

# Power Systems Operation and Control

## Lecture 4

### Optimal Unit Commitment for Economic Power Operation

**Lecturer:** Teshome Goa (Assist. Prof.)

#### ***Lecture learning outcomes:***

At the end of this lecture, you will be able to:

- i. Understand the importance of optimal unit commitment.
- ii. Investigate the Unit Commitment Solution Techniques
- iii. Prepare the UC Priority Table Using Priority List Method.

# Content

1. Introduction
2. Importance of unit commitment
3. Optimal Unit Commitments (OUC)
4. Comparison of UC Vs ELD(UC Vs ELD)
5. Objective of UC(OUC)
6. UC Problem Formulation
7. UC Solution Techniques

**Summary**

**References**

# 1. Introduction

- The power system's overall load fluctuates during the day and reaches a distinct peak value every day
- It is not constant, follows a particular hourly load cycle over a day. Each period will have distinct load levels that vary, as indicated in Fig.1[1].

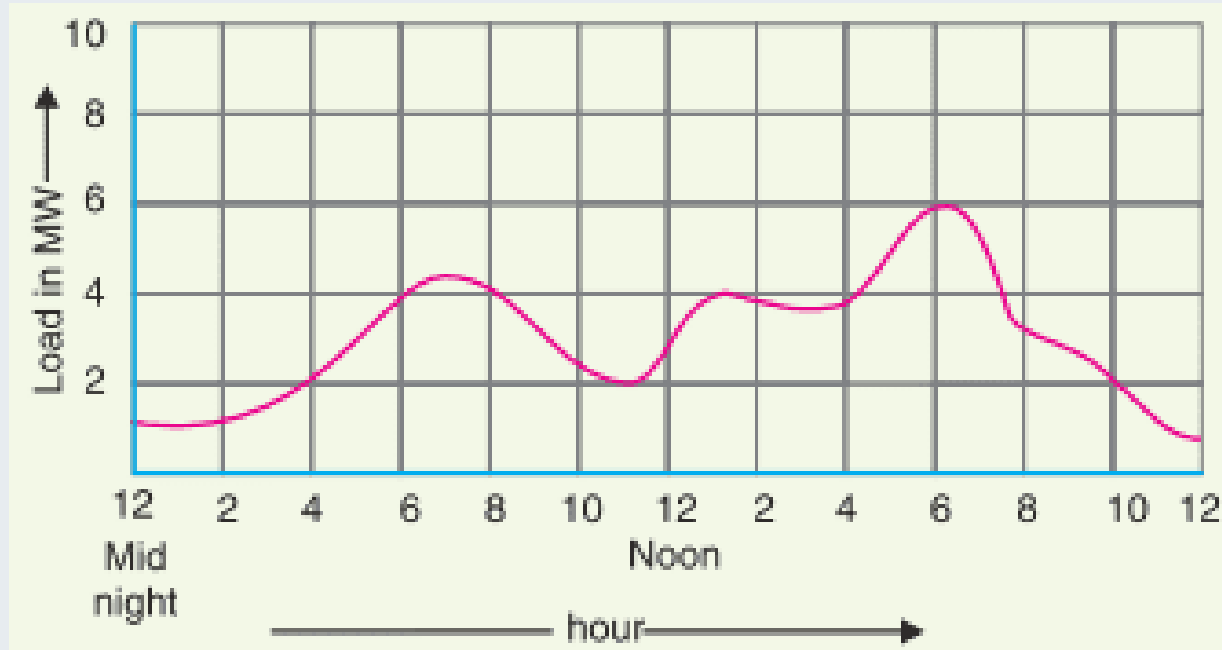


Figure 1. Load variation curve per day. Url: <https://www.electricalengineeringinfo.com/wp-content/uploads/2014/12/Variable2Bload2Bon2Bpower2Bstations.png>

# Introduction

# Cont....

- Accordingly, it is not advisable to run all available units continuously.
- Instead, it is important to plan ahead and choose which generators to start, when to connect them to the network, what order to shut down the operating units, and for how long.
- Unit commitment (UC) is the computational process used to make these decisions
- Thus, a unit is considered committed when it is scheduled to connect to the system based on the unit commitment process
- Unit commitment (UC) is a crucial optimization problem in power system operations.

