Theory of Structures - I

Chapter 1. Introduction

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Chapter 1. Introduction

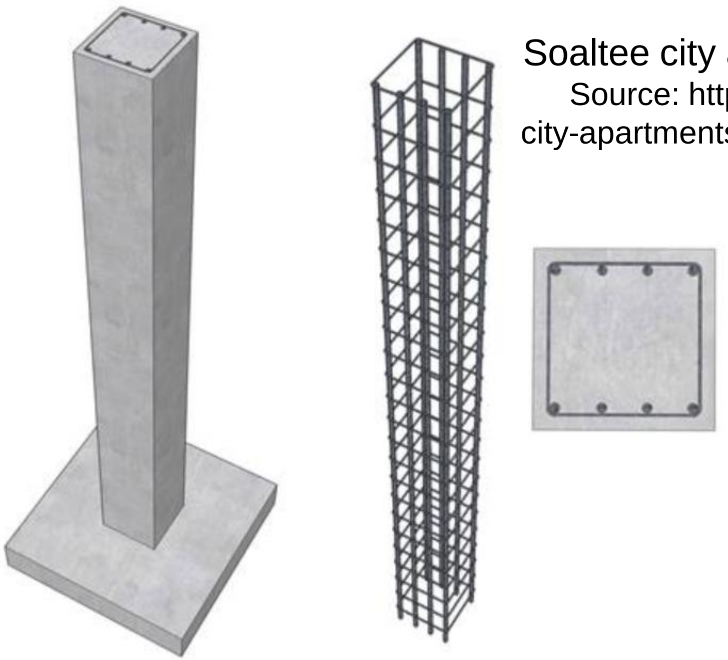
1.1 Types of structures
1.2 Structural mechanics
1.3 Two basic approaches of structural analysis
1.4 Linearly elastic structures
1.5 Non-linearity in Structural Analysis
1.6 Computer-based methods
1.7 Principle of superposition

1.1 Types of structures: based on materials used

[A] RCC (Reinforced Cement Concrete) structures:

- Cement concrete reinforced with steel bar, hence the name *reinforced* concrete structures
- Resilient to weather and environmental conditions
- Heavy structures

Soaltee city apartments, Kathmandu, Nepal
Source: https://basobaas.com/property/soaltee-city-apartments-apartment-for-sale-in-ravi-bhawan-kathmandu-6010



Source: https://www.structurepoint.org/pdfs/Interaction-Diagram-Tied-Reinforced-Concrete-Column-Symmetrical-ACI318-14.html

1.1 Types of structures: based on materials used

[B] Steel structures:

- Light-weight structures
- High strength to width ratio
- Skyscrapers : Steel frame

Kyoto Tower
Source: https://en.wikipedia.org/wiki/Kyoto_Tower#/media/File:Kyoto_Tower_at_dusk,_from_railway_station.jpg

Tokyo Gate Bridge
Source: https://www.roadtraffic-technology.com/projects/tokyo-gate-bridge/

A General Steel Framework
Source: https://www.bharatsteels.in/blog/common-steel-frame-construction-types-and-their-components/

1.1 Types of structures: based on materials used

[C] Timber structures:

- Vernacular construction
- Timber provides ductility to certain extent unlike brittle masonry structures

Horyuji, Nara, Japan
Source: https://www.interactiongreen.com/horyuj-world-oldest-wooden-architecture/

Modern timber frame building
Source: https://www.timber2udirect.co.uk/5-benefits-of-using-a-timber-frame-for-your-home-construction/

1.1 Types of structures: based on materials used

[D] Brick Masonry structures:
- Traditional construction
- Locally manufactured bricks
- Very good in compression, very weak in tension
- Brittle failure during seismic events
- 3-4 stories max without any additional reinforcement

A traditional Newari house in Kathmandu, Nepal

Source:
Romão, X., Paupério, E., & Menon, A. (2015). Traditional construction in high seismic zones: A losing battle? The case of the 2015 Nepal earthquake. *Seismic retrofitting: Learning from vernacular architecture*, 93-100.

1.1 Types of structures: based on materials used

[E] Stone Masonry structures:

- Traditional construction, especially in higher Himalayan region in case of Nepal, where stones are found in abundance rather than soil suitable for bricks
- Very good in compression
- 1-2 stories

A semi-dressed dry stacked stone masonry building in mid-mountains of Nepal

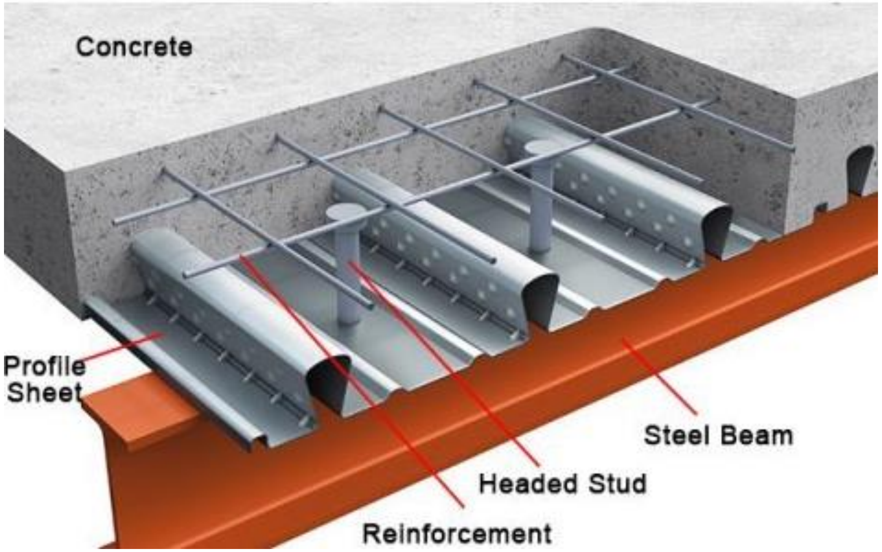
Source:

Bothara, J., Ingham, J., & Dizhur, D. (2018, December). Understanding, experience and research on seismic safety of low-strength loadbearing masonry buildings. In *16th Symposium on Earthquake Engineering, Paper No.: Kn2* (pp. 1-26).

1.1 Types of structures: based on materials used

[F] Composite structures:

- Combination of more than one material for construction
- For example, steel-concrete composite structures

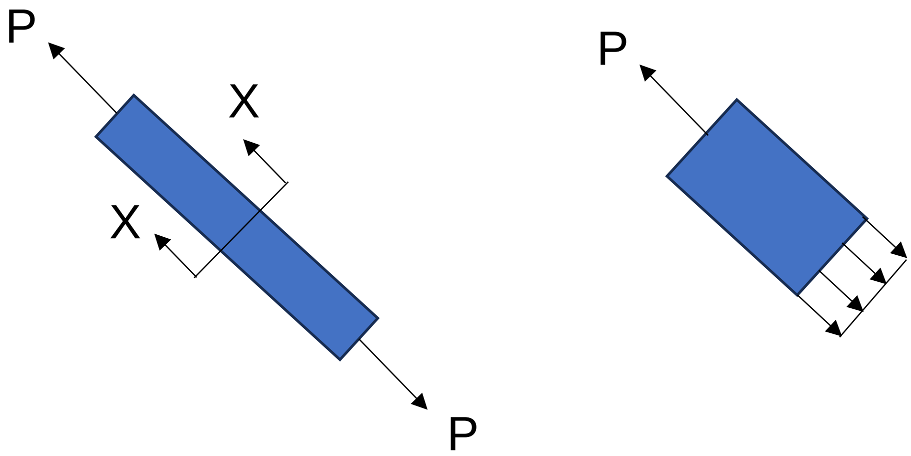


Source: https://www.ibeehivesteelstructures.com/what-are-steel-concrete-composite-structures/

1.1 Types of structures: based on internal stress developed

[A] Uniform stress members:

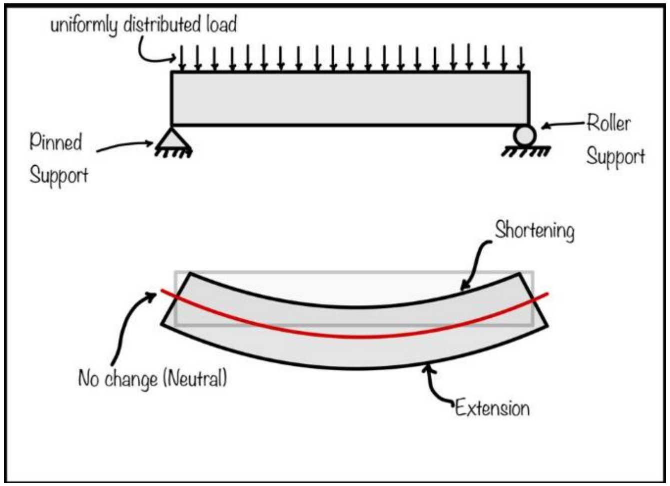
- Internal stress developed along the depth of the members is uniform
- For example, truss, arch, cable, etc.



