Applied Mechanics

Chapter 10

Kinetics of Particles and Rigid Body

Lecture 16 (week 16)

Newton's Second Law of Motion and momentum, Angular Momentum and Rate of Change, Equation of Motion-Rectilinear and Curvilinear

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Learning Objectives:

- Understanding Newton's Second Law of Motion:
- Define Newton's Second Law of Motion.
- Describe how Newton's Second Law applies to variable mass systems.
- Understand and apply the concept of impulse.
- Define angular momentum for a particle and a rigid body.
- Understand the concept of rate of change of angular momentum.

9.1 Newton's Second Law of Motion and momentum

9.1.1 Newton's 2nd law of motion

Statement: If the resultant force acting on particle is not zero the particle will have an acceleration proportional to the magnitude of resultant force and in the direction of this resultant force.

i.e. when the particle of mass 'm' is subjected to different forces $\vec{F_1}$, $\vec{F_2}$,...... $\vec{F_n}$ different times and corresponding acceleration $\vec{a_1}$, $\vec{a_2}$.......$\vec{a_n}$ etc.

Then, according to statement

$F \alpha a$

$F = ma$

$$\frac{F}{a} = m \text{ (constant)}$$

$$\text{Then, } \left[\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = m \text{ (constant)}\right]$$

$$\left(\begin{array}{l} F_x = ma_x \\ F_y = ma_y \\ F_2 = ma_z \end{array}\right)$$

In general term

$\vec{F} = m\vec{a}$

$F_x (\vec{i}) + F_y \vec{j} + F_2 \vec{k} = m (a_x \vec{i} + a_y \vec{j} + a_z \vec{k})$

Equating coefficients, we get

9.1.2 Linear momentum:

If the particle having mass m and velocity $\vec{v}$ at a time t at any position p then the quantity $m\vec{v}$ is called momentum or linear momentum of the particle. It is generally denoted by H or G and its direction is equals to the direction of velocity and magnitude is equal to (mass × velocity)

i.e. $\vec{H} = m\vec{v}$ [its unit kg-m/sec or Ns

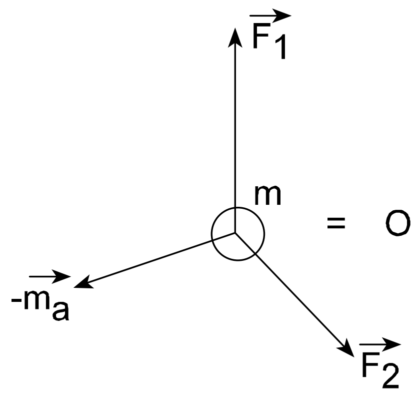
9.2 Equation of Motion and Dynamic Equilibrium

9.2.1 Equation of motion

Equation of motion

Let us consider a particle having mass 'm' is acted upon by a several forces producing the acceleration 'a' then

Form Newton's 2nd law of motion.



$$\Sigma \vec{F} = m\vec{a}$$

Using rectangular components of these system

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = ma_y$$

$$\Sigma F_z = ma_z$$

Are the equation of motion

9.2.2 Dynamic equilibrium

Considering a particle of mass 'm' which is acted upon by several forces then Newton's law gives

$$\Sigma\vec{F} = m\vec{a}$$

If we consider inertia vector also called reversible vector ($m\vec{a}$) is adding to the system or in other words by adding a reversible force $m\vec{a}$ to the system of forces, the system will be in equilibrium.

i.e. $\Sigma\vec{F} - m\vec{a} = 0$ [Which is the general expression for dynamic equilibrium]

Practically the particle is not at rest in this condition but actually moving and we termed it dynamic equilibrium.

In rectangular components we can write

$\Sigma F_x = max, \quad \Sigma F_y = ma_y \quad \Sigma F_z = ma_z$

Also,

$\Sigma F_x = m\ddot{x} \quad \Sigma F_y = m\ddot{y} \quad \Sigma F_z = m\ddot{z}$

Where $a = \dfrac{\left(\dfrac{dx}{dt}\right)}{dt} = \ddot{x}$

9.3 Angular Momentum and Rate of Change, Equation of Motion-Rectilinear and Curvilinear

9.3.1 Angular momentum and Rate of change

The product of mass of the particle and the velocity vector is known as momentum of the particle (i.e. linear momentum). And the moment of the linear momentum is called angular momentum of the particle.

