Theory of Structures - I

Chapter 3. Analysis by Virtual Work Method [Part II of II]

Lecturer: Dr. Sanjeema Bajracharya

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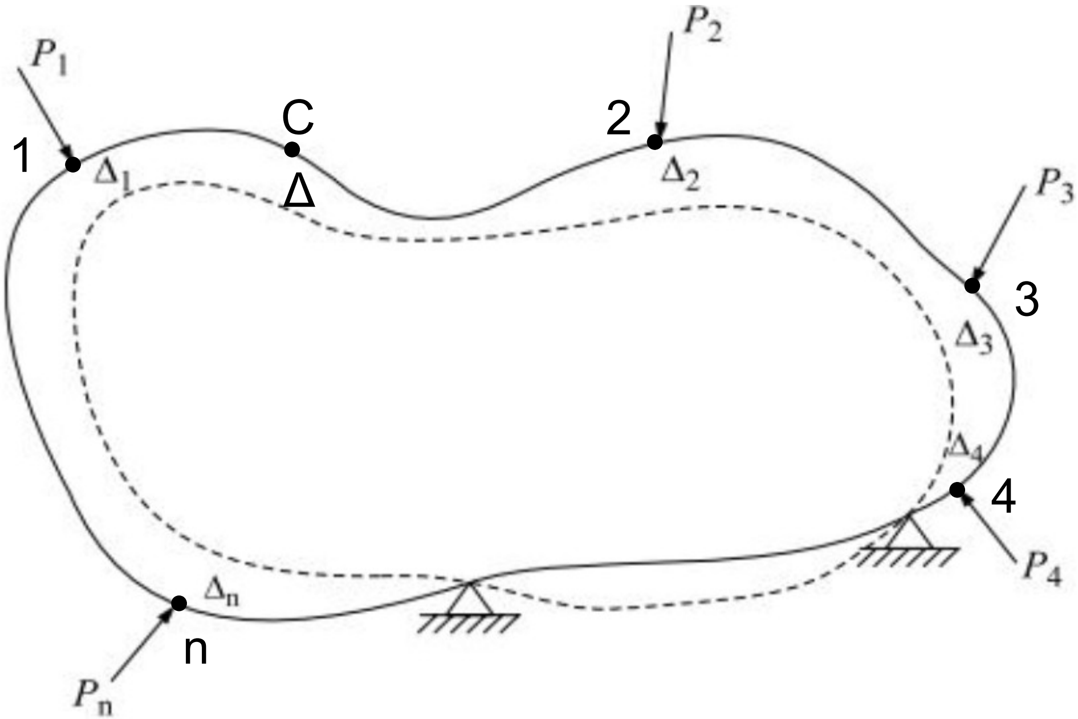
3.4 Displacement by Method of Virtual Work: Unit load method



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Let us consider a body subjected to forces P_1, P_2, P_3, P_4, ….P_n applied gradually.

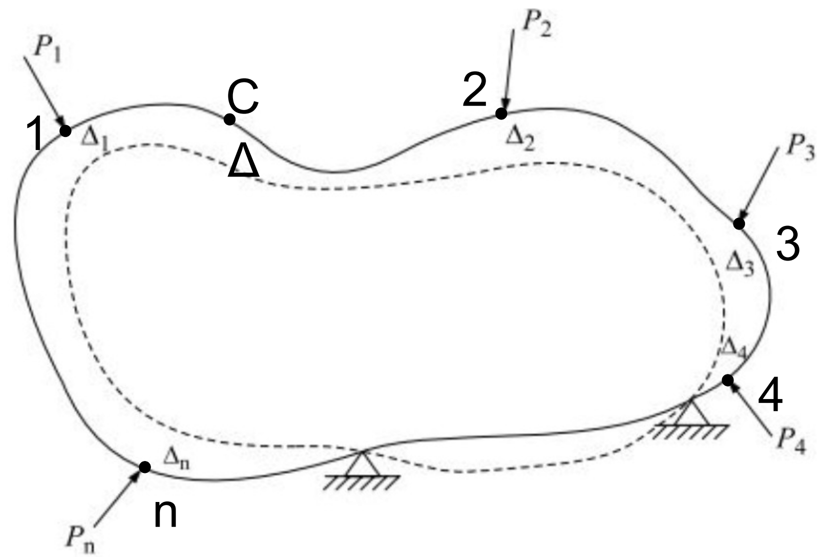
Let the displacement under the loads at points be Δ_1,Δ_2, Δ_3, Δ_4,….Δ_n, and at point C be Δ.

Then,

External work done=

$$\frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + \frac{1}{2}\Delta_3 P_3 + \ldots\ldots\ldots\ldots\ldots + \frac{1}{2}\Delta_n P_n$$

3.4 Displacement by Method of Virtual Work: Unit load method



And,

Strain Energy Stored = $\int \frac{1}{2} pe\, dv$

Where,

p=stress in the element considered

e=strain in the element considered

We have,

External work done = Strain energy stored

$$\frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + \frac{1}{2}\Delta_3 P_3 + \ldots\ldots\ldots\ldots\ldots + \frac{1}{2}\Delta_n P_n = \int \frac{1}{2} pe\, dv - - - - (1)$$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

3.4 Displacement by Method of Virtual Work: Unit load method

Now, consider the same body subjected to a unit load applied gradually at C, when it is free of system of P-forces.

Let the displacements at 1, 2, 3, ….,n be δ_1, δ_2, δ_3,….,δ_n respectively and the displacement at C be δ.

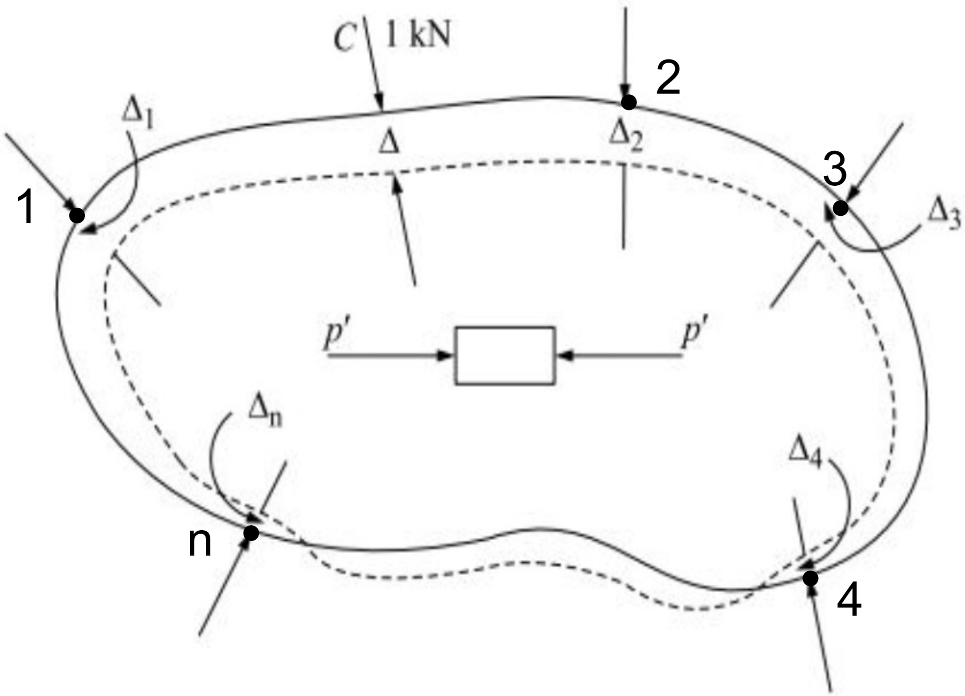
Let the stress produced in the element be p' and the strain be e'.

Then,
External Work Done = Internal Work Done

Or, $$\frac{1}{2} * 1 * \delta = \int \frac{1}{2} p' e' dv ----(2)$$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

3.4 Displacement by Method of Virtual Work: Unit load method



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

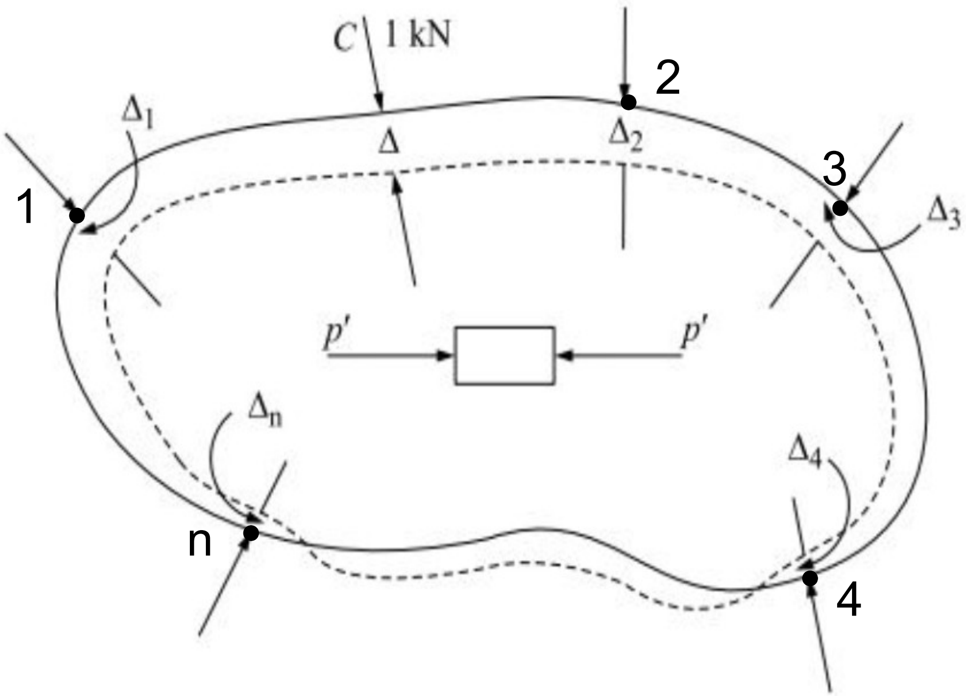
Now,
If P system of forces is applied to the body as shown in the adjacent figure,
Then,
External work done =

$$\frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + \frac{1}{2}\Delta_3 P_3 + \ldots\ldots$$

$$\ldots\ldots + \frac{1}{2}\Delta_n P_n + 1 * \Delta$$

Since the unit load is already acting

3.4 Displacement by Method of Virtual Work: Unit load method



And,
Internal work done =

$$\int \frac{1}{2} pe\,dv + \int p'e\,dv$$

Since stress p' is acting throughout the deformation.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Since,
External work done = Internal work done

$$\frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + \frac{1}{2}\Delta_3 P_3 + \ldots\ldots \frac{1}{2}\Delta_n P_n + 1 * \Delta = \int \frac{1}{2} pe\,dv + \int p'e\,dv \;---(3)$$