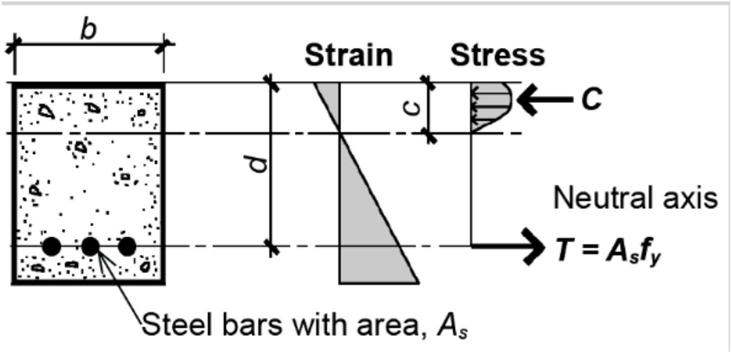
1.1 Types of structures: based on stress distribution

[B] Non-Uniform stress members:

- Internal stress developed along the depth of the members is not uniform
- For example, beam



Source: https://www.eigenplus.com/stresses-in-beam/

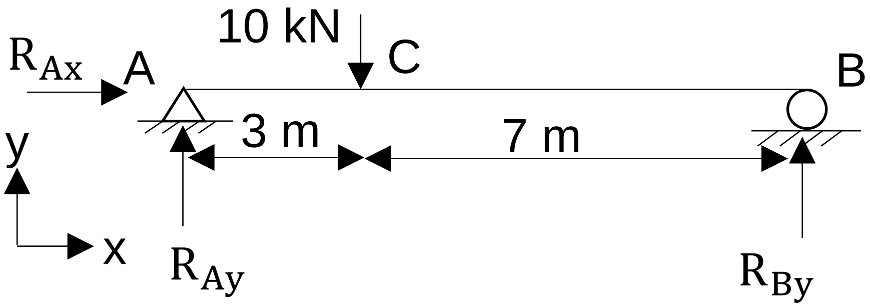


Source: https://jonochshorn.com/structuralelements/book/5.07-beams.html

1.1 Types of structures: based on method of analysis

[A] Determinate structures:

- That can be analyzed using three equations of static equilibrium only.
- For example, simply supported beam, cantilever beam



Source:
Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

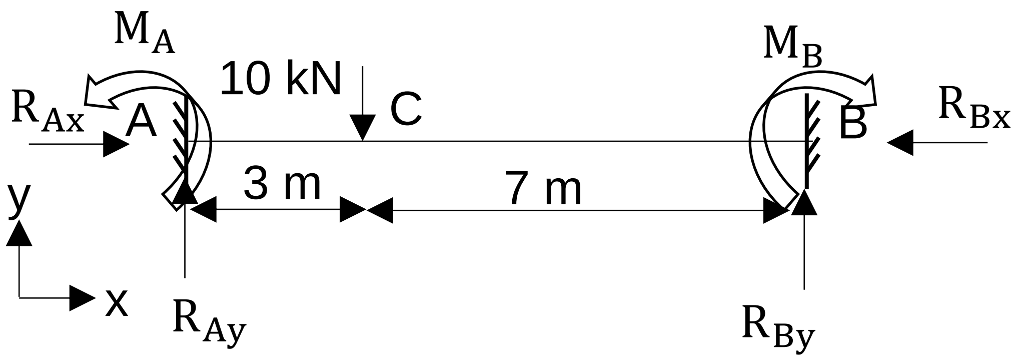
Here,

- 3 unknowns – R_{Ax}, R_{Ay}, and R_{By}
- 3 available equations of static equilibrium
- Can be solved

1.1 Types of structures: based on method of analysis

[B] Indeterminate structures:

- That cannot be analyzed using three equations of static equilibrium only.
- For example, propped cantilever beam, fixed beam



Source:
Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

Here,

- 6 unknowns – R_{Ax}, R_{Ay}, R_{Bx}, R_{By}, M_A and M_B
- 3 available equations of static equilibrium
- Cannot be solved by using equilibrium equations only

1.1 Types of structures: based on method of analysis

[B] Indeterminate structures:

- Degree of Static Indeterminacy (DSI) = The no. of equations required over and above the equations of static equilibrium for the analysis of a structure.

Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraAw Hill.
Parajuli, H.R. & Neupane, S. (2024). *A textbook of Engineering Mechanics.* Kathmandu: Heritage Publishers & Distributors.

- $DSI = DSI_{(external)} + DSI_{(internal)}$

↓ Due to support conditions ↓ Due to members

- $DSI_{(external)} = r - 3$

Where, r = no. of support reactions
3 = no. of available equations of static equilibrium

1.1 Types of structures: based on method of analysis

[B] Indeterminate structures:

Degree of Static Indeterminacy (DSI)

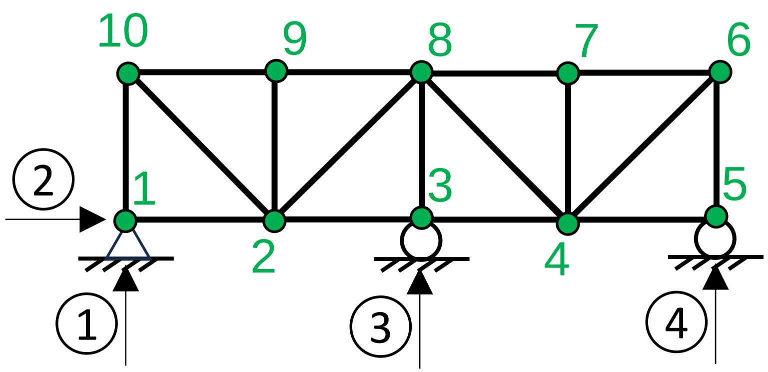
- DSI (For Truss) = m+r-2j
- DSI (For Beam) = 3m+r-2j
- DSI (For 2D Frame) = 3m+r-3j
- DSI (For 3D Frame) = 6m+r-6j

Where, m = no. of members
r = no. of support reactions
j = no. of joints

Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraAw Hill.
Parajuli, H.R. & Neupane, S. (2024). *A textbook of Engineering Mechanics.* Kathmandu: Heritage Publishers & Distributors.

1.1 Types of structures: based on method of analysis

[B] Indeterminate structures: DSI



$DSI_{(external)} = 4-3 = 1$

$DSI_{(total)} = 17+4-2*10 = 1$

Static Indeterminacy due to support conditions.

- $DSI_{(external)} = r-3$
- DSI (Truss) = m+r-2j
- DSI (Beam) = 3m+r-2j
- DSI (2D Frame) = 3m+r-3j
- DSI (3D Frame) = 6m+r-6j

Where,
m = no. of members
r = no. of support reactions
j = no. of joints

Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraAw Hill.
Parajuli, H.R. & Neupane, S. (2024). *A textbook of Engineering Mechanics.* Kathmandu: Heritage Publishers & Distributors.

1.1 Types of structures: based on method of analysis

[B] Indeterminate structures:

Degree of Kinematic Indeterminacy (DKI) is the no. of equilibrium conditions needed to find the displacement components of all joints of the structure.

- DKI (For Truss, 2D) = 2j-r
- DKI (For Truss, 3D) = 3j-r
- DKI (For 2D Plane Frame) = 3j-r
- DKI (For 3D Space Frame) = 6j-r

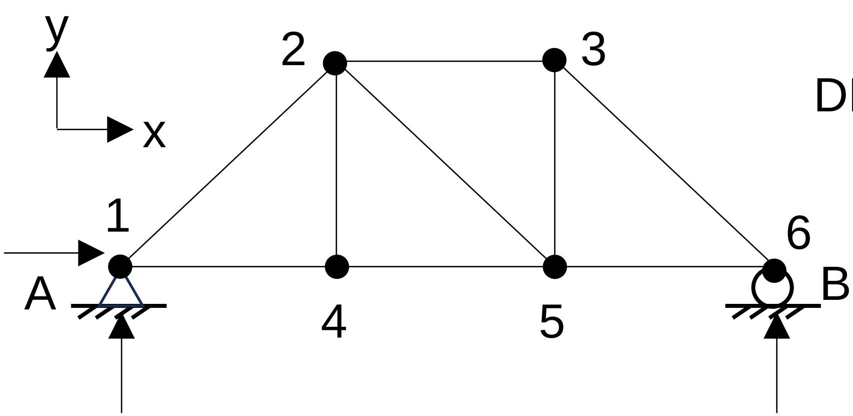
Where, r = no. of support reactions
j = no. of joints

Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraAw Hill.
Parajuli, H.R. & Neupane, S. (2024). *A textbook of Engineering Mechanics.* Kathmandu: Heritage Publishers & Distributors.