Q. Prove that the rate of change of angular momentum is equal to the moment of the force acting on the particle about the same point.

Let us consider a particle having mass 'm' moving with vector velocity $\vec{v}$ in x-y plane .And the momentum of this particles be $m\vec{v}$ and the components of $m\vec{v}$, are $m\vec{v}$ x and $m\vec{v}$ y in x and y direction respectively.

We know,

Angular momentum = moment of momentum

$Ho = m\vec{v}_x \times y - m\vec{v}_y \times x$ [moment about point o and take clockwise moment positive]

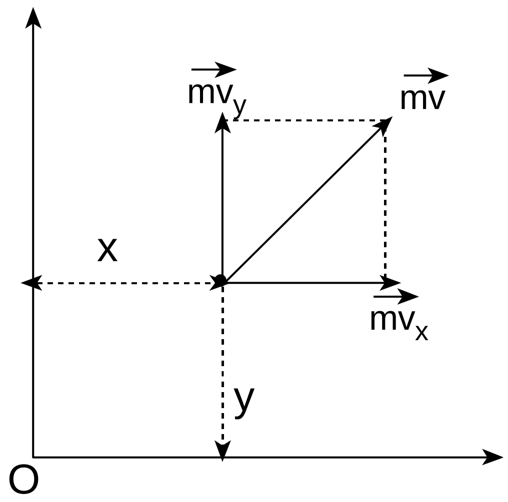
$[Ho = m(v_x \times y - V_y \times x]$

Rate of change of angular momentum means derivatives of angular momentum w.r.t time so,

$$\frac{dH_o}{dt} = \dot{H} = m.\frac{d}{dt}(v_x.y - V_y.x)$$

$$= m\left(d\frac{v_x}{dt}.y + \frac{dy}{dt}v_x - \left(\frac{dv_y}{dt}x + Vy.\frac{dx}{dt}\right)\right.$$

$$\dot{H}_o = m\left(\dot{V}_x xy + \dot{y}\,Vx - \dot{v}\,yx - \dot{x}\,Vy\right)..........(1)$$



And we know

$$\dot{x} = \frac{dx}{dt} = v_x$$

$$\dot{y} = \frac{dy}{dt} = v_y$$

$$\dot{v}_x = \frac{dv_x}{dt} = a_x$$

$$\dot{v}_y = \frac{dv_y}{dt} = ay$$

so from equation (1)

$$\dot{H}_o = m\,(a_x \times y + V_y \times V_x - a_y \times x - V_x \times V_y)$$

$$\dot{H}_o = m\,(a_x \times y - a_y x)$$

$$\dot{H}_o = ma_x \times y - ma_y x$$

$$[\dot{H}_o = F_x \times y - F_y \times x]$$

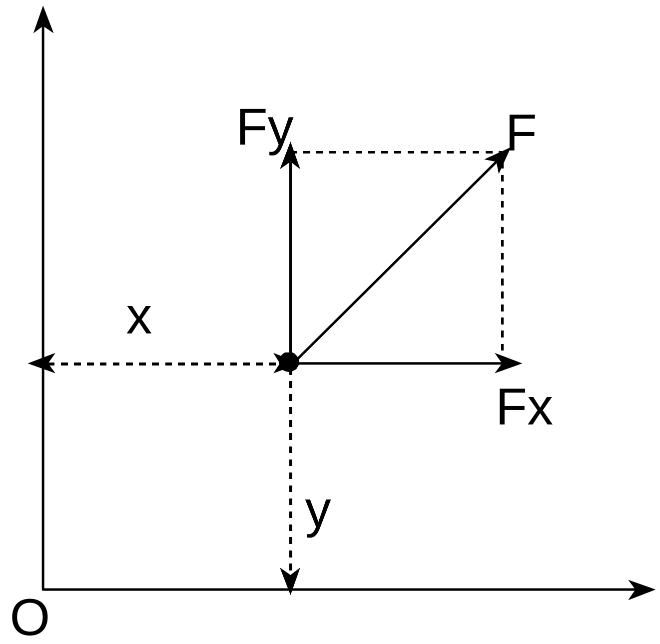


Figure for force

From second figure

Moment of the force about same point 0

$(MF_o) = Fx \times y - Fy \times x$ this equal to $\dot{H}_o$

So we can conclude that the rate of change of angular momentum of the particle about any point is equal to the moment of the fore ($\vec{F}$) acting on that particle about the same point. i.e. [$\dot{H}_o = M.Fo$]