3.4 Displacement by Method of Virtual Work: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Subtracting equation (1) from equation (3), we get,

$$1 * \Delta = \int p' e \, dv - - - (4)$$

Where,

Δ= deflection at point where unit load is applied and measured in the direction of unit load

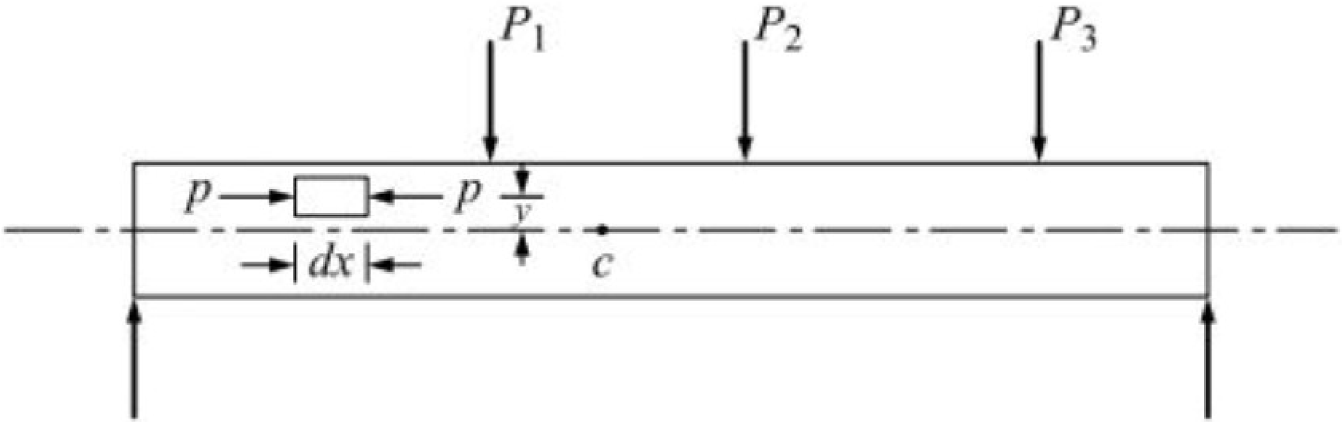
p’= stress in an element due to unit load

e= strain the element due to the given load system

This equation (4) is the basis for unit load method.

3.5 Displacement in beams: Unit load method

Let us consider a beam shown in figure subjected to a system of P-forces, whereby p is the stress developed in the element of length dx.



Then, stress in the element at a distance y from Neutral Axis is: $p=\frac{M}{I}*y$

Where, M is the moment acting at the section.

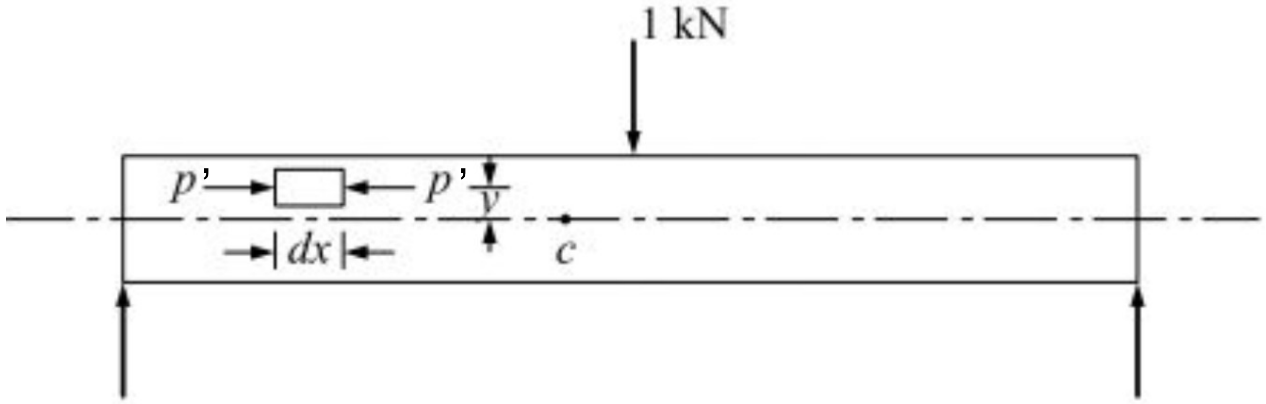
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

And,
Corresponding strain in the element is:

$$e=\frac{M}{EI}*y$$

3.5 Displacement in beams: Unit load method

Let us consider another typical beam subjected to unit load as shown in the following figure, whereby p’ is the stress developed in the element.



Then, stress in the element is: $p' = \frac{m}{I} * y$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Where, m is the moment acting at the section.

Then, from equation (4), we have,

$$1 * \Delta = \int p' e \, dv$$

$$\Delta = \int \frac{m}{I} * y \frac{M}{EI} * y \, dv$$

3.5 Displacement in beams: Unit load method

Then, from equation (4), we have,

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

$$1 * \Delta = \int p' e \, dv$$

$$\Delta = \int \frac{m}{I} * y \frac{M}{EI} * y \, dv$$

$$\Delta = \int_0^L \frac{Mm}{EI^2} \left(\int_0^A y^2 \, dA \right) dx$$

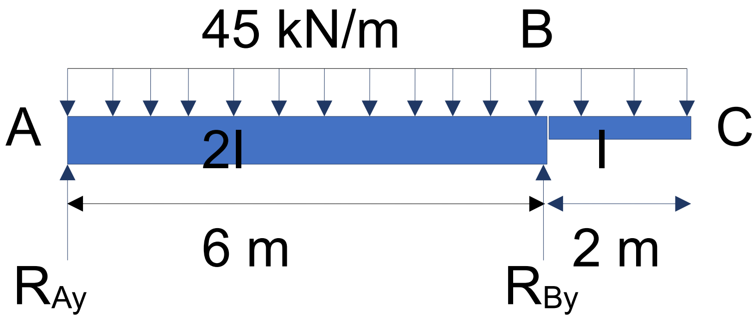
$$\Delta = \int_0^L \frac{Mm}{EI^2} I \, dx$$

$$\Delta = \int_0^L \frac{Mm}{EI} dx - - - - - (5)$$

From equation (5), the deflection at any point C can be found.

3.5 Displacement in beams: Unit load method

Numerical#1. Determine the deflection at free end of the overhanging beam. Use unit load method.



Solution:

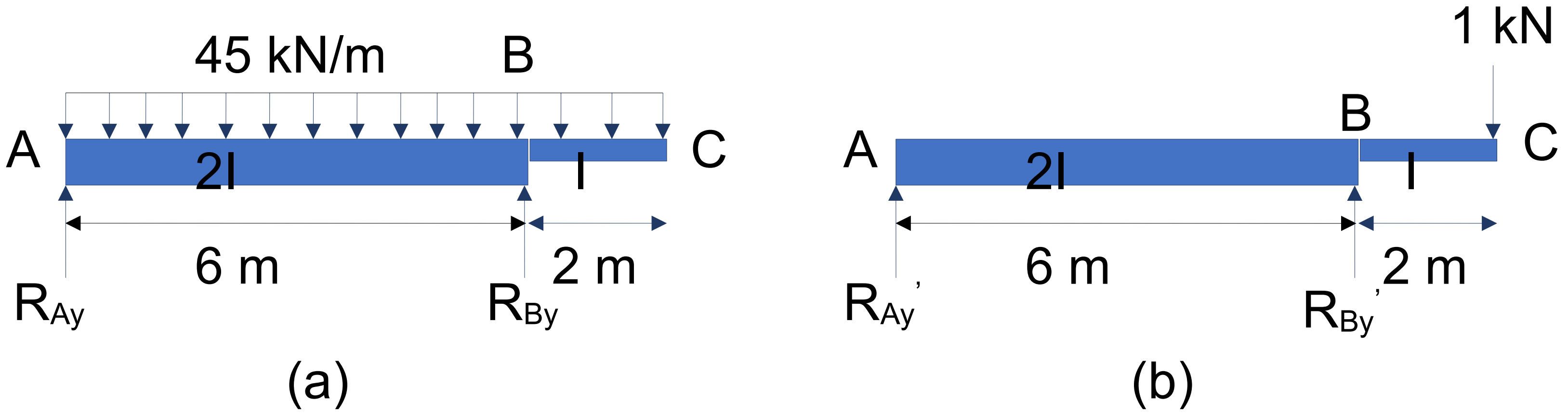
Step 1: Consider (a) Given beam subjected to given loading and (b) Given beam subjected to unit load (1kN) at the point where deformation parameters (displacement or rotation) are required.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

3.5 Displacement in beams: Unit load method

Numerical#1.

