Power Systems Operation and Control
Lecture 3
Economic Load Dispatch (ELD)

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Lecture learning outcomes:

At the end of this lecture, you will be able to:

i. Know the Impact of transmission loss in Economic Operation of Power System

ii. Understand the Economic Load Dispatch with the Transmission Line Loss

iii. Knows how to model ELD with Transmission Loss consideration

iv. Identify the ELD Solution Techniques

Content

1. Introduction

- Economic Load Dispatch (ELD) is the process of determining the optimal generation levels for various power plants in a way that minimizes the total cost of electricity generation while meeting the demand for power.
- This involves allocating generation among available units based on their cost characteristics and system constraints, such as capacity limits and transmission losses[1].
- Transmission line losses significantly impact the economic operation of a power system in several ways[2]:
- ***Increased generation costs***: Losses necessitate additional generation to meet demand. Utilities must generate more power to compensate for losses, leading to higher fuel and operational costs.

Introduction Cont.…

- ***Capacity Planning***: Losses influence the sizing of generation and transmission capacity. Utilities mus t overestimate capacity needs to account for losses.
- ***Additional tariff*** : Higher losses can result in increased transmission tariffs, affecting end-user prices and overall system affordability.
- ***Affects the operational efficiency***. Thus, minimizing losses through optimal routing and the use of h igh-efficiency conductors can enhance overall system efficiency, reducing costs.
- Violate the regulatory agreement: Utilities face regulations on loss limits, which can lead to penalties or required investments in upgrades, impacting financial performance.
- Thus, managing transmission line losses is crucial for maintaining economic viability in power system s, affecting generation costs, infrastructure investments, and overall operational efficiency.

2. Importance of ELD with Line Loss (IELDLL)

- Importance of ELD with the transmission line loss consideration are;
- **Minimizes the cost:** ELD aims to minimize fuel costs and operational expenses while ensuring reliable power supply.
- By including transmission losses, the economic implications of generation decisions are more accurately assessed as;
- For the consideration of efficiency in power delivery, including transmission losses in ELD ensures:
- the generated power is sufficient to meet demand at the load points

IELDLL Cont…..

- Enhances efficiency: Considering the power lost in transmission line enhances the overall **system efficiency.**
- **Improve the system reliability:** Properly accounting for transmission losses aids in maintaining sy stem stability and reliability, ensuring that generation matches actual demand despite losses in the network.
- **Improves** the positive impact on environmental Impact
- Thus, optimizing generation with losses in account can also lead to a reduction in emissions by mini mizing the need for excess generation while using excess inputs

IELDLL Cont.….

- It also encourages the use of more *efficient plants* over using inefficient systems.
- ***Provides better decision making***: the power utilities operators can make better operational and planning decisions by understanding the economic impact of transmission losses
- This leads to more resilient and cost-effective grid operation.
- Enhances global marketing/ profitability of the utility:
- ELD with loss consideration ensures fair competition and pricing, influencing market dynamics and promoting renewable integration.

3. Objective of ELD With Line loss consideration

The main objective function is the cost reduction as given by[3]:

- To minimize the total generation cost while meeting the electrical demand, including the costs assoc iated with transmission losses and generation limits.
- Optimal distribution of generation for the required demand, which is:
- To determine the most efficient distribution of load among available generation units, ensuring that each unit operates within its optimal output range.

Objective Cont.….

- Accurate demand satisfaction, ensure the actual power delivered to consumers meets the demand, factoring in losses incurred during transmission.
- Proper utilization of resource : to maximize the use of available resources by dispatching generatio n units that can operate at the lowest marginal cost while accounting for efficiency losses.
- Enhances the system reliability: To maintain system stability and reliability by effectively managing generation and minimizing the risk of overloading transmission lines.
- Ensuring environmental sustainability: to reduce emissions and environmental impacts by optimizin g the use of cleaner, more efficient energy sources while considering the losses in the system.

4. Impact of Line Loss in ELD

The most common negative impact of increased line loss in ELD are;

- Increased cost of generation to compensate the power lost in the line
- Unnecessary cost of additional fuel , thus to compensate for losses, generators must produce more ele ctricity, leading to higher fuel costs and operational expenses.
- Leads to inefficient dispatch, if transmission losses are not accurately considered, the optimal dispatch may be compromised, leading to increased costs.

