

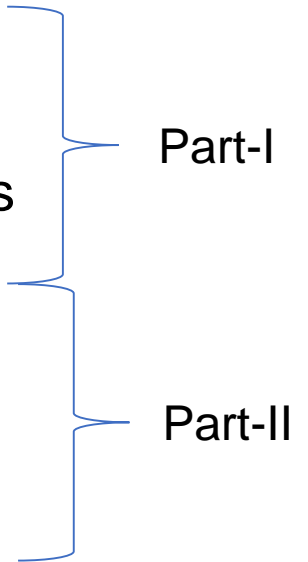
Theory of Structures - I

Chapter 5. Influence Lines for
Simple Structures [Part II of III]

Lecturer: Dr. Sanjeema Bajracharya

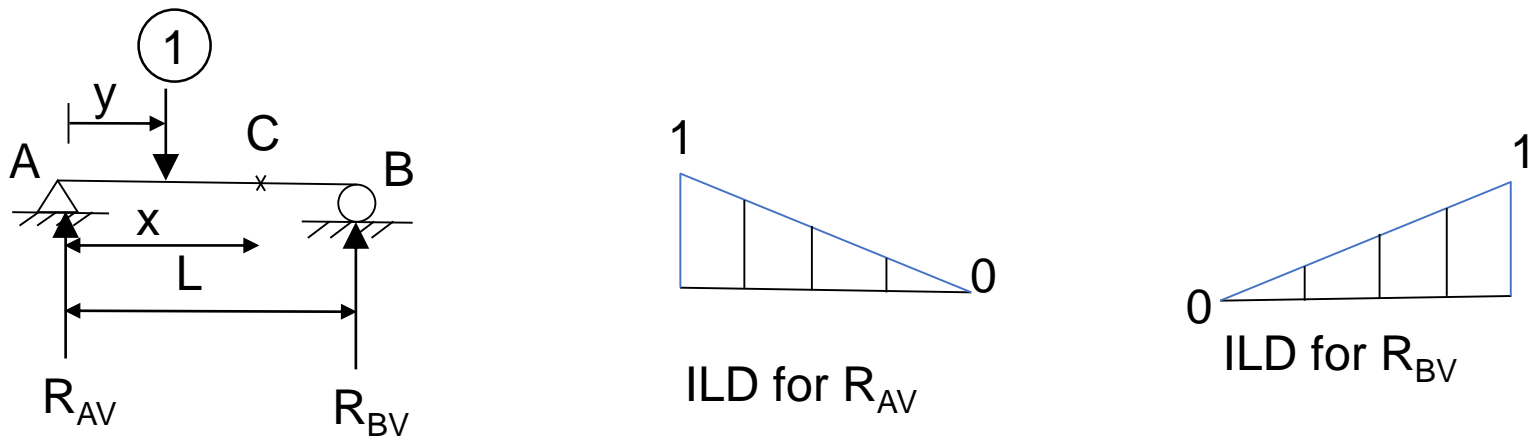
Contents

- 5.1 Moving Static Loads and Influence Lines
- 5.2 Influence Lines for Statically Determinate Structures
- 5.3 Moving Loads and ILDs on Statically Determinate Beams
- 5.4 Influence Lines for Statically Determinate Trusses
- 5.5 Influence Lines for Support Reactions
- 5.6 Influence Lines for Support Moment
- 5.7 Influence Lines for Shear Force
- 5.8 Influence Lines for Bending Moment



5.5 Influence Lines for Support Reactions

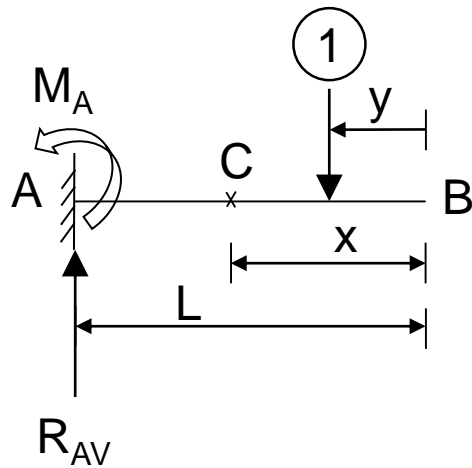
We have seen in the lecture 7, the Influence lines for support reactions R_{AV} and R_{BV} for a simply supported beam as unit point load moves across its span, as follows:



Now, we will look at how influence lines vary for different support conditions – e.g. a cantilever beam, overhanging beam, etc.

5.5 Influence Lines for Support Reactions

[A] Cantilever beam:

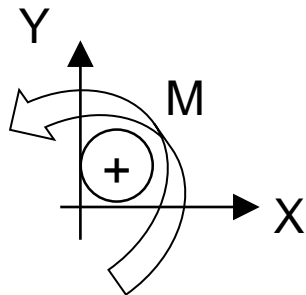


Using one of the equations of static equilibrium,

$$\Sigma F_y = 0$$

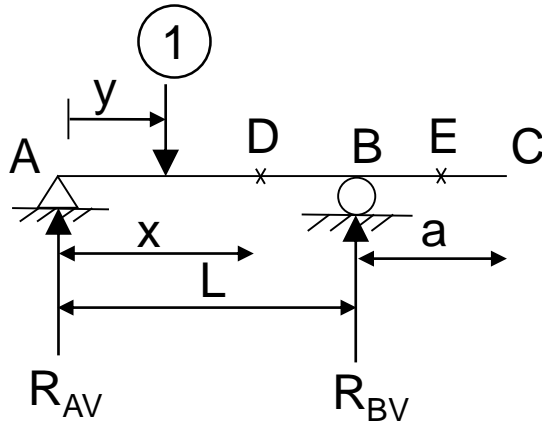
$$\text{Or, } R_{AV} - 1 = 0$$

$$\therefore R_{AV} = 1$$



5.5 Influence Lines for Support Reactions

[B] Single overhang beam:



Using equations of static equilibrium,

$$\Sigma F_y = 0$$

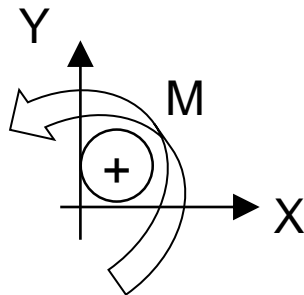
$$\Sigma M_B = 0$$

$$\text{Or, } R_{AV} - 1 + R_{BV} = 0 \quad \text{Or, } R_{AV} * L - 1 * (L - y) = 0$$

$$\therefore R_{AV} + R_{BV} = 1$$

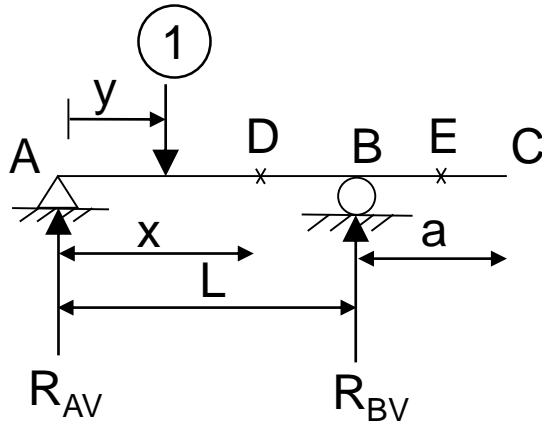
$$\therefore R_{AV} = 1 - \frac{y}{L}$$

$$\therefore R_{BV} = \frac{y}{L}$$



5.5 Influence Lines for Support Reactions

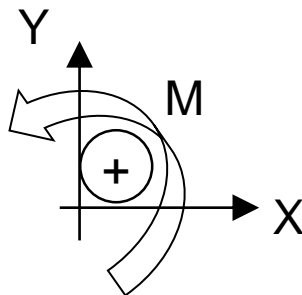
[B] Single overhang beam:



$$\therefore R_{AV} = 1 - \frac{y}{L}$$

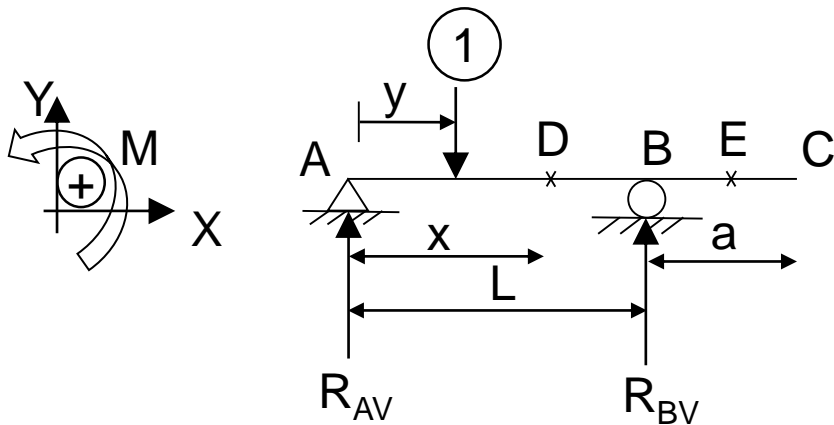
$$\therefore R_{BV} = \frac{y}{L}$$

	R_{AV}	R_{BV}
$y=0$	1	0
$y=x$	$1 - \frac{x}{L}$	$\frac{x}{L}$
$y=L$	0	1
$y=L+a$	$-\frac{a}{L}$	$1 + \frac{a}{L}$

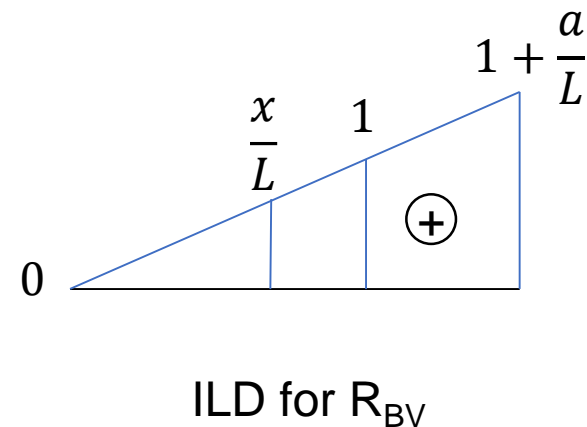
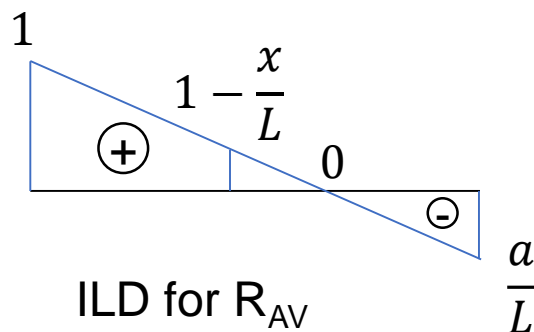


5.5 Influence Lines for Support Reactions

[B] Single overhang beam:

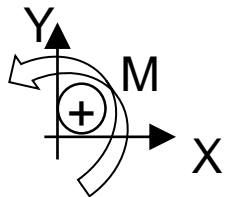
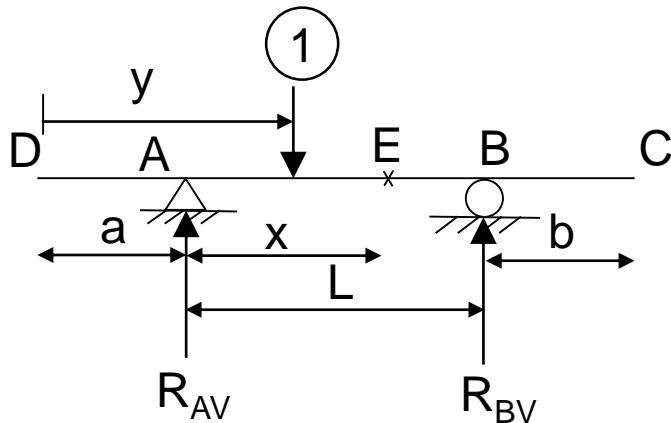


	R_{AV}	R_{BV}
$y=0$	1	0
$y=x$	$1 - \frac{x}{L}$	$\frac{x}{L}$
$y=L$	0	1
$y=L+a$	$-\frac{a}{L}$	$1 + \frac{a}{L}$



5.5 Influence Lines for Support Reactions

[C] Double overhang beam:



Using equations of static equilibrium,

$$\Sigma F_y = 0$$

$$\text{Or, } R_{AV} - 1 + R_{BV} = 0$$

$$\therefore R_{AV} + R_{BV} = 1$$

$$\Sigma M_B = 0$$

$$\text{Or, } R_{AV} * L - 1 * (L + a - y) = 0$$

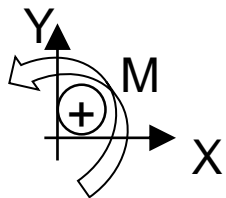
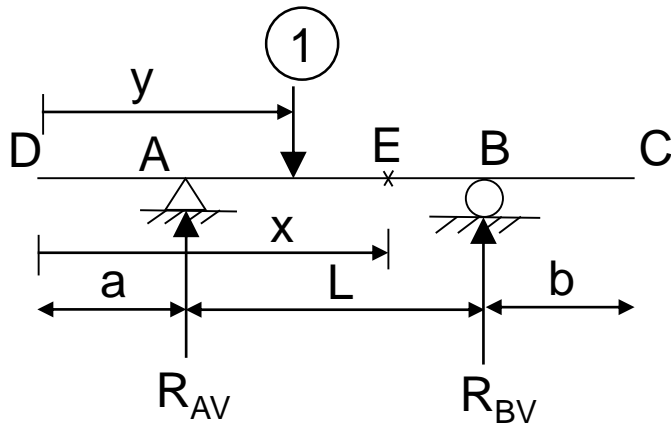
$$\therefore R_{AV} = \frac{L + a - y}{L}$$

$$\text{Or, } R_{BV} = 1 - \frac{L + a - y}{L}$$

$$\therefore R_{BV} = \frac{y - a}{L}$$

5.5 Influence Lines for Support Reactions

[C] Double overhang beam:



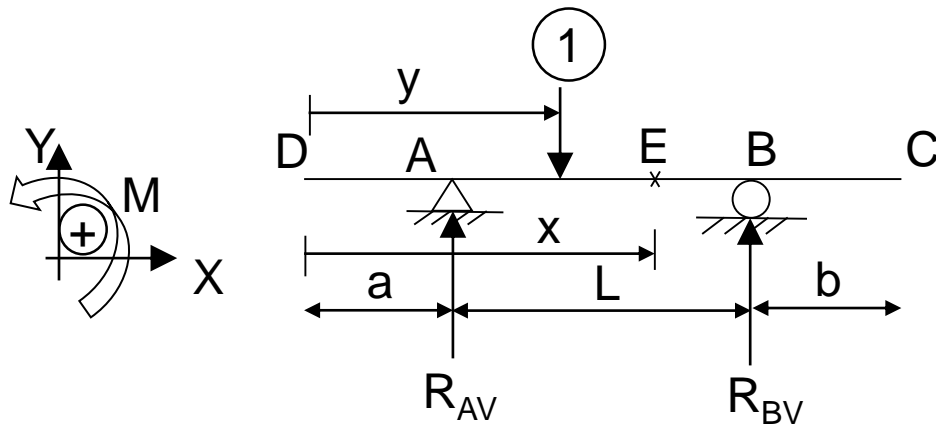
$$\therefore R_{AV} = \frac{L + a - y}{L}$$