Solution:
Step 1: Consider (a) Given beam subjected to given loading and (b) Given beam subjected to unit load (1kN) at the point C, where deflection is required. **Note:** Here, we have assumed downward deflection at point C, hence the downward acting 1 kN load.

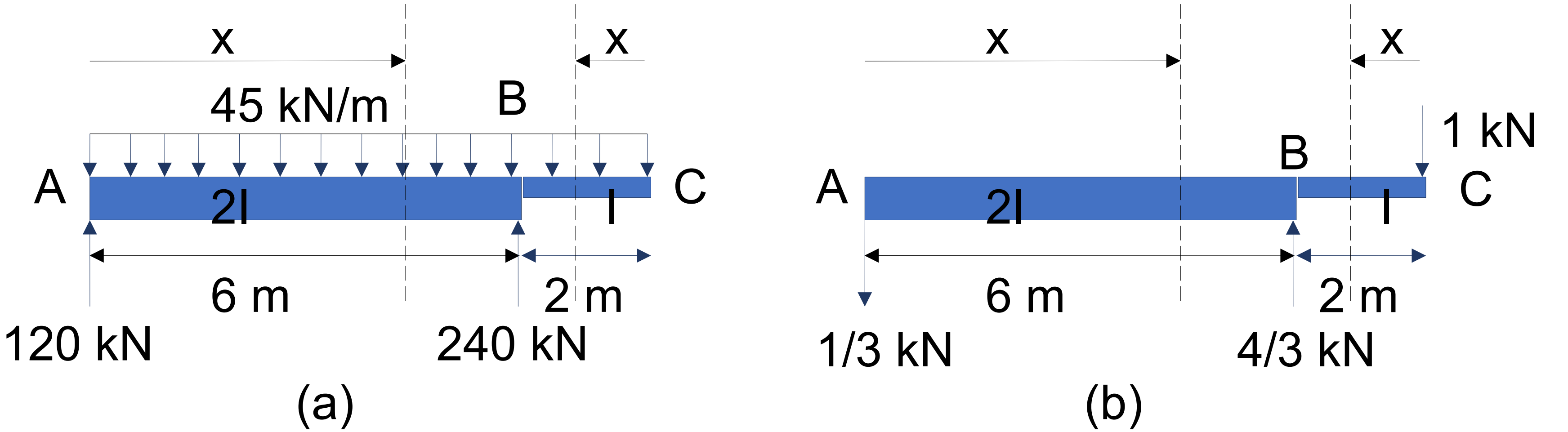


Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

3.5 Displacement in beams: Unit load method

Numerical#1.

Step 2: Solve each beam to find Moment at any section at a distance x from A or C (Origin): M for beam(a) and m for beam (b).



Portion	Origin	Limits	M	m	I
AB	A	$0 \le x \le 6$	$120x - 45x^2/2$	-1/3*x	2I
BC	C	$0 \le x \le 2$	$-45x^2/2$	-x	I

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

3.5 Displacement in beams: Unit load method

Numerical#1.

Step 3: We have, deflection at any point by unit load method:

$$\Delta = \int_0^L \frac{Mm}{EI} dx$$

Portion	Origin	Limits	M	m	I
AB	A	0≤x≤6	120x-45x²/2	-1/3*x	2I
BC	C	0≤x≤2	-45x²/2	-x	I

Then,

$$\Delta_C = \int_0^6 \frac{\left(120x - \frac{45x^2}{2}\right)\left(\frac{-x}{3}\right)}{E(2I)} dx + \int_0^2 \frac{\left(\frac{-45x^2}{2}\right)(-x)}{E(I)} dx$$

Portion AB Portion BC

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

$$\Delta_C = \frac{1}{2EI} \int_0^6 \left(-40x^2 + \frac{15x^3}{2}\right) dx + \frac{1}{EI} \int_0^2 \left(\frac{45x^3}{2}\right) dx$$

$$\Delta_C = \frac{1}{2EI} \left(-40\frac{x^3}{3} + \frac{15x^4}{(2*4)}\right)\Bigg|_0^6 + \frac{1}{EI} \frac{45x^4}{(2*4)}\Bigg|_0^2$$

3.5 Displacement in beams: Unit load method

Numerical#1.

Step 3: We have, deflection at any point by unit load method:

$$\Delta_C = \frac{1}{2EI}\left(-40\frac{x^3}{3} + \frac{15x^4}{(2*4)}\right)\Bigg|_0^6 + \frac{1}{EI}\frac{45x^4}{(2*4)}\Bigg|_0^2$$

$$\Delta_C = \frac{1}{2EI}\left(-40\frac{6^3}{3} + \frac{15*6^4}{(2*4)}\right) + \frac{1}{EI}\frac{45*2^4}{(2*4)}$$

$$\Delta_C = \frac{-225}{EI} + \frac{90}{EI}$$

$$\Delta_C = \frac{-135}{EI}$$

$$\Delta_C = \frac{135}{EI}(\uparrow)$$

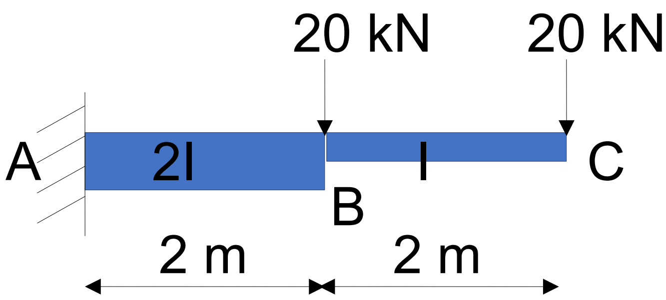
The negative sign shows that the deflection is upward.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

3.5 Displacement in beams: Unit load method

Numerical#2. Determine the deflection and rotation at the free end of the cantilever beam shown in figure. Use unit load method.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

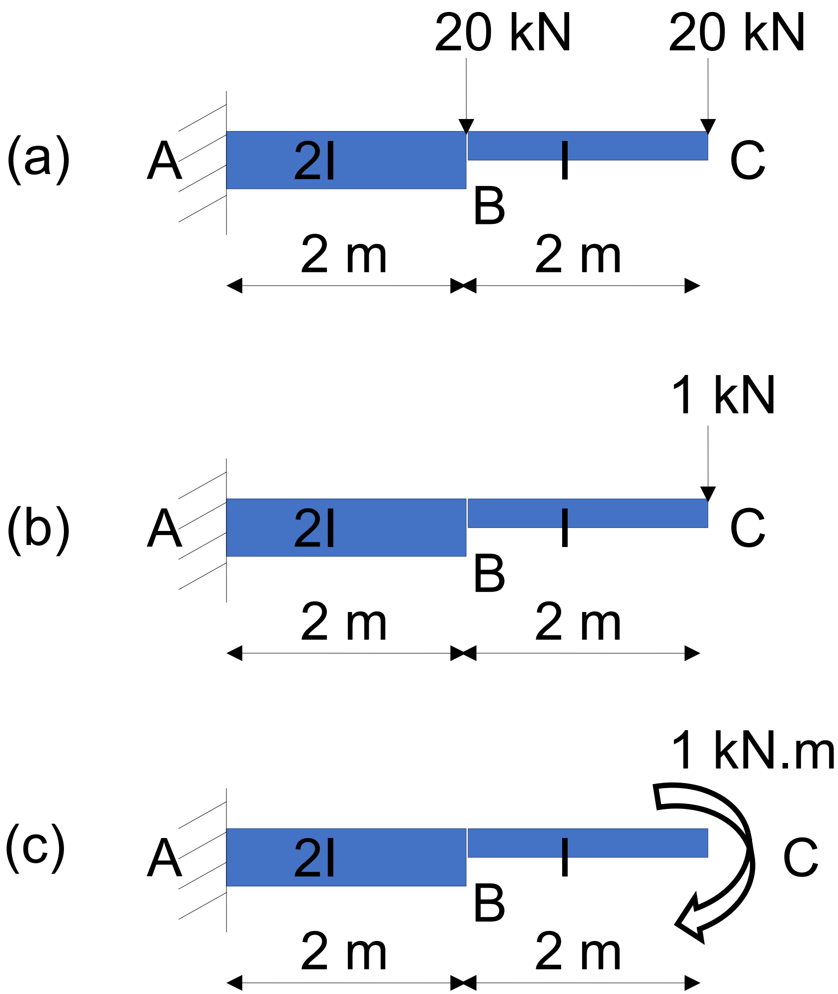


Solution:
Here, we need to consider (a) Given beam subjected to given loading,
(b) Given beam subjected to unit load (1kN) at the point C, where deflection is required.
(c) Given beam subjected to unit moment (1kN.m) at the point C, where rotation is required.