Impact of Line Loss Cont.…

- May alters the optimal generation limits
- Relocation of Load, the losses can shift the optimal output levels of generation units, causing less efficient plants to be utilized more than necessary .
- Non-linearity in dispatch. the relationship between generation output and losses can lead to non-li near cost functions, complicating the dispatch optimization.
- Leads to system modeling complexity

Impact of Line Loss Cont.…

- May needs more complex and sophisticated algorithms to find out optimal solutions
- Dynamic adjustments: continuous adjustments may be needed to account for fluctuating line losses due to changing load conditions and network configurations.
- Risk of overloading: If not properly accounted for, transmission losses can lead to overloading of lin es, increasing the risk of outages and system instability.
- Voltage drop issues: higher losses can result in significant voltage drops, impacting the reliability of power delivery to consumers.

Impact of Line Loss Cont.…

- Increased emissions: higher generation levels due to losses can lead to increased emissions, contra-dicting sustainability goals.
- Inefficient use of RE: Losses may deter the integration of renewable energy sources, which could othe rwise help mitigate costs and emissions.
- Affects the market dynamics: In competitive markets, not accounting for losses can lead to unfair prici ng and market distortions, affecting both suppliers and consumers.
- Affects the investment decisions: utilities may need to invest in more infrastructure or generation capa city to mitigate the effects of transmission losses, impacting overall economic efficiency

5. Modeling of ELD with Transmission Loss

- Economic Load Dispatch (ELD) with transmission line losses can be represented using an optimization model.
- The general approach involves minimizing the total generation cost while meeting demand and accoun ting for transmission losses.
- The objective is to minimize the total generation cost as given by:

$$C_T = \sum_{i=1}^{N} C(P_{Gi}) \qquad \text{eqn.(1)}$$

Where:

C_T is the total cost , $C(P_{Gi})$ is the cost function of each generator and N is number of generating units

- The cost of each generation unit is given using quadratic equation, given as;

$$F_i = C(P\text{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}{}^2 Rs/h \qquad \text{eqn.(2)}$$

Modeling of ELD Cont.….

- **Constraint**: a. **Demand constraint is given by**:

- Based on energy conservation, the total power generated must meet the total demand and loss as given as; $$\sum_{i=1}^{N}(P_{Gi}) = P_D + P_L$$ eqn.(3)

- The power transmission line losses can be modeled using a loss formula, represented as:

$$P_L = \sum_{I-1}^{N}\sum_{J=1}^{N} L_{ij}P_iP_j$$ eqn.(4)

Where, L_{ij} is the transmission line loss coefficients between tow generators, generator i and j.

- Lij refers to the measure of energy loss in transmission lines due the line parameters.
- It helps evaluate how efficiently electricity is transmitted from one point to another, ultimately affecting the overall performance of the distribution system

Modeling of ELD Cont.….

- The loss coefficient (L_{ij} or B_{ij}) requires the knowledge of bus admittance matrix as given by for four bus system
- It’s normally refers the diagonal elements of Y-bus matrix ;

$$Y = \begin{vmatrix} Y11 & Y12 & Y13 & Y14 \\ Y21 & Y22 & Y23 & Y24 \\ Y31 & Y32 & Y33 & Y34 \\ Y41 & Y42 & Y43 & Y44 \end{vmatrix}$$

Modeling of ELD Cont…..

- The line loss is approximated as:

$$P_L = \sum_{i=1}^{N} k_i P_i^2 \qquad \text{eqn.(5)}$$

Where, k_i is the loss coefficient of generation i and given as;

$$k_i = \frac{\partial P_L}{\partial P_i} \qquad \text{eqn.(6)}$$

- b. **The generation Constraints**: economic load dispatch should consider the generation limit constraints as given by;

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max}$$

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \qquad \text{eqn.(7)}$$

Then, the overall optimization model could be:

$$\text{Minimize } C_T = \sum_{i=1}^{N} C(P_{Gi}) \qquad \text{s.t:} \qquad \sum_{i=1}^{N} P_i = P_D + \sum_{i=1}^{N} k_i P_i^2 \qquad \text{eqn.(8)}$$

$$P_{i\min} \le P_i \le P_{i\max}$$

6. ELD Solution Methods

To solve this optimization problem, various methods can be used, including[4]:

- Lagrange Multiplier Method: Useful for handling equality constraints.
- Lambda iteration method: useful unconstrained methods
- Dynamic Programming: Breaks the problem into simpler sub-problems.
- Linear Programming and Non-linear Programming: Depending on the nature of the cost and loss functions.