$$\therefore R_{BV} = \frac{y - a}{L}$$

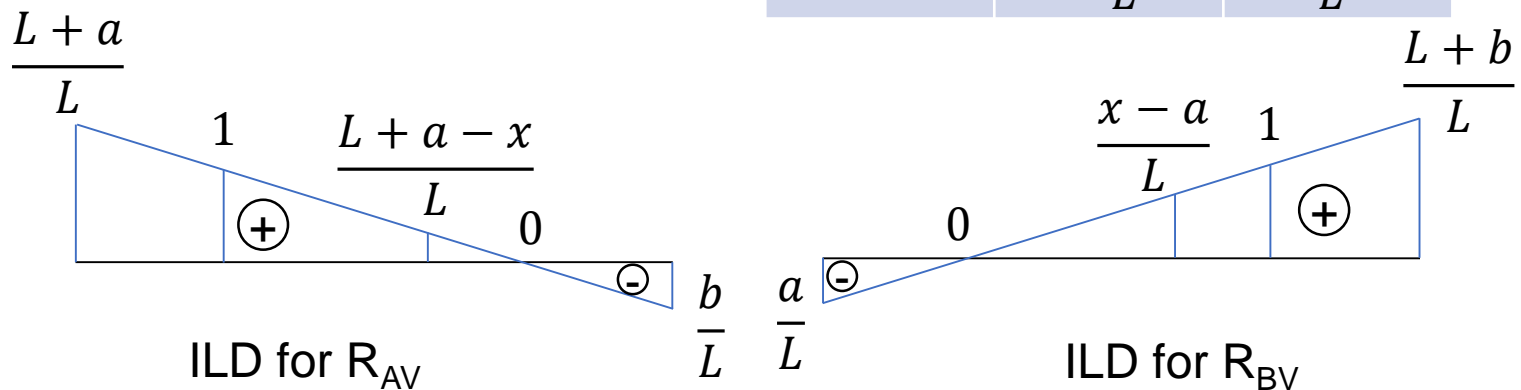
	R_{AV}	R_{BV}
$y=0$	$\frac{L + a}{L}$	$-\frac{a}{L}$
$y=a$	1	0
$y=x$	$\frac{L + a - x}{L}$	$\frac{x - a}{L}$
$y=L+a$	0	1
$y=L+a+b$	$-\frac{b}{L}$	$\frac{L + b}{L}$

5.5 Influence Lines for Support Reactions

[C] Double overhang beam:

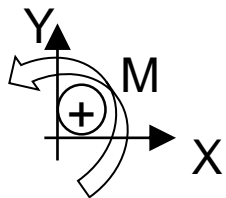
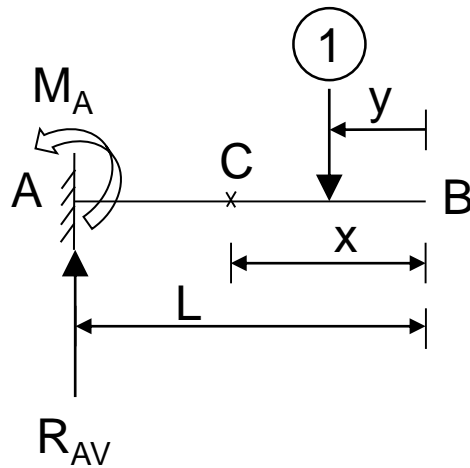


	R_{AV}	R_{BV}
$y=0$	$\frac{L+a}{L}$	$-\frac{a}{L}$
$y=a$	1	0
$y=x$	$\frac{L+a-x}{L}$	$\frac{x-a}{L}$
$y=L+a$	0	1
$y=L+a+b$	$-\frac{b}{L}$	$\frac{L+b}{L}$



5.6 Influence Lines for Support Moments

Cantilever beam:



Using equation of static equilibrium,

$$\Sigma M_B = 0$$

$$\text{Or, } M_A - R_{AV}L + 1 * y = 0$$

$$\text{We have, } R_{AV} = 1$$

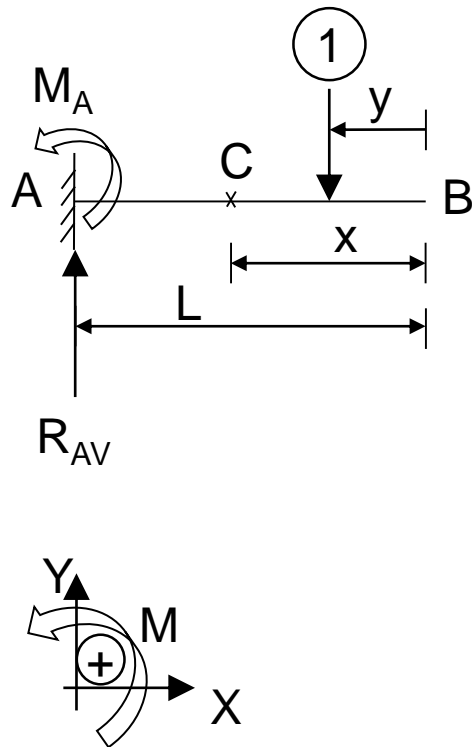
$$\text{Or, } M_A - 1 * L + y = 0$$

$$\therefore M_A = L - y$$

	$y = 0$	$y = x$	$y = L$
M_A	L	L-x	0

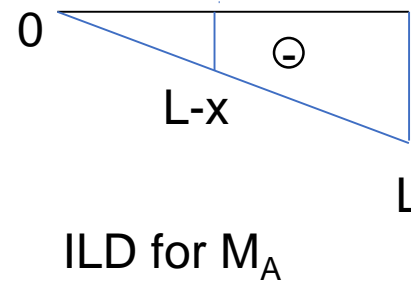
5.6 Influence Lines for Support Moments

Cantilever beam:



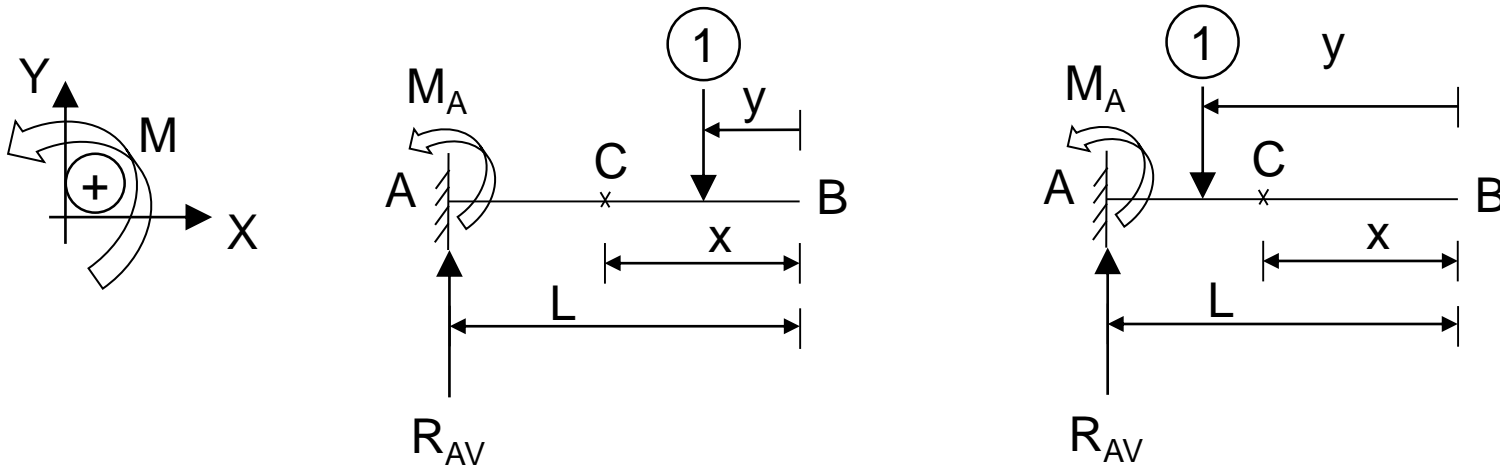
$$\therefore M_A = L - y$$

	$y = 0$	$y = x$	$y = L$
M_A	L	L-x	0



5.7 Influence Lines for Shear Force

[A] Cantilever beam (SF at C):