3.5 Displacement in beams: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#2.



Solution:

Step 1: Consider (a) Given beam subjected to given loading,

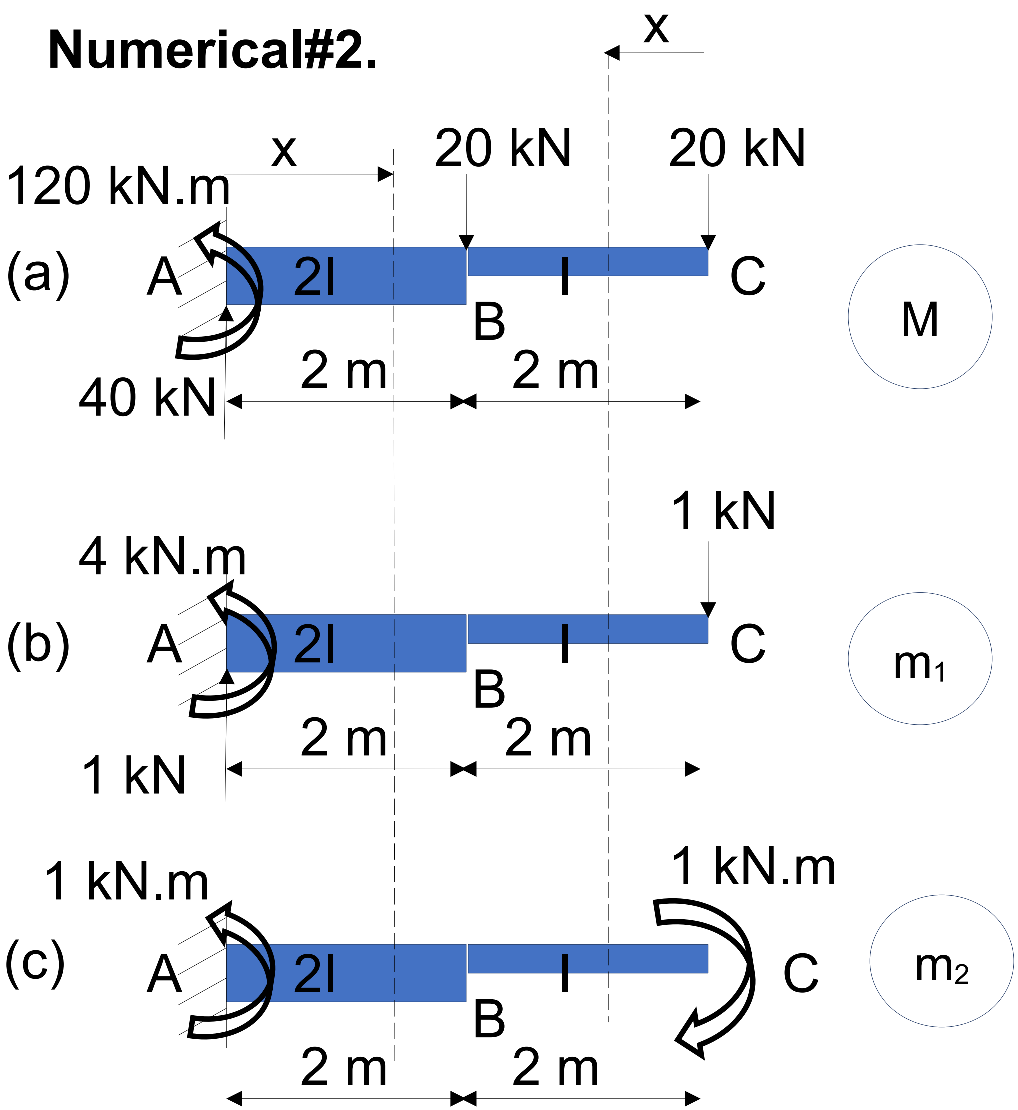
(b) Given beam subjected to unit load (1kN) at the point C, where deflection is required.

(c) Given beam subjected to unit moment (1kN.m) at the point C, where rotation is required.

3.5 Displacement in beams: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#2.



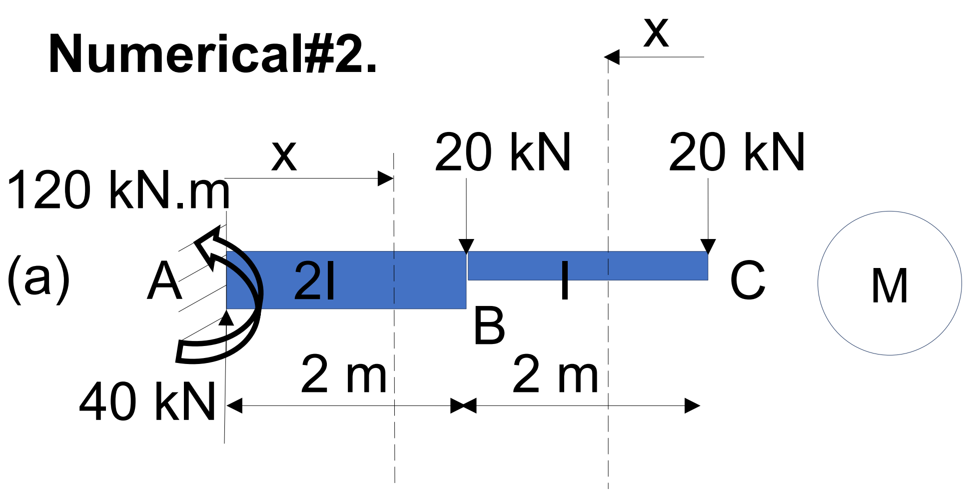
Solution:

Step 2: Solve each beam to find Moment at any section: M for beam(a), m_1 for beam (b), and m_2 for beam (c).

3.5 Displacement in beams: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#2.



Solution:

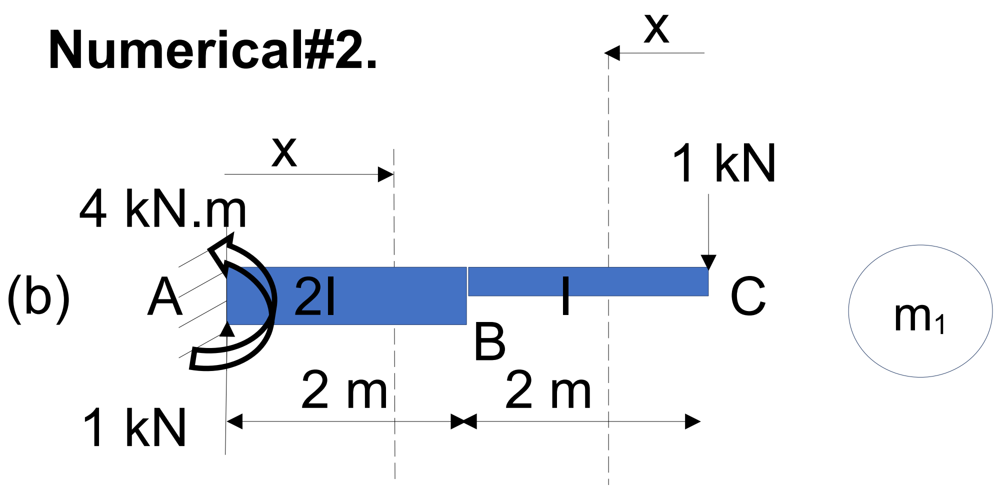
Step 2.1: Calculate M for beam(a).

Portion	Origin	Limits	M	I
AB	A	0≤x≤2	-120+40x	2I
BC	C	0≤x≤2	-20x	I

3.5 Displacement in beams: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#2.



Solution:

Step 2.1: Calculate m_1 for beam(b).

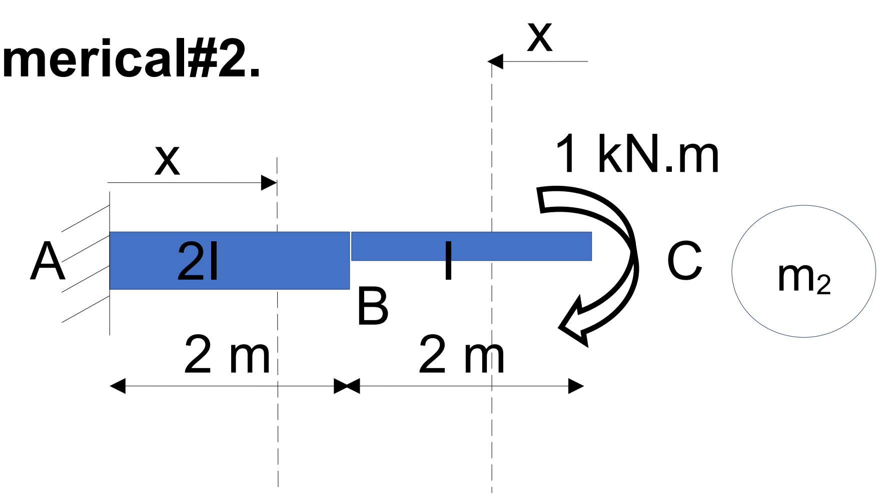
Portion	Origin	Limits	M	m_1	I
AB	A	0≤x≤2	-120+40x	-4+x	2I
BC	C	0≤x≤2	-20x	-x	I

3.5 Displacement in beams: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#2.

(c)



Solution:

Step 2.1: Calculate m_2 for beam(c).