# Introduction

# Cont....

- A power system's daily load profile varies significantly between off-peak and peak demand hours.
- Scheduling generators based on daily peak demand can result in lower power levels during off-peak hours.
- It will raise the cost of energy generation, which can be prevented by utilizing efficient scheduling.
- As a result, it is the responsibility of a grid operator to identify cost-effective generation.
- Schedules, often known as a Unit Commitment (UC) problem, a mathematical optimization its purpose is to coordinate the generation of a series of generators to meet the demand with the expected time slots as presented in Fig.2.

# Introduction

# Cont....

- The Colored boxes indicate the committed generators in the corresponding-colored time intervals

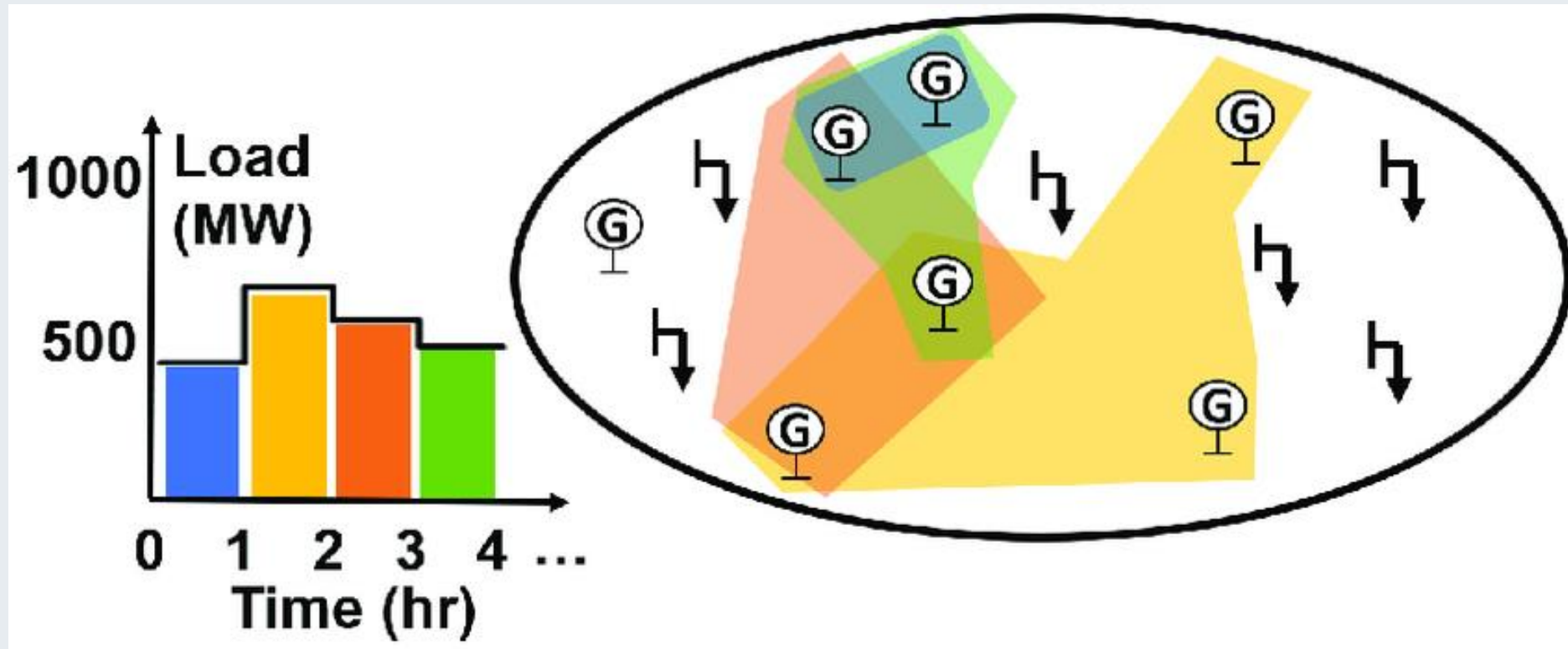


Figure 2. The unit commitment operation of Generators[2].

url: <https://www.researchgate.net/publication/364591709/figure/fig1/AS:11431281091274530@1666360995294/Unit-Commitment-Problem-Illustration-Colored-boxes-indicate-the-committed-generators-in.png>