We have,

$$R_{AV} = 1$$

$$M_A = L - y$$

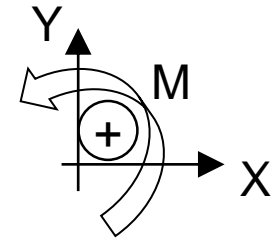
$$(i) 0 \leq y \leq x$$

$$\text{SF at C} = 1$$

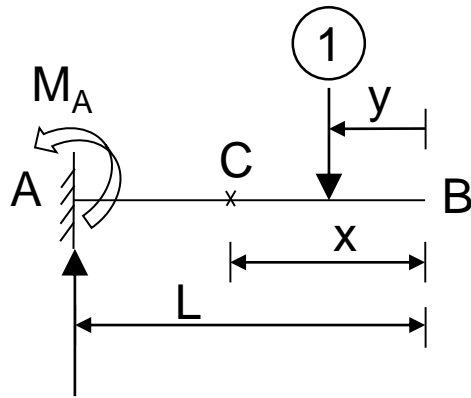
$$(ii) x \leq y \leq L$$

$$\text{SF at C} = 0$$

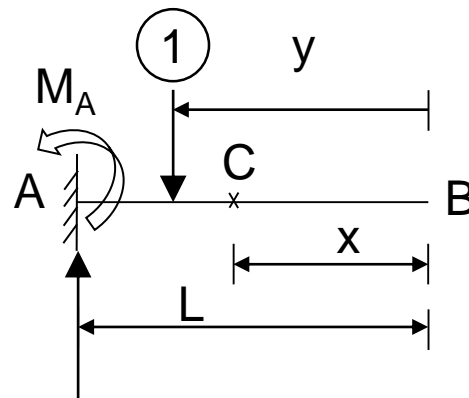
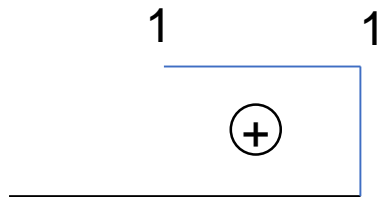
5.7 Influence Lines for Shear Force



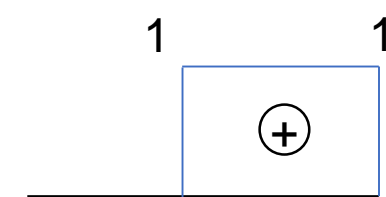
[A] Cantilever beam (SF at C):



R_{AV} (i) $0 \leq y \leq x$
SF at C = 1



R_{AV} (ii) $x \leq y \leq L$
SF at C = 0



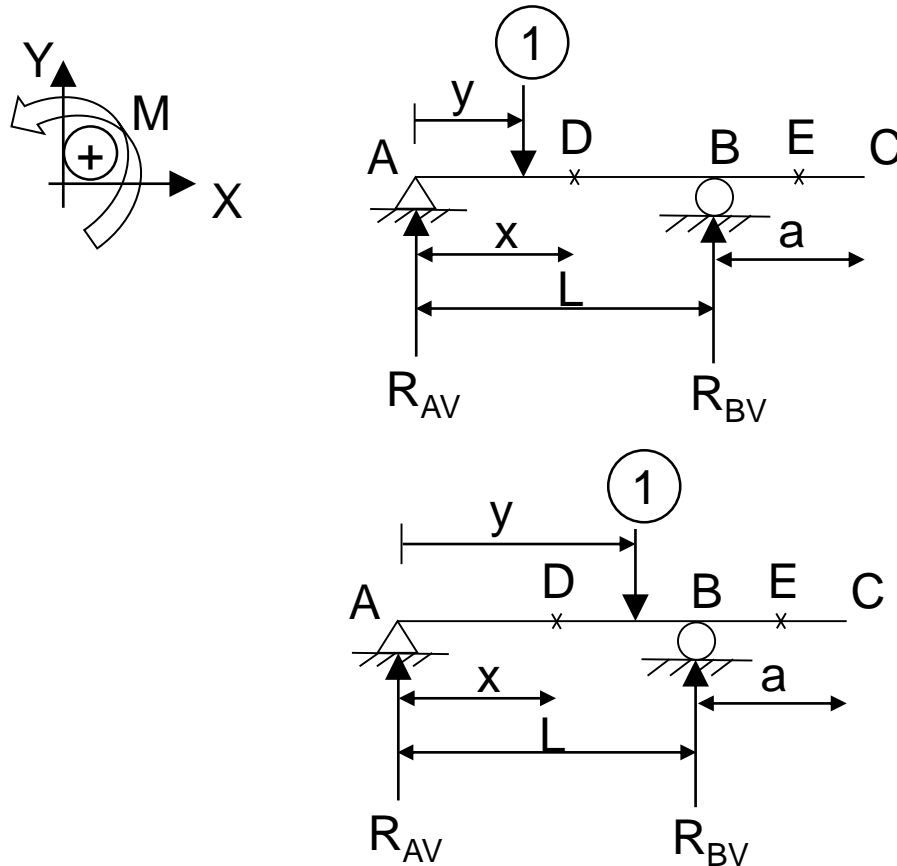
ILD for SF at C

	$0 \leq y \leq x$	$x \leq y \leq L$
$y = 0$	1	-
$y = x$	1	0
$y = L$	-	0

Plotting the ordinates, we get

5.7 Influence Lines for Shear Force

[B] Single Overhang Beam (SF at D):



We have,

$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$

(i) $0 \leq y \leq x$

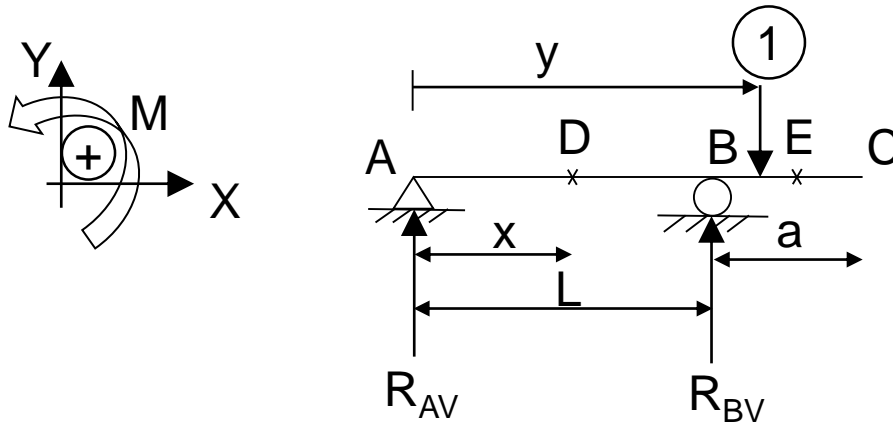
$$\begin{aligned} SF \text{ at } D &= R_{AV} - 1 \\ &= 1 - \frac{y}{L} - 1 \\ &= -\frac{y}{L} \end{aligned}$$