ELD Solution Methods Cont.…

- Lagrange method: The generation dispatch using Lagrange method with the inclusion of transmission losses is given by;

$$L(P_G,\lambda)=\sum_{i=1}^{n}C(P_{Gi})+\lambda\left(P_{Dtotal}+P_L(P_G)-\sum_{i=1}^{N}P_{Gi})\right)$$

$$\frac{\partial L(.)}{\partial P_{Gi}}=0 \quad ,\frac{\partial L(.)}{\partial \lambda}=0$$

eqn.(9)

- In general, using generators closer to the load results in lower losses
- This impact on losses should be included when doing the economic dispatch

$$\frac{\partial L(P_G,\lambda)}{\partial P_{Gi}}=\sum_{i=1}^{N}C_i(P_{Gi})+\lambda(P_D+P_L(P_G)-\sum_{i=1}^{N}P_{Gi})$$

eqn.(10)

Based on eqn.9, the two equations are given as;

$$\frac{\partial L(P_G,\lambda)}{\partial P_{Gi}}=\frac{\partial C_i(P_{Gi})}{\partial P_{Gi}}-\lambda(1-\frac{\partial P_L(P_G)}{\partial P_{Gi}})=0$$

eqn.(11)

eqn.(12)

$$P_D+P_L(P_{G)}-\sum_{i=1}^{N}P_{Gi}=0$$

ELD Solution Methods Cont.…

- From eqn.11, lambda can be determined as:

$$\frac{\partial L(P_G,\lambda)}{\partial P_{Gi}} = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} - \lambda\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right) = 0$$

$$\lambda = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)} * \left[\frac{\partial C_i(P_{Gi})}{\partial P_{Gi}}\right]$$

eqn.(13)

Define the Penalty factor L_i for each generator as given by :

$$Li = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$$

eqn.(14)

ELD Solution Methods Cont.…

- The power system penalty factor is the amount that has to be multiplied by the plant's incremental cost of power output in order to account for transmission losses.
- It can also be defined as the plant's power generation divided by the actual power required to meet the load following transmission loss.
- As penalty factor is the ratio of power generated to power fed to the load
- Cost of Power Generated increase with penalty factor, which is:

$$C(P_i) = \quad = \text{IFC x Penalty Factor} \qquad \text{eqn.(15)}$$

- Which indicates that the penalty factor increase overall costs
- The penalty factor at the slack bus is always unity!

ELD Solution Methods Cont.…

- Then, the basic condition for optimal dispatch is :

$$L_1 IFC_1(P_{G1}) = L_2 IFC_2(P_{G2}) \ldots\ldots\ldots\ldots L_N IFC_N(P_{GN}) = \lambda$$ eqn.(16)

Accordingly, if the generation power (P_{Gi}) increase, power loss increase, which is given by:

$$P_L = \frac{\partial P_L(P_G)}{\partial P_i} > 0 \Rightarrow L_i > 1$$ eqn.(17)

- Which leads to the given generation more expensive and if Li less than unity means the intended generation is less expensive

ELD Solution Methods Cont.…

Calculation of penalty factor

- The penalty factor for N-bus power network can be determined using power flow techniques using:

$$\Delta P_i = P_{i,sched} - P_{i,calculated} \Rightarrow (P_{Gi} - P_{Di}) - P_{i,calculted} \quad \text{eqn.(18)}$$

$$\Delta Q_i = Q_{i,sched} - Q_{i,calculated} \Rightarrow (Q_{Gi} - Q_{Di}) - Q_{i,calculted}$$

- The reason is that small change in power flow impacts the entire network operation cost.
- You need to know the types of bus, power network and the change in power from power flow techniqu es. Which is given by;

$$L_i = \frac{1}{(1 - \frac{\Delta P_L(P_G)}{\Delta P_{Gi}})} \quad \text{eqn.(19)}$$

ELD Solution Methods Cont.…

- For all generators, the total of the incremental production costs of power at any plant plus the incremental transmission losses at bus i from generating Pi, at the rate of lambda, must be constant and equal to lambda.
- The incremental cost of the received power is represented by this constant, lambda.
- Lii or B-coefficients are a crucial, straightforward, and approximate approach of representing transmission loss in relation to generating powers.