Portion	Origin	Limits	M	m_1	m_2	I
AB	A	0≤x≤2	-120+40x	-4+x	-1	2I
BC	C	0≤x≤2	-20x	-x	-1	I

3.5 Displacement in beams: Unit load method

Numerical#2.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Solution:

Step 3: We have, deflection at any point by unit load method:

$$\Delta = \int_0^L \frac{M m_1}{EI} dx$$

Portion	Origin	Limits	M	m_1	m_2	I
AB	A	0≤x≤2	-120+40x	-4+x	-1	2I
BC	C	0≤x≤2	-20x	-x	-1	I

Then,

$$\Delta_C = \int_0^2 \frac{(-120+40x)(-4+x)}{E(2I)} dx + \int_0^2 \frac{(-20x)(-x)}{E(I)} dx$$

Portion AB Portion BC

$$\Delta_C = \frac{1}{2EI} \int_0^2 (480 - 160x - 120x + 40x^2) dx + \frac{1}{EI} \int_0^2 (20x^2) dx$$

$$\Delta_C = \frac{40}{2EI} \int_0^2 (12 - 7x + x^2) dx + \frac{20}{EI} \int_0^2 (x^2) dx$$

3.5 Displacement in beams: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#2.

Solution:

Step 3:

Portion	Origin	Limits	M	m_1	m_2	I
AB	A	0≤x≤2	-120+40x	-4+x	-1	2I
BC	C	0≤x≤2	-20x	-x	-1	I

Then,

$$\Delta_C = \frac{40}{2EI}\int_0^2 \left(12 - 7x + x^2\right)dx + \frac{20}{EI}\int_0^2 \left(x^2\right)dx$$

$$\Delta_C = \frac{20}{EI}\left(12x - 7\frac{x^2}{2} + \frac{x^3}{3}\right)\Bigg|_0^2 + \frac{20}{EI}\left(\frac{x^3}{3}\right)\Bigg|_0^2$$

$$\Delta_C = \frac{20}{EI}\left(12 * 2 - \frac{7 * 2^2}{2} + \frac{7^3}{3}\right) + \frac{20}{EI}\left(\frac{2^3}{3}\right)$$

$$\Delta_C = \frac{920}{3EI} = \frac{306.67}{EI}$$

Ans.

3.5 Displacement in beams: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#2.

Solution:

Step 4: We have, rotation at any point by unit load method:

$$\theta = \int_0^L \frac{Mm_2}{EI} dx$$

Then,

Portion	Origin	Limits	M	m_1	m_2	I
AB	A	0≤x≤2	-120+40x	-4+x	-1	2I
BC	C	0≤x≤2	-20x	-x	-1	I

$$\theta_C = \int_0^2 \frac{(-120+40x)(-1)}{E(2I)} dx + \int_0^2 \frac{(-20x)(-1)}{E(I)} dx$$

$$\theta_C = \int_0^2 \frac{(120-40x)}{E(2I)} dx + \int_0^2 \frac{(20x)}{E(I)} dx$$

$$\theta_C = \frac{1}{2EI}\left(120x - 40\frac{x^2}{2}\right)\Bigg|_0^2 + \frac{20}{EI}\left(\frac{x^2}{2}\right)\Bigg|_0^2$$

3.5 Displacement in beams: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#2.

Solution:

Step 4:

Then,

Portion	Origin	Limits	M	m_1	m_2	I
AB	A	0≤x≤2	-120+40x	-4+x	-1	2I
BC	C	0≤x≤2	-20x	-x	-1	I

$$\theta_C = \frac{1}{2EI}\left(120x - 40\frac{x^2}{2}\right)\Bigg|_0^2 + \frac{20}{EI}\left(\frac{x^2}{2}\right)\Bigg|_0^2$$

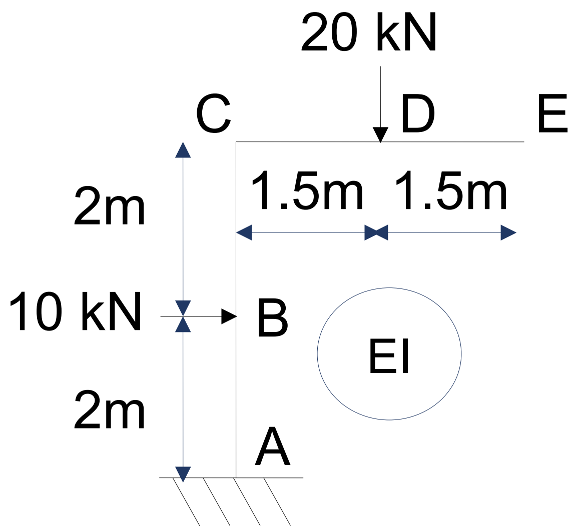
$$\theta_C = \frac{1}{2EI}\left(120*2 - \frac{40*2^2}{2}\right) + \frac{20}{EI}\left(\frac{2^2}{2}\right)$$

$$\theta_C = \frac{120}{EI} \curvearrowright$$ Ans.

3.5 Displacement in frames: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4^{th} ed.).New Delhi: Vikas Publishing House.

Numerical#3. Determine the vertical and horizontal deflection at the free end of the frame shown in figure. Assume uniform flexural rigidity EI throughout.



Solution:

Here, we need to consider (a) Given frame subjected to given loading,

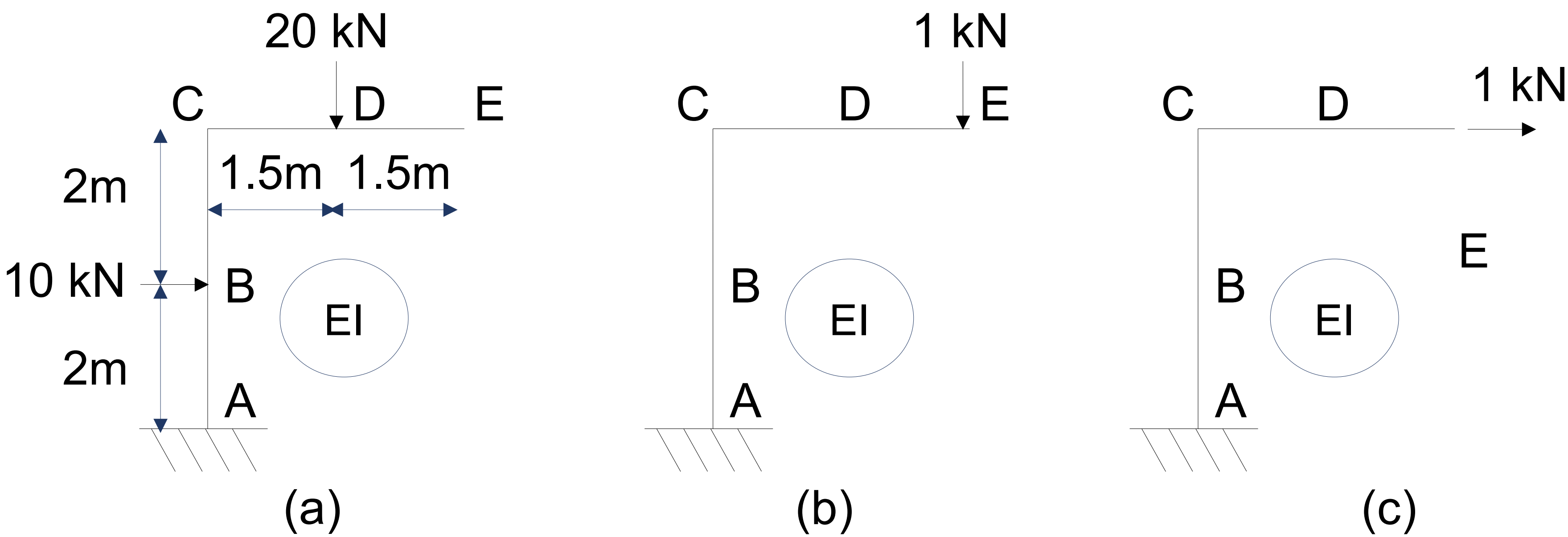
(b) Given frame subjected to vertical unit load (1kN) at the point E, where vertical deflection is required.

(c) Given frame subjected to horizontal unit load (1kN) at the point E, where horizontal deflection is required.

3.5 Displacement in frames: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#3.

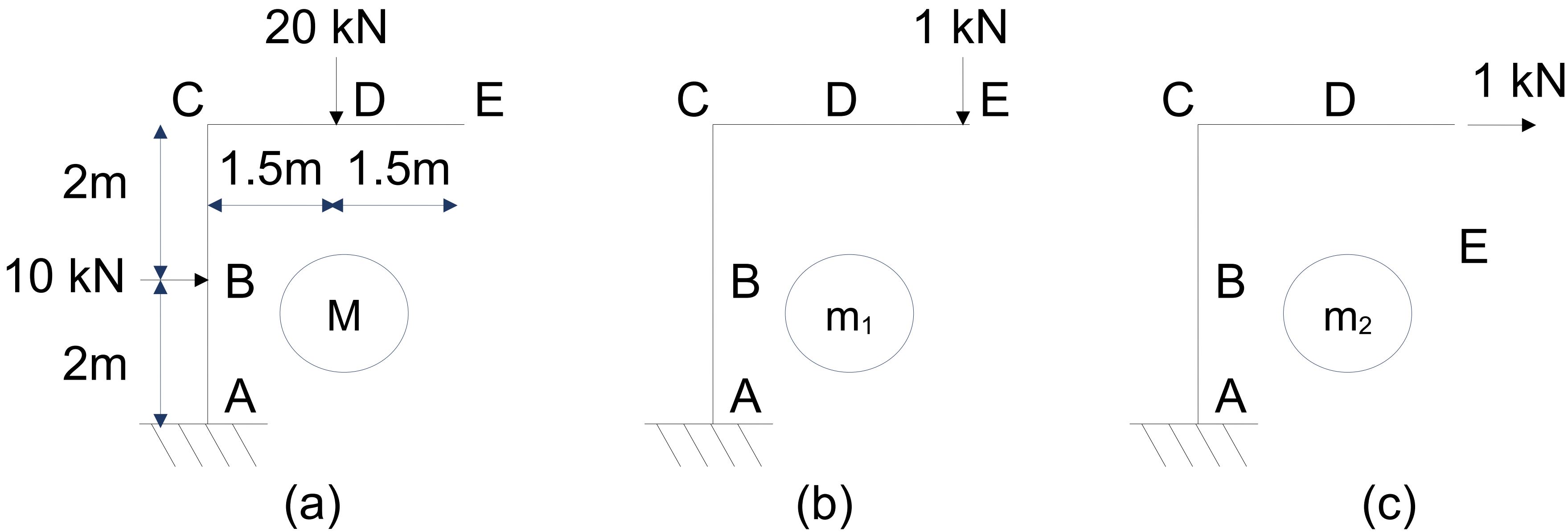


Step 1: Consider (a) Given frame subjected to given loading,
(b) Given frame subjected to vertical unit load (1kN) at the point E, where vertical deflection is required.
(c) Given frame subjected to horizontal unit load (1kN) at the point E, where horizontal deflection is required.