(ii) $x \leq y \leq L$

$$\begin{aligned} SF \text{ at } D &= R_{AV} \\ &= 1 - \frac{y}{L} \end{aligned}$$

5.7 Influence Lines for Shear Force

[B] Single Overhang Beam (SF at D):



We have,

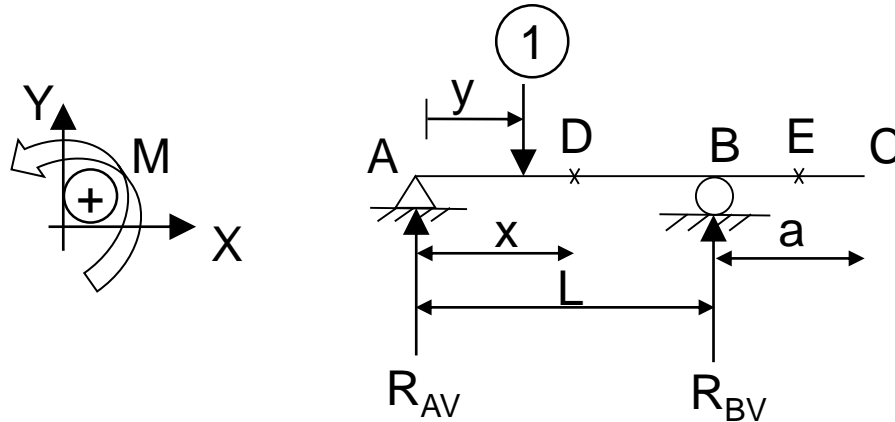
$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$

(iii) $L \leq y \leq L+a$

$$\begin{aligned} SF \text{ at } D &= R_{AV} \\ &= 1 - \frac{y}{L} \end{aligned}$$

5.7 Influence Lines for Shear Force

[B] Single Overhang Beam (SF at D):



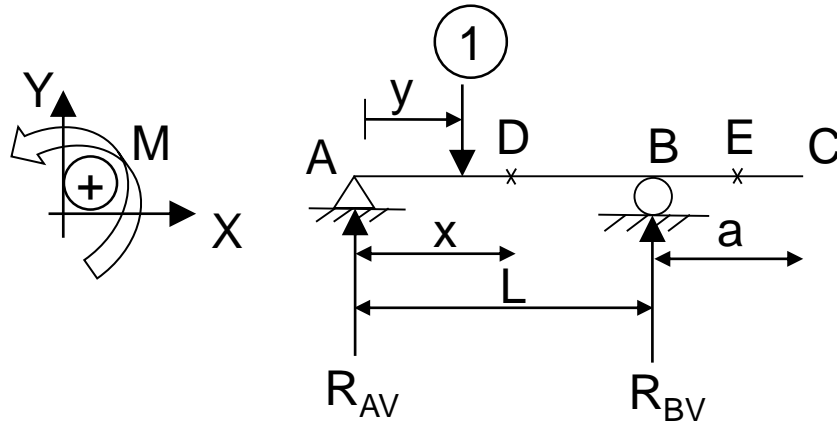
We have,

$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$

Unit load	SF at D	y=0	y=x	y=L	y=L+a
$0 \leq y \leq x$	$R_{AV} - 1 = -\frac{y}{L}$	0	$-\frac{x}{L}$	-	-
$x \leq y \leq L$	$R_{AV} = 1 - \frac{y}{L}$	-	$1 - \frac{x}{L}$	0	-
$L \leq y \leq L+a$	$R_{AV} = 1 - \frac{y}{L}$	-	-	0	$1 - \frac{L+a}{L}$ $= -\frac{a}{L}$

5.7 Influence Lines for Shear Force

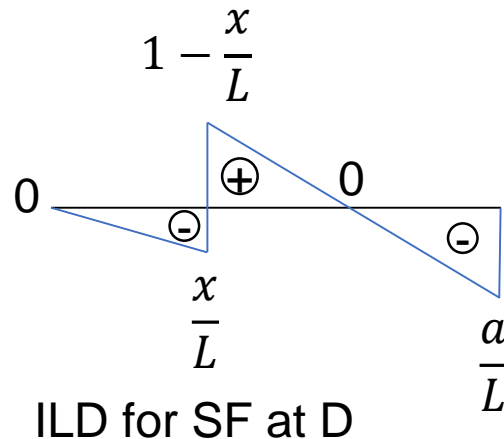
[B] Single Overhang Beam (SF at D):



We have,

$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$

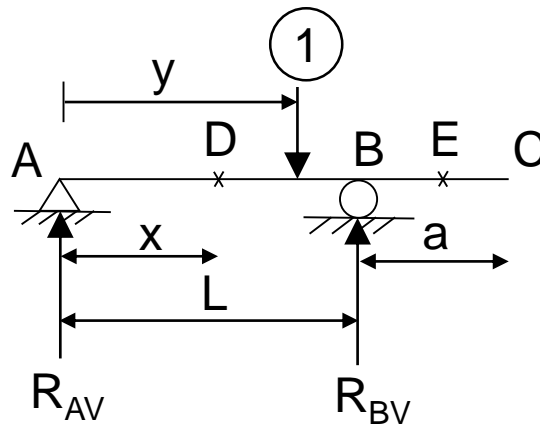
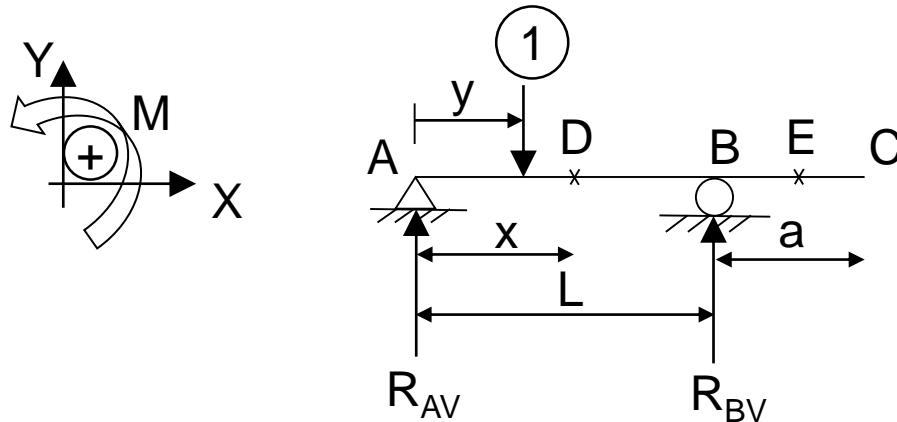
Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.



$y=0$	0		
$y=x$	$-\frac{x}{L}$	$1 - \frac{x}{L}$	
$y=L$		0	0
$y=L+a$			$-\frac{a}{L}$

5.7 Influence Lines for Shear Force

[B] Single Overhang Beam (SF at E):