- Also called Degree of Freedom (DoF)

1.1 Types of structures: based on method of analysis

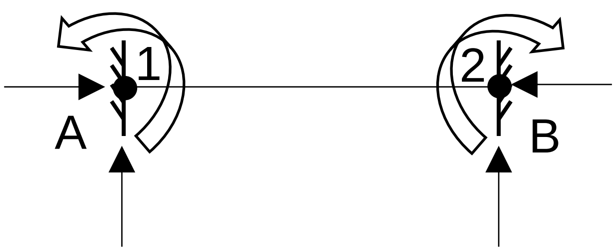
[B] Indeterminate structures: DKI



DKI = 2*j – r

= 2*6-3

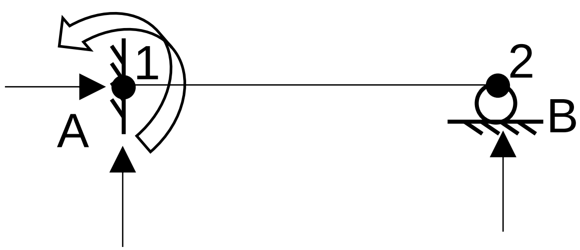
= 9



DKI = 3*j-r

=3*2-6

= 0



DKI = 3*2-4

= 2

- DKI (Truss, 2D) = 2j-r
- DKI (Truss, 3D) = 3j-r
- DKI (2D Frame) = 3j-r
- DKI (3D Frame) = 6j-r

Where,

r = no. of support reactions

j = no. of joints

Source:

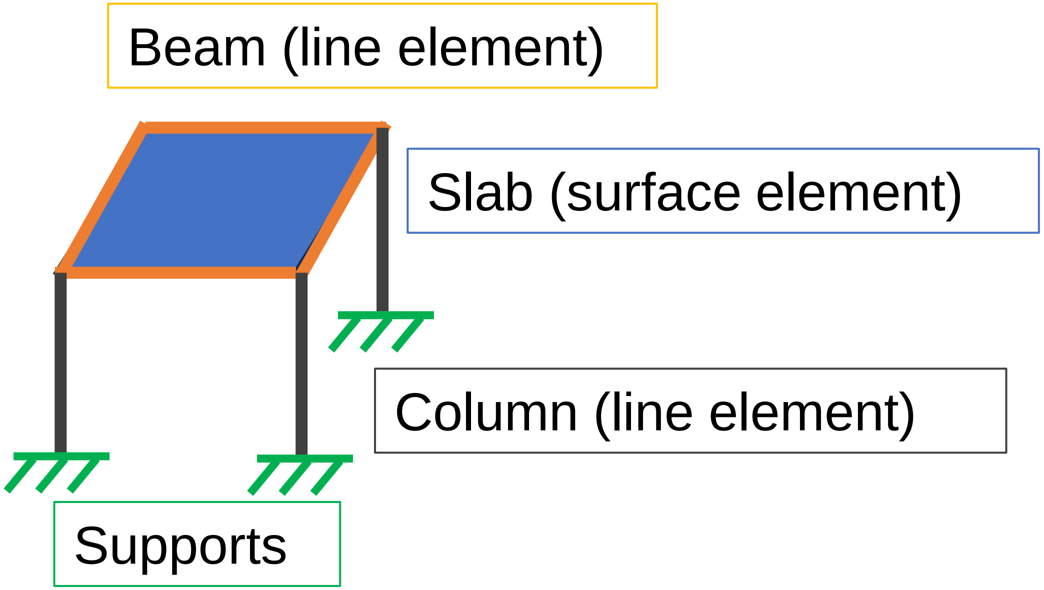
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraAw Hill.

Parajuli, H.R. & Neupane, S. (2024). *A textbook of Engineering Mechanics.* Kathmandu: Heritage Publishers & Distributors.

1.2 Structural Mechanics

Structure:
It is a combination of line elements, surface elements, plate and shell elements provided with discrete supports at base joints

Structural Mechanics:
It is a science concerned with strength, stiffness, and stability of engineering structures and their components.

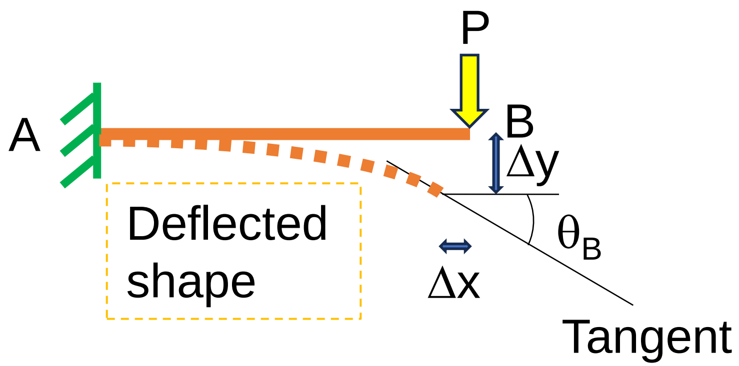


Source:
Thapa, K.B. (2019). *Theory of Structures-I (Determinate).* Kathmandu: Pratibha Pustak Sadan & Stationary

1.2 Structural Mechanics

Analysis:

- To check the stability of the structure
- To find support reactions
- To determine internal stresses namely axial force, shear force, and bending moment at any point in the structure
- To determine the displacement parameters (Δx, Δy, and θ)



Cantilever beam AB with point load, P at free end

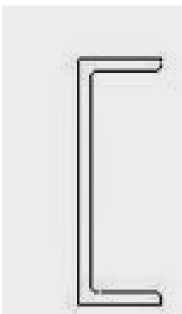
Source:
Thapa, K.B. (2019). *Theory of Structures-I (Determinate).* Kathmandu: Pratibha Pustak Sadan & Stationary

1.2 Structural Mechanics

Design:

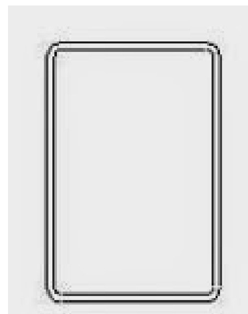
- Selection of section
- Sizing of elements based on analysis
- Check: Developed stresses and deformations ≤ Permissible values

Steel sections

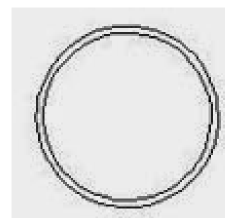


Channel section

I or H section



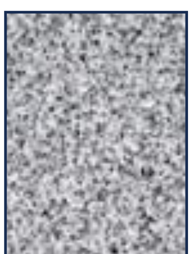
Rectangular Hollow section



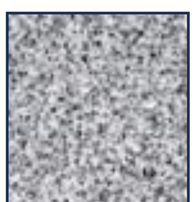
Circular Hollow section

Source: https://engineeringgravity.blogspot.com/2014/05/structural-steel-sections.html

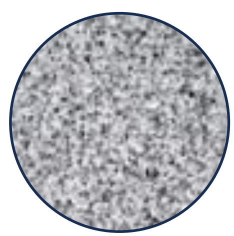
Concrete sections



Rectangular



Square



Circular

1.3 Two basic approaches of structural analysis

To find the magnitude of the redundant forces (forces which are not absolutely essential for the stability of the structure), we can use two basic approaches of structural analysis:

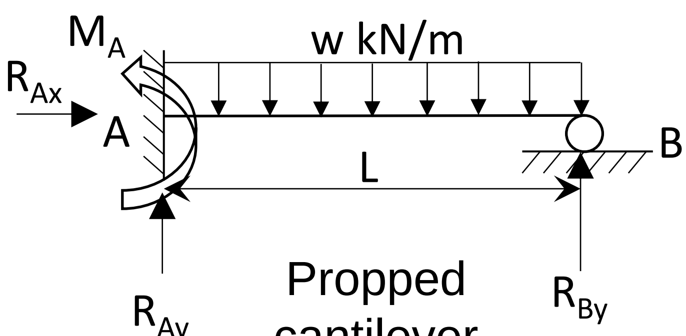
1. Force method
2. Displacement method

Source:

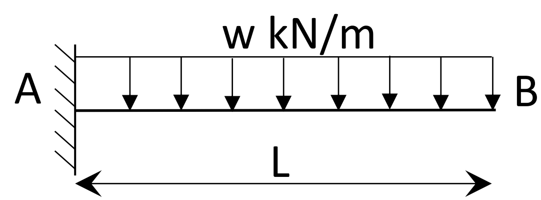
Thapa, K.B. (2019). *Theory of Structures-I (Determinate).* Kathmandu: Pratibha Pustak Sadan & Stationary

1.3 Two basic approaches of structural analysis: Force method

- Also called flexibility methods or compatibility methods
- In this method, redundant forces are removed from an indeterminate structure to produce a stable, determinate released structure.