3.5 Displacement in frames: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#3.



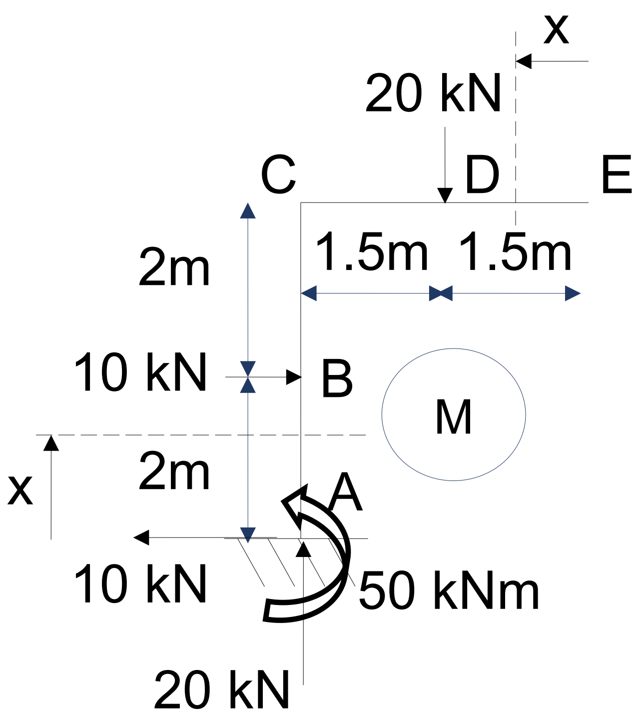
Step 2: Solve each frame to find Moment at any section: M for frame (a), m_1 for frame (b), and m_2 for frame (c).

Here, we have assumed downward vertical deflection and right horizontal deflection.

3.5 Displacement in frames: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#3.



(a)

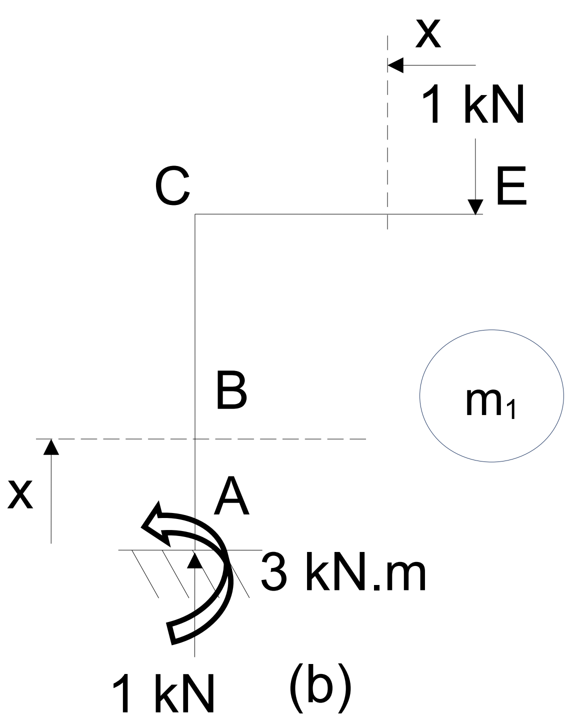
Step 2.1: Calculate M for frame (a) at different sections at a distance x from A or E.

Portion	Origin	Limits	M
AB	A	0≤x≤2	10x-50
BC	B	2≤x≤4	-30
CD	D	1.5≤x≤3	-20(x-1.5)
DE	E	0≤x≤1.5	0

3.5 Displacement in frames: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#3.



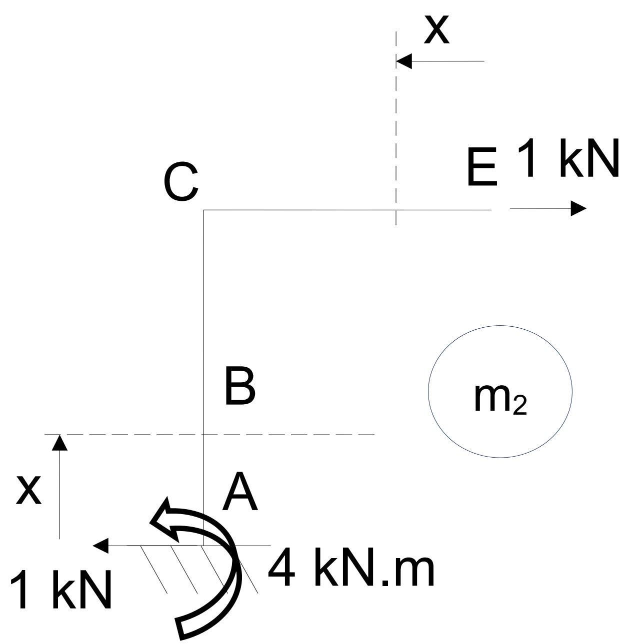
Step 2.2: Calculate m_1 for frame (b) at different sections at a distance x from A or E.

Portion	Origin	Limits	m_1
AB	A	$0 \le x \le 2$	-3
BC	B	$2 \le x \le 4$	-3
CD	D	$1.5 \le x \le 3$	-x
DE	E	$0 \le x \le 1.5$	-x

3.5 Displacement in frames: Unit load method

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Numerical#3.



(c)

Step 2.3: Calculate m_2 for frame (c) at different sections at a distance x from A or E.

Portion	Origin	Limits	m_2
AB	A	0≤x≤2	x-4
BC	B	2≤x≤4	x-4
CD	D	1.5≤x≤3	0
DE	E	0≤x≤1.5	0

3.5 Displacement in frames: Unit load method

Numerical#3.

Portion	Origin	Limits	M	m_1	m_2	EI
AB	A	0≤x≤2	10x-50	-3	x-4	EI
BC	B	2≤x≤4	-30	-3	x-4	EI
CD	D	1.5≤x≤3	-20(x-1.5)	-x	0	EI
DE	E	0≤x≤1.5	0	-x	0	EI

Step 3: Calculate the vertical deflection at E as:

$$\Delta_{VE} = \int_0^L \frac{Mm_1}{EI} dx$$

$$\Delta_{VE} = \int_0^2 \frac{(10x-50)(-3)}{EI} dx + \int_2^4 \frac{(-30)(-3)}{EI} dx + \int_{1.5}^3 \frac{-20(x-1.5)(-x)}{EI} dx + \int_0^{1.5} \frac{(0)(-x)}{EI} dx$$

Portion AB | Portion BC | Portion CD | Portion DE

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

3.5 Displacement in frames: Unit load method

Numerical#3.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Step 3: Calculate the vertical deflection at E as:

$$\Delta_{VE}=\int_0^2 \frac{(10x-50)(-3)}{EI}dx+\int_2^4 \frac{(-30)(-3)}{EI}dx+\int_{1.5}^3 \frac{-20(x-1.5)(-x)}{EI}dx+\int_0^{1.5} \frac{(0)(-x)}{EI}dx$$

$$\Delta_{VE}=\frac{1}{EI}\left[\int_0^2 (150-30x)dx+\int_2^4 90\,dx+\int_{1.5}^3 (20x^2-30x)dx+0\right]$$

$$\Delta_{VE}=\frac{1}{EI}\left[\left(150x-30\frac{x^2}{2}\right)\Big|_0^2+\left(90\frac{x^2}{2}\right)\Big|_2^4+\left(20\frac{x^3}{3}-30\frac{x^2}{2}\right)\Big|_{1.5}^3\right]$$

$$\Delta_{VE}=\frac{1}{EI}\left[\left(150*2-\frac{30*2^2}{2}\right)+\left(\frac{90*4^2}{2}-\frac{90*2^2}{2}\right)+\left(\frac{20*3^3}{3}-\frac{30*3^2}{2}-\frac{20*1.5^3}{3}+\frac{30*1.5^2}{2}\right)\right]$$

$$\Delta_{VE}=\frac{476.25}{EI}$$ Ans.

3.5 Displacement in frames: Unit load method

Numerical#3.

Step 4: Calculate the horizontal deflection at E as:

$$\Delta_{HE}=\int_0^L \frac{Mm_2}{EI}dx$$

Portion	Origin	Limits	M	m_1	m_2	EI
AB	A	0≤x≤2	10x-50	-3	x-4	EI
BC	B	2≤x≤4	-30	-3	x-4	EI
CD	D	1.5≤x≤3	-20(x-1.5)	-x	0	EI
DE	E	0≤x≤1.5	0	-x	0	EI

$$\Delta_{HE}=\int_0^2 \frac{(10x-50)(x-4)}{EI}dx+\int_2^4 \frac{(-30)(x-4)}{EI}dx+\int_{1.5}^3 \frac{-20(x-1.5)(0)}{EI}dx+\int_0^{1.5} \frac{(0)(0)}{EI}dx$$

Portion AB | Portion BC | Portion CD | Portion DE

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

3.5 Displacement in frames: Unit load method

Numerical#3.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

Step 4: Calculate the horizontal deflection at E as:

$$\Delta_{HE}=\int_0^2 \frac{(10x-50)(x-4)}{EI}dx+\int_2^4 \frac{(-30)(x-4)}{EI}dx+\int_{1.5}^3 \frac{-20(x-1.5)(0)}{EI}dx+\int_0^{1.5} \frac{(0)(0)}{EI}dx$$

$$\Delta_{HE}=\frac{1}{EI}\left[\int_0^2 (10x^2-50x-40x+200)\,dx+\int_2^4 (120-30x)\,dx+0+0\right]$$

$$\Delta_{HE}=\frac{1}{EI}[(\frac{10*x^3}{3}-90\frac{x^2}{2}+200*x)\Big|_0^2+(120x-30\frac{x^2}{2})\Big|_2^4]$$

$$\Delta_{HE}=\frac{1}{EI}[(\frac{10*2^3}{3}-\frac{90*2^2}{2}+200*2)+(120*4-\frac{30*4^2}{2}-120*2+\frac{30*2^2}{2})]$$

$$\Delta_{HE}=\frac{306.67}{EI}$$

Ans.