# Introduction

# Cont....

- **It involves determining** which generating units to turn **ON or OFF** over a specific time horizon to meet electricity demand reliably and cost-effectively while satisfying various operational constraints[3].
- **Accordingly**, UC is a process used in power system operations to determine the optimal schedule for generating units over a specified time horizon, typically within a day or a week.
- Its goal is to ***meet forecasted electricity*** demand while minimizing costs and ensuring reliability

## The UC process involves[4]:

- ***Forecasting the electricity***
- **Selecting Generating Units:** Deciding **which** power plants to operate and **when** based on their availability, operational constraints, and costs.

# Introduction

# Cont....

- ***Selecting the time horizon***: usually considers multiple time intervals (hourly scheduling) throughout the scheduling period
- ***Identifying the constraints***: Incorporating various constraints, such as ramp rates, minimum up and down times, and fuel availability.
- ***Minimizing the overall cost***: Balancing generation costs against demand while considering start-up and shut-down costs of units.
- ***Providing reliable supply***: Ensuring that sufficient generation capacity is available to meet demand and maintain system stability.

## 2. Importance of unit commitment

- Unit Commitment (UC) plays a crucial role in power system operations for several reasons:
- Cost efficiency: by optimizing the schedule of generating units, UC minimizes operational costs, including fuel, maintenance, and start-up expenses, which is vital for utility companies and consumers.
- Reliability: UC ensures that sufficient generation capacity **is available to meet** expected electricity demand. This helps prevent outages and maintains system stability, particularly during peak demand periods.
- Integration of Renewables: As more renewable energy sources, like wind and solar, are integrated into the grid, **UC helps to manage their variability and uncertainty, ensuring that reliable backup generation** is available when renewable output is low.

# Importance of UC

# Cont.....

- Environmental considerations: Effective UC can help reduce greenhouse gas emissions by optimizing the use of cleaner energy sources and minimizing reliance on less efficient, higher-emission plants.
- Operational Flexibility: UC provides the ability to respond to real-time changes in demand and generation conditions, allowing for quick adjustments to the power supply as needed.
- Grid Stability: UC helps to maintain the balance between supply and demand, which is critical for grid stability and preventing frequency fluctuations.
- Regulatory compliance: Many power systems are subject to regulatory requirements that mandate certain reliability and operational standards, which UC helps to fulfill

### 3. Optimal Unit Commitments (OUC)

- In order to meet demand for energy while reducing costs, generation units are strategically scheduled as part of the optimal unit commitment (OUC) for economic power operation.

**The main objective is to reduce the overall cost of producing electricity, which comprises:**

- Fuel expenses related to power generation.
- Power plant start-up and shutdown costs.
- Operating expenses, which could include variable and maintenance expenditures.
- Ramp rates: the speed at which a device can change its output.
- Schedules for maintenance.

## 4. Comparison of UC Vs ELD(UC Vs. ELD)

- The two most important processes in power system management are Economic Load Dispatch (ELD) and Unit Commitment (UC), which is compared based on[5]:

### Objective function:

- UC involves selecting which generation units (power plants) to turn ON or OFF over a defined period of time (often a day or more).
- Economic Load Dispatch (ELD) seeks to minimize total operating costs while meeting generation constraints and meeting demand while all units are working

### Time Horizon :

- **UC usually** takes into account a longer time horizon (e.g., planning one day in advance), determining which units to commit for a given period.

# UC Vs ELD

# Cont.....

- ELD: focuses **on the immediate output of already committed units** and operates on a shorter time period (such as hourly or real-time).

## UC Inputs and Constraints:

- Demand and load characteristics predicted.
- Features of the unit (prices, capacity, ramp rates).
- Rules pertaining to the environment and other operational limitations.

## ELD Input Sources:

- Current generation mix and system load.
- Functions of committed units' generation costs.
- Units' physical limitations (max/min output).

# 5. Objective of UC(OUC)

Accordingly, the general objective of UC is to:

- Meet the Demand through different generation units: Ensure that total generation meets or exceeds the load forecast.
- Minimize Costs: Minimize operational costs, including fuel costs, startup costs, and maintenance costs
- Maintain Reliability: Ensure that the system can respond to contingencies and maintain system reliability.
- Meet the constraints:

## **Generation Units are :**

- Thermal Units: Use fuel (coal, natural gas, oil) to produce electricity.
- Renewable Units: Solar and wind, which are variable and depend on environmental conditions.
- Hydroelectric Units: Can adjust output quickly but depend on water availability.