Propped cantilever beam



Primary structure (Determinate + stable)

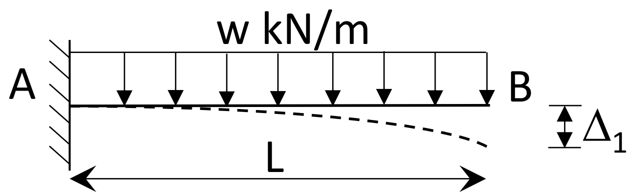
DSI = r-3 = 4-3 =1

Indeterminate

Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.
https://www.youtube.com/watch?v=mktQ61X-hGo&t=50s

1.3 Two basic approaches of structural analysis: Force method

- Then, the displacements – translations and rotations are determined at all releases in the direction of releases, for the primary structure under given loading.



Primary structure
(Determinate + stable)

$$\Delta_1 = \frac{wL^4}{8EI}$$

Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.
https://www.youtube.com/watch?v=mktQ61X-hGo&t=50s

1.3 Two basic approaches of structural analysis: Force method

- Next, the displacements at the released coordinates due to redundant forces are determined, one-by-one (if more than one redundant).

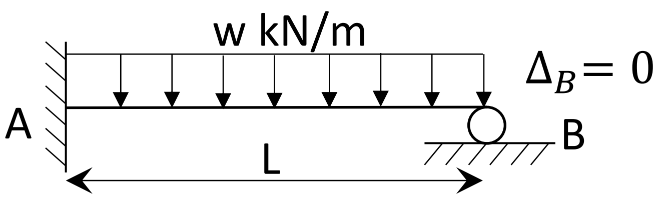
Applying Redundant force, R_{By}, at B (released coordinate)

$$\Delta_2 = -\frac{PL^3}{3EI} = -\frac{R_{By}L^3}{3EI}$$

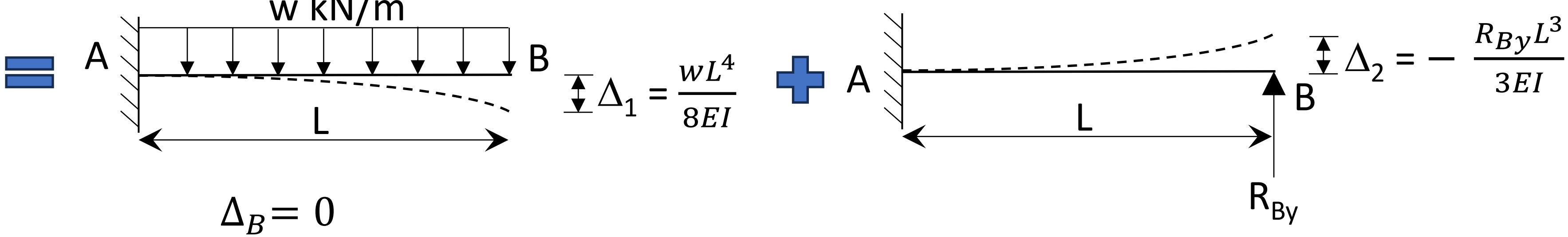
Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.
https://www.youtube.com/watch?v=mktQ61X-hGo&t=50s

1.3 Two basic approaches of structural analysis: Force method

- To satisfy the compatibility criteria, displacement at support B should be zero, i.e. $\Delta_B = 0$.



Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.
https://www.youtube.com/watch?v=mktQ61X-hGo&t=50s



$$\Delta_B = 0$$

Or, $\Delta_1 + \Delta_2 = 0$

Or, $\frac{wL^4}{8EI} - \frac{R_{By}L^3}{3EI} = 0$

$\therefore R_{By} = \frac{3wL}{8}\ kN\ (\uparrow)$

Hence, the unknown redundant force is obtained.

1.3 Two basic approaches of structural analysis: Displacement method

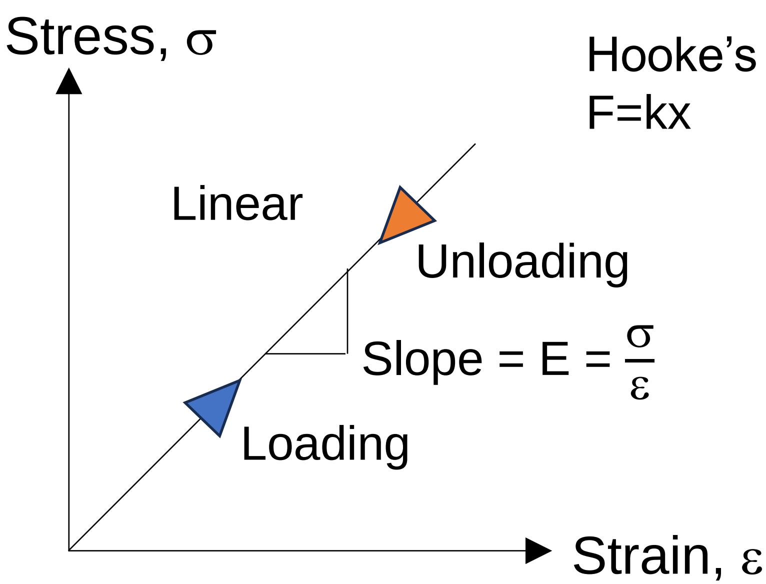
- Also called stiffness method
- In this method, the displacement of the joints – rotations and translations are treated as unknowns.
- Equilibrium equations are written for each joint of the structure in terms of (i) applied loads, (ii) properties of the members framing into the joint, and (iii) unknown displacements.
- A set of linear, algebraic equations equal in number to the unknown joint displacements is produced.
- These equations are then simultaneously solved to find the joint displacements.

Source:
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

1.4 Linearly Elastic Structures

A structure is called linearly elastic if

- its material has a *linear stress-strain relationship*, obeys *Hooke's law*,
- loading and unloading follows the same path (*elastic*), and
- has a *small deflection*, which does not affect the geometry of the structural member and therefore, can be *neglected*.



Hooke's law:
F=kx

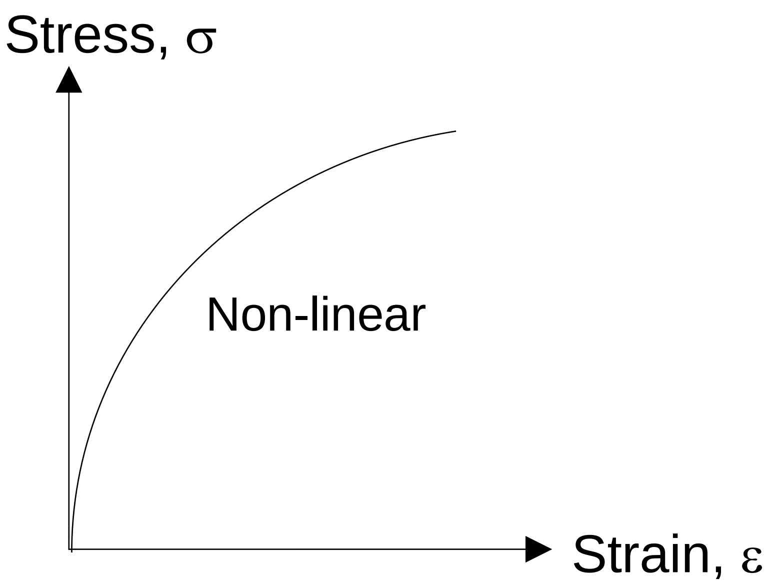
Source:
Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

1.5 Non-linearity in Structural Analysis

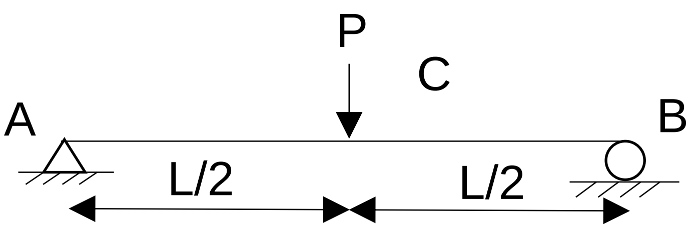
Non-linearity occurs in structures due to following two primary reasons

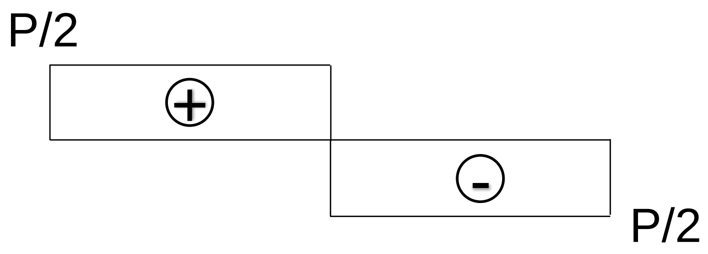
- Material non-linearity: has a ***non-linear*** *stress-strain relationship*, i.e. doesn't obey *Hooke's law*, and
- Geometric non-linearity: has a *large deformation* such that the changes in the geometry *cannot be neglected* in the structural analysis.

