

Theory of Structures - I

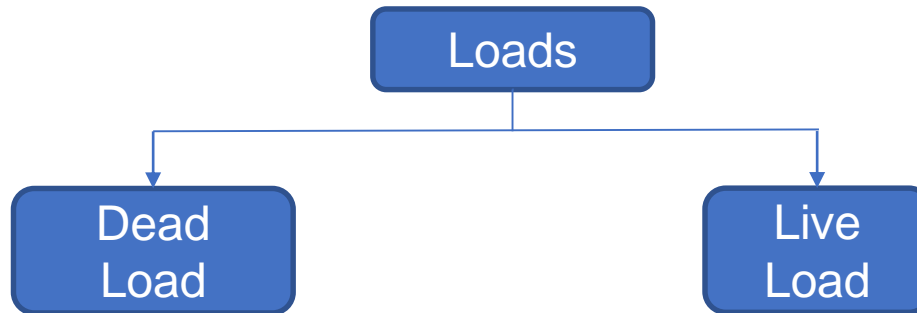
Chapter 5. Influence Lines for
Simple Structures [Part I of III]

Lecturer: Dr. Sanjeema Bajracharya

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- 5.4 Influence Lines for Statically Determinate Trusses

5.1 Moving Static Loads and Influence Lines



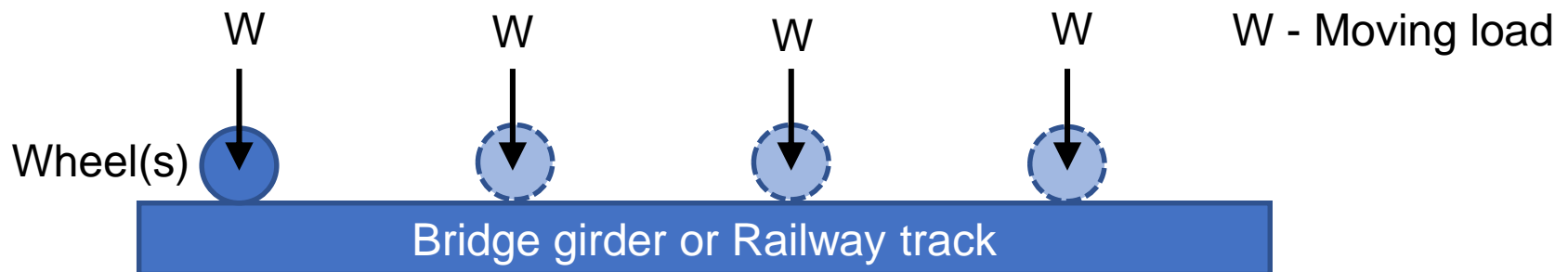
- Loads that do not change their position during the lifetime of a structural member. e.g. self-weight
- Fixed in magnitude and position
- Loads that change their position during the lifetime of a structural member. e.g. traffic loads in bridge girders, railway tracks, etc.
- Fixed in magnitude but can have a variety of positions on a structural member.

Source: Timoshenko, S.P. and Young, D.H. (1965). Theory of Structures (2nd ed.). Singapore: McGraw-Hill Book Co. Singapore.

5.1 Moving Static Loads and Influence Lines

Bridge girders – Are intended to carry loads which roll over them from end to another (trucks, automobiles, etc.)

Railway tracks – Are intended to carry loads from train wheels that roll over them from one end to another

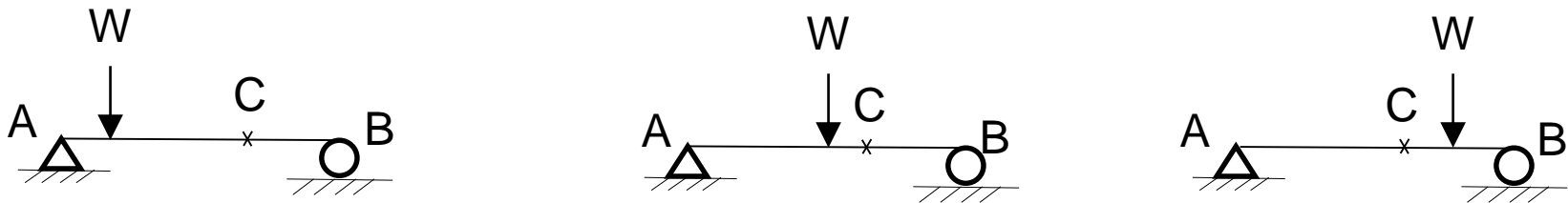


Therefore, we need to analyze the response of the structural member for moving loads. For this purpose, **Influence lines** are used.

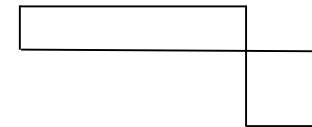
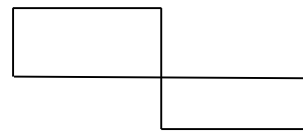
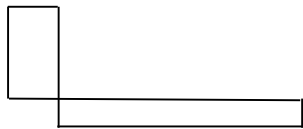
Source: Timoshenko, S.P. and Young, D.H. (1965). Theory of Structures (2nd ed.). Singapore: McGraw-Hill Book Co. Singapore.

5.1 Moving Static Loads and Influence Lines

Influence lines show graphically how the *changing the position of a single load* on a structure or structural member *influences* the response of the structure such as *reactions, bending moments, shear forces, and deflections*.



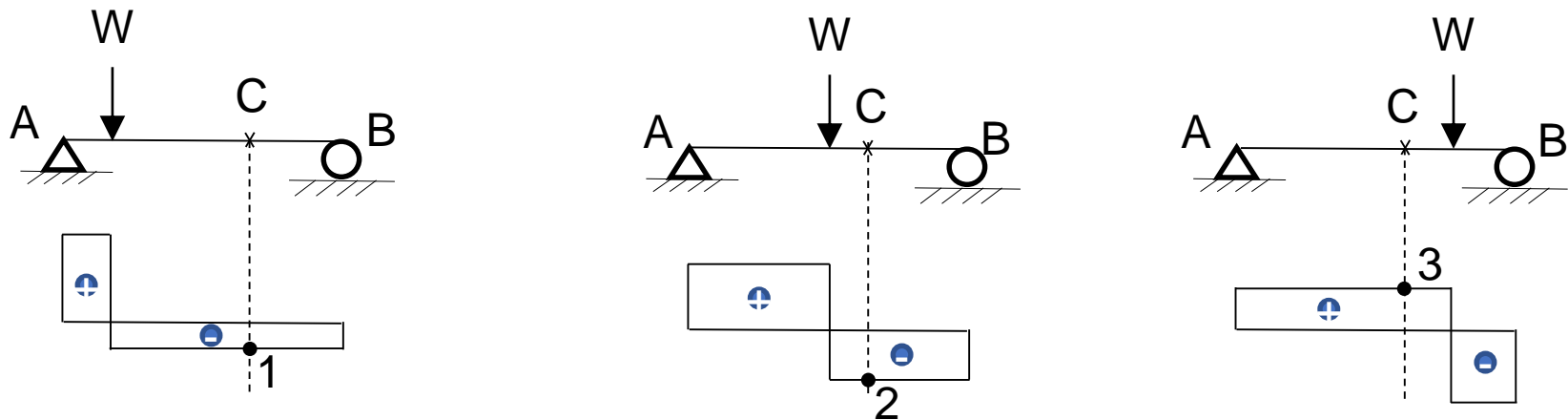
Source: Timoshenko, S.P. and Young, D.H. (1965). Theory of Structures (2nd ed.). Singapore: McGraw-Hill Book Co. Singapore.



Shear Force Diagrams for different positions of point load W

Q# For what location of W , shear force at point C in the simply supported beam AB is maximum/critical? → We use influence lines for this.

5.2 Influence Line Diagrams for Statically Determinate Structures

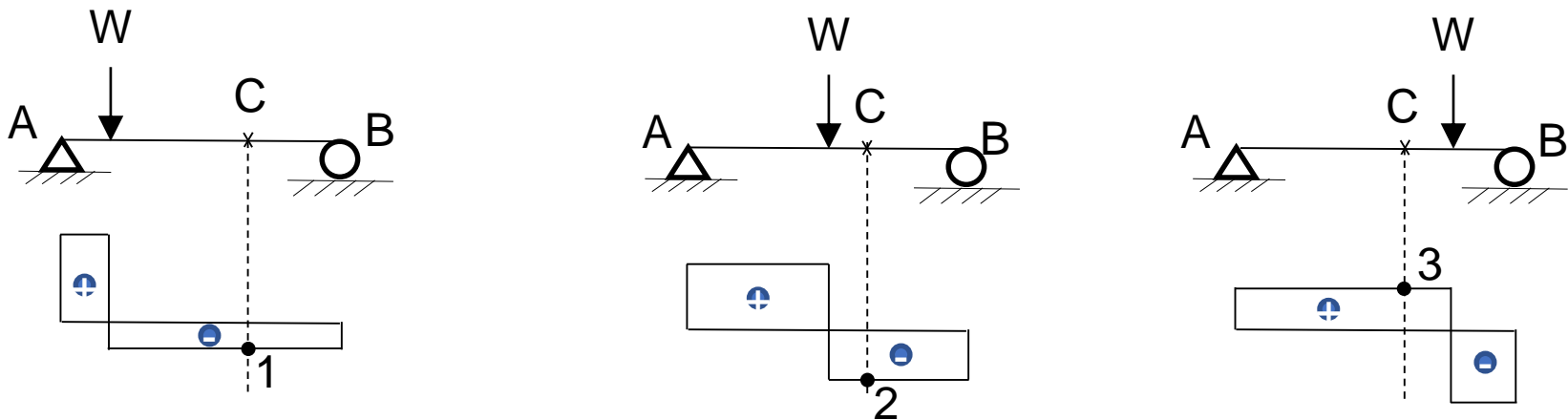


Shear Force Diagrams for different positions of point load W

Let us consider this statically determinate, simply supported beam AB with a point load W acting at different locations. Here, as an example, let us plot influence line diagram for shear force at location C of the beam.

Source: Timoshenko, S.P. and Young, D.H. (1965). Theory of Structures (2nd ed.). Singapore: McGraw-Hill Book Co. Singapore.

