Course: Compiler Construction

Week 3: Finite Automata (FA)

Lecturer: Martha Gichuki

Course Description

- The course begins with a coverage of basic Principles in Compiler Design with an introduction to Formal programming language translation and compiler design concepts; compilers and interpreters.
- A coverage of Language theory, Parsing Context-Free Languages (Top - down and bottom - up parsing), translation specifications and machine- independent code optimization will then follow.

Course Description …

- The main phases of compilation i.e. Lexical, Syntax and Semantic analysis will be taught.
- Code generation and Optimization, Symbol table design, Program compilation, Loading and execution will then be covered.
- Compilation techniques, Optimization, Design of a simple complete compiler. will culminate the course.

Learning outcomes Week 3: Finite Automata

At the end of the lecture, you will be able to:

(i) Define Regular Expressions and Regular Languages

(ii) Describe how Deterministic FA (DFA) and Non-deterministic FA (NFA) computes

(iii)Construct DFAs and NFAs using regular expressions.

Finite Automata

- ✓A Finite Automaton is a recognizer used to identify the tokens occurring in an input stream.
- ✓Simply, it is a **machine with a finite number of states and a finite number of transitions** between them[1]

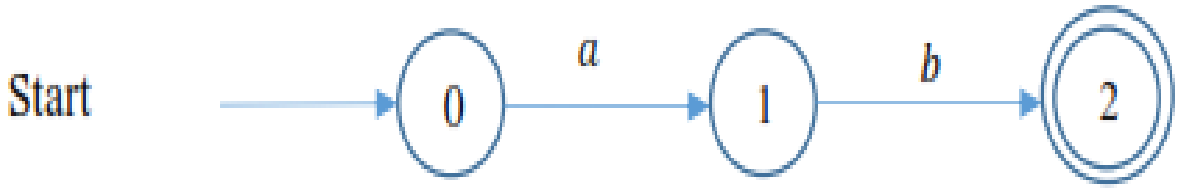
1. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 172

Representing Finite Automata[2]

- ✓ There is a distinctive start state where the machine starts.
- ✓ Between the states, there are transitions labelled by the alphabets of the language
- ✓ **Example;** if the machine is at state $\mathbf{S_i}$ and there is a transition labelled by alphabet $\boldsymbol{a_k} \in \boldsymbol{\Sigma}$ to the state $\mathbf{S_j}$ then the machine can perform such a transition provided that the current symbol is $\boldsymbol{a_k}$.

2. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison-Wesley Pub Co, ISBN: 0201100886 (2007) page 172

- ✓ Starting from the **start state**, the machine finally reaches some state after exhausting all the input symbols.
- ✓ The **final state** reachable for all the input streams (tokens) is called the **acceptor/final state** e.g. the following is a finite automata for the string ab^3.



3. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison-Wesley Pub Co, ISBN: 0201100886 (2007) page 173

- ✓If the machine is in the state $\mathbf{S_i}$ and there does not exist any possible transition from $\mathbf{S_i}$ to the next input symbol then the input is **rejected**[4].

4. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison-Wesley Pub Co, ISBN: 0201100886 (2007) page 174

There are two types of Finite Automata[5]

i. Non-deterministic Finite Automata (NFA)
ii. Deterministic Finite Automata (DFA)

5. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 172

1. *Non-deterministic Finite Automata (NFA)*

- ✓This is a finite automata that has ability of **more than one possible transitions** from a state on the same input symbol.
- ✓The actual transition occurring is selected **non-deterministically** by the machine[6].

6. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison-Wesley Pub Co, ISBN: 0201100886 (2007) page 172

- ✓ Another non-determinism lies within the empty transitions **(e-transition).**
- ✓ If there is an **e-transition** from state $\mathbf{S_i}$ to state $\mathbf{S_j}$ then the machine can do such a transition without consuming any other input[7].