Source:
Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

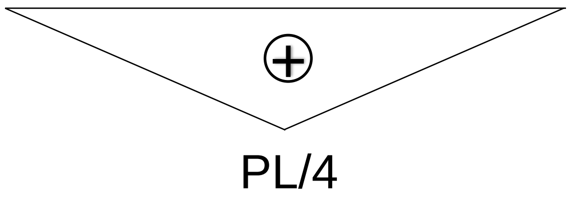


1.6 Computer based methods





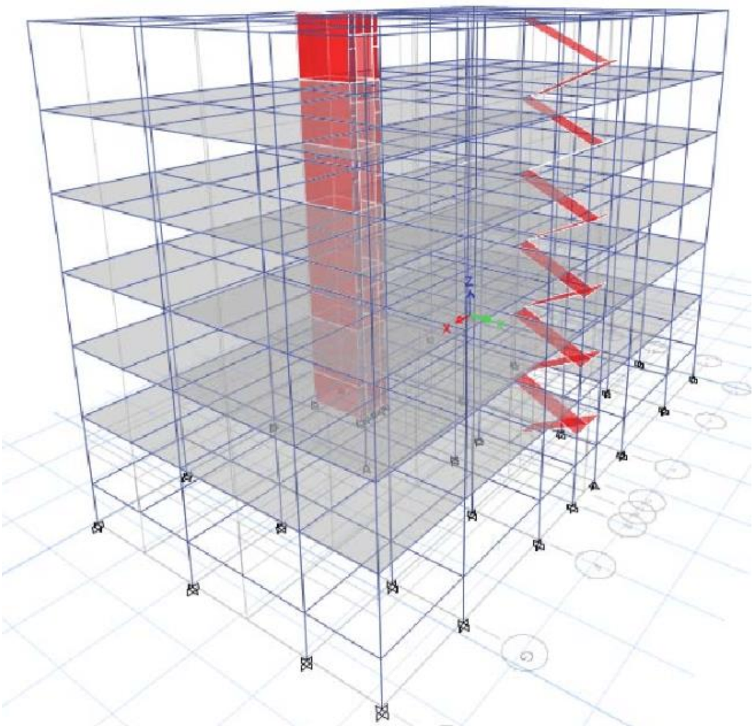
Shear Force Diagram (SFD)



Bending Moment Diagram (BMD)

- Simple structures
- Few loads
- Manual calculation for internal stresses such as shear force and bending moment

1.6 Computer based methods



3D model of finished building in ETABS 15

- Complex structures
- Multiple loads
- Matrix method
- High speed computers
- SAP2000, ETABS, STAADPRO, etc.

Source:
Thapa, K.B. (2019). *Theory of Structures-I (Determinate).* Kathmandu: Pratibha Pustak Sadan & Stationary
Das, S.S., Roy, S., & Tahsin, A. (2018). *Pushover Analysis of an OMRF building located in Dhaka.* Global Journal of Researches in Engineering: Civil and Structural Engineering, 18 *(1)*, pp. 51-58.

1.7 Principle of Superposition

The principle of superposition holds good when the material

- is assumed to be perfectly elastic, and
- obeys Hooke's law

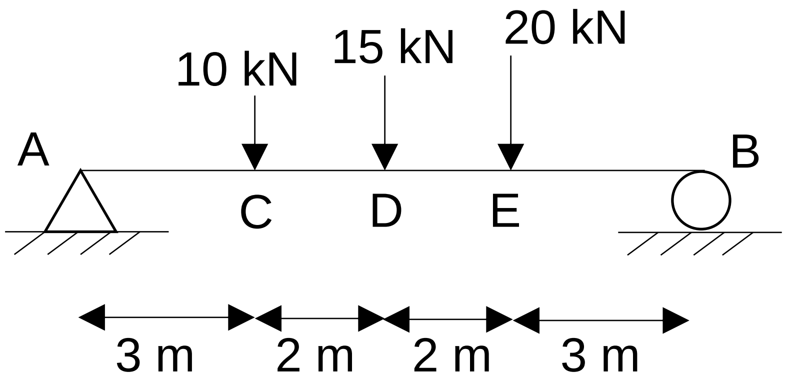
for the range of loads considered.

It means that '<u>the structure can be analyzed for different loads separately and the results be superposed to get the final results due to different combination of loading</u>'.

Source:
Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

1.7 Principle of Superposition

Example:
Let us consider a simply supported beam with 3 point loadings as shown below, for which bending moment diagram is to be computed for the given loadings:

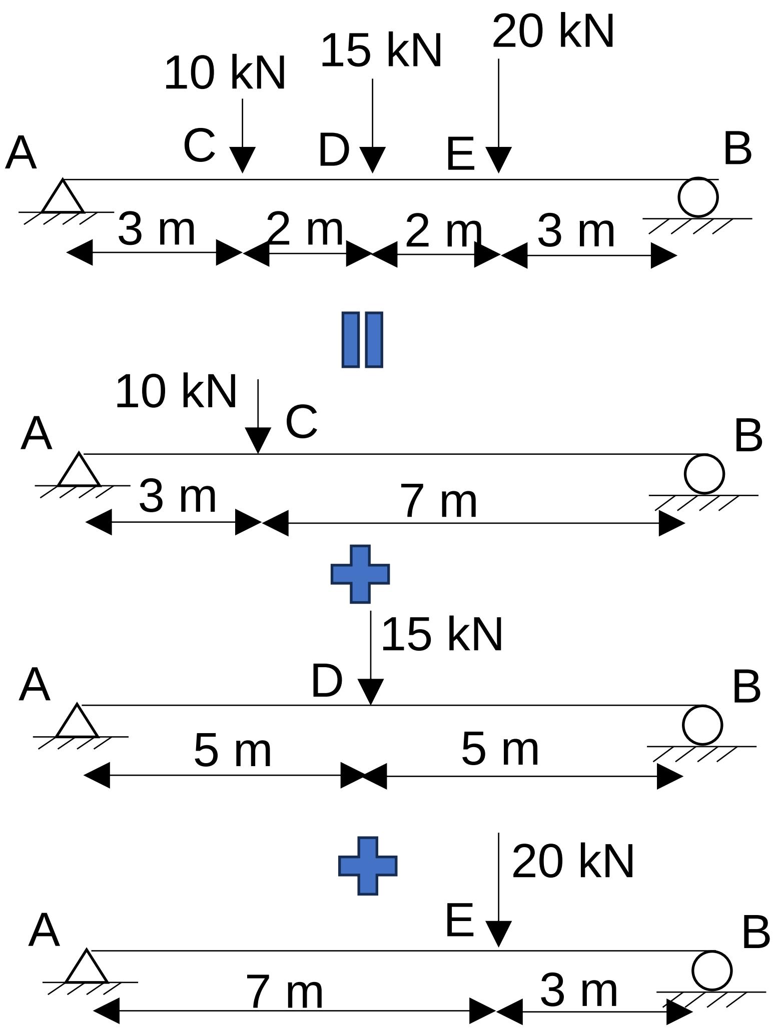


- Approach 1: Calculate the bending moments at points along the beam by taking all the loads at once. (Applied Mechanics: Statics)

- Approach 2: Calculate the bending moment for each load and add them to obtain the combined bending moment due to all 3 loads. (Principle of Superposition)

1.7 Principle of Superposition

Example:



Step 1: Apply only one loading at a time to the beam

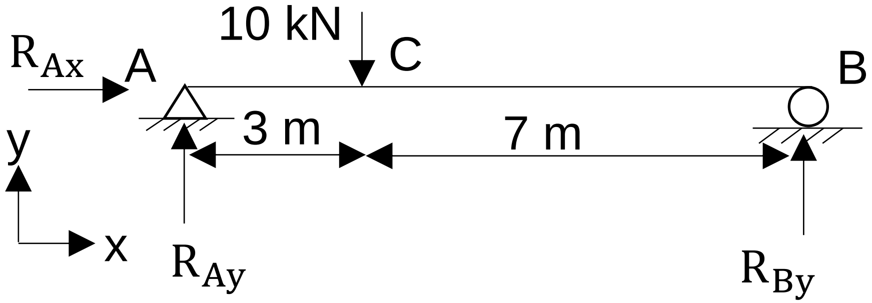
Beam 1

Beam 2

Beam 3

1.7 Principle of Superposition

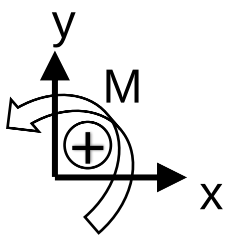
Demonstrated: BMD for Beam 1



Step 2: Solve each beam for its Bending Moment Diagram (BMD)

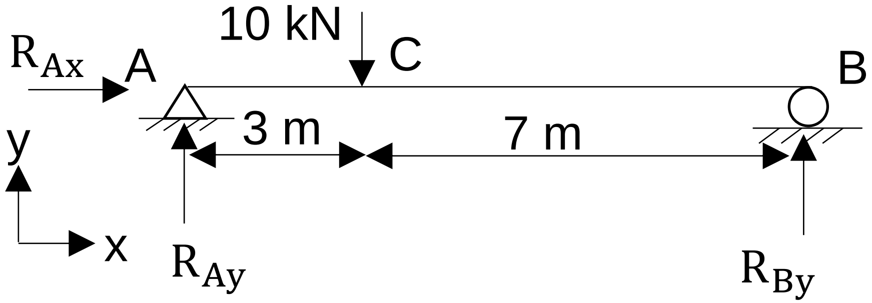
From equations of static equilibrium,
$\sum F_x = 0;\ \sum F_y = 0;\ \sum M_{A\ or\ B} = 0$

Taking following positive sign conventions,



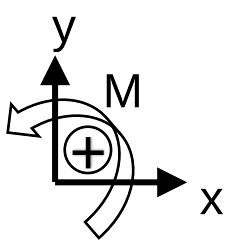
1.7 Principle of Superposition

Demonstrated: BMD for Beam 1



Step 2: Solve each beam for its Bending Moment Diagram (BMD)

Positive sign conventions



$\sum F_x = 0 \Rightarrow R_{Ax} = 0$ kN

$\sum F_y = 0 \Rightarrow R_{Ay} + R_{By} = 10$ kN -----(i)

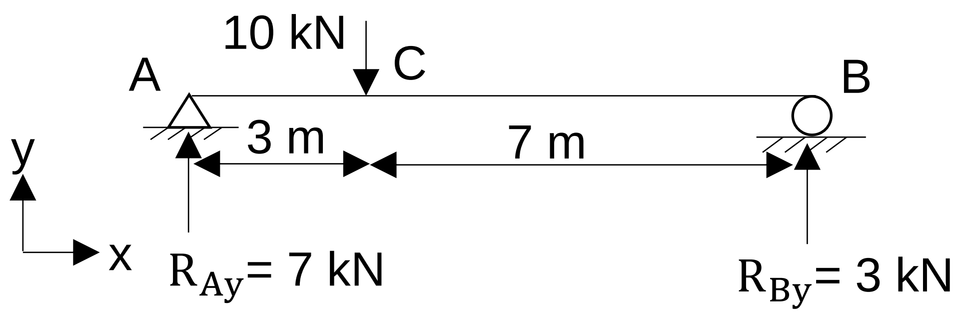
$$\sum M_A = 0 \Rightarrow R_{By} * (3 + 7) - 10 * (3) = 0$$

$$\Rightarrow R_{By} = \frac{30}{10} = 3\ kN\ (\uparrow)$$

From equation (i), R_{Ay}= 7 kN (↑)

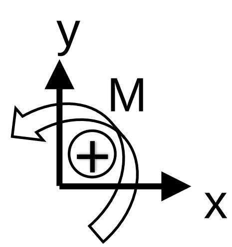
1.7 Principle of Superposition

Demonstrated: BMD for Beam 1



Step 2: Solve each beam for its Bending Moment Diagram (BMD)

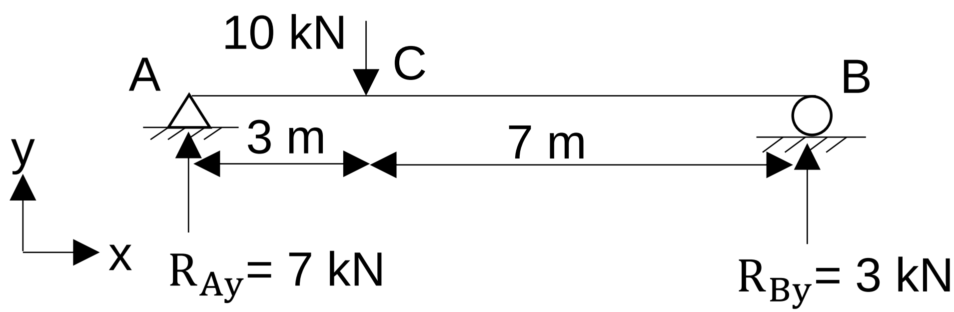
Positive sign conventions

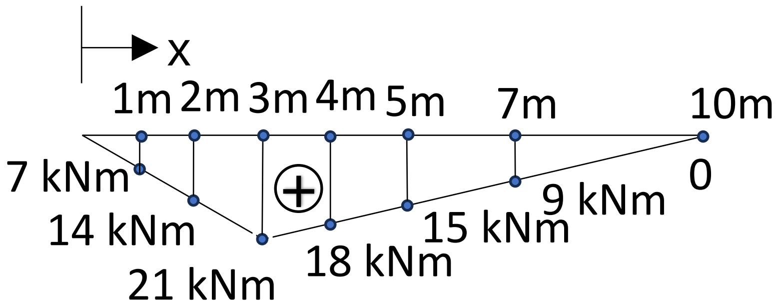


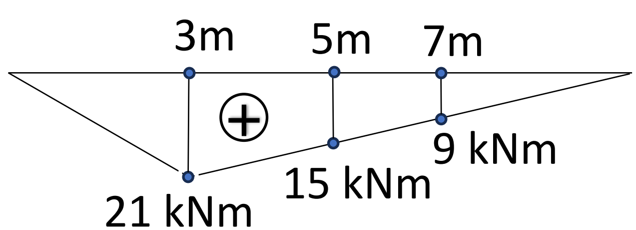
Distance from A, x (m)	Bending Moment, B. M. at x (m) [kN-m]
1	7*1 = 7 kN-m
2	7*2 = 14 kN-m
3	7*3 = 21 kN-m
4	7*4 – 10*(4-3) = 7*4 – 10*1 =18 kN-m
5	7*5 – 10*(5-3) = 15 kN-m
7	7*7 – 10*(7-3) = 9 kN-m
10	7*10-10*(10-3) = 0 kN-m

1.7 Principle of Superposition

Demonstrated: BMD for Beam 1







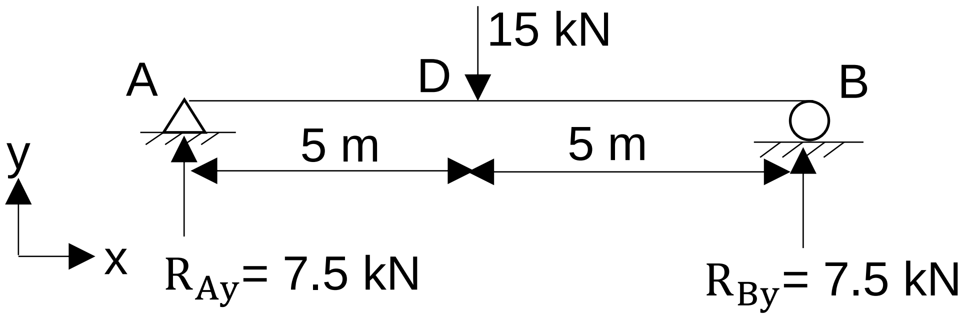
Step 3: Construct BMD for each beam

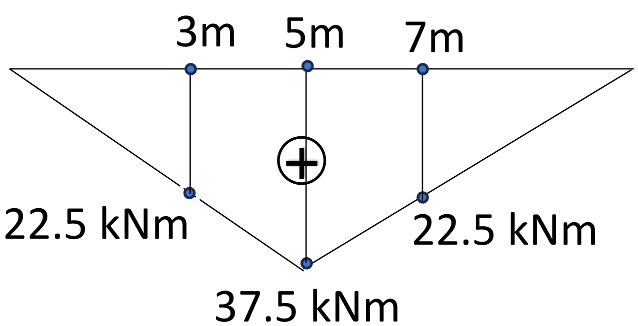
x (m)	B. M. at x (m) [kN-m]
1	7 kN-m
2	14 kN-m
3	21 kN-m
4	18 kN-m
5	15 kN-m
7	9 kN-m
10	0 kN-m