## **Constraints are:**

- Power Balance: Total generation must equal total demand plus losses.
- Generation Limits: Each unit has minimum and maximum output levels

- Ramp Rates: Restrictions on how quickly a unit can increase or decrease output.
- Start-Up and Shut-Down Times: Time required to turn units on or off.
- Spinning reserve
- Minimum Up/Down Times: Minimum duration a unit must remain on or off.

## **Time Horizon**

- UC problems can be solved for different time horizons, such as hourly, daily, or weekly.

# 6. UC Problem Formulation

The Unit Commitment (UC) problem can be formulated as a mathematical optimization problem while identifying the variables like[6]:

- Decision Variable
- Objective Function and
- Constraints and Time Horizon

## A. Decision Variables:

- In unit commitment problems, **decision variables are essential** for determining which generation units to operate over a specific time horizon to meet demand while minimizing costs and satisfying operational constraints.
- The common decision variables are **grouped into binary and continuous variables** as given:
  - **Binary Variables:** It is a variable that indicates whether unit ' $i$ ' is committed or not at time ' $t$ ',  $U_{it}$ .
  - **Power Output Variables:** The power output of unit ' $i$ ' at time ' $t$ ', which can be continuous and may be subject to the unit's capacity, presented as  $P_{it}$ , which is also known as continuous variables

# UC Problem Formulation

# Cont....

- Start-Up Variables: A variable that indicates whether unit ' $i$ ' is started up at time ' $t$ ' (1 if starting up, '0' otherwise, denoted as  $S_{it}$ )
- Shut-Down Variables: it indicates whether unit ' $i$ ' is shut down at time ' $t$ ' (1 if shutting down, 0 otherwise, as indicated by  $d_{it}$ ).
- Minimum Up/Down Time Variables: which is the number of consecutive time periods that unit ' $i$ ' must remain ON once it is committed,  $M_{it}$  and the number of consecutive time periods that unit ' $i$ ' must remain OFF once it is recommitted,  $N_{it}$ .
- Ramp Rate Variables: the rate at which unit ' $i$ ' can increase or decrease its output at time ' $t$ ', reflecting the physical limitations of the unit,  $r_{it}$ .

# UC Problem Formulation

# Cont....

- Fuel Consumption Variables: The fuel consumed by unit 'i' at time 't' ( $f_{it}$ ), which may depend on the output level.
- Startup Costs: The cost incurred for starting unit  $i$ ,  $C_{\text{start}, i}$
- Maintenance and Availability: A variable indicating whether unit 'i' is available for operation at time 't' ( $a_{it}$ )
- Storage Variables: The energy stored in unit 'i' at time 't' ( $e_{it}$ ) and the charging or discharging state of storage unit 'i' ( $S_{it}$ ).
- Generally, the decision variables help to formulate the constraints and objective functions necessary to solve unit commitment **problems, balancing operational efficiency, reliability, and cost-effectiveness** in power generation.

# UC Problem Formulation

# Cont....

- A unit commitment problem's objective function usually seeks to minimize the entire operational expenses related to **producing power over a certain time horizon**.
- This function may involve a number of expenses, including fuel costs, start-up costs, and generation costs. It is given as:

The objective function is:

$$\text{Minimize } C_T = \sum_{i=1}^T \left( \sum_{i=1}^N C_i(P_{it}) + \sum_{i=1}^N C_{start,i} * (S_{it}) \right) \quad \text{eqn.(1).}$$

Where,  $C_i(P_{it})$  is the cost of generation unit 'i' at time 't', which is given as the quadratic function as:

$$C(P_{it}) = a_i + b_i * P_{it} + c_i P_{it}^2 \text{ Rs/h} \quad \text{eqn.(2)}$$

# UC Problem Formulation

# Cont....

- $C_{\text{start},i}$  is the fixed cost incurred for starting unit 'i', which is multiplied by the binary start-up variable  $s_{it}$
- The objective function in eqn.1 can be further extended if other costs like the fuel costs, environmental costs or emissions and etc are considered .
- The overall goal is to minimize the total costs while satisfying constraints related to demand, generation limits, minimum up and down times, and ramp rates as presented by:

# UC Problem Formulation

Cont....