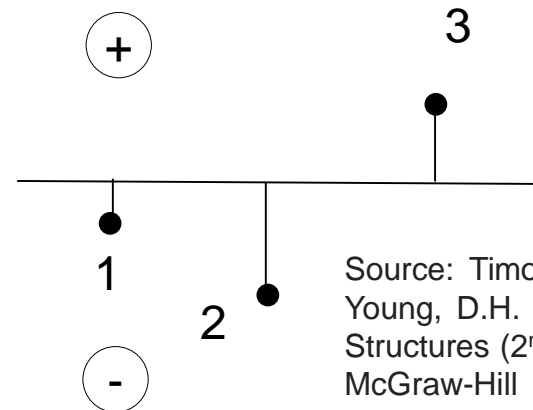
5.2 Influence Line Diagrams for Statically Determinate Structures



Shear Force Diagram (SFD)s for different positions of point load W

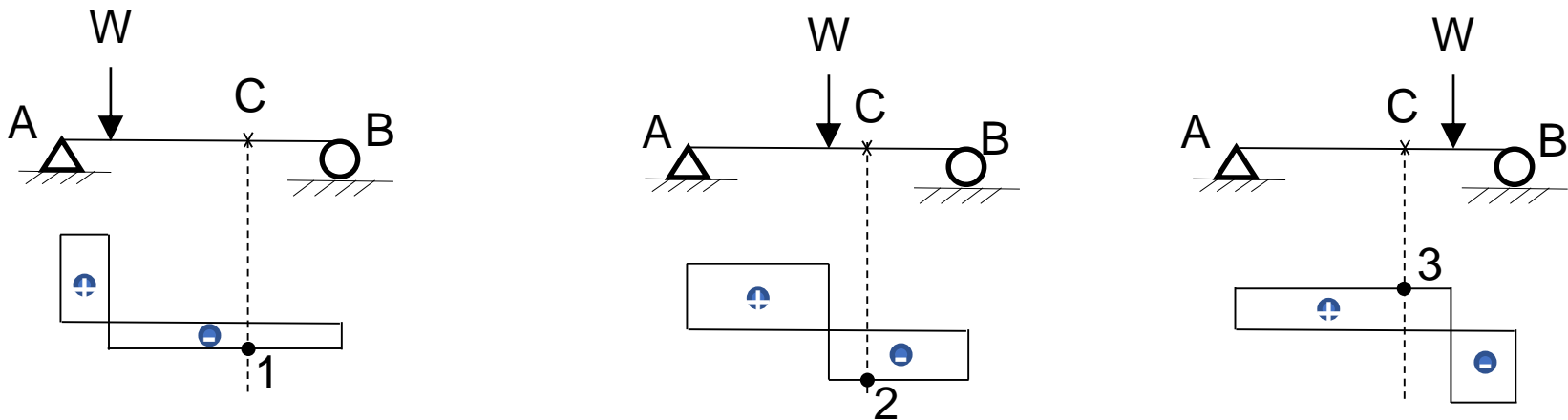
Step 1: Plot the SFDs for each location of point load W .

Step 2: Plot the ordinate of SFD at C . Positive ordinates are plotted above the line, and negative below the line.



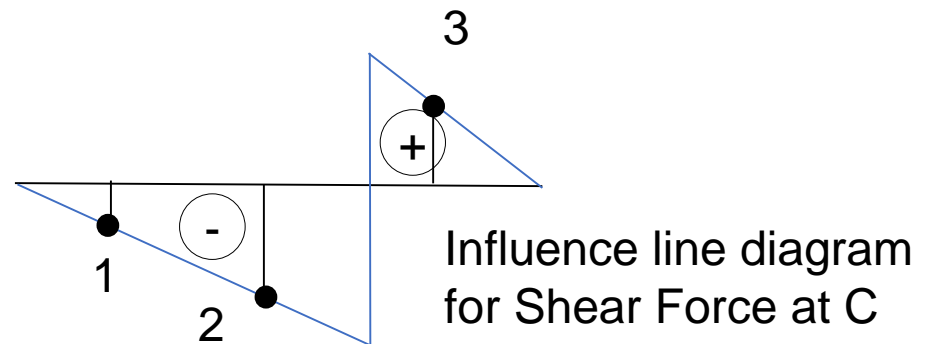
Source: Timoshenko, S.P. and Young, D.H. (1965). Theory of Structures (2nd ed.). Singapore: McGraw-Hill Book Co. Singapore.

5.2 Influence Line Diagrams for Statically Determinate Structures



Shear Force Diagram (SFD)s for different positions of point load W

Step 3: Join these ordinates to get the Influence Line Diagram.



5.2 Influence Line Diagrams for Statically Determinate Structures

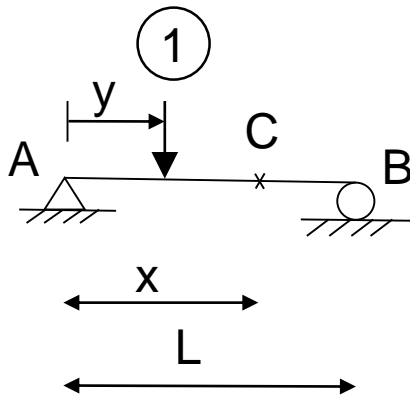
ILD (Influence Line Diagram):

- Drawn for various stress resultants –
 - Reaction
 - Shear Force
 - Bending Moment
- It is a graphical representation of variation of stress resultant at a required section of the structural member when a unit load moves along its span.
- ILD for a stress resultant is the one in which the ordinate represents the value of stress resultant for the position of unit load at the corresponding abscissa.

Source: Bhavikatti, S. S. (2011).
Structural Analysis –I (4th ed.). New
Delhi: Vikas Publishing House.

5.3 Moving loads and ILDs for Statically Determinate Beams

Let us make ILDs for the reaction, shear force (SF), and bending moment (BM) at a point C at a distance x from support A of a simply supported beam AB of span L .



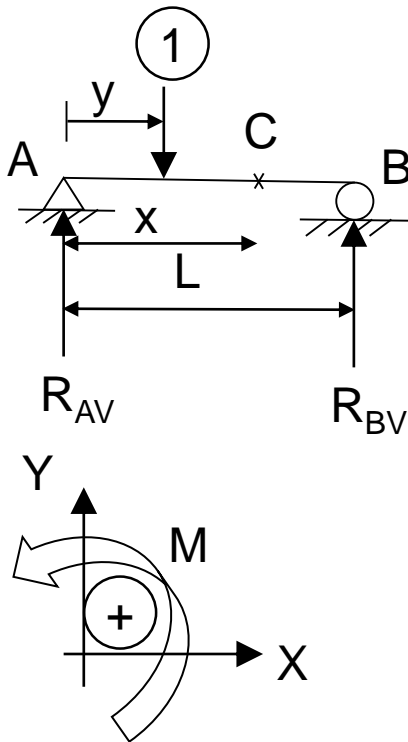
Note:

1. We consider a unit load (1 kN) moving along the span of the beam AB.
2. In the adjacent figure, let us consider, at the moment, the unit load is at a distance of y from support A.
3. Here, y is variable, x is fixed for location of C, and L is fixed (span of the beam AB).

Source: Bhavikatti, S. S. (2011).
Structural Analysis – I (4th ed.). New
 Delhi: Vikas Publishing House.

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for reaction at support A, R_{AV}



Taking moment about support B [one of the equations of static equilibrium],

$$\Sigma M_B = 0$$

$$\text{Or, } -R_{AV} * L + 1 * (L - y) = 0$$

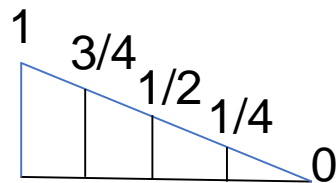
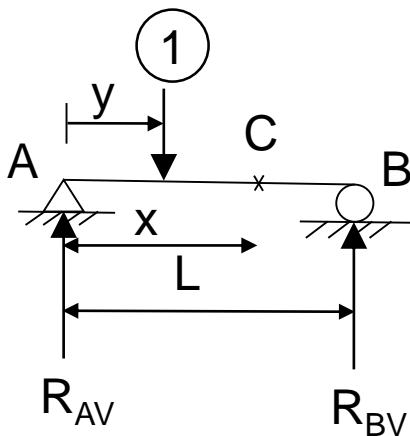
$$\text{Or, } R_{AV} = \frac{L - y}{L}$$

$$\therefore R_{AV} = 1 - \frac{y}{L}$$

This expression gives us the ordinates for ILD for R_{AV} as y changes, i.e. as unit load moves along the span.

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for reaction at support A, R_{AV}



ILD for R_{AV}

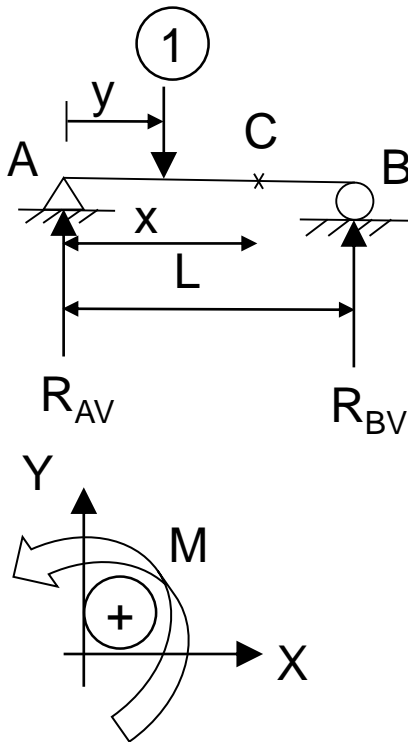
Now, let us plot the ordinates of influence lines for R_{AV} using the obtained expression for different values of y .

$$\therefore R_{AV} = 1 - \frac{y}{L}$$

y	0	$L/4$	$L/2$	$3L/4$	L
R_{AV}	1	$3/4$	$1/2$	$1/4$	0

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for reaction at support B, R_{BV}



Taking moment about support A [one of the equations of static equilibrium],

$$\Sigma M_A = 0$$

$$\text{Or, } R_{BV} * L - 1 * (y) = 0$$

$$\text{Or, } R_{BV} = \frac{y}{L}$$

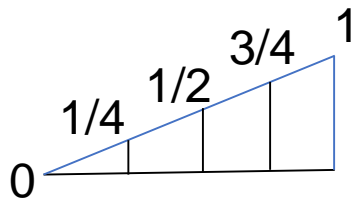
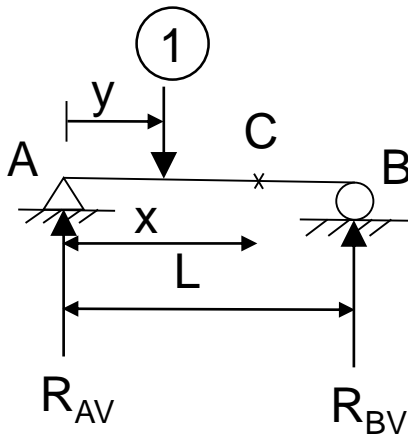
$$\therefore R_{BV} = \frac{y}{L}$$

Source: Bhavikatti, S. S. (2011).
Structural Analysis – I (4th ed.). New
 Delhi: Vikas Publishing House.