1.7 Principle of Superposition

Similarly, BMD for Beam 2

Step 3: Construct BMD for each beam

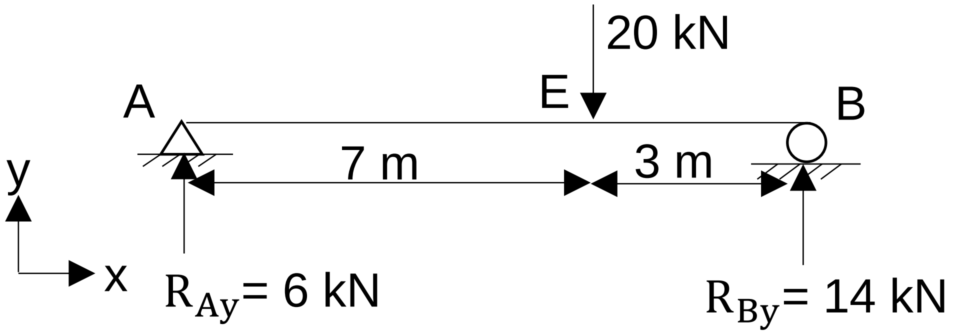


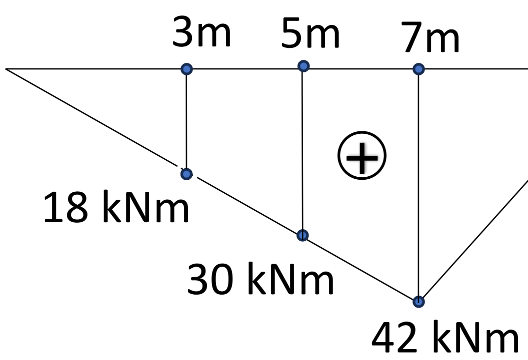


1.7 Principle of Superposition

Similarly, BMD for Beam 3

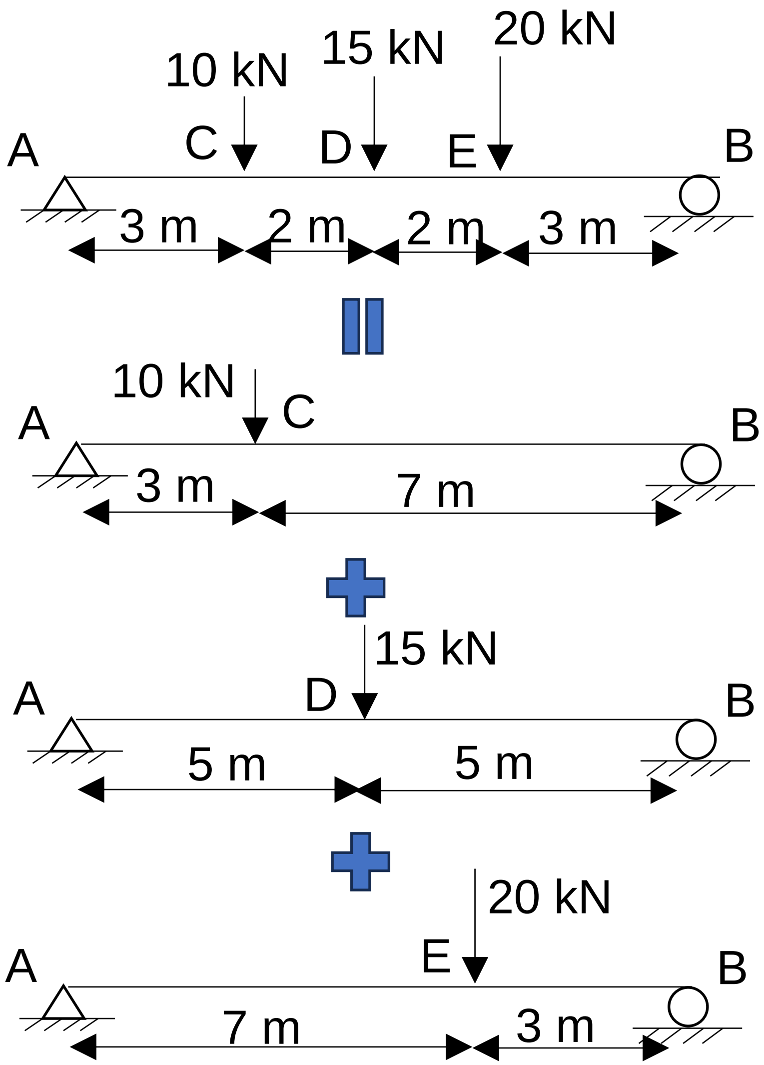
Step 3: Construct BMD for each beam





1.7 Principle of Superposition

Step 4: Add corresponding of each of the beams to get the total B.M. at the parent beam



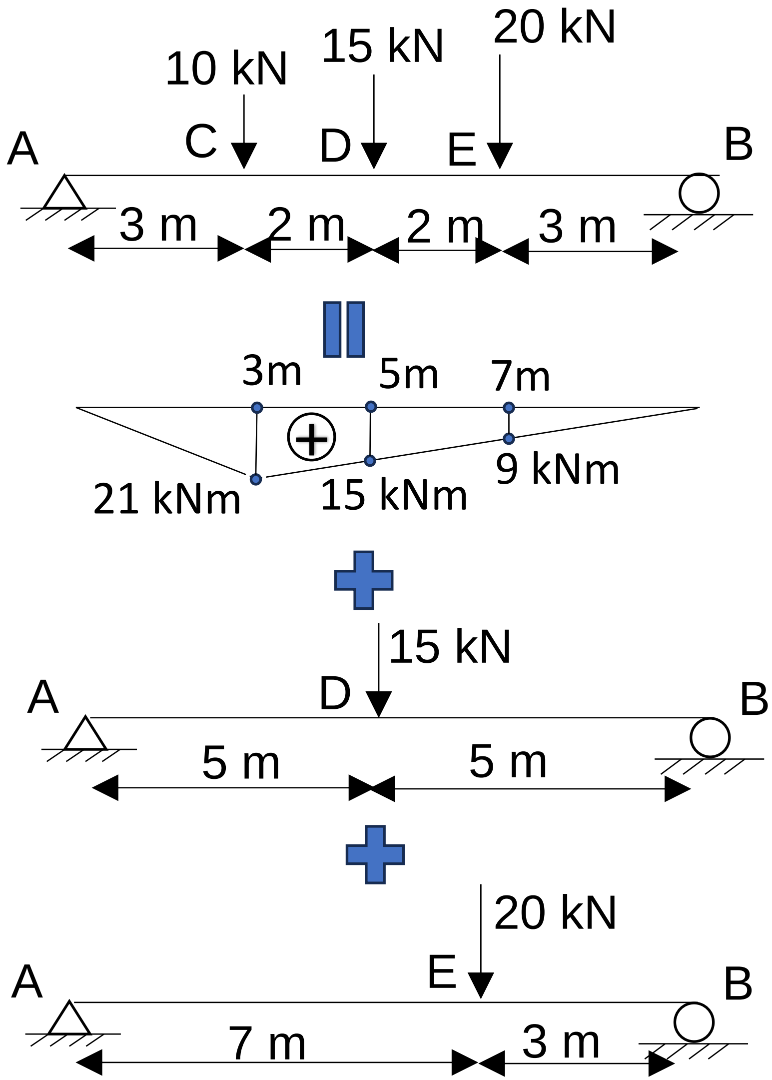
Beam 1

Beam 2

Beam 3

1.7 Principle of Superposition

Step 4: Add corresponding of each of the beams to get the total B.M. at the parent beam