- **Constraint: a. Demand constraint is given by:**

- Based on energy conservation, the total power generated must meet the total demand and loss as given

as;

$$\sum_{i=1}^N (P_{Gi}) = P_D + P_L \quad \text{eqn.(3)}$$

- The power transmission line losses can be modeled using a loss formula, represented as:

$$P_L = \sum_{I=1}^N \sum_{J=1}^N L_{ij} P_i P_j \quad \text{eqn.(4)}$$

Where,  $L_{ij}$  is the transmission line loss coefficients between two generators, generator  $i$  and  $j$ .

- $L_{ij}$  refers to the measure of energy loss in transmission lines due the line parameters.
- It helps evaluate how efficiently electricity is transmitted from one point to another, ultimately affecting the overall performance of the distribution system

# UC Problem Formulation

Cont....

- The line loss is approximated as:

$$P_L = \sum_{i=1}^N k_i P_i^2 \quad \text{eqn.(5)}$$

Where,  $k_i$  is the loss coefficient of generation  $i$  and given as;

$$k_i = \frac{\partial P_L}{\partial P_i} \quad \text{eqn.(6)}$$

- **b. The generation Constraints:** economic load dispatch should consider the generation limit constraints as given by;

$$\begin{aligned} P_{Gi}^{\min} &\leq P_{Gi} \leq P_{Gi}^{\max} \\ Q_{Gi}^{\min} &\leq Q_{Gi} \leq Q_{Gi}^{\max} \end{aligned} \quad \text{eqn.(7)}$$

# UC Problem Formulation

Cont....

c. Start-Up and Shut-Down Constraints:

- Verify that the device must be turned ON for a minimum amount of uptime and turned OFF for a minimum amount of downtime as given as:

- The up-time is given as:

$$\sum_{j=t}^{t+M_i-1} u_{ij} \geq M_i * u_{it} \quad \forall i \in \{1,2,\dots,M\}, \forall t \in \{1,2,\dots,T - M_i + 1\} \quad \text{eqn.(8)}$$

- The down-time is represented as:

$$\sum_{j=t}^{t+N_i-1} (1-u_{ij}) \geq N_i * u_{it} \quad \forall i \in \{1,2,\dots,N\}, \forall t \in \{1,2,\dots,T - N_i + 1\} \quad \text{eqn.(9)}$$

Then, the overall optimization model could be:

$$\text{Minimize } C_T = \sum_{i=1}^T \left( \sum_{i=1}^N C_i(P_{it}) + \sum_{i=1}^N C_{start,i} * (S_{it}) \right) \quad \text{s.t: the a, b \& constraints} \quad \text{eqn.(10)}$$

# 7. UC Solution Techniques

- UC is an important issue in power system operation, where the objective is to decide which **generation units to turn on** or off in order to satisfy demand while keeping costs as low as possible.
- The following are some typical unit commitment problem-solving strategies
  1. Priority-list schemes.
  2. Dynamic programming (DP) method.
  3. Lagrange's relaxation (LR) method.

## 7.1 Priority List method of UC (PLM)

- The **Priority List Method is a heuristic approach used in Unit** Commitment (UC) problems to simplify the scheduling of generating units.
- This method prioritizes units based on **specific criteria** to determine their commitment status.

Prioritization criteria include:

**Cost of Generation:** Units are ranked according to their variable operating costs (fuel, maintenance, etc

- **Units with lower costs** are prioritized for commitment
- **Units with longer minimum up/down times** may be given higher priority in order to prevent frequent cycling; and
- **units with higher availability** and reliability may be given higher priority in order to maintain system stability.

## Steps:

### A. Data Collection:

- Gather relevant data, including: Load forecast for the planning period (hourly or sub-hourly)
- Generation capacities and cost functions of available units.
- Start-up and shut-down times of units.
- Minimum up and down times.
- System constraints (e.g., reserve requirements, ramp rates).

B. Cost Calculation: Determine production costs for each generating unit.

This is based on the unit's fuel cost characteristics and may include variable expenses (marginal cost).