This expression gives us the ordinates for ILD for R_{BV} as y changes, i.e. as unit load moves along the span.

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for reaction at support B, R_{BV}



ILD for R_{BV}

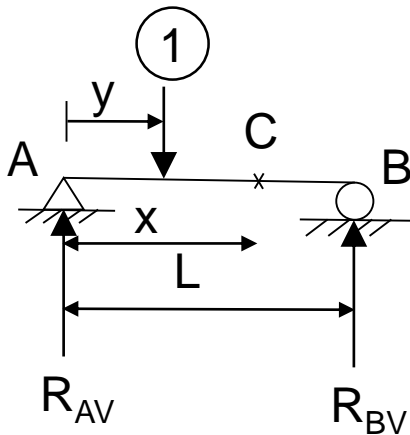
Now, let us plot the ordinates of influence lines for R_{BV} using the obtained expression for different values of y .

$$\therefore R_{BV} = \frac{y}{L}$$

y	0	$L/4$	$L/2$	$3L/4$	L
R_{BV}	0	$1/4$	$1/2$	$3/4$	1

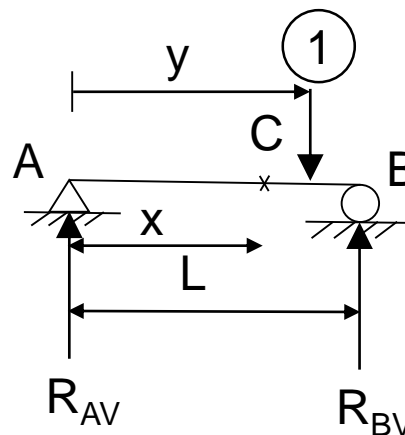
5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for SF at C



(i) $0 \leq y \leq x$

$$\begin{aligned} SF \text{ at } C &= R_{AV} - 1 \\ &= \left(1 - \frac{y}{L}\right) - 1 \\ &= -\frac{y}{L} \end{aligned}$$



(ii) $x \leq y \leq L$

$$SF \text{ at } C = R_{AV} = 1 - \frac{y}{L}$$

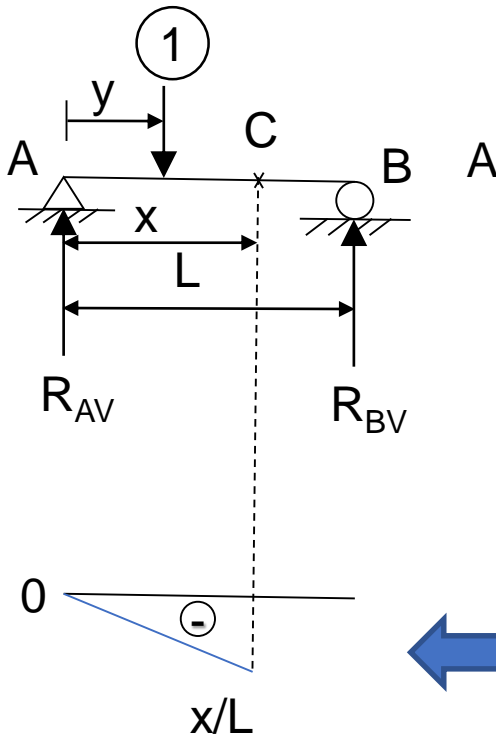
We have,

$$R_{AV} = 1 - \frac{y}{L}$$

$$R_{BV} = \frac{y}{L}$$

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for SF at C



$$(i) 0 \leq y \leq x$$

$$SF \text{ at } C = -\frac{y}{L}$$

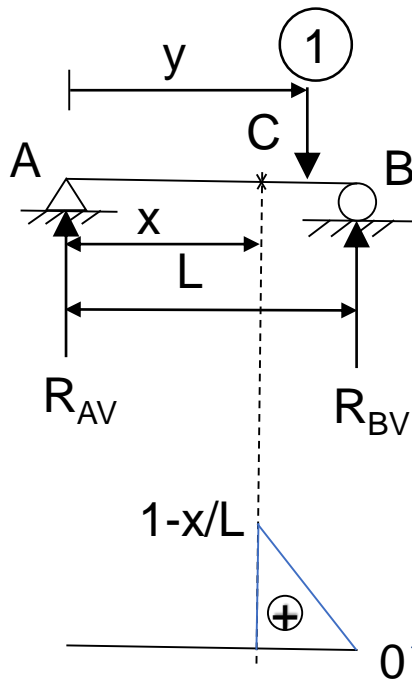
Now, determining the ordinates of influence lines for SF at C using above expression for values of y between 0 and x .

	$y = 0$	$y = x$	$y = L$
SF at C	0	$-x/L$	Not applicable

Plotting the ordinates,

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for SF at C



$$(ii) x \leq y \leq L$$

$$SF \text{ at } C = 1 - \frac{y}{L}$$

Now, determining the ordinates of influence lines for SF at C using above expression for values of y between x and L .

	$y = 0$	$y = x$	$y = L$
SF at C	Not applicable	$1 - x/L$	0

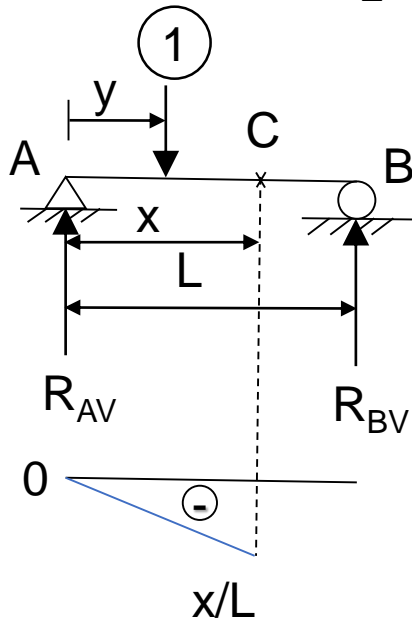
Plotting the ordinates,

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for SF at C

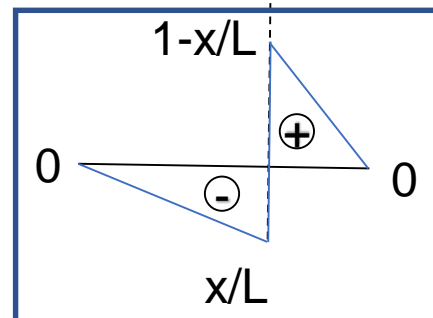
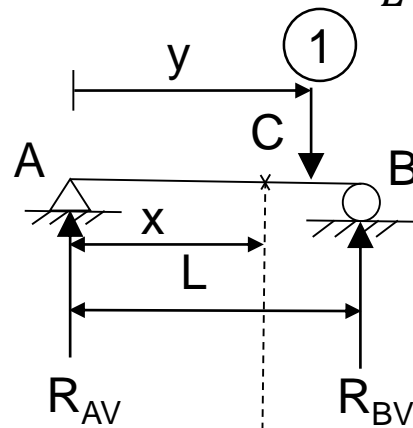
(i) $0 \leq y \leq x$

$$SF \text{ at } C = -\frac{y}{L}$$



(ii) $x \leq y \leq L$

$$SF \text{ at } C = 1 - \frac{y}{L}$$



Combining the results,

	$0 \leq y \leq x$	$x \leq y \leq L$
$y = 0$	0	
$y = x$	$-x/L$	$1-x/L$
$y = L$		0



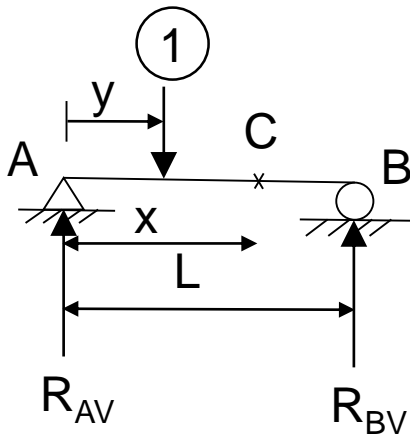
ILD for SF at C for a simply supported beam when a unit load moves along its span.

Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

5.3 Moving loads and ILDs for Statically Determinate Beams

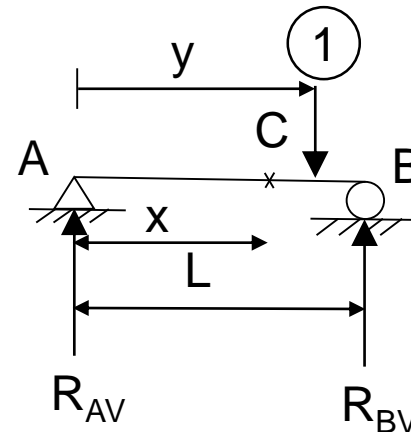
ILD for BM at C

We have, $R_{AV} = 1 - \frac{y}{L}$ $R_{BV} = \frac{y}{L}$



(i) $0 \leq y \leq x$

$$\begin{aligned} BM \text{ at } C &= R_{AV} * x - 1 * (x - y) \\ &= \left(1 - \frac{y}{L}\right) x - x + y \\ &= \frac{y}{L} (L - x) \end{aligned}$$

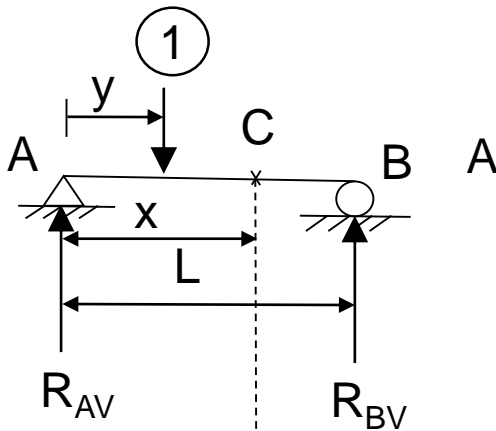


(ii) $x \leq y \leq L$

$$\begin{aligned} BM \text{ at } C &= R_{AV} * x \\ &= \left(1 - \frac{y}{L}\right) x \\ &= \frac{x(L-y)}{L} \end{aligned}$$

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for BM at C

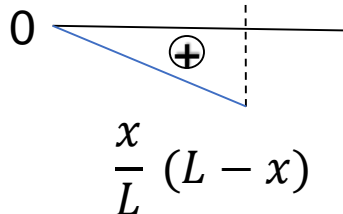


$$(i) 0 \leq y \leq x$$

$$BM \text{ at } C = \frac{y}{L} (L - x)$$

Now, determining the ordinates of influence lines for BM at C using above expression for values of y between 0 and x .