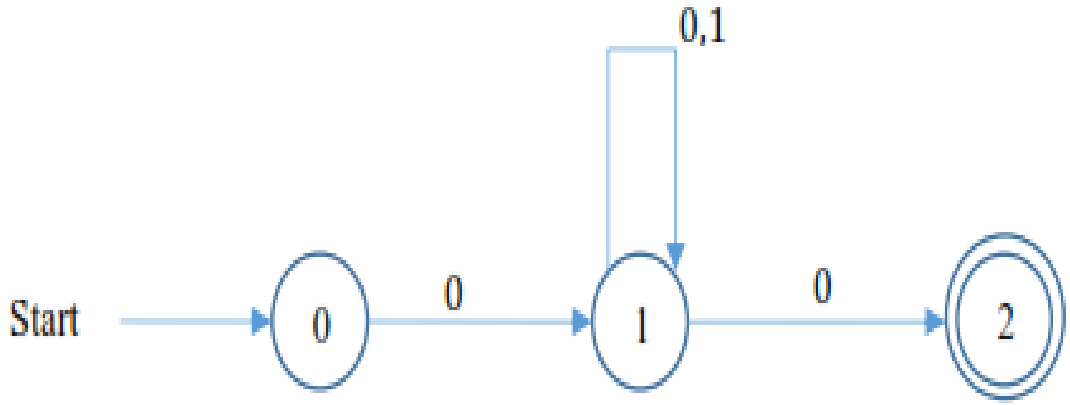
7. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 173

- ✓ An input is said to be **accepted/recognized** by the automata if there exists at least one path from the start state of the machine to the final state whose transitions are governed by the input stream[8].

8. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 173

Examples:

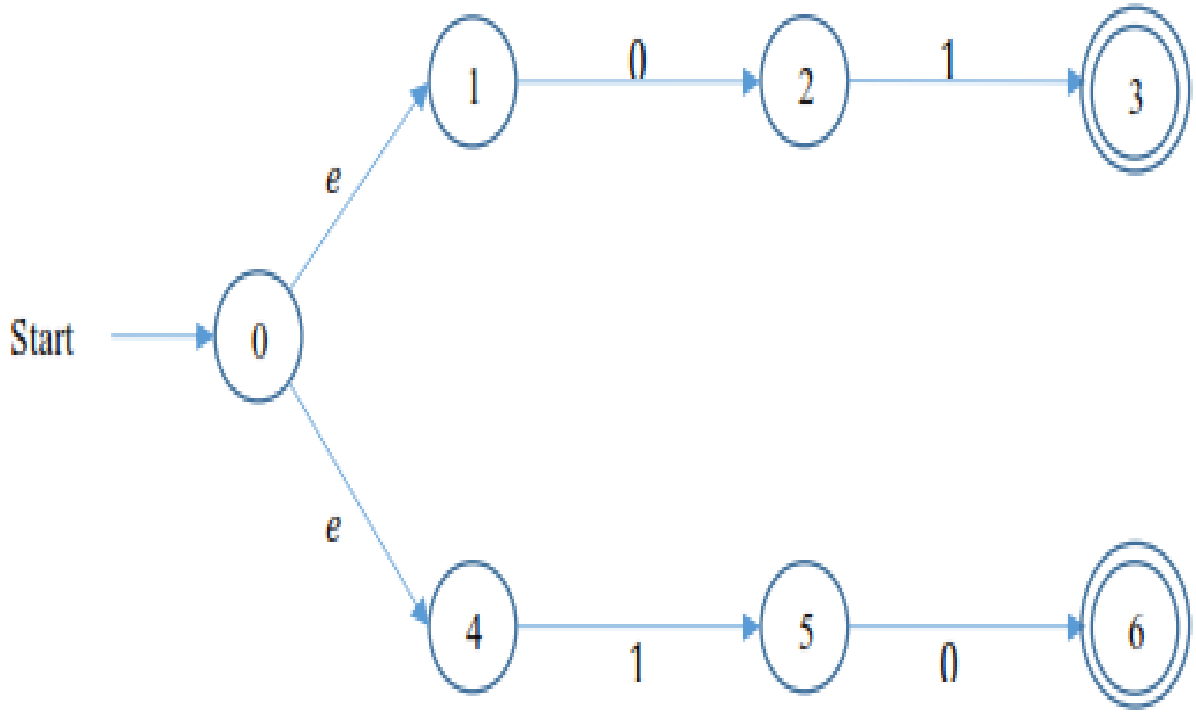
i. Construct an NFA for the regular expression 0(0|1)*0



Note: from **state 1**, there are two transitions labelled by **0**, one to **state 1** itself (loop) and the other to **state 2**[9].

9. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 172

ii) Construct NFA for the regular expression 01|10[10]



10. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison-Wesley Pub Co, ISBN: 0201100886 (2007) page 172

II). Deterministic Finite Automata (DFA)

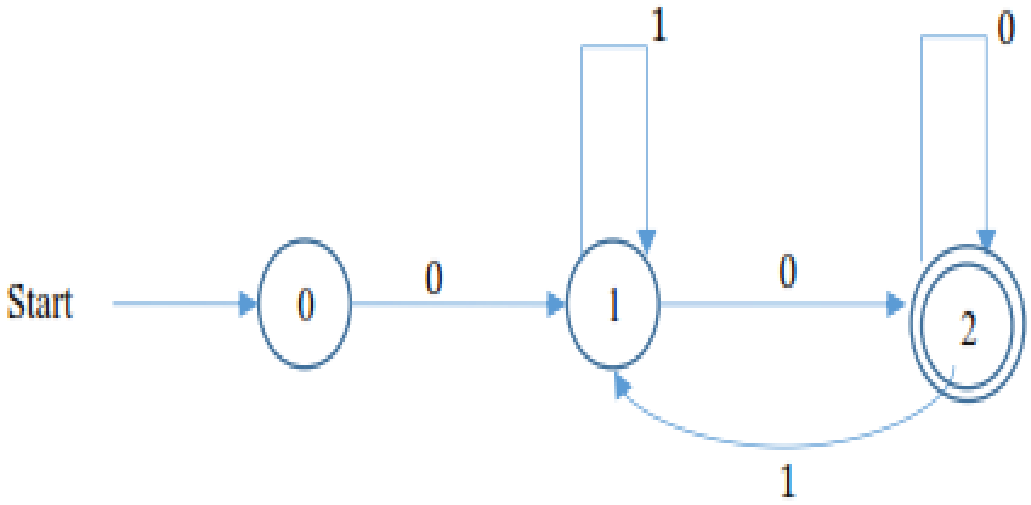
- ✓This is a finite automata that cannot have more than one transition emanating from the same state labelled by the same input symbol.
- ✓Also, there cannot be any empty transitions[11].

11. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 174

Examples

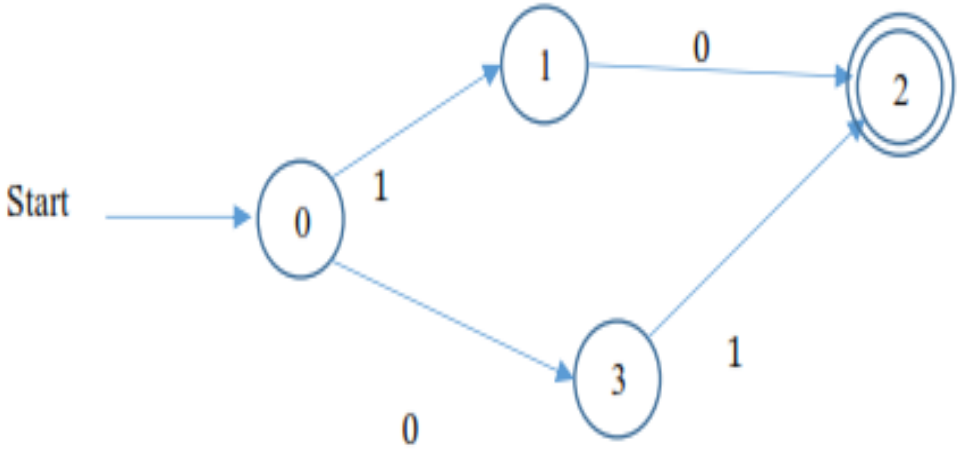
i. Construct a DFA for the following regular expressions[12]:

a) 0(0|1)*0



12. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 176

b) 01|10



Note:

- ✓Every non-deterministic alternative of an NFA gets converted into an explicit path from **start to final state** in a DFA.
- ✓This results in an explosion in the number of states in a DFA[13].

13. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 176

The following is an algorithm to test whether an input stream is accepted by a DFA or not:

Algorithm: DFA-test

Input: a DFA with a start state S_0.

An input stream

Output: "yes" is accepted, "no" otherwise[14].

14. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 175

Begin

$S=S_0$

While not end-of-input do

let c = next input symbol

if there is a transition on c from S to S_1 then

$S=S_1$

end while

If S is a final state and c=end-of-input then

return "yes"

else

return "no[15]*"*

15. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 176

Note:

- ✓ the existence of non-determinism makes the task of testing if an input stream is acceptable difficult.
- ✓ this is because it requires *trying all possible alternatives* that may occur in non-deterministic transition of states[16]

16. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 177

Converting Regular Expressions to NFA

- ✓**Three steps** are involved: -

(i) Break down the regular expression into the simplest sub-expressions,

(ii) Construct the corresponding NFAs,

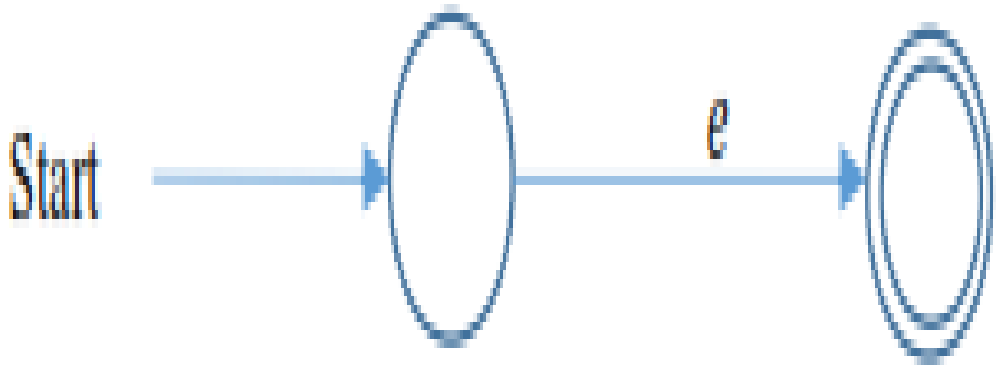
(iii) Combine these small NFAs guided by the operations of the **regular expression**[17].

- ✓This construction is known as **Thompson's construction rules;** which are as follows:-

17. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 177

Thompson's construction rules…

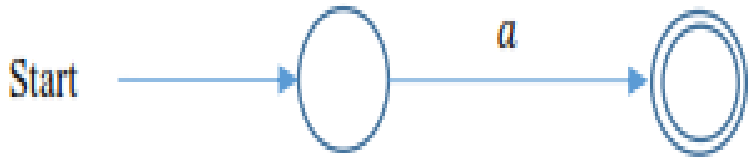
i) For ***e***, the NFA consists of two states (start and final state) [18]. The transition is labelled by (***e***).



18. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 184

Thompson's construction rules…

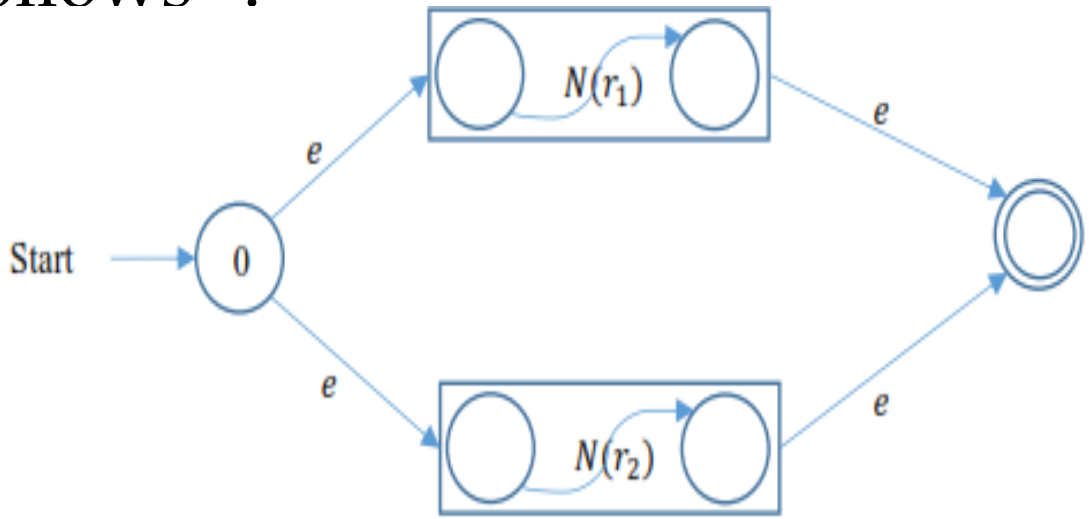
ii) For any alphabet symbol '**a**' in the alphabet **set Σ**, the NFA also consists of two states with transition labelled by ***a***[18]



18. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 184

Thompson's construction rules…

iii) For the regular expression $\mathbf{r_1 | r_2}$, if $\mathbf{N(r_1)}$ is NFA for $\mathbf{r_1}$ and $\mathbf{N(r_2)}$ is NFA of $\mathbf{r_2}$ then NFA of $\mathbf{N(r_1 | r_2)}$ is constructed as follows[19]:



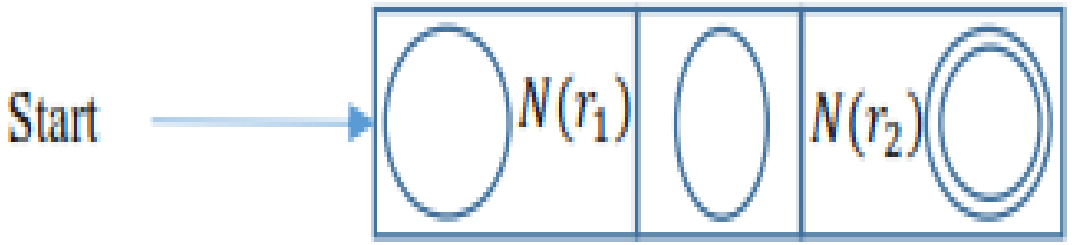
19. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison-Wesley Pub Co, ISBN: 0201100886 (2007) page 185

- ✓This means that the ***e*-transitions** are introduced from the new start state to the start states of $\mathbf{N(r_1)}$ and $\mathbf{N(r_2)}$ and similarly from the final state of $\mathbf{N(r_1)}$ and $\mathbf{N(r_2)}$ to the newly created final state[20].

20. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison-Wesley Pub Co, ISBN: 0201100886 (2007) page 185

Thompson's construction rules…

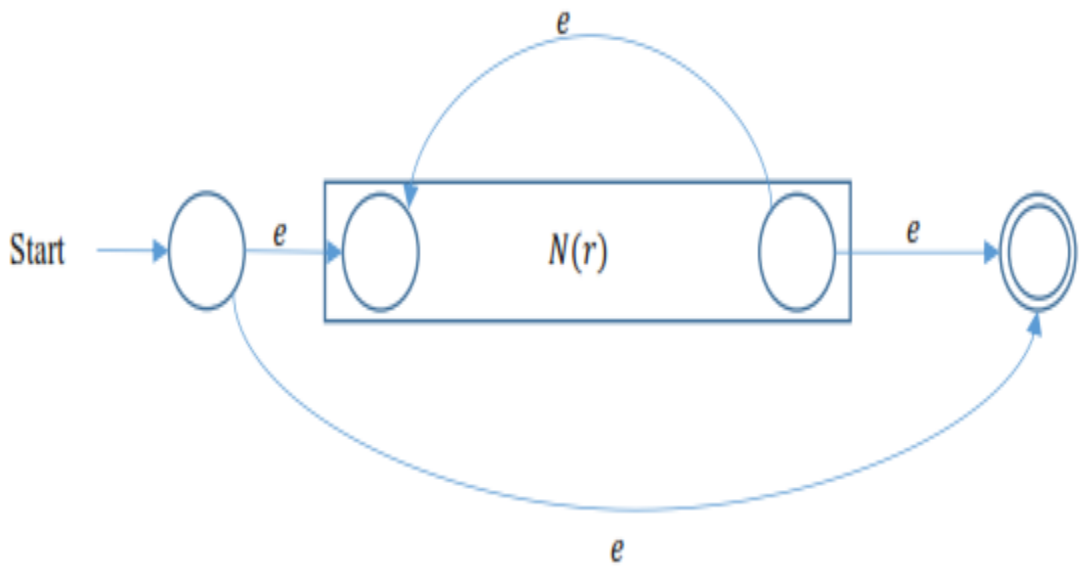
iv) For the regular expression $\mathbf{r_1 r_2}$, the NFA $\mathbf{N(r_1 r_2)}$ is constructed by **merging** the NFAs $\mathbf{N(r_1)}$ and $\mathbf{N(r_2)}$ i.e. the final state of $\mathbf{N(r_1)}$ is merged with the start state of $\mathbf{N(r_2)}$ as shown[21]: -



21. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 160

Thompson's construction rules…

v) For the regular expression **r*, N(r*)** is constructed as follows[22]:-



22. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 186

- ✓This means that **e-transitions** from the new start state to the new final state correspond to **zero occurences of r**, whereas, from the final state to the initial state of **N(r)** corresponds to the repeated occurrence of $\boldsymbol{r}$ [23]

23. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 186

Thompson's construction rules…

vi) If **N(r)** is NFA for a regular expression, it is also NFA for the parenthesized expression **(r)** [23].