We have,

$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$

(i) $0 \leq y \leq x$

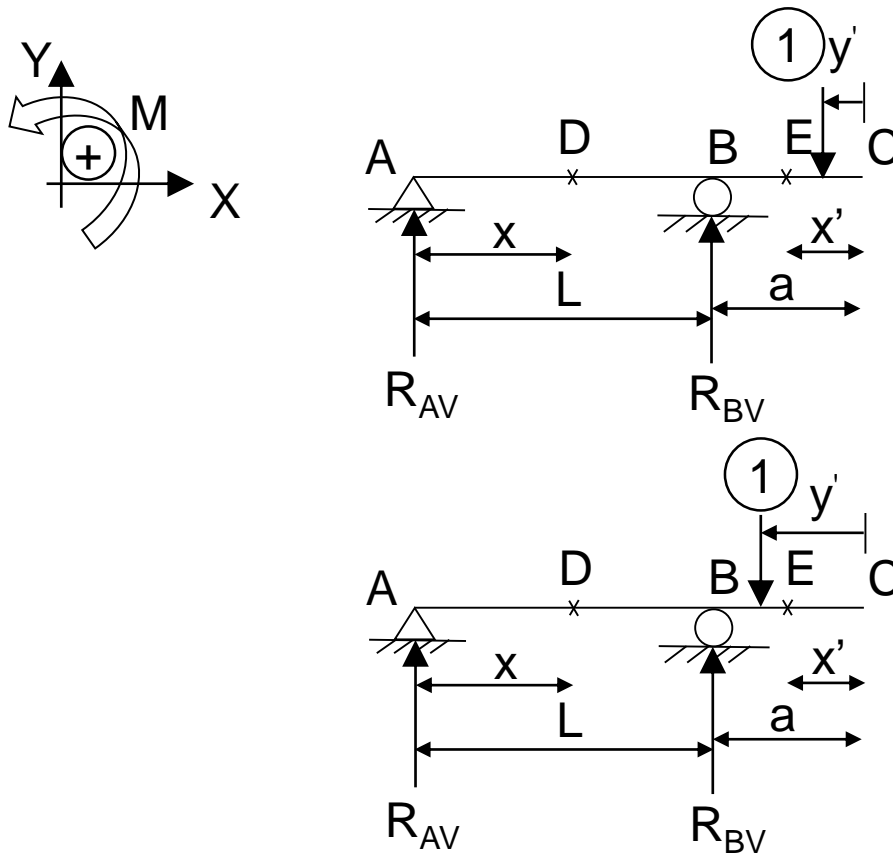
$$\begin{aligned} SF \text{ at } E &= R_{AV} - 1 + R_{BV} \\ &= 1 - \frac{y}{L} - 1 + \frac{y}{L} \\ &= 0 \end{aligned}$$

(ii) $x \leq y \leq L$

$$\begin{aligned} SF \text{ at } E &= R_{AV} - 1 + R_{BV} \\ &= 1 - \frac{y}{L} - 1 + \frac{y}{L} \\ &= 0 \end{aligned}$$

5.7 Influence Lines for Shear Force

[B] Single Overhang Beam (SF at E):



(iii) $0 \leq y' \leq x'$

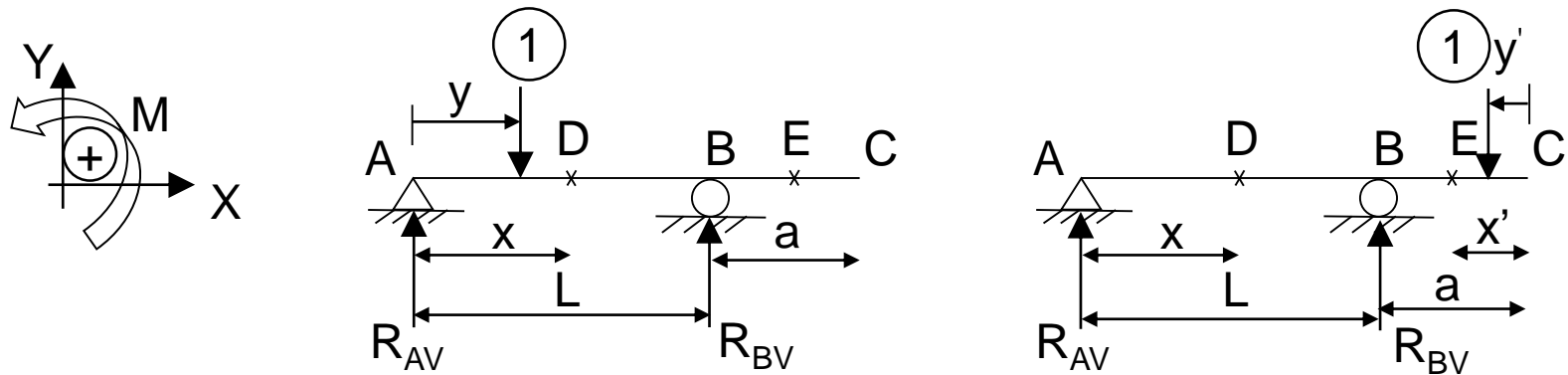
SF at E = 1

(iv) $x' \leq y' \leq a$

SF at E = 0

5.7 Influence Lines for Shear Force

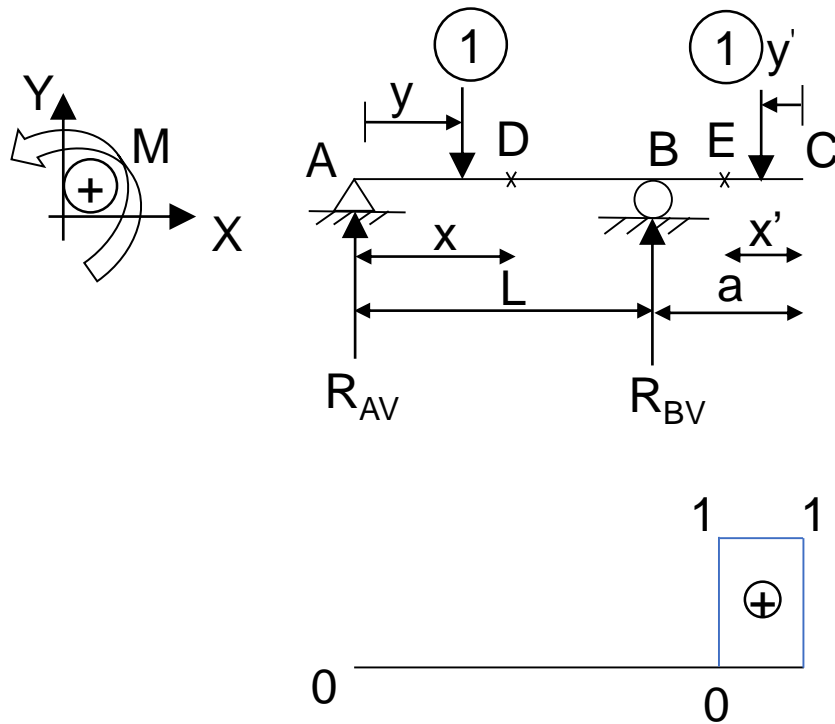
[B] Single Overhang Beam (SF at E):



Unit load	SF at E	$y=0$	$y=x$	$y=L$	$y'=0$	$y'=x'$	$y'=a$
$0 \leq y \leq x$	0	0	0	-	-	-	-
$x \leq y \leq L$	0	-	0	0	-	-	-
$0 \leq y' \leq x'$	1	-	-	-	1	1	-
$x' \leq y' \leq a$	0	-	-	-	-	0	0

5.7 Influence Lines for Shear Force

[B] Single Overhang Beam (SF at E):



y=0	0			
y=x	0	0		
y=L		0		
y'=0			1	
y'=x'			1	0
y'=a				0

ILD for SF at E

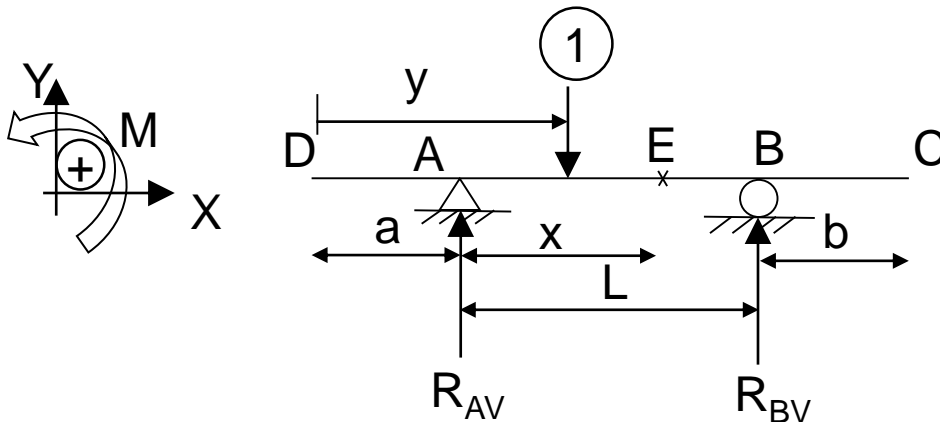
5.7 Influence Lines for Shear Force

[C] Double Overhang Beam (SF at E):