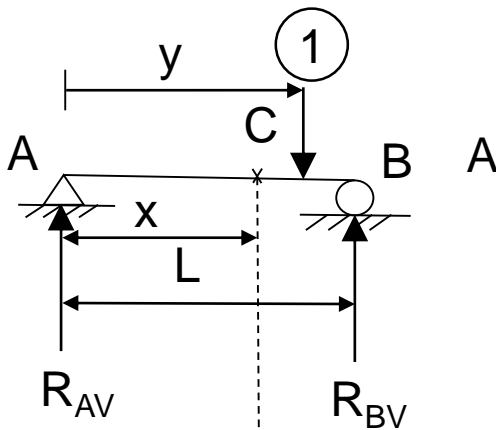
	$y = 0$	$y = x$	$y = L$
BM at C	0	$x(L-x)/L$	Not applicable



Plotting the ordinates,

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for BM at C



$$(ii) \quad x \leq y \leq L$$

$$BM \text{ at } C = \frac{x(L - y)}{L}$$

Now, determining the ordinates of influence lines for BM at C using above expression for values of y between 0 and x .

	$y = 0$	$y = x$	$y = L$
BM at C	Not applicable	$x(L-x)/L$	0

$$\frac{x}{L} (L - x)$$



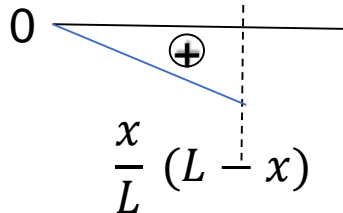
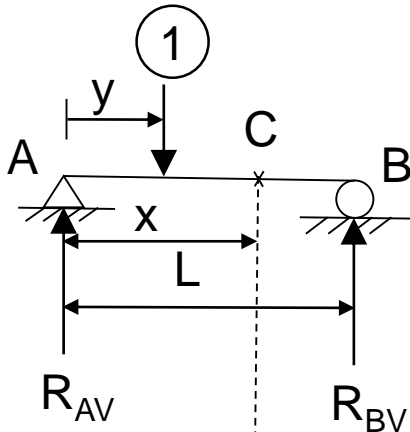
Plotting the ordinates,

5.3 Moving loads and ILDs for Statically Determinate Beams

ILD for BM at C

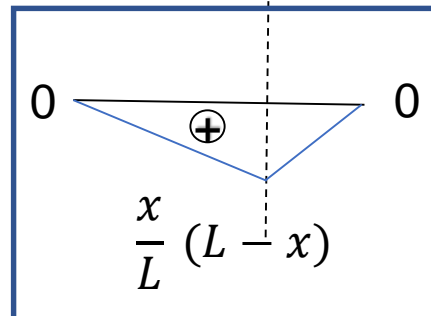
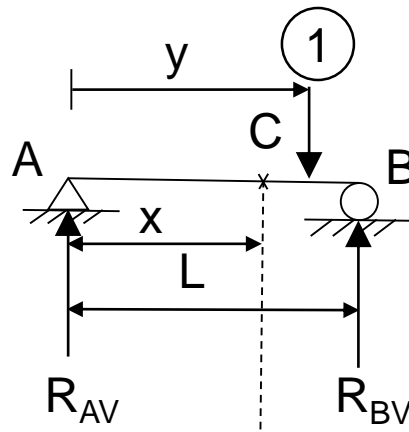
(i) $0 \leq y \leq x$

$$BM \text{ at } C = \frac{y}{L} (L - x)$$



(ii) $x \leq y \leq L$

$$BM \text{ at } C = \frac{x(L - y)}{L}$$



Combining the results,

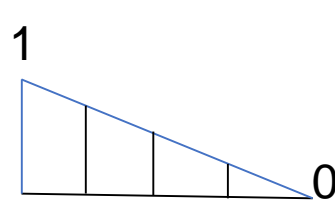
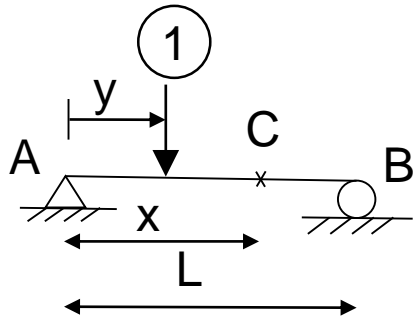
	$0 \leq y \leq x$	$x \leq y \leq L$
$y = 0$	0	
$y = x$	$x(L-x)/L$	$x(L-x)/L$
$y = L$		0



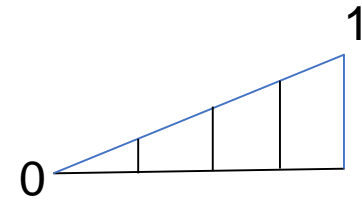
ILD for BM at C for a simply supported beam when a unit load moves along its span.

5.3 Moving loads and ILDs for Statically Determinate Beams

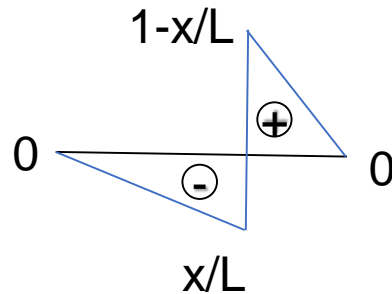
Summary for ILDs for a Simply Supported Beam with unit load moving along its span.



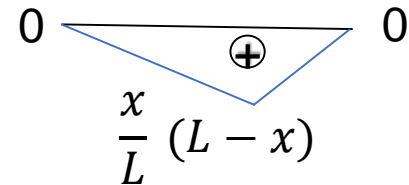
ILD for R_{AV}



ILD for R_{BV}



ILD for SF at C



ILD for BM at C

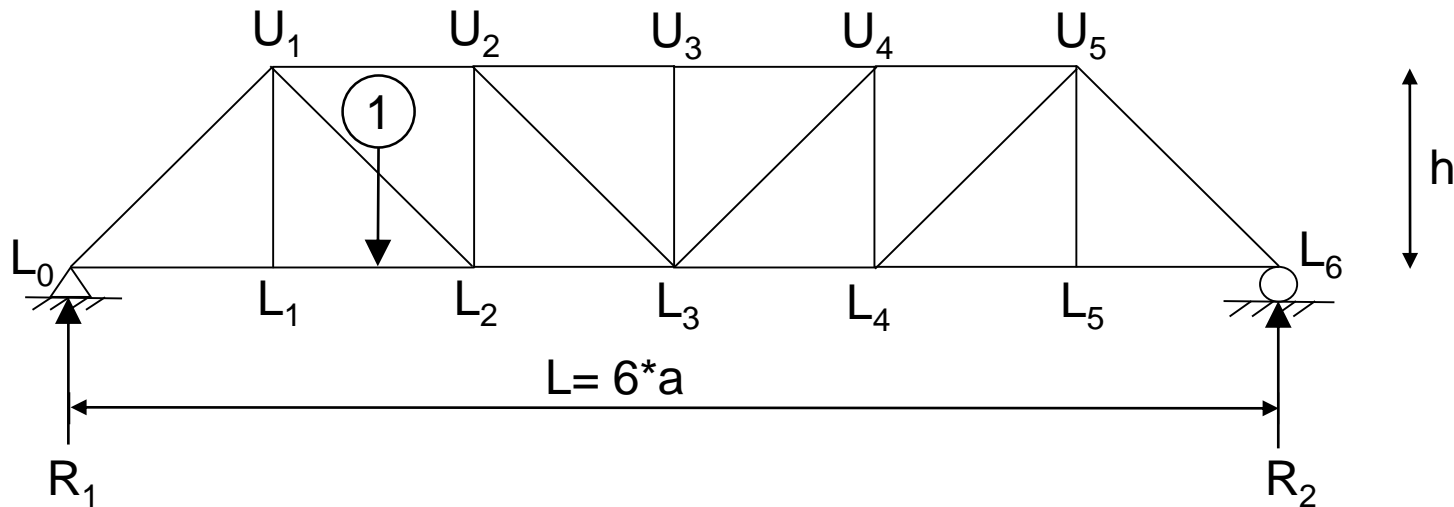
5.4 Influence Lines for Statically Determinate Trusses

In the design of a truss, we must find the most unfavorable position of live load for each member and then evaluate the corresponding axial force in that member.

For such investigation, the influence lines can be used.

Source: Timoshenko, S.P. and Young, D.H. (1965). Theory of Structures (2nd ed.). Singapore: McGraw-Hill Book Co. Singapore.