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

We have, deflection at the point and in the direction of unit load applied,

$$\Delta = \int p' e\, dv ----(4)$$

Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

Where,
p'=stress due to unit load
e=strain due to applied load

This equation holds good irrespective of the type of structure.

In case of pin-jointed frames, there is only one type of stress, i.e. direct stress. Hence,

$$\int p' e\, dv = \Sigma\, p' eAL ---(6)$$

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

$$\int p' e \, dv = \Sigma p' eAL --- (6)$$

Where,
A= cross-sectional area of the member
L= length of the member
p'=stress due to unit load
p'= **k**/A -----(7)
where, **k** is the force in the member *due to unit load*
e=strain due to applied load
e= σ/E = **P**/AE ------(8)
where, **P** is the force in the member *due to given loading*

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

Substituting equations (6), (7), and (8) into equation (4), we get:

$$\Delta = \int p' e\, dv = \Sigma p' eAL = \Sigma \frac{k}{A} * \frac{P}{AE} * AL$$

$$\Delta = \Sigma \frac{kPL}{AE} \text{-------} (9)$$

Or,

$$\Delta = \Sigma k \delta L \text{-------} (10)$$

Where,

$\delta L = \dfrac{PL}{AE}$; change in length (extension or shortening) of the member due to external loads only

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

We have,

$$\Delta = \Sigma \frac{kPL}{AE} = \Sigma\, k\, \delta L$$

Here,

$\delta L = \frac{PL}{AE}$; change in length (extension or shortening) of the member due to external loads only

Now,

δf = change in length due to lack of fit (**misfit**: fabrication error)
 = taken as positive if longer length
 = taken as negative if shorter length

δt = change in length due to **temperature**
 =α*Δt*L

Where,

α = coefficient of thermal expansion, /ºC

Δt = change in temperature
 = taken as positive if increase in temperature
 = taken as negative if decrease in temperature

L = original length of the member

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Source: Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.

We have,

$$\Delta = \Sigma \frac{kPL}{AE} = \Sigma\, k\, \delta L ------ (10)$$

Here,
δL = change in length of the member due to external loads only = PL/AE

δf = change in length due to lack of fit (**misfit**: fabrication error)

δt = change in length due to **temperature** =α*Δt*L

Then,
Change in length due to external loads, misfit and temperature (**Combined effects**),
δ = δL+δf+δt

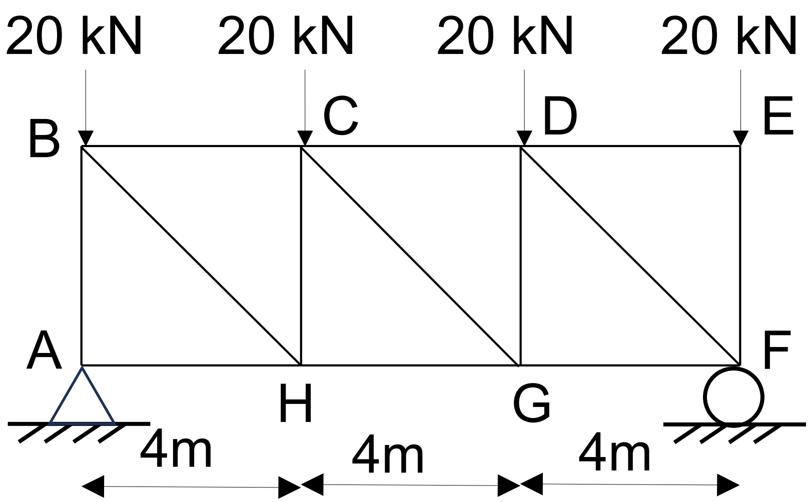
Therefore,
Equation (10) becomes, $$\Delta = \Sigma\, k\, \delta ----- (11)$$

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Numerical#4. Determine the vertical deflection of the joint H of the truss shown in the figure. Area of cross-section of each member is 2000 mm^2. Take E = 200 kN/mm^2.

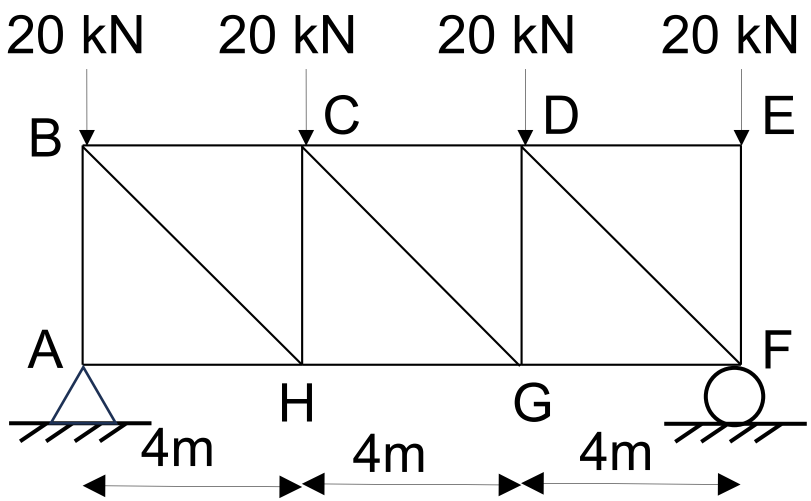
If the temperature of the bottom chord members go up by 20ºC, what will be the additional deflection of joint H? Given = $12*10^{-6}/^oC$?

If the diagonal members are 10 mm too short before fabrication, what will be the deflection of joint due to lack of fit alone?



3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Numerical#4.



Solution:

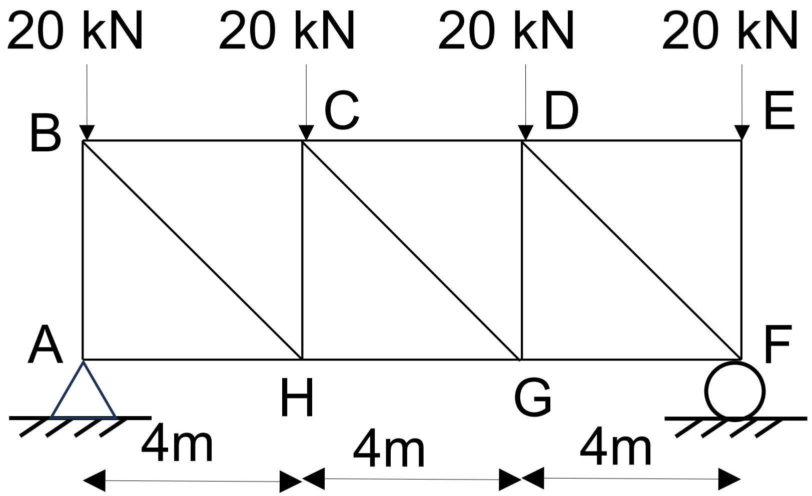
Here, we need to consider (a) Given truss subjected to given loading (**P-system**), (b) Given truss subjected to vertical unit load (1kN) at the point H (**k-system**), where vertical deflection is required.

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

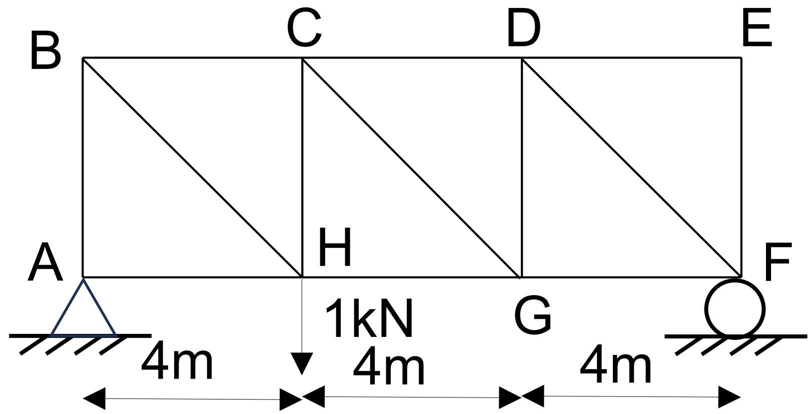
Numerical#4.

Solution:

Step 1: Consider (a) Given truss subjected to given loading (**P-system**),
(b) Given truss subjected to vertical unit load (1kN) at the point H (**k-system**), where vertical deflection is required.