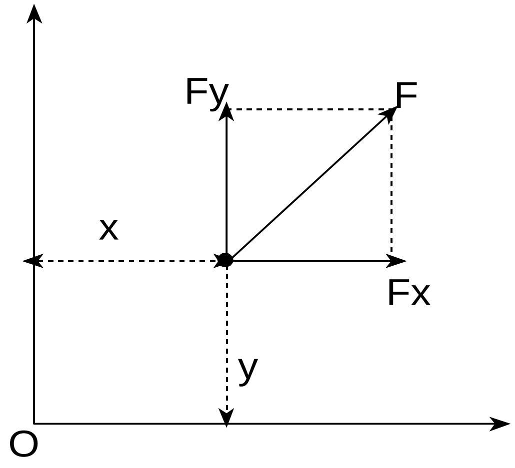


Figure for force

Rate change of momentum and impulse momentum principle

Let us consider a particle having mass m which is acted upon by force $\vec{F}$ then from Newton's 2^{nd} law.

$$\vec{F} = m\vec{a} = m.\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \left[\vec{a} = \frac{d\vec{v}}{dt}\right]$$

[Where $m\vec{v}$ is called linear momentum]

From above expression we can say that the rate of change of linear momentum is equal to the applied force.

Now, $\vec{F} \times dt = d(m\vec{v})$

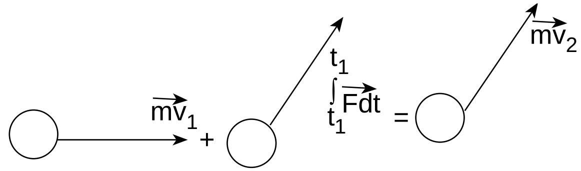
Integrating w.r.to time from t_1 to t_2 and velocity from v_1 to v_2

$$\int_{t_1}^{t_2} \vec{F} \times dt = \int_{\vec{v_1}}^{\vec{v_2}} d(m\vec{v})$$

$$\int_{t_1}^{t_2} \vec{F} \times dt = (m\vec{v_2} - m\vec{v_1})$$

or, $m\vec{v_1} + \int_{t_1}^{t_2} \vec{F} \times dt = m\vec{v_2}$ **,The term $\int_{t_1}^{t_2}$ F.dt is called impulse of the force during the time interval $(t_2 - t_1)$**

or, [Initial momentum + $\vec{Imp}$ = final momentum]



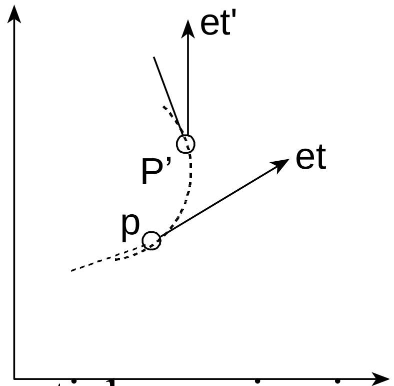
From the above expression, we conclude that when an impulse of the force $\vec{F}$ is acted or (added) to the system of initial momentum $(m\vec{v_1})$, then it change into final momentum $(m\vec{v_2})$.

This is the required expression for the impulse momentum principle

9.4 Rectangular: Tangential and Normal Components and Polar Coordinates: Radial and Transverse Components with examples

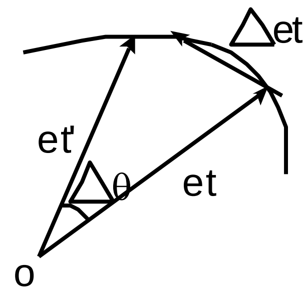
9.4.1 Tangential and Normal components

We know that velocity of particle is the vector tangent to the path of the particle but the acceleration is not. So, the acceleration is resolve into two components directed respectively along the tangent and normal to the path of the particle.