5.4 Influence Lines for Statically Determinate Trusses

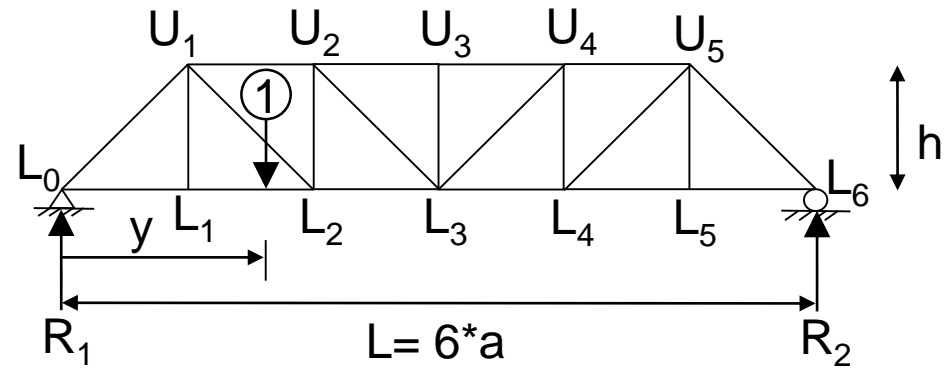


Let us consider the above simply supported, symmetric truss, with L_0, L_1, \dots, L_6 denoting lower joints and U_1, U_2, \dots, U_5 denoting upper joints. Correspondingly, L_0L_1, L_1L_2, \dots are lower chord members, U_1U_2, U_2U_3, \dots are upper chord members and L_0U_1, U_1L_1, \dots are the web members of the truss. Its span is $6a$ and height is h .

Let R_1 and R_2 be the reactions at the L_0 and L_6 supports as a unit load moves along the span of the truss.

5.4 Influence Lines for Statically Determinate Trusses

Let us start with drawing **ILD for support reactions R_1 and R_2** when a unit load moves along the span of the truss.



Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

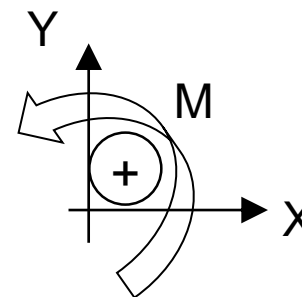
Taking moment about support L_6 [one of the equations of static equilibrium],

$$\Sigma M_{L_6} = 0$$

$$\text{Or, } -R_1 * L + 1 * (L - y) = 0$$

$$\text{Or, } R_1 = \frac{L - y}{L}$$

$$\therefore R_1 = 1 - \frac{y}{L}$$



This expression gives us the ordinates for ILD for R_1 as y changes, i.e. as unit load moves along the span.

5.4 Influence Lines for Statically Determinate Trusses

Similarly, taking moment about support L_0 [one of the equations of static equilibrium],

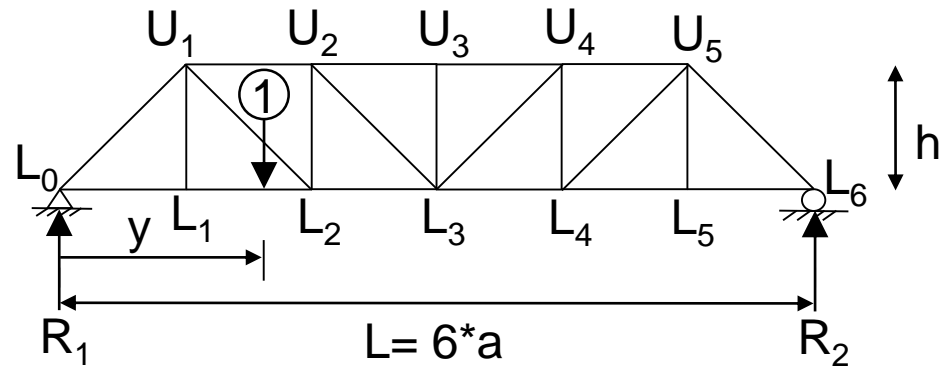
$$\Sigma M_{L_0} = 0$$

$$\text{Or, } R_2 * L - 1 * (y) = 0$$

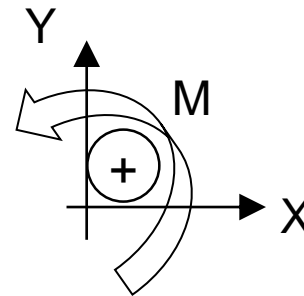
$$\text{Or, } R_2 = \frac{y}{L}$$

$$\therefore R_2 = \frac{y}{L}$$

This expression gives us the ordinates for ILD for R_2 as y changes, i.e. as unit load moves along the span.



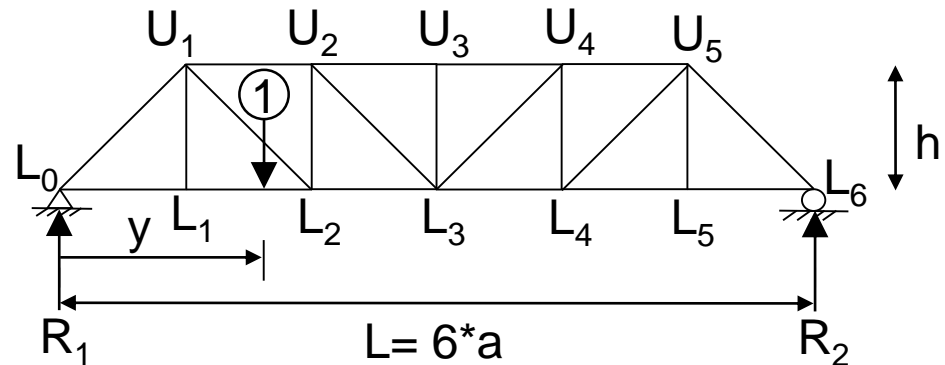
Source: Bhavikatti, S. S. (2011).
Structural Analysis - I (4th ed.). New
 Delhi: Vikas Publishing House.



5.4 Influence Lines for Statically Determinate Trusses

Now, plotting the ordinates of influence lines for R_1 and R_2 ,

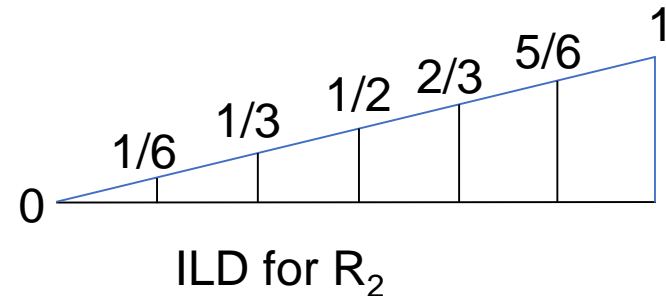
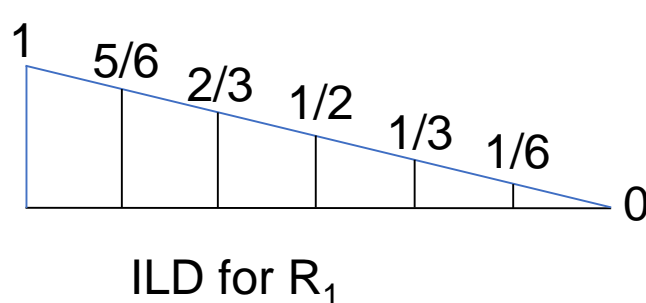
$$R_1 = 1 - \frac{y}{L} \quad ; \quad R_2 = \frac{y}{L}$$



Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

y	0	a	2a	3a	4a	5a	6a
R_1	1	5/6	2/3	1/2	1/3	1/6	0
R_2	0	1/6	1/3	1/2	2/3	5/6	1

Where, $L = 6*a$

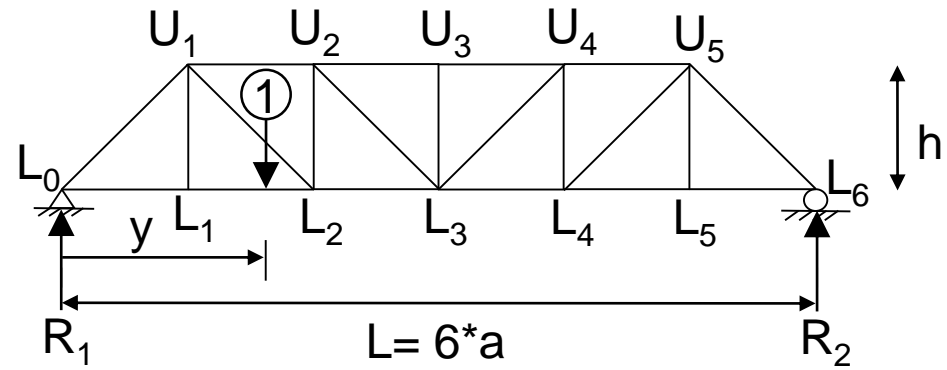


5.4 Influence Lines for Statically Determinate Trusses

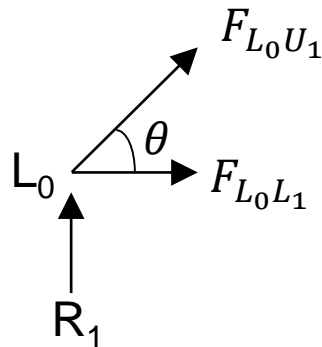
Now, plotting ILD for forces in members L_0U_1 , L_0L_1 , U_1U_2 .