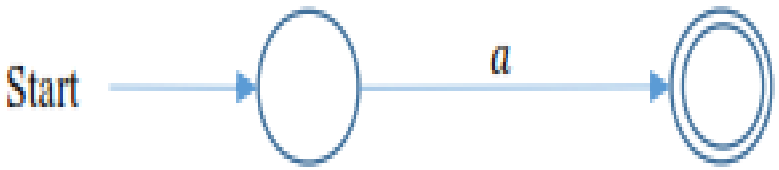
23. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 159

Example:

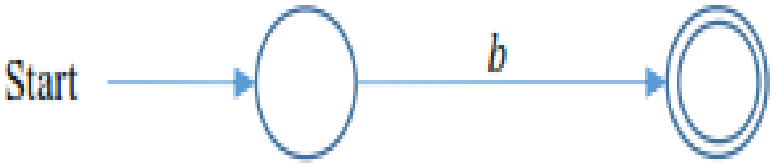
Use Thompson construction rules to come up with the NFA of the following regular expression **a(a|b)*ab**

The sub-expressions are[24]:-

1. **a**

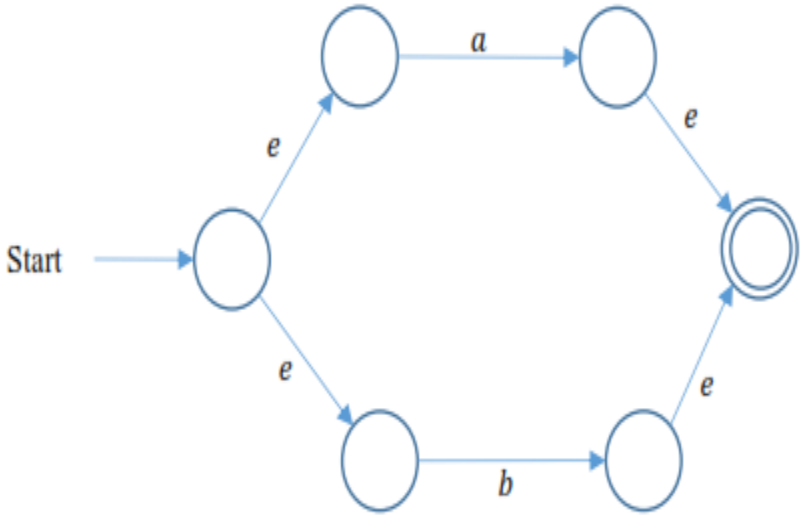


2. **b**

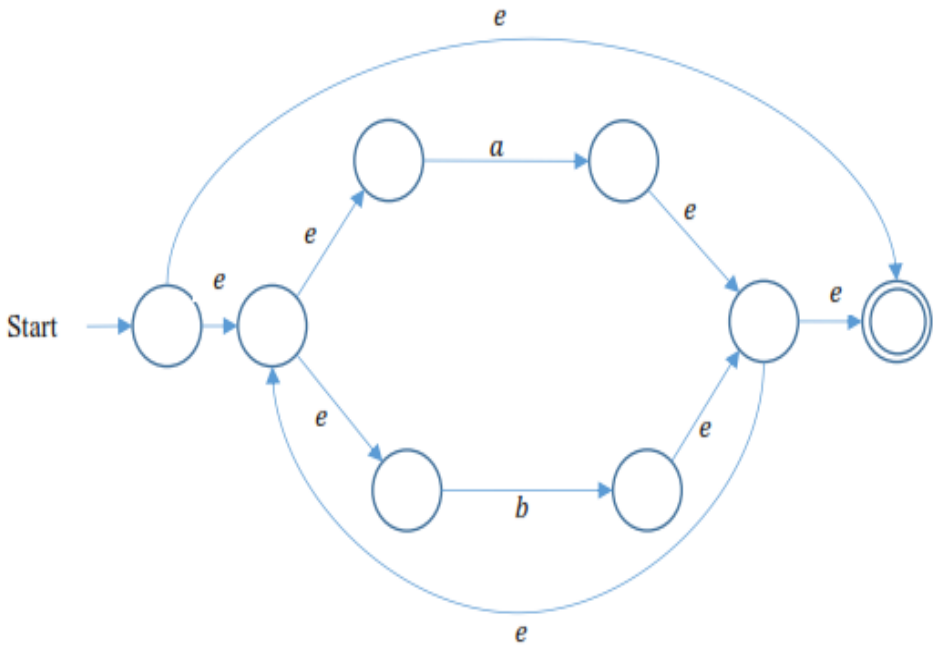


24. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 160

3. a|b...[25]