(a) P-system



(b) k-system

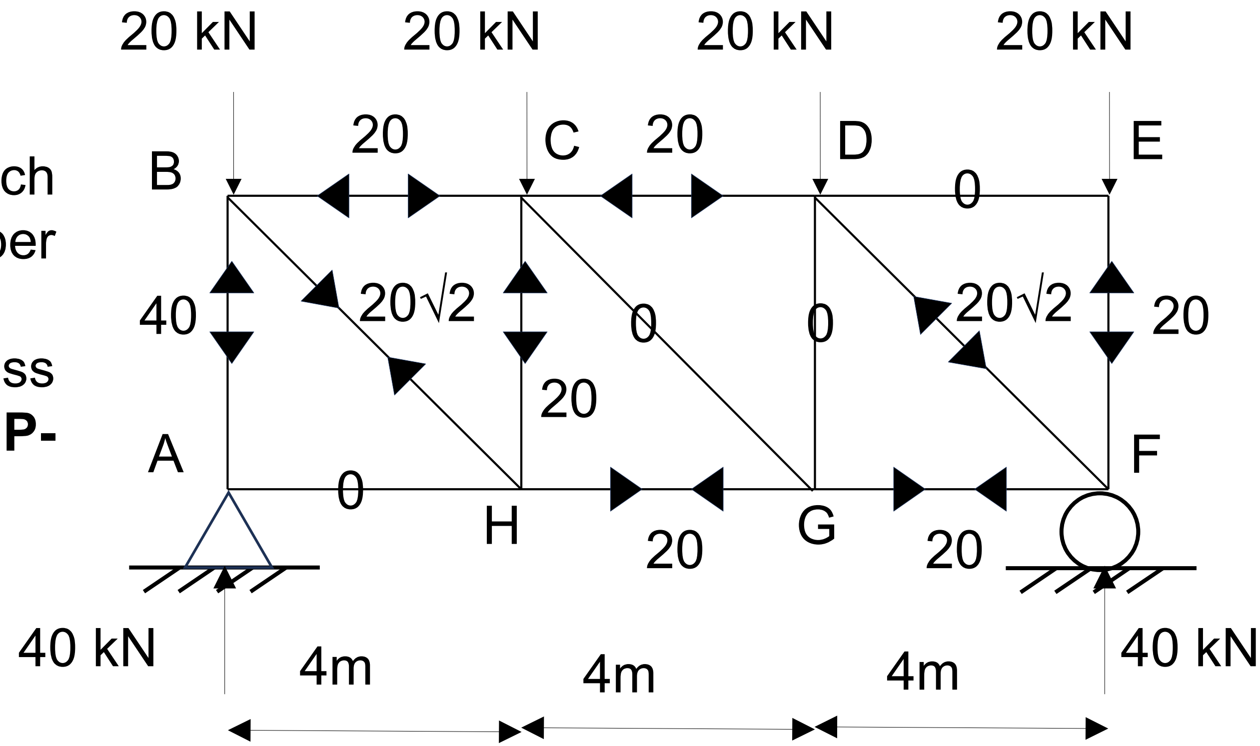
3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Numerical#4.

Solution:

Step 2: Solve each truss for member forces:

2.1 (a) Solving truss subjected for **P-system** of loading



(a) P-system

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Numerical#4.

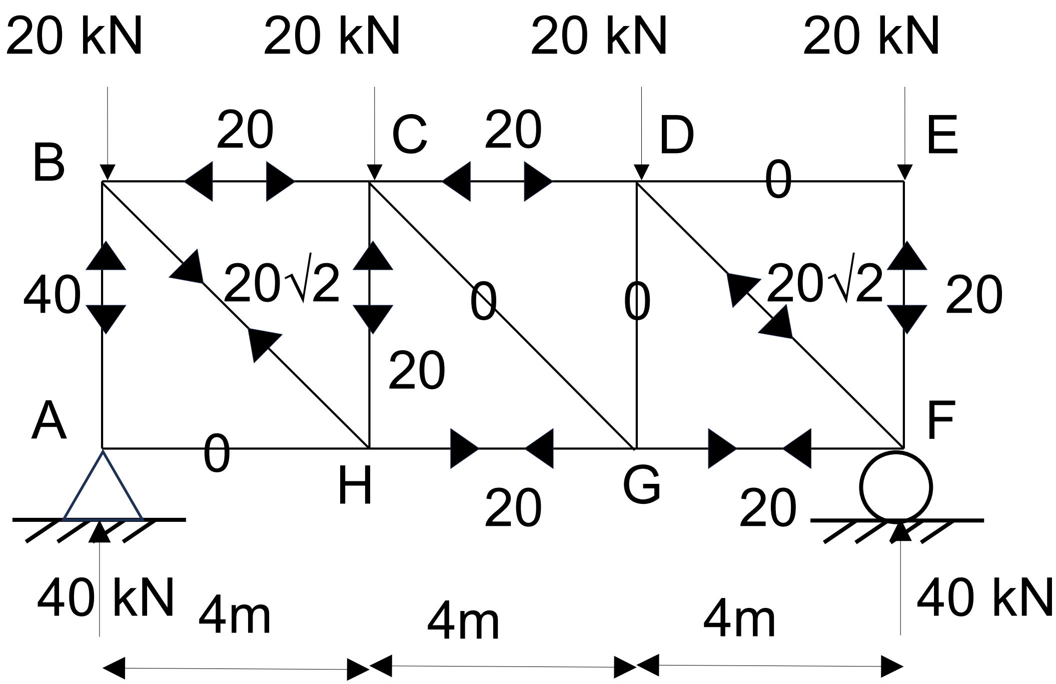
Solution:

Step 2.1 (a) Solving truss subjected for **P-system** of loading

Here,

The internal forces developed in each truss member is denoted by ▶◀ for Tension and ◀▶ for Compression ; the units are in kN.

The —0— members are *zero-force members.*



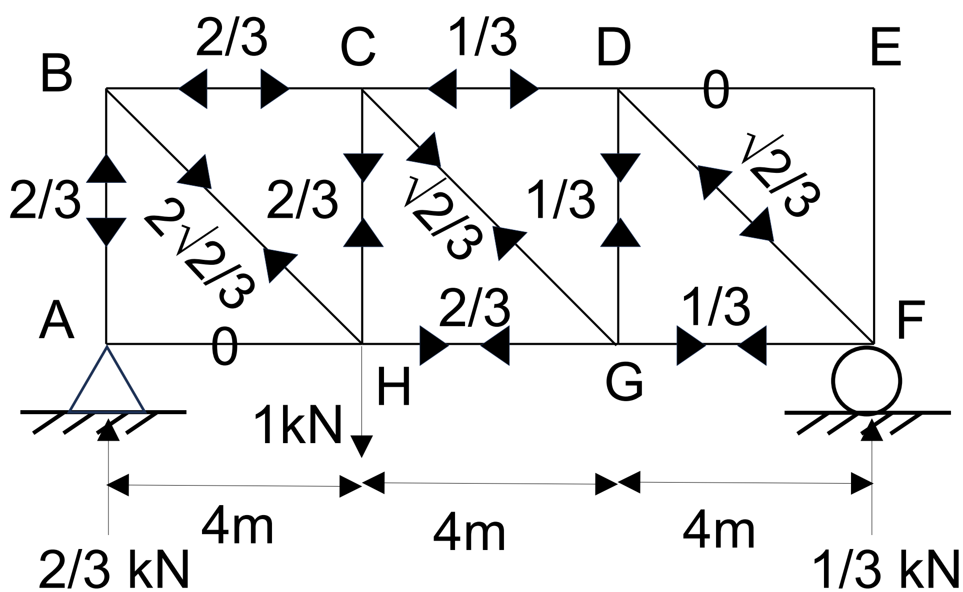
(a) P-system

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Numerical#4.

Solution:

Step 2.2: Similarly, (b) Solving truss subjected for k**-system** of loading



(b) k-system

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Numerical#4.

Solution:

Step 3: Calculate

$$\Delta = \Sigma k\,\delta L$$

$$\Delta = 1.165\,mm$$

Which is deflection at H due to given loading.

Member	P-forces (kN)	k-forces (kN)	Length (mm)	AE	KδL =kPL/AE
AB	-40	-2/3	4000	$4*10^5$	0.267
BC	-20	-2/3	"	"	0.133
CD	-20	-1/3	"	"	0.067
DE	0	0	"	"	0
EF	-20	0	"	"	0
FG	20	1/3	"	"	0.067
GH	20	2/3	"	"	0.133
AH	0	0	"	"	0
BH	20√2	2√2/3	4000√2	"	0.377
CH	-20	1/3	4000	"	-0.067
CG	0	-√2/3	4000√2	"	0
DG	0	1/3	4000	"	0
DF	-20√2	-√2/3	4000√2	"	0.188
				Σ k δl =	1.165 mm

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Numerical#4.

Solution:
Step 4: Calculate

$$\Delta = \Sigma k \delta f$$

$$\Delta = 0$$

Which is deflection at H due to misfit.

Member	k-forces (kN)	δf	kδf
AB	-2/3		
BC	-2/3		
CD	-1/3		
DE	0		
EF	0		
FG	1/3		
GH	2/3		
AH	0		
BH	2√2/3	-10	-9.428
CH	1/3		
CG	-√2/3	-10	4.714
DG	1/3		
DF	-√2/3	-10	4.714
		Σ k δf =	0 mm

3.6 Adjustments and Misfits in Truss Elements and Temperature Effects

Numerical#4.

Solution:
Step 5: Calculate

$$\Delta = \Sigma k \, \delta t$$

$$\Delta = 0$$

Which is deflection at H due to temperature changes.

Member	k-forces (kN)	δt =α*Δt*L	kδt
AB	-2/3		
BC	-2/3		
CD	-1/3		
DE	0		
EF	0		
FG	1/3	0.96	0.32
GH	2/3	0.96	0.64
AH	0	0.96	0
BH	2√2/3		
CH	1/3		
CG	-√2/3		
DG	1/3		
DF	-√2/3		
		Σ k δf =	0.96 mm

-------------End of Lecture#4-------------------

-------------End of Part II of II for Chapter3--------------

References

[1] Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.
[2] Reddy, C.S. (2011). *Basic Structural Analysis (3rd ed.).* New Delhi: Tata McGraw Hill.