ILD for force in member L_0U_1

Using joint method of analysis of truss,



Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

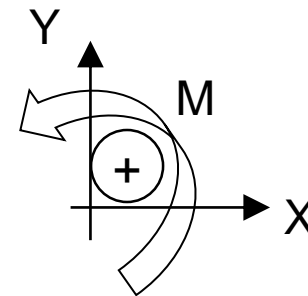


$$\sum F_y = 0$$

$$\text{Or, } R_1 + F_{L_0U_1} \sin\theta = 0$$

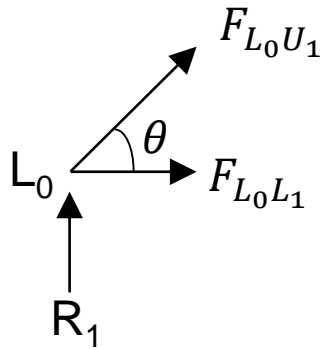
$$\text{Or, } F_{L_0U_1} = -R_1 \operatorname{cosec}\theta$$

$$\therefore F_{L_0U_1} = R_1 \operatorname{cosec}\theta \text{ (C)}$$

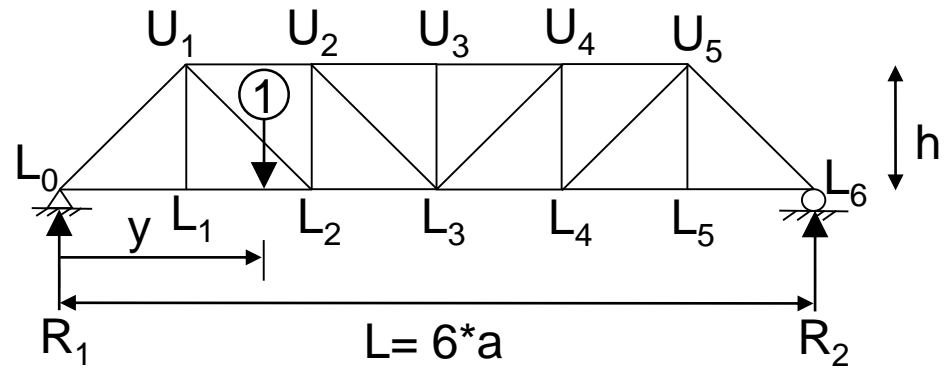


5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member L_0U_1



$$\therefore F_{L_0U_1} = R_1 \operatorname{cosec}\theta (C)$$



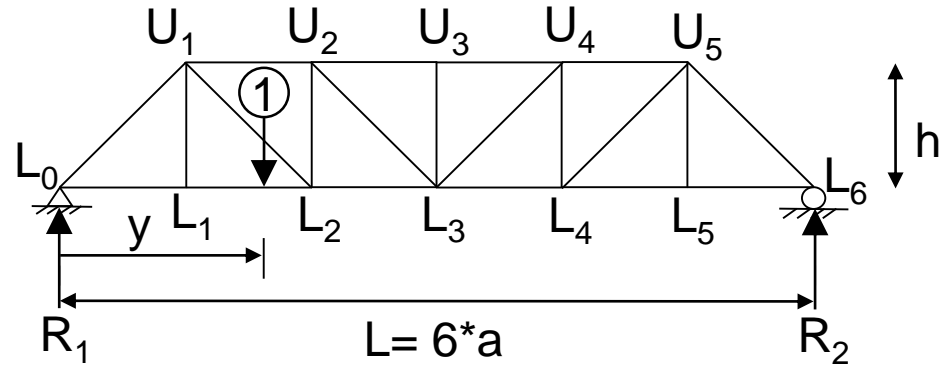
Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

y	0	a	2a	3a	4a	5a	6a
R_1	1	5/6	2/3	1/2	1/3	1/6	0
$F_{L_0U_1}$	$\operatorname{cosec}\theta$	$5/6 \operatorname{cosec}\theta$	$2/3 \operatorname{cosec}\theta$	$1/2 \operatorname{cosec}\theta$	$1/3 \operatorname{cosec}\theta$	$1/6 \operatorname{cosec}\theta$	0

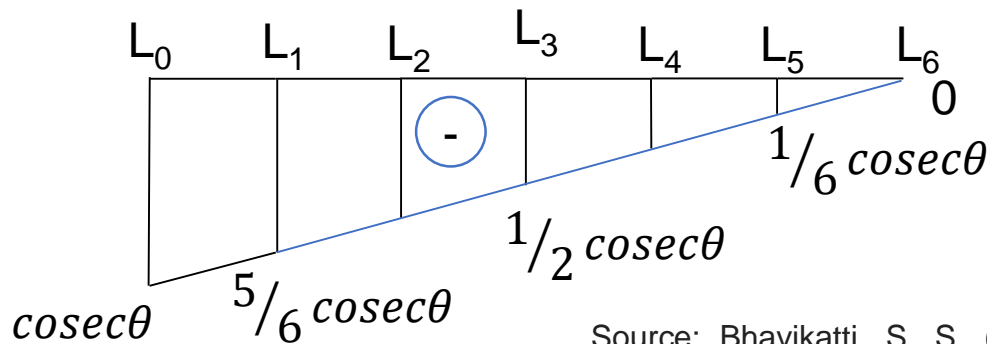
5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member L_0U_1

$$\therefore F_{L_0U_1} = R_1 \operatorname{cosec}\theta (C)$$



y	0	a	2a	3a	4a	5a	6a
$F_{L_0U_1}$	$\operatorname{cosec}\theta$	$5/6 \operatorname{cosec}\theta$	$2/3 \operatorname{cosec}\theta$	$1/2 \operatorname{cosec}\theta$	$1/3 \operatorname{cosec}\theta$	$1/6 \operatorname{cosec}\theta$	0

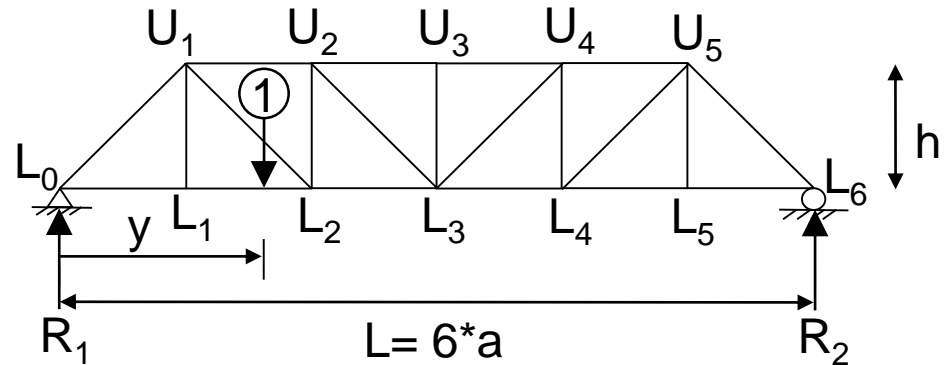


Source: Bhavikatti, S. S. (2011).
 Structural Analysis – I (4th ed.). New
 Delhi: Vikas Publishing House.

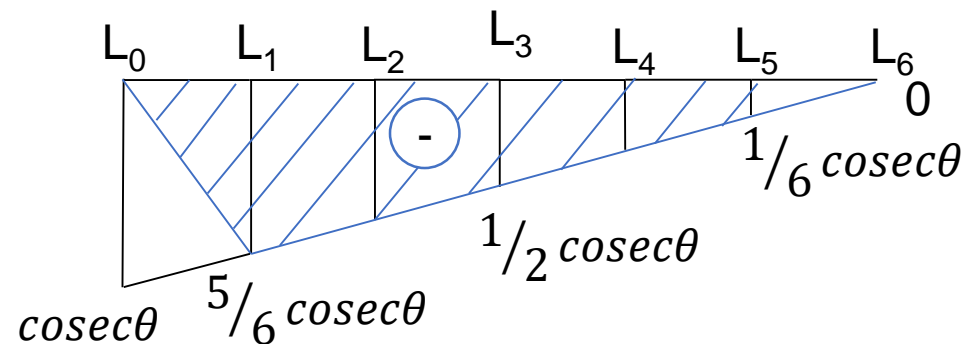
5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member L_0U_1

- When unit load is between L_0 and L_1 , it gets transferred partly to joint L_0 and partly to joint L_1 .
- Load transferred to L_0 goes to the support without giving rise to axial forces in the member.
- When the load is exactly on L_0 , no force develops in the member L_0U_1 .
- Hence, influence line for force in L_0U_1 varies linearly from zero at L_0 to $\frac{5}{6} \operatorname{cosec} \theta$ at L_1 .



Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

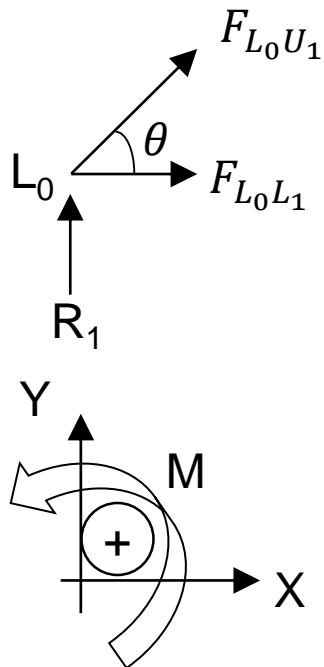
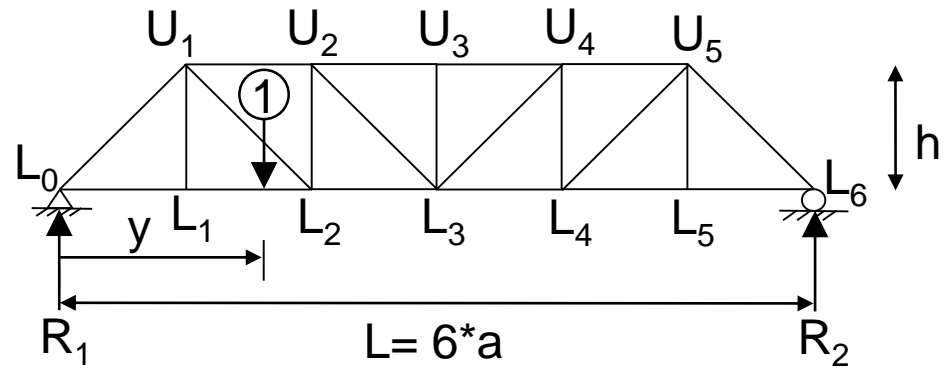


The shaded region shows the ILD for the force in the member L_0U_1 as unit load moves along the span.