So let us consider a two instant for a particle moving in a curved path at position P and P' and the e_t and et' are the respective unit vector corresponding to that position.

$$[a = \frac{dv}{dt} e_t + \frac{V^2}{\rho} e_n]$$



Thus scalar components of the acceleration are

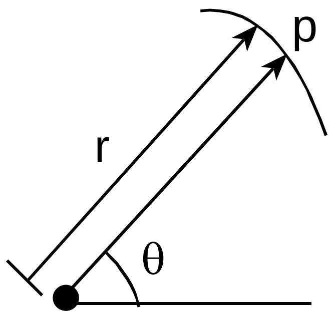
$$\left[at = \frac{dv}{dt} \qquad an = \frac{v2}{r} \right]$$

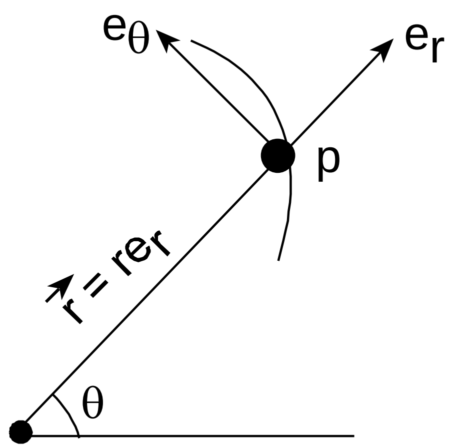
(Thus we can say that the tangential component of the acceleration is equal to the rate of change of speed of the particle i.e at = $\frac{dv}{dt}$)

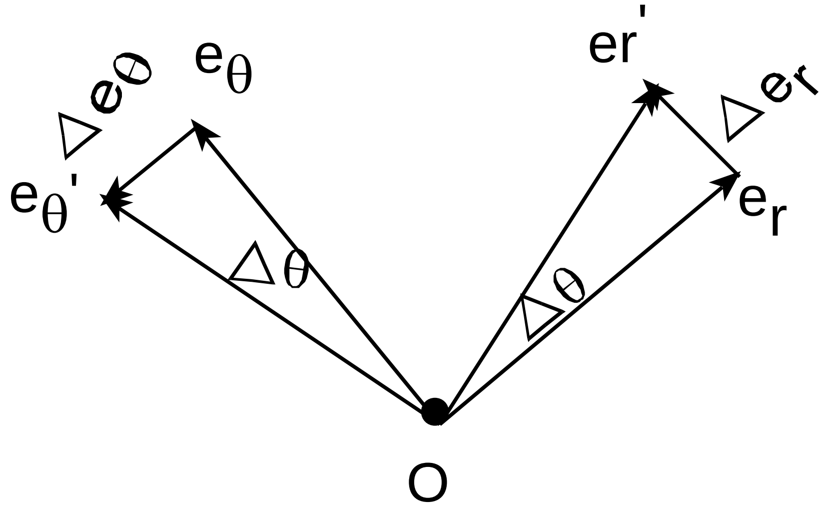
The normal component of the acceleration is equal to square of the speed divided by the radius of the curvature of the path. i.e. $a_n = \frac{v^2}{\rho}$

9.4.2 Radial and transverse components

Let e_r and e_θ be the unit vector along the radial direction (particle P, moves if r increased at θ constant) and the unit vector along transverse direction (P would moves if θ increases at r kept constant) respectively.







And from the limit derivation

$$v = \dot{r}\, e_r + r\dot{\theta}\, e_\theta \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

again,

$$a = \frac{dv}{dt} = \ddot{r}\, e_r + \dot{r}\, \dot{e}_r + \dot{r}\, \dot{\theta}\, e_\theta + r\ddot{\theta}\, \dot{e}_\theta + r\dot{\theta}\, \dot{e}_\theta$$

Substituting the value of $\dot{e}_r$ and $\dot{e}_\theta$ we get

$$a = \ddot{r}\, e_r + \dot{r}\, \dot{\theta}\, e_\theta + \dot{r}\, \dot{\theta}\, e_\theta + \dot{r}\, \dot{\theta}\, e_\theta + r\ddot{\theta}\, e_\theta + r\dot{\theta}\, (-\dot{\theta}\, e_r)$$

$$[a = (\ddot{r} - r\dot{\theta}^2)\, e_r + (r\ddot{\theta} + 2\dot{r}\, \dot{\theta})\, e_\theta] \ldots\ldots\ldots (2)$$