We have,

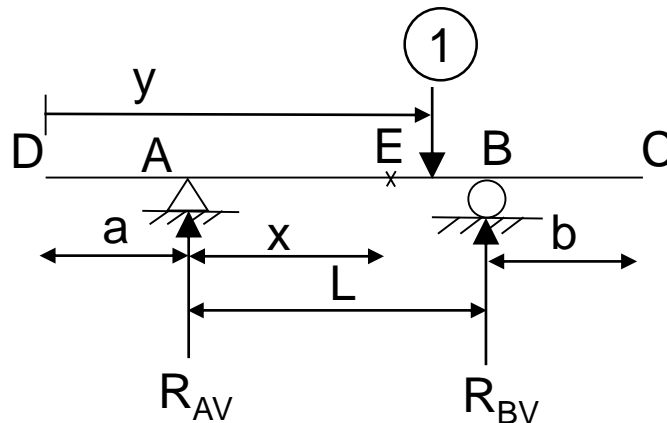
$$\therefore R_{AV} = \frac{L + a - y}{L}$$

$$\therefore R_{BV} = \frac{y - a}{L}$$



(i) $0 \leq y \leq a+x$

$$\begin{aligned} SF \text{ at } E &= R_{AV} - 1 \\ &= \frac{L+a-y}{L} - 1 \\ &= \frac{a-y}{L} \end{aligned}$$

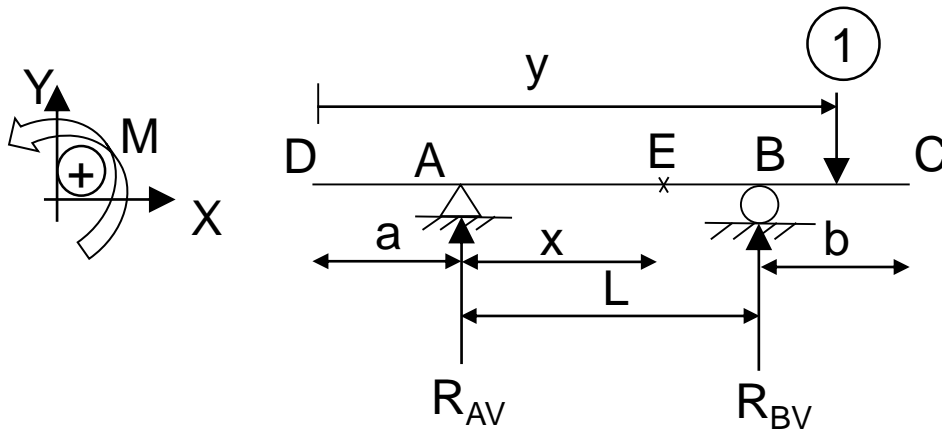


(ii) $a+x \leq y \leq L+a$

$$\begin{aligned} SF \text{ at } E &= R_{AV} \\ &= \frac{L+a-y}{L} \end{aligned}$$

5.7 Influence Lines for Shear Force

[C] Double Overhang Beam (SF at E):



We have,

$$\therefore R_{AV} = \frac{L + a - y}{L}$$

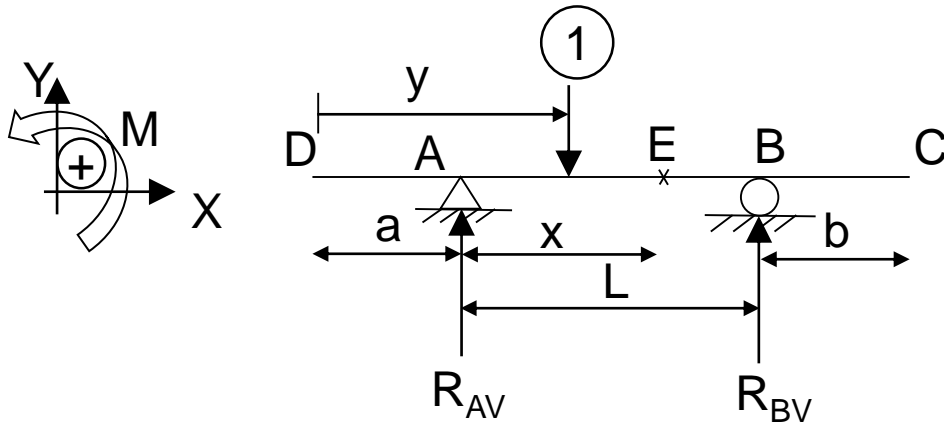
$$\therefore R_{BV} = \frac{y - a}{L}$$

(iii) $L + a \leq y \leq L + a + b$

$$\begin{aligned} SF \text{ at } E &= R_{AV} \\ &= \frac{L + a - y}{L} \end{aligned}$$

5.7 Influence Lines for Shear Force

[C] Double Overhang Beam (SF at E):



We have,

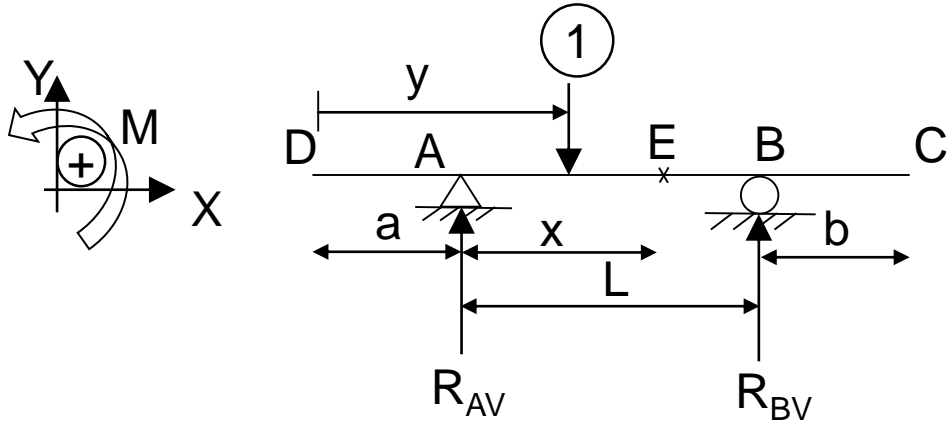
$$R_{AV} = \frac{L + a - y}{L}$$

$$R_{BV} = \frac{y - a}{L}$$

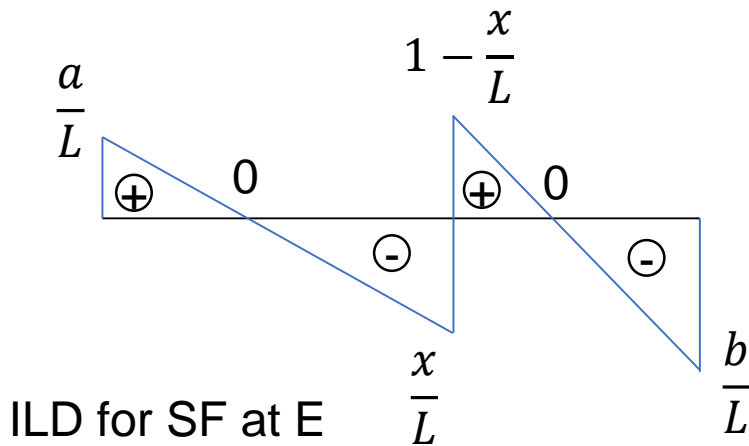
Unit load	SF at E	y=0	y=a	y=a+x	y=L+a	y=L+a+b
$0 \leq y \leq a$	$\frac{a - y}{L}$	$\frac{a}{L}$	0	-	-	-
$a \leq y \leq x+a$	$\frac{a - y}{L}$	-	0	$-\frac{x}{L}$	-	-
$x+a \leq y \leq L+a$	$\frac{L + a - y}{L}$	-	-	$1 - \frac{x}{L}$	0	-
$L+a \leq y \leq L+a+b$	$\frac{L + a - y}{L}$	-	-	-	0	$-\frac{b}{L}$