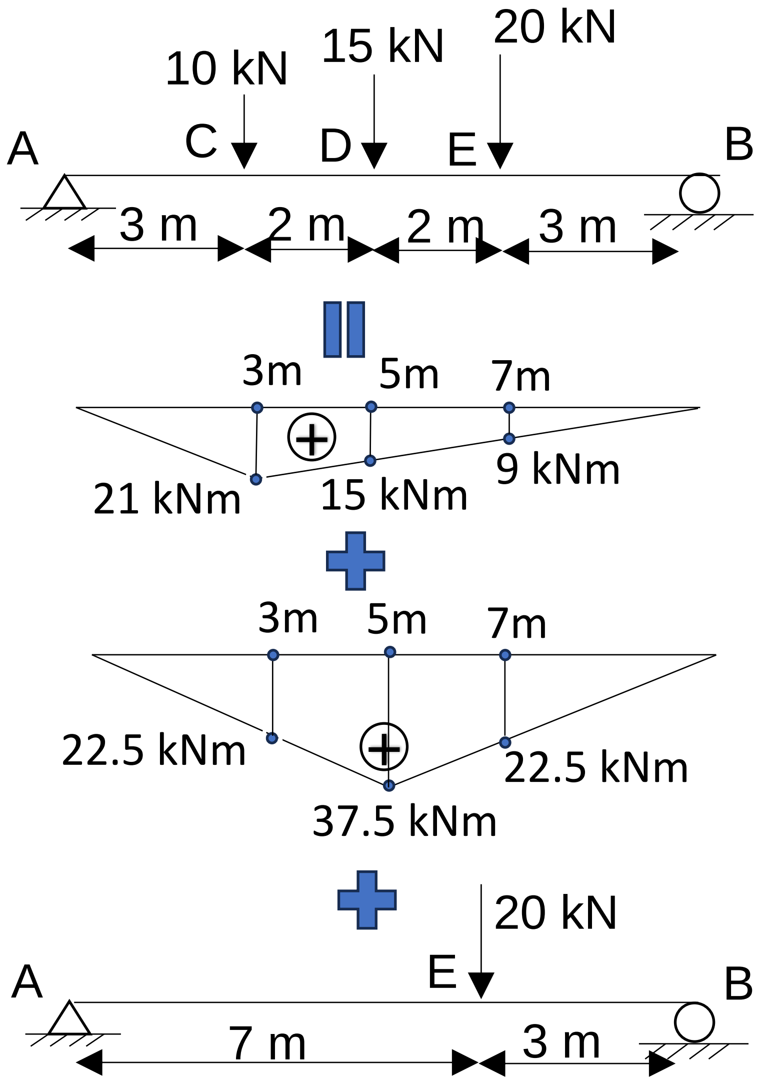
Beam 1

Beam 2

Beam 3

1.7 Principle of Superposition

Step 4: Add corresponding of each of the beams to get the total B.M. at the parent beam

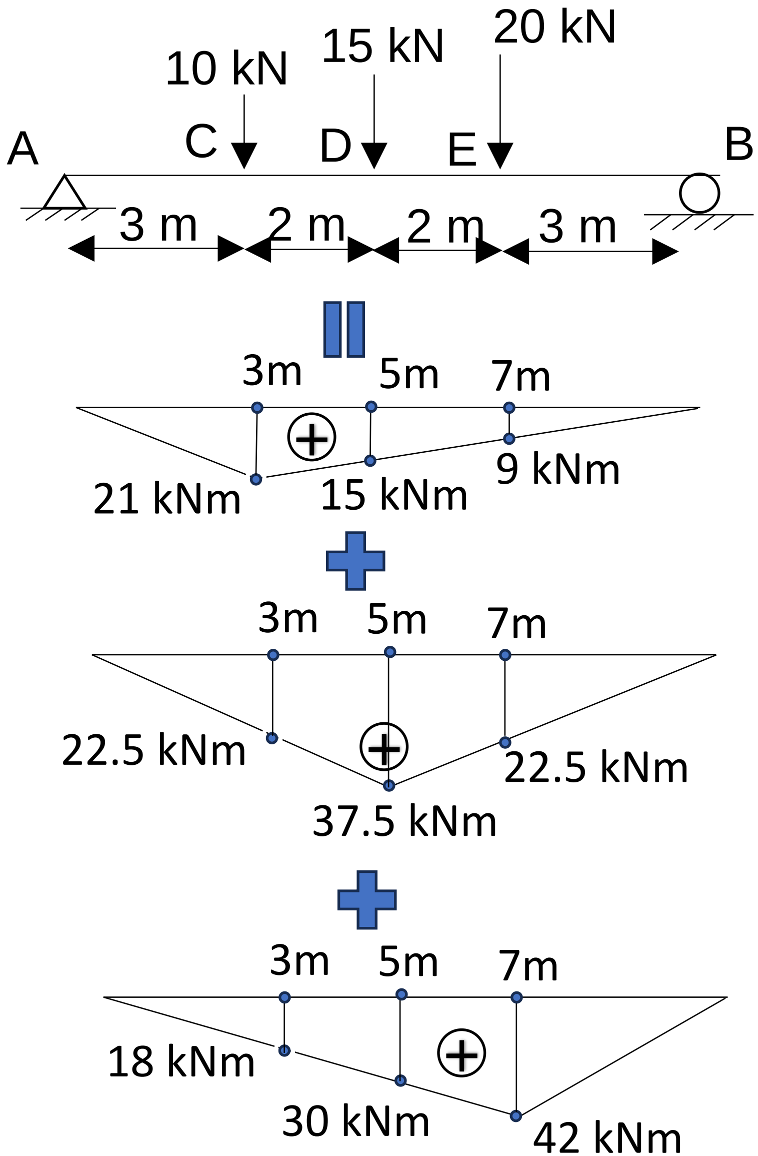


Beam 1

Beam 2

Beam 3

1.7 Principle of Superposition



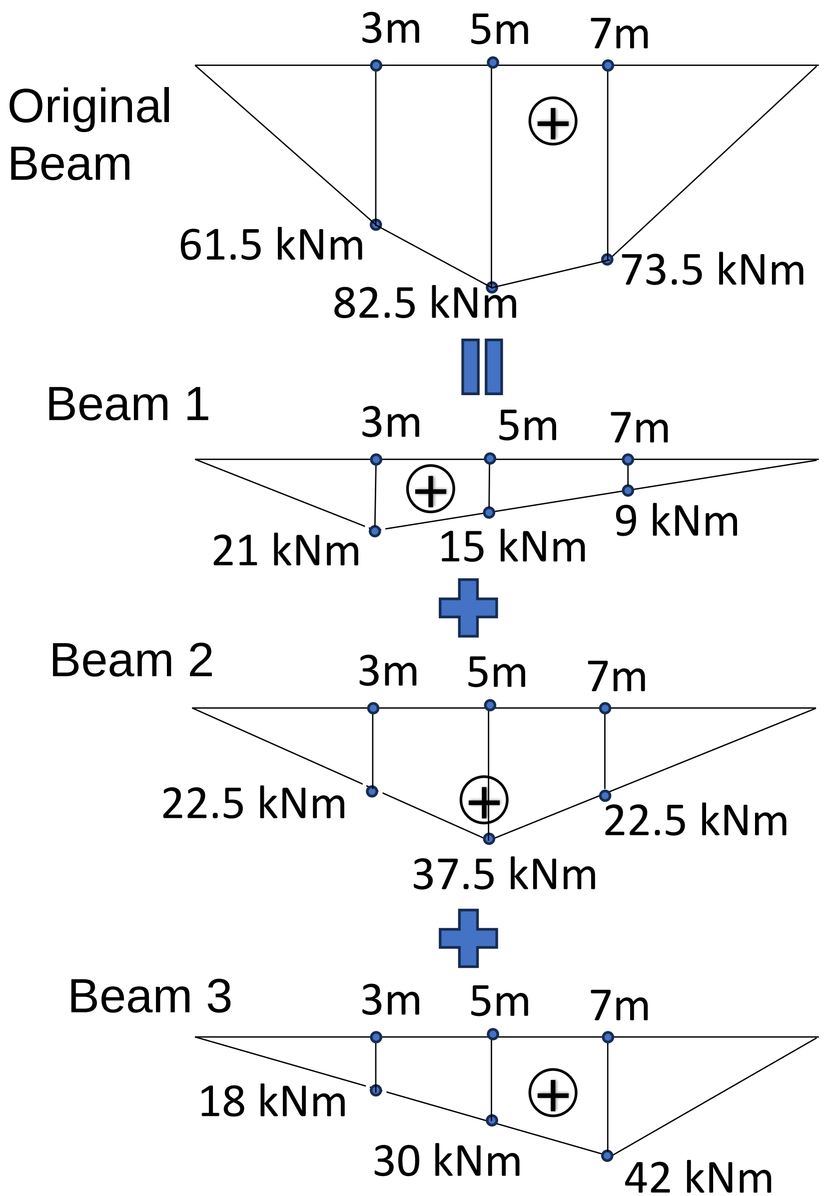
Step 4: Add corresponding of each of the beams to get the total B.M. at the parent beam

Beam 1

Beam 2

Beam 3

1.7 Principle of Superposition



Step 5: Plot BMD for the original beam

x	B.M.
3m	21+22.5+18 = 61.5 kNm
5m	15+37.5+30=82.5 kNm
7m	9+22.5+42=73.5 kNm

-------------End of chapter 1------------------

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