ELD Solution Methods Cont.…

- The transmission loss in the injected bus is quadratic under typical operating conditions, which is utilized by this method.

$$P_L = \sum_i \sum_j P_i B_{ij} P_j \quad \longrightarrow \quad \frac{\partial P_{loss}}{\partial P_{Gi}} = \sum_j 2 B_{ij} P_j \qquad \text{eqn.(20)}$$

where Pi and Pj are real power injection at bus i and j, and B_{ij} is loss coefficients.

- Substituting this to the previous first derivative eqn.11, we get:

eqn.(21)

$$\frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} = \lambda(1 - \sum_j 2 B_{ij} P_j); \quad i = 1,\ldots.n$$

ELD Solution Methods Cont.…

- If the incremental costs are represented by a linear relationship following a quadratic characteristics, then $IC_i(P_{Gi}) = C_i(P_i) + P_{di;}$

$$C_i(P_{Gi}) + P_{Di} + 2\lambda B_{ii} P_i + 2\lambda \sum_{\substack{i=1 \\ i \neq j}}^{n} B_{ij} P_j = \lambda$$

eqn.(22)

- Substituting *PGi = Pi + PDi*, and simplifying for Pi

$$P_i = \frac{1 - \frac{P_{Di}}{\lambda} + 2\frac{c_i}{\lambda} P_{Di} - 2\sum_{\substack{i=1 \\ i \neq j}}^{n} B_{ij} P_j}{\left(\frac{c_i}{\lambda} + B_{ii}\right)}; \quad \mathrm{i} = 1,2,\ldots \mathrm{n}$$

eqn.(22)

- For any particular value of λ, the above equation can be solved iteratively by assuming initial values of Pi. It should be understood that losses can be considered not only as a constraint but also as objective function.

ELD Solution Methods Cont.…

Liner Programing ELD method : Economic load dispatch is carried out by linear programming in cases when the generator cost functions are linear.

- The fundamental idea is to minimize the overall cost of generation while taking demand and generation limitations into account. It needs the following points

a. The objective functions. Cost function

b. Power balance: energy conservation should met

c. Generation limits should met

d. Then, you can use different MATLAB source codes or modern optimization techniques

ELD Solution Methods Cont.…

Dynamic Programming for Economic Load Dispatch (ELD)

- One effective optimization method for resolving complicated issues is dynamic programming (DP)
- which divides larger issues into smaller ones.
- DP can be used in the context of Economic Load Dispatch (ELD) to find the best generation schedu le for a number of power plants while reducing costs.

The following points should be considered first:

- State Definition: The index of the generator under consideration and the total power generated up t o that point can be used to define each state.
- Decision Variable: Each unit's power generation is represented by the decision variable.
- Cost Function: Each generator's cost function, which shows the connection between cost and power output, is usually quadratic.

ELD Solution Methods Cont.…

Basic steps are:

1. Develop cost function for each generator

2. Describe the State and Stage of units

- Let A be the total power generated.

- Let k be the index of the generator currently being considered.

3. · Relation: The total cost at each state can be calculated as:

$$C(A,k) = \underset{P_{Gi}}{Min}(C(A-P_k, k-1) + C_k(P_k) + P_{los}(A)) \qquad \text{eqn.(23)}$$

4. For base case, C(0,0)=0;

5. Iteratively perform the relation until optimal solution is reached

7. Important Points for Economic Operations

- For better satisfaction of power operation the generation, voltage and lines should met as;

$$i.\ \ P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max}, \quad i = 1,2.....N$$

eqn.(24)

$$ii.\ \ Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max}, \quad i = 1,2.....N$$

$$iii.\ V_i^{\min} \le V_i \le V_i^{\max}, \quad i = 1,2.....N$$

$$iv.\ \ P_{Gij} \le P_{Gij}^{\max}, \textit{ for all lines}$$

$$v.\ \ Q_{Gij} \le Q_{Gij}^{\max}, \textit{ for all lines}$$

Important Points for Economic Cont.….

- The load flow equation should be satisfied since they are equality constraint
- The lower and upper limit of generations should not be violated
- The voltages should be within the acceptable standard limits
- The transmission line thermal and stability limit should not be violated
- The objective function minimization should takes these all in account and hence it can be performed using different iterative methods

Summary

- The optimal strategy to reduce the present generator operating costs is determined by economic dispatch.
- An effective strategy for resolving the economic dispatch problem is discussed in this lecture
- Limits on generators are readily managed and different power system constraints are mandatory while doing ELD
- The impact of losses is taken into account using penalty factors.
- The transmission system restrictions are neglected in the basic version of economic dispatch, but it's mandatory for actual power system operation.

References

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Thank you !