Fixed costs (where relevant).

C. Priority List Formation: Rank units based on operational costs.

This can be accomplished by constructing a priority list that takes into account the incremental cost of generating.

Other benefits include dependability, adaptability, and startup costs.

## D. Load Forecasting:

- Predict the expected load for the scheduling period, typically in hourly intervals.
- E. Initial Commitment:
  - Begin with the highest-priority unit and commit units based on their rank until the forecasted load is met.
  - Ensure that committed units meet the minimum generation requirements and operational constraints.

F. Adjustments for limits: Consider limits such as minimum up/down times, reserve requirements, and fuel limitations. Adjust the commitment as needed, either by re-evaluating the priority list or exploring alternate units.

G. Cost minimization: Determine the total cost of committed units. If applicable, iterate over the priority list to investigate various combinations of committed units that could minimize costs while still satisfying demand.

- Finalize the commitment schedule for the planning period based on the revisions and cost analysis.
- Ensure that all operational limitations and reliability requirements are met.
- Documentation and Reviews: Document the judgments made, as well as any assumptions and limits that were considered.
- Examine the commitment schedule for compliance with system requirements and performance indicators.
- Implementation and Monitoring: Execute the commitment schedule in real-time operations.
- Continuously monitor system performance and demand, and alter commitments as needed based on real-time data.

# PLM

# Cont....

- **Example:** Perform the optimal unit commitment for the three generators using the priority list method. The three generators having their cost function ,and limits as given below are expected to feed the demand of 600 MW.

Cost function

$$C_1 = 20 + 2.5 * P_1 + 0.002 * P_1^2 \text{ MW / hr / USD}$$

$$C_2 = 22 + 1.5 * P_2 + 0.003 * P_2^2 \text{ MW / hr / USD}$$

$$C_3 = 23 + 1.5 * P_3 + 0.004 * P_3^2 \text{ MW / hr / USD}$$

Generation limits

$$300 \leq P1 \leq 900 \text{ MW}$$

$$150 \leq P2 \leq 850 \text{ MW}$$

$$100 \leq P3 \leq 400 \text{ MW}$$

- **Solution** :To supply a total load of 600 MW most economically, the combinations of units and their generation status, there are  $2^n$  Combinations ,  $2^3 = 8$  , options.
- The first step is calculating the production cost as follows:
- Calculate the production cost for  $P_1$  considering the maximum generation limits(900MW, 850MW and 400 MW) and given as:

$$C(P_1) = (20 + 2.5 * 900 + 0.002 * 900 * 900MW / hr / USD) / 900$$

$$C(P_1) = 4.32USD/hr.$$

$$C(P_2) = 4.07 USD/hr.$$

$$C(P_3) = 3.16USD/hr.$$

# PLM

# Cont....

- The Switching priority order based on the costs and its operation combination are given in Table 1. a&b

Table 1: a. Priority ON/OFF status

Units	USD/MW h	$P_{Gmin}$	PGmax
3	3.16	100	400
2	4.07	150	850
1	4.32	300	900

b. the combination of ON/OFF status

No.	COMBINATIONS			Remarks
	PG1	PG2	PG3	
1	0	0	0	Not feasible
2	0	0	1	Not feasible
3	0	1	0	Feasible
4	0	1	1	Feasible
5	1	0	0	Feasible
6	1	0	1	Feasible
7	1	1	0	Feasible
8	1	1	1	Feasible

- Then, the share of each generation to meet the demand is calculated based on ELD given below

# PLM

- When unit 1 and 2 is on to supply 600 load, the equation can be rewritten as:

$$P_1 + P_2 = 600 \quad \text{eqn.1}$$

$$0.004P_1 + 2.5 = \lambda$$

$$0.006P_2 + 1.5 = \lambda$$

$$\Leftrightarrow 0.006P_2 + 1.5 = 0.004P_1 + 2.5$$

$$\Rightarrow -0.004P_1 + 0.006P_2 = 1 \quad \text{eqn.2}$$

Then,

$$P_1 + P_2 = 600$$

$$-0.004P_1 + 0.006P_2 = 1$$

$$-0.004(600 - P_2) + 0.006P_2 = 1$$

$$P_2 = \frac{3.4}{0.01} = 340 \text{ MW}$$

$$P_1 = 600 - 340 = 260 \text{ MW}$$

# Cont....

- When unit 1 and 3 are ON to supply 600 load, the equation can be rewritten as:

$$0.004P_1 + 2.5 = 0.008P_3 + 1.5$$

$$0.004P_1 - 0.008P_3 = -1 \quad \text{eqn.(1)}$$

$$P_1 + P_3 = 600 \quad \text{eqn.(2)}$$

$$P_1 = 600 - P_3$$

$$\Leftrightarrow 0.004(600 - P_3) - 0.008P_3 = -1$$

$$-0.012P_3 = -3.4$$

$$P_3 = \frac{3.4}{0.012} = 283.33$$

$$P_1 = 316.67$$

# PLM

# Cont....

- When unit 2 and 3 are ON to supply 600 load, the equation can be rewritten as:

$$0.006P_2 + 1.5 = 0.08P_3 + 1.5$$

$$0.006P_2 - 0.008P_3 = 0 \quad \text{eqn.(1)}$$

$$P_2 + P_3 = 600 \quad \text{eqn.(2)}$$

$$P_2 = 600 - P_3$$

$$\Leftrightarrow 0.006(600 - P_3) - 0.008P_3 = 0$$

$$-0.014P_3 = -3.6$$

$$P_3 = \frac{3.6}{0.014} = 257.14$$

$$P_2 = 342.86 \text{ MW}$$

- When all units are ON to supply 600 load, the iterative solution considering unit constraints are applied and their share is given as :

- $P_1 = 300 \text{ MW}$
- $P_2 = 150 \text{ MW}$
- $P_3 = 150 \text{ MW}$

- Finally, the overall priority order based the operation combination presented in Table 1.b is presented in Table 2.

Table 2. The overall priority List combination three Unit operation to meet 600 MW load

No.	Combination, status of generations			PG, max	PG, min	PG1	PG2	PG3	C1	C2	C3	CT
1	0	0	0	0	0	inf	inf	inf	-	-	-	-
2	0	0	1	400	100	inf	inf	inf	-	-	-	
3	0	1	0	850	150	0	600	0	0	932.8	0	932.8
4	0	1	1	1250	250	0	322.86	257.14	0	819.00	673.19	1,492.2
5	1	0	0	900	300	600	0	0	1,527.2	0	0	1,527.2
6	1	0	1	1300	450	316.67	0	283.33	1,012.23	0	769.09	1,781.3
7	1	1	0	1250	250	260	340	0	805.2	878.8	0	1,684
8	1	1	1	2150	550	300	150	150	950	314.5	338	1,602.5

- Table 2, presents the share of each generator, its cost and the overall cost of generations to satisfy the required load demands

## 7.2 Dynamic Programming

- Dynamic **Programming (DP)** is a **systematic approach for solving** the Unit Commitment (UC) problem by breaking it down into smaller, manageable sub-problems.

### Steps are :

- State Representation:
  - Define the state at each time period ' $t$ ' as a combination of  $i^{\text{th}}$  current time period.
  - The commitment status of each unit (on/off).

The cumulative cost incurred so far.

- Decision Variables: For each unit, decide whether to commit (on) or decommit (off) based on operational constraints.
- Cost Calculation: Include both operating costs and start-up costs for units being turned on.
- Recursion: Transition from one state to another based on the decisions made.

## 7.3 Other UC Solution Techniques

- Mixed-Integer Linear Programming (MILP):
- Lagrangian Relaxation:
- Genetic Algorithms:
- Tabu Search:
- Simulated Annealing:
- ANN
- ANFIS
- etc

# Summary

- Efficiently controlling the commitment of generation units provides system dependability and cost effectiveness
- Numerous solution strategies available, ranging from accurate methods like priority list method, Dynamic Programming and Mixed-Integer Linear Programming etc.
- It's observed that the operation of real-world scenarios include a variety of limitations, such as operational limits, regulatory requirements, and the integration of renewable energy sources.
- The importance of committing and recommitting certain power plants/units for better, and cost-effective operation is discussed well in this lecture
- Finally, it's observed that the problem formulations (formulation of objective functions, decision variables, constraints and control variables) takes paramount while performing the unit commitment using different modern optimization techniques

# References

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Thank you !