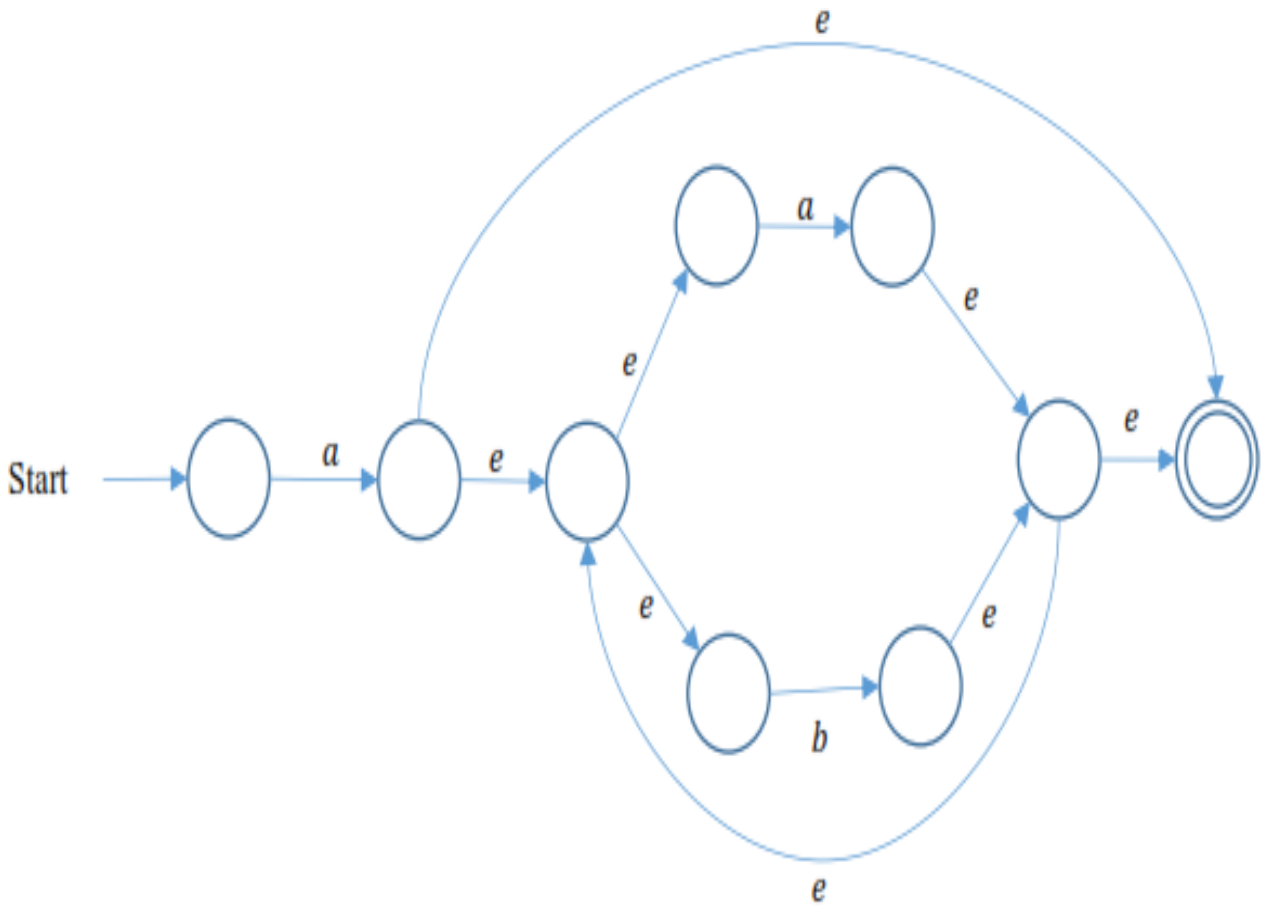


4. (a|b)*



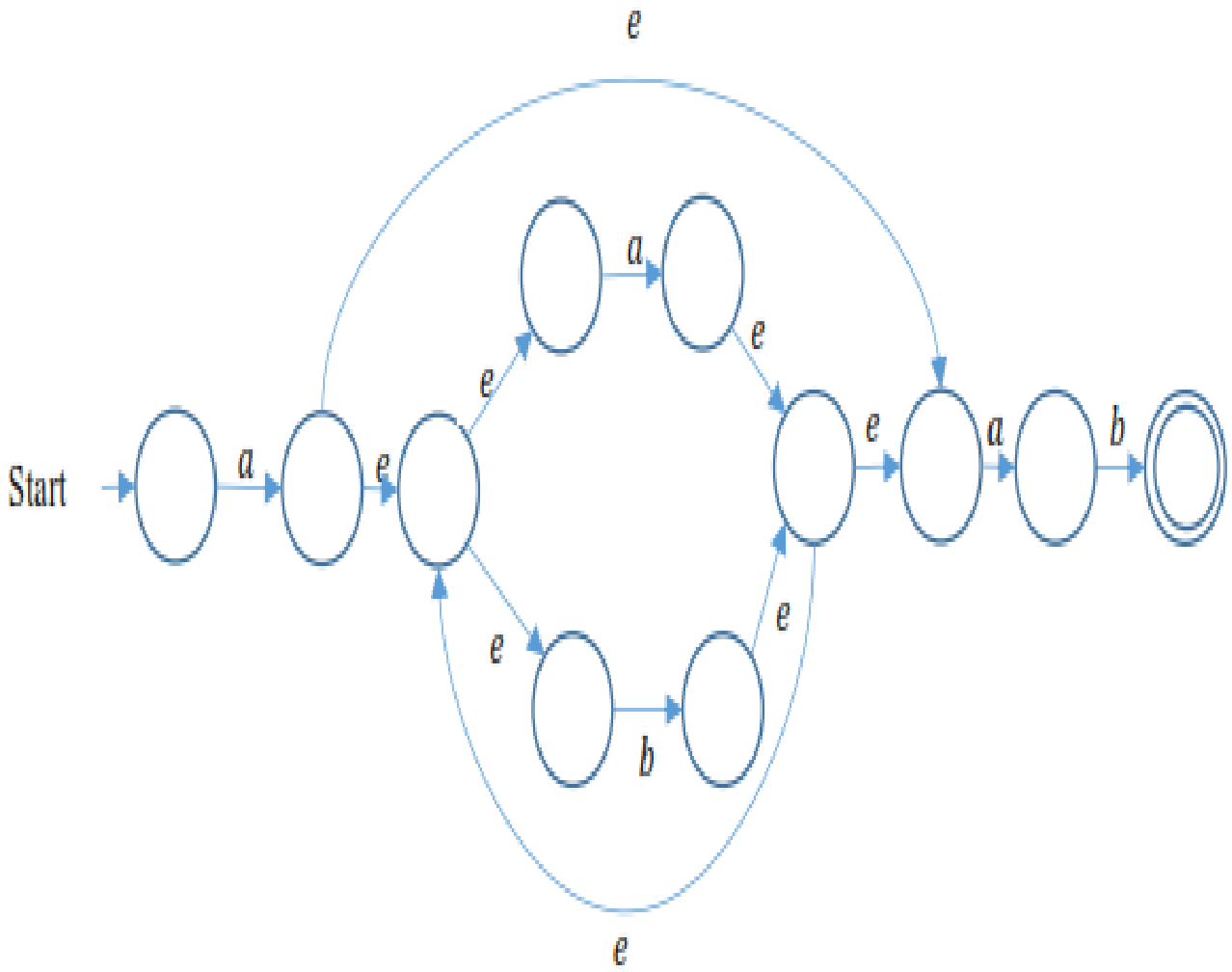
25. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 184

5. a(a|b)* …[26]



26. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 188

6. a(a|b)*ab…[27]



27. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; Addison- Wesley Pub Co, ISBN: 0201100886 (2007) page 188

Content Covered in Week 3: Finite Automata

At the end of the lecture, we were able to:

(i)Define regular expressions and regular Languages

(ii) Describe how DFA and NFA computes

(iii)Construct DFAs and NFAs using regular expressions.

Course Text Books

1. Compilers: Principles, Techniques and Tools, Aho, Lam, Sethi, and Ullman; *Addison- Wesley Pub Co,* ISBN: 0201100886 (2007))
2. Compiler Construction: Principles and practices; Kenneth C. Louden; *Cengage Learning*; 1st edition, ISBN-10 : 0534939724 (1997)
3. Basics of Compiler Design: Torben Mogensen; *DIKU University of Copenhagen Universitetsparken 1 DK-2100 Copenhagen DENMARK* (2007)
4. Engineering a Compiler: 2nd edition; Keith D. Cooper and Linda Torczon; *Morgan Kaufmann Publishers* ISBN: 978-0-12-088478-0 (2003)
5. Compiler Design: Santanu Chattopadhyay; *PHI Learning Publishers*, ISBN 812032725X, 9788120327252 (2005)