\vec{V} = \frac{d\vec{r}}{dt}$ and $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$ [1]

If x and y be the rectangular coordinates of the point A then its position vector $\vec{r}$ can be expressed as;

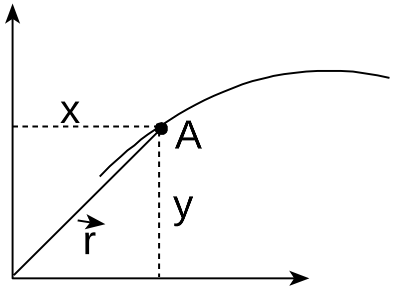
$$\vec{r} = x\vec{i} + y\vec{j}$$

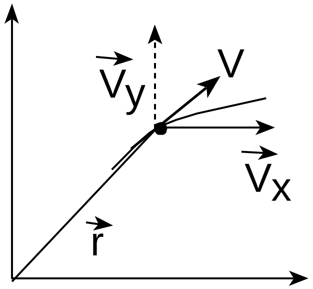
Velocity vector

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx\,\vec{i}}{dt} + \frac{dy}{dt}\vec{j}, \qquad \left[\vec{V} = V_x\,\vec{i} + Vy\,\vec{j}\right]$$

∴ The magnitude and direction of instantaneous velocity

Figure





$$V = \sqrt{V_x^{\,2} + vy^2}$$

$$\theta_r = \tan^{-1}\left[\frac{Vy}{Vx}\right]$$

$$\vec{a} = a_x\,\vec{i} + a\,y\,\vec{j}$$

$$a = \sqrt{ax^2 + ay^2}$$

$$\theta_a = \tan^{-1}\left[\frac{ay}{ax}\right]$$

Q. A particle moves along a curvilinear path defined by $y = ax^2$ where x and y are in meters. The velocity and acceleration of the particle at a point (5m, 2.5m) are respectively 5m/s and $2 m/s^2$. Determine the total acceleration of the particle at that point.

Solution: The equation of curvilinear path of the particle $y = ax^2$

For the value of constant, putting given condition in this equation

When x = 5m, y = 2.5m, so

$2.5 = a \times 5^2$

$a = 0.1$

The equation becomes

$[y = 0.1x^2]$

we know the normal component of acceleration is an = $\frac{v^2}{\rho}$

so first finding the radius of curvature (ρ) [Since it is not given]

We know, (ρ) = $\frac{[1 + \left(\frac{dy}{dx}\right)^2]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$ [It is from mathematical derivation]

So, $\frac{dy}{dx} = 0.1 \times 2x = 0.2x$

And $\frac{d^2y}{dx^2} = 0.2$

At given point (5m, 2.5m)

$\frac{dy}{dx} = 0.2 \times 5 = 1$

$$\frac{d^2y}{dx^2} = 0.2$$

$$\therefore \rho = \frac{[1 + 12]3/2}{0.2} = 14.14m$$

$$\text{And } a_n = \frac{v^2}{\rho} = \frac{52}{14.14} = 1.77m/s2$$

And the given acceleration $2m/s^2$ is tangential acceleration

$$at = 2m/s^2$$

$\therefore$ Therefore total acceleration is given as

$$a = \sqrt{at^2 + an^2} = \sqrt{2^2 + 1.77^2}$$

$$[a = 2.67m/s^2]$$

References

[1] Kumar, D. (2019). *Engineering Mechanics.* New delhi: S.K Kataria and Sons.

Thank You!!!