The scalar components of the velocity and the acceleration in the radial and traverse direction are

$$V_r = \dot{r}\ ,\ v_\theta = r\dot{\theta}$$

$$a_r = \ddot{r} - r\dot{\theta}^2,\ a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta}$$

[Note: a_r is not equal to time derivate of v_r, a_θ is not equal to time derivate of v_θ]

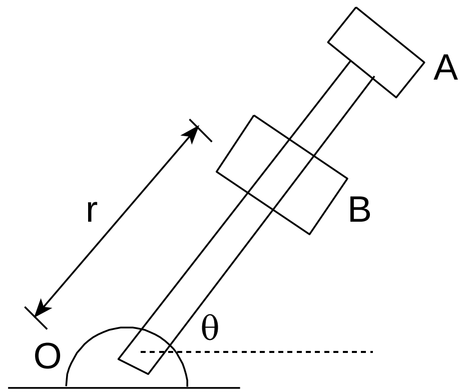
Note: *(Tangential and normal component) and (radial and transverse component) both can be analyzed in kinematic as well kinetic that depends on considering force or not*

Worked Out Examples

1. The rotation of the 0.9m arm OA about O is defined by the relation $\theta = 0.15t^2$, where θ is expressed in radians and t in seconds. Collar B slides along the arm in such a way that its distance from o is $r = 0.9 - 0.12t^2$, where r is expressed in m at t in sec after the arm OA has rotated through 30° determine.

(a) total velocity of the collar

(b) the total acceleration of the collar



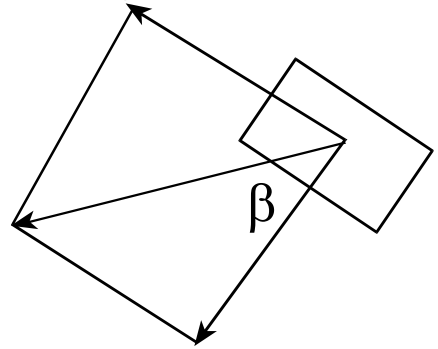
Solution:

Rotation $(\theta) = 0.15t^2$

(r) $= 0.9 - 0.12t^2$

Velocity of the collar (v) = ?

The system has two component radial and transverse so we know, Total velocity (v) = $\sqrt{(v_r)^2 + (V_\theta)^2}$



and Vr = $\frac{dr}{dt}$ = ṙ = – 0.12 × 2 × t ....(i)

$V_\theta = r\dot{\theta}$ = (0.9 – 0.12t^2) × 0.15 × 2 × t ....(ii)

After the arm OA has rotated through 30° So

θ = 30°, t = ?,   180° = π

30° = $\left(\frac{\pi}{180} \times 30°\right)$ = 0.524 rad

So, θ = 0.524 = 0.15t^2 $\therefore$ t = 1.869 sec

so, t = 1.869 sec at θ = 30°

so, equation (i) and (ii) becomes

V_r = – 0.12 × 2 × 1.869 = – 0.449 m/s

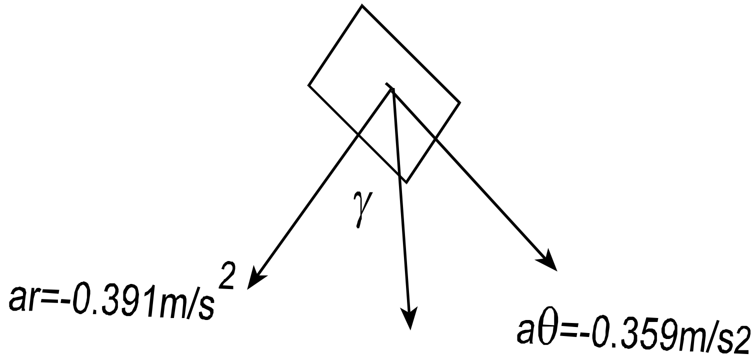
V_θ = (0.9 – 0.12 × 1.869^2) × 0.15 × 2 × 1.869 = 0.27m/s

(a) so the magnitude of total velocity

$$(V) = \sqrt{(v_r)^2 + (v_\theta)^2} = \sqrt{(-0.449)^2 + (0.27)^2}$$

$$(V) = 0.524 \text{ m/s}, \quad \text{Direction } (\beta) = \tan^{-1}\left(\frac{v_\theta}{v_r}\right) = 31.0°$$