5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member L_0L_1

Using joint method of analysis of truss,



$$\Sigma F_x = 0$$

$$\text{Or, } F_{L_0U_1} \cos\theta + F_{L_0L_1} = 0$$

$$\text{Or, } F_{L_0L_1} = -F_{L_0U_1} \cos\theta = R_1 \operatorname{cosec}\theta \cos\theta$$

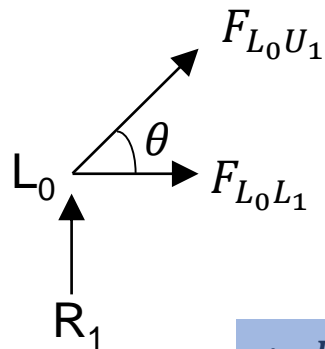
$$\text{Or, } F_{L_0L_1} = R_1 \frac{1}{\sin\theta} \cos\theta$$

$$\therefore F_{L_0L_1} = R_1 \cot\theta \text{ (T)}$$

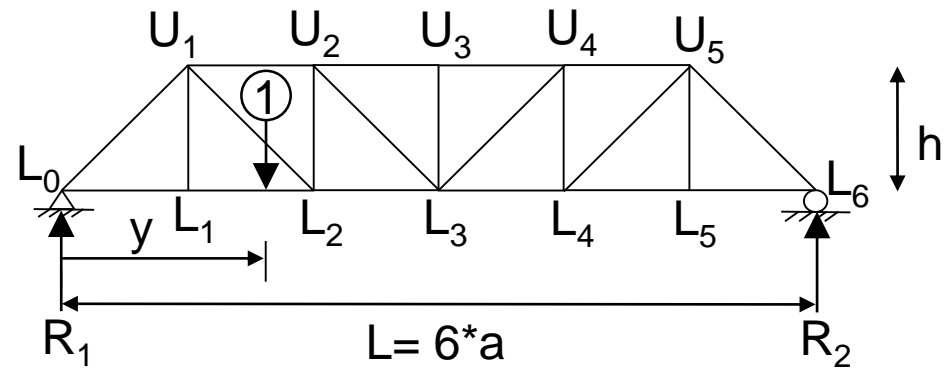
Source: Bhavikatti, S. S. (2011). *Structural Analysis -I* (4th ed.). New Delhi: Vikas Publishing House.

5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member L_0L_1



$$\therefore F_{L_0L_1} = R_1 \cot\theta \quad (T)$$



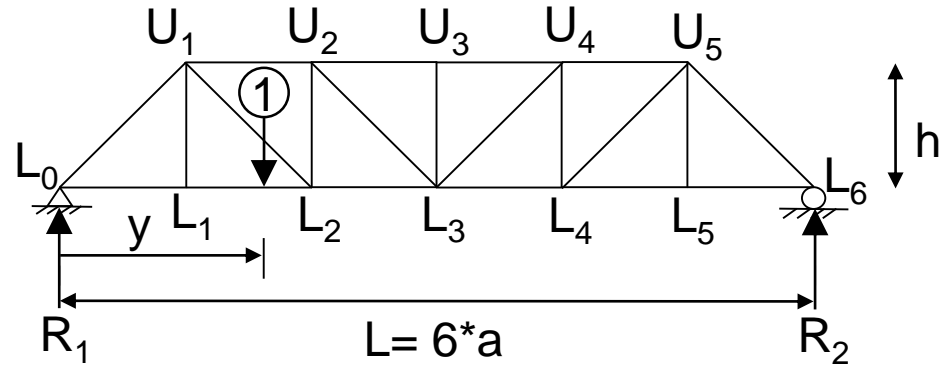
Source: Bhavikatti, S. S. (2011).
Structural Analysis – I (4th ed.). New
 Delhi: Vikas Publishing House.

y	0	a	2a	3a	4a	5a	6a
R_1	1	5/6	2/3	1/2	1/3	1/6	0
$F_{L_0L_1}$	$\cot\theta$	$5/6 \cot\theta$	$2/3 \cot\theta$	$1/2 \cot\theta$	$1/3 \cot\theta$	$1/6 \cot\theta$	0

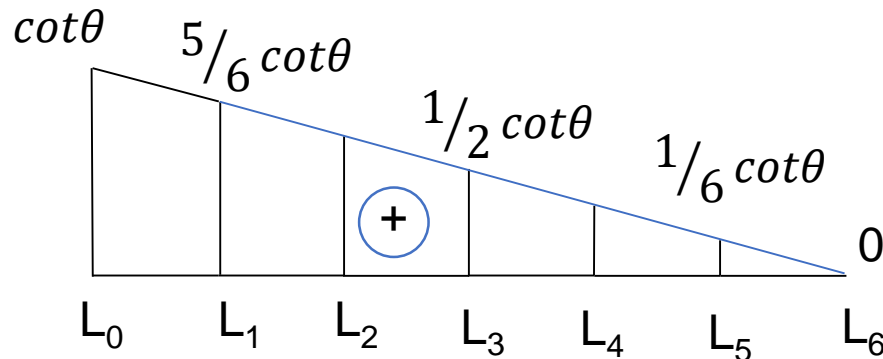
5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member L_0L_1

$$\therefore F_{L_0L_1} = R_1 \cot\theta (T)$$



y	0	a	2a	3a	4a	5a	6a
$F_{L_0L_1}$	$\cot\theta$	$5/6 \cot\theta$	$2/3 \cot\theta$	$1/2 \cot\theta$	$1/3 \cot\theta$	$1/6 \cot\theta$	0

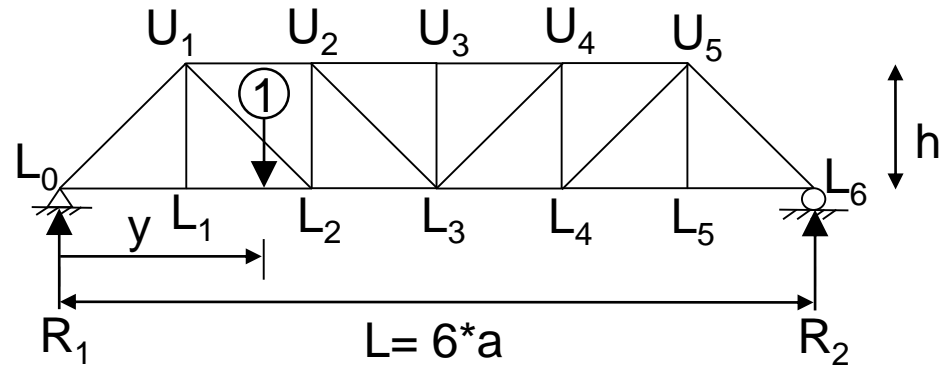


Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

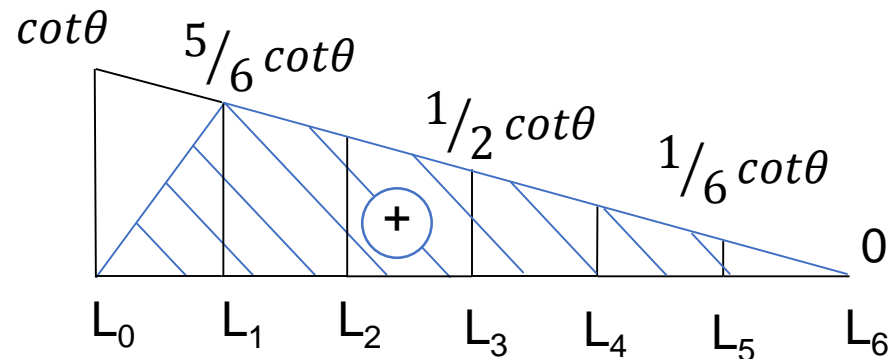
5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member L_0L_1

- When unit load is between L_0 and L_1 , it gets transferred partly to joint L_0 and partly to joint L_1 .
- Load transferred to L_0 goes to the support without giving rise to axial forces in the member.
- When the load is exactly on L_0 , no force develops in the member L_0L_1 .
- Hence, influence line for force in L_0L_1 varies linearly from zero at L_0 to $\frac{5}{6} \cot\theta$ at L_1 .



Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

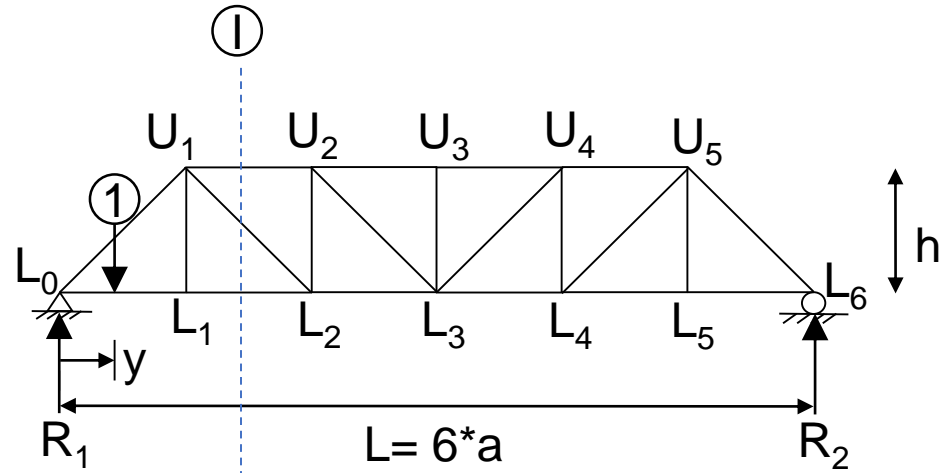


The shaded region shows the ILD for the force in the member L_0L_1 as unit load moves along the span.

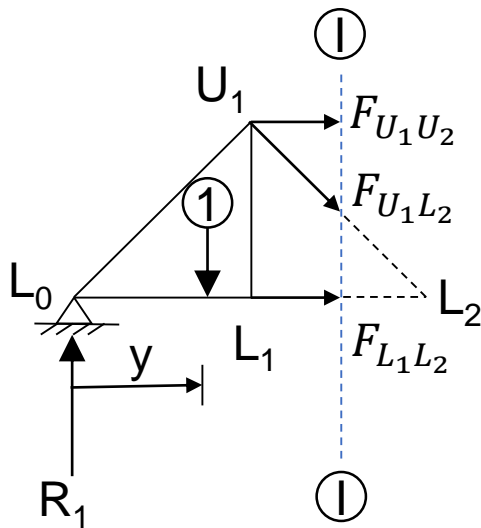
5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member U₁U₂

Using section method of analysis of truss, taking section at I-I,



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.). New Delhi: Vikas Publishing House.



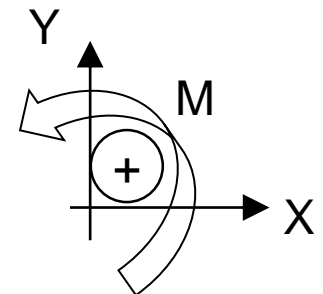
Unit load between L₀ and L₂

$$\sum M_{L_2} = 0$$

$$\text{Or, } -R_1 * 2a + 1 * (2a - y) - F_{U_1U_2} * h = 0$$

$$\text{Or, } F_{U_1U_2} = (1 - R_1) * \frac{2a}{h} - \frac{y}{h}$$

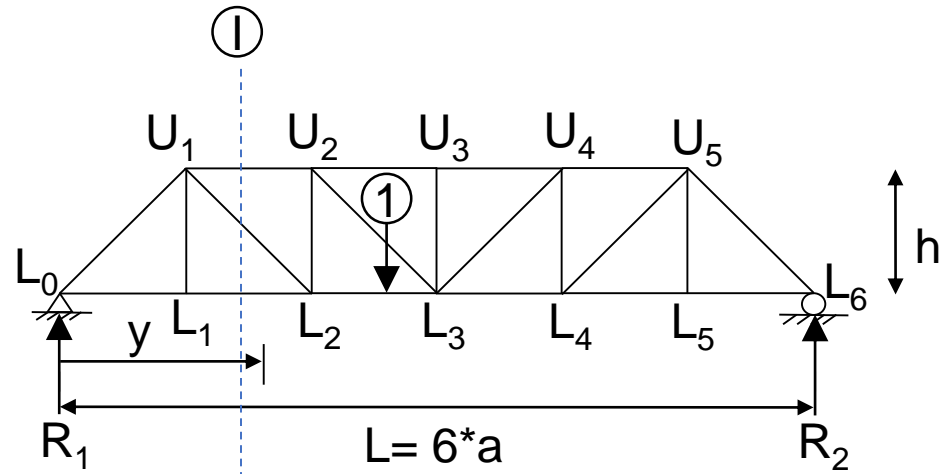
$$\therefore F_{U_1U_2} = (1 - R_1) * \frac{2a}{h} - \frac{y}{h}$$



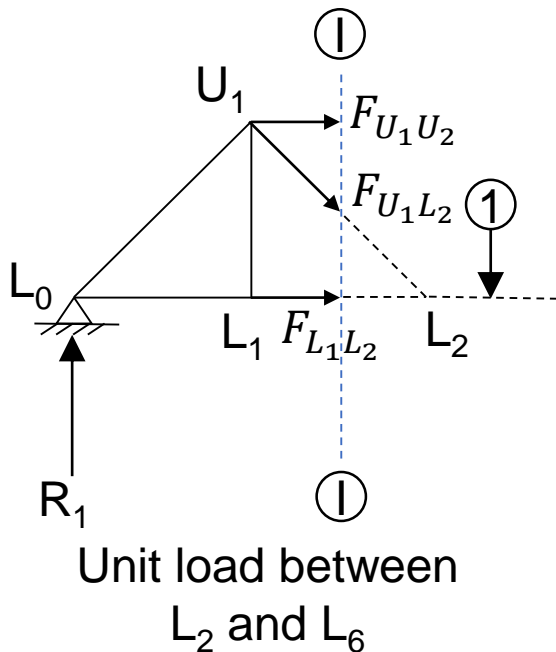
5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member U_1U_2

Using section method of analysis of truss, taking section at I-I,



Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

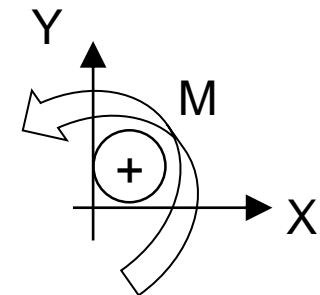


$$\Sigma M_{L_2} = 0$$

$$\text{Or, } -R_1 * 2a - F_{U_1U_2} * h = 0$$

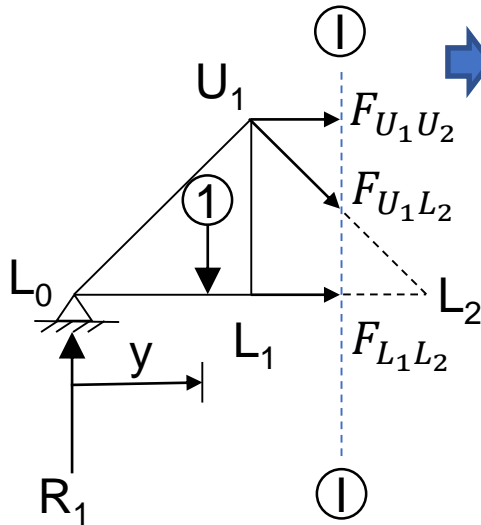
$$\text{Or, } F_{U_1U_2} = -R_1 * \frac{2a}{h}$$

$$\therefore F_{U_1U_2} = R_1 * \frac{2a}{h} \text{ (C)}$$



5.4 Influence Lines for Statically Determinate Trusses

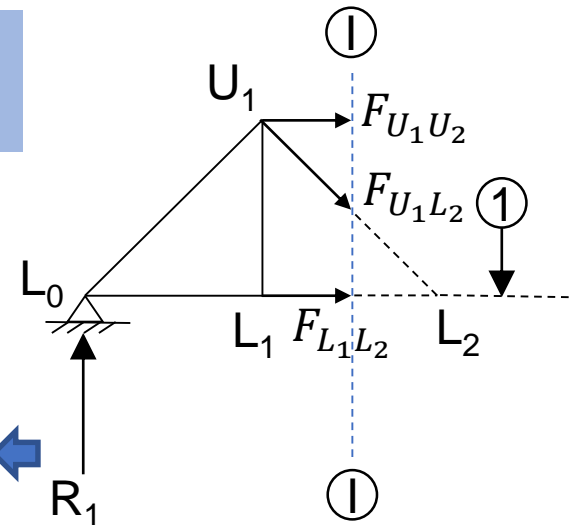
ILD for force in member U_1U_2



$$\therefore F_{U_1U_2} = (1 - R_1) * \frac{2a}{h} - \frac{y}{h}$$

Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

$$\therefore F_{U_1U_2} = R_1 * \frac{2a}{h} \quad (C)$$



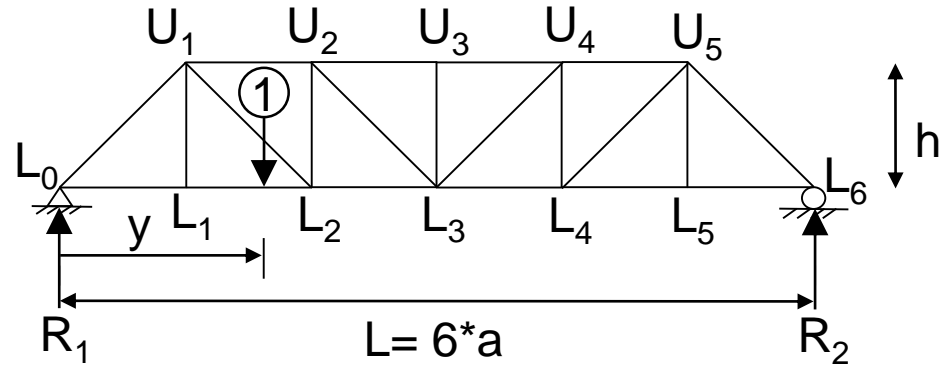
y	0	a	2a	3a	4a	5a	6a
R_1	1	5/6	2/3	1/2	1/3	1/6	0
$F_{L_0U_1}$	0	$-2a/3h$	$-4a/3h$	$-a/h$	$-2a/3h$	$-a/3h$	0

Unit load between L_0 and L_2

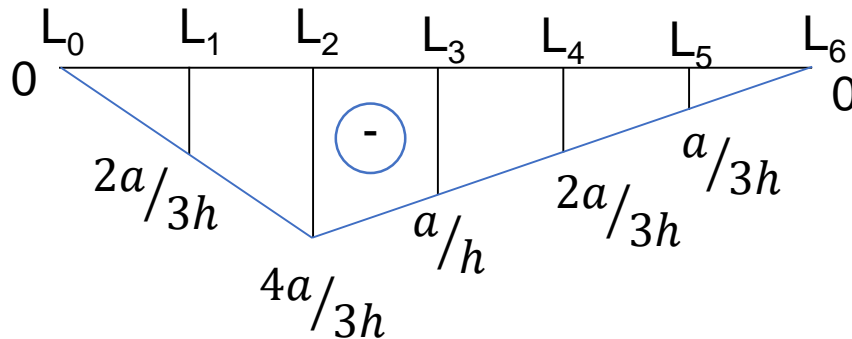
Unit load between L_2 and L_6

5.4 Influence Lines for Statically Determinate Trusses

ILD for force in member U_1U_2



y	0	a	2a	3a	4a	5a	6a
$F_{U_1U_2}$	0	$-2a/3h$	$-4a/3h$	$-a/h$	$-2a/3h$	$-a/3h$	0

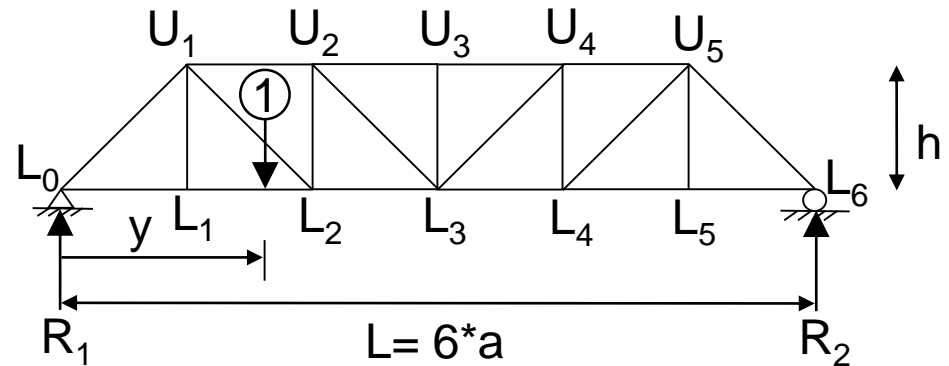


ILD for the force in the member U_1U_2 as unit load moves along the span.

5.4 Influence Lines for Statically Determinate Trusses

Similarly, Influence lines and ILDs can be constructed for the members U_1U_3 , L_1L_2 , U_1L_2 ,

Since the truss is symmetric, ILD need to be computed only for the half the truss members. It will be the same for the corresponding symmetric truss member.



Source: Bhavikatti, S. S. (2011).
Structural Analysis – I (4th ed.). New
 Delhi: Vikas Publishing House.

-----End of Lecture#7-----

-----End of Part I of III for Chapter5-----

References

- [1] Timoshenko, S.P. and Young, D.H. (1965). Theory of Structures (2nd ed.). Singapore: McGraw-Hill Book Co. Singapore.
- [2] Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.