(b) Total acceleration of collar : $a = \sqrt{(a_r)^2 + (a_\theta)^2}$



$$a_r = \ddot{r} - r\dot{\theta}^2 \quad \text{..........(iii)}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \quad \text{...........(iv)}$$

$$r = 0.9 - 0.12t^2 = 0.48\text{m}$$

$$\dot{r} = -0.24t = -0.449\text{m/s}$$

$$\ddot{r} = -0.24 = -0.240\text{m/s}^2$$

$$\theta = 0.15t^2 = 0.524 \text{ rad}$$

$$\dot{\theta} = 0.3\ 0t = 0.561 \text{ rad}$$

$$\ddot{\theta} = 0.30 = 0.30 \text{ rad/s}^2$$

putting the above value in equations (iii) and (iv) we get

$a_r = -0.240 - 0.481 \times 0.561^2 = -0.391 \text{ m/s}^2$

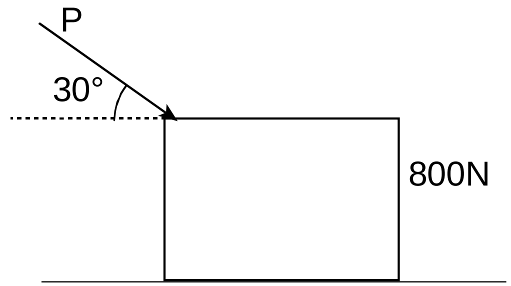
$a_\theta = 0.481 \times 0.30 + 2 \times (-0.449) \times (0.561) = -0.359 \text{m/s}^2$

So, total acceleration (a) = $\sqrt{(-0.391)^2 + (-0.359)^2}$

$= 0.531 \text{ m/s}^2$

And angle $\gamma = 42.6°$

2. A 800N block rest on a horizontal plane as shown in figure. Find the magnitude of the force P required to give the block an acceleration of $2 m/sec^2$ to the right. The coefficient of friction between the block and the plane is 0.25.[1]



Solution:

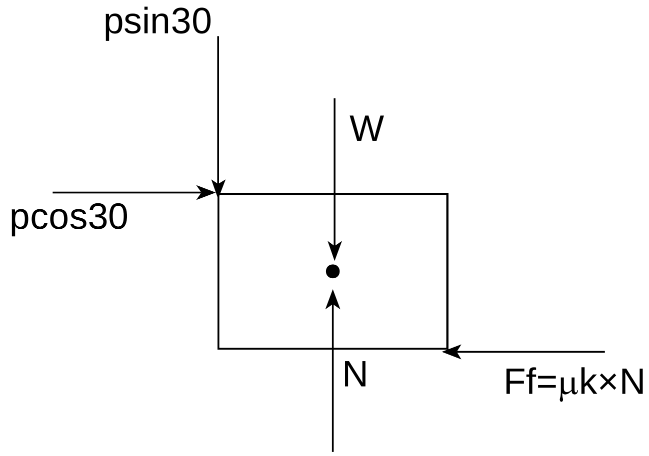
Given, wt. of the block = 800N

Acceleration to be produced = $2 m/sec^2$

Coefficient of friction (μ_k) = 0.25

Mass of block (m) = $\frac{800}{9.81} = 81.55 kg$

Draw the F.B.D of the system



Applying dynamic equilibrium

$\overset{+}{\rightarrow}$

$\Sigma Fx = ma_x$

$p\cos 30° - \mu_k N = ma_x$

$p\cos30° - 0.25N = 81.55 \times 2$

$0.866p - 0.25N = 163.1$ ……………..(1)

(+ ↑) $F_y = 0$ [Here motion produced only in x direction]

$N - W - P \sin 30 = 0$

$N - 800 - P \times 0.5 = 0$

$0.5P + N = 800$…………..(2)

on solving (i) and (2) we get

P = 366.39 Newton,   N = 616.80 Newton.

Hence, 366.39 N force is required to give the block an acceleration of $2 m/sec^2$

References

[1] Kumar, D. (2019). *Engineering Mechanics.* New delhi: S.K Kataria and Sons.

Thank You!!!