5.7 Influence Lines for Shear Force

[C] Double Overhang Beam (SF at E):

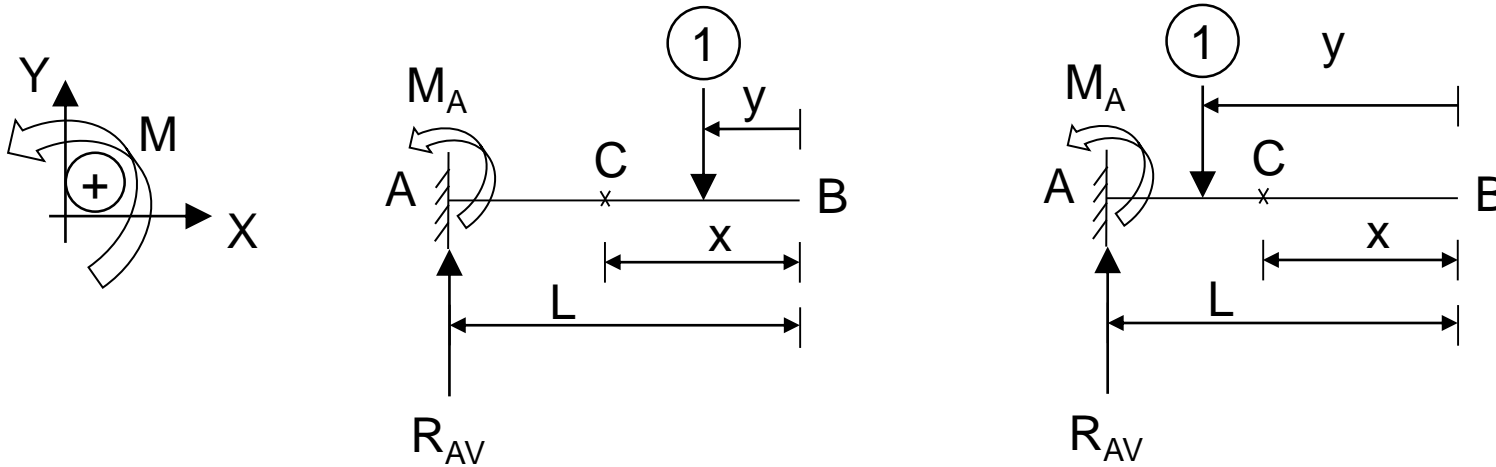


y=0	$\frac{a}{L}$			
y=a	0	0		
y=a+x		$-\frac{x}{L}$	$1 - \frac{x}{L}$	
y=L+a			0	0
y=L+a+b				$-\frac{b}{L}$



5.8 Influence Lines for Bending Moment

[A] Cantilever beam (BM at C):



We have,

$$R_{AV} = 1$$

$$M_A = L - y$$

(i) $0 \leq y \leq x$

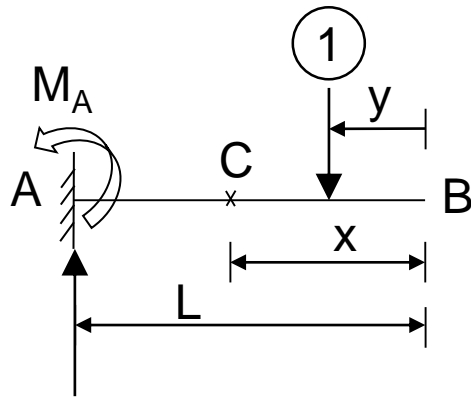
BM at C = $y - x$

(ii) $x \leq y \leq L$

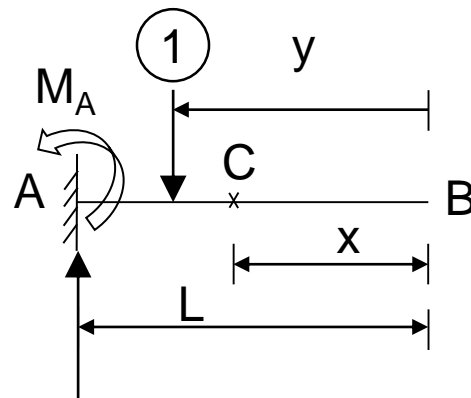
BM at C = 0

5.8 Influence Lines for Bending Moment

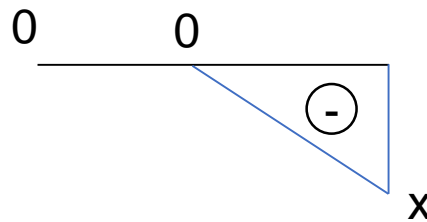
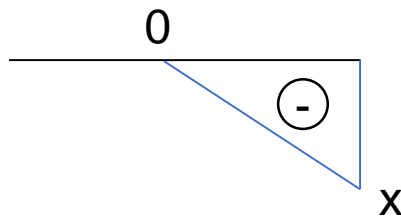
[A] Cantilever beam (BM at C):



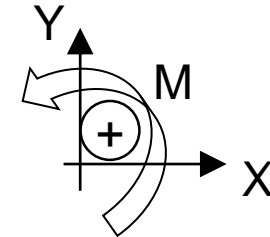
R_{AV} (i) $0 \leq y \leq x$
 BM at C = $y-x$



R_{AV} (ii) $x \leq y \leq L$
 BM at C = 0



ILD for BM at C



	$0 \leq y \leq x$	$x \leq y \leq L$
$y = 0$	$-x$	-
$y = x$	0	0
$y = L$	-	0

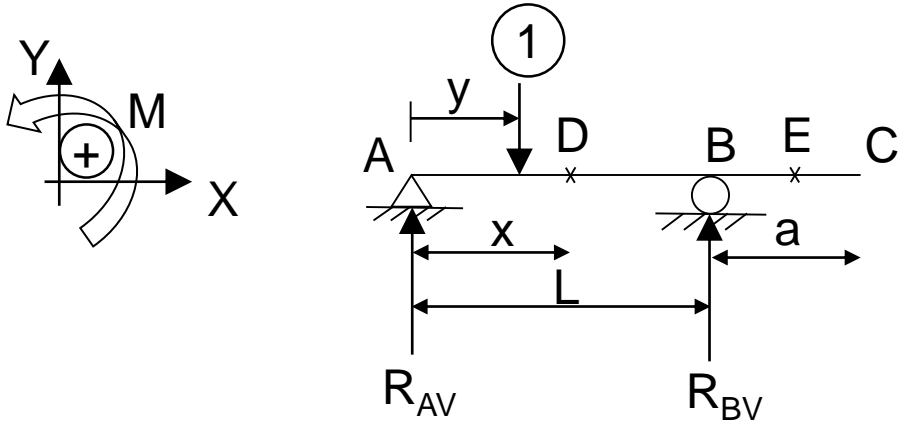
Plotting the ordinates, we get

5.8 Influence Lines for Bending Moment

[B] Single Overhang Beam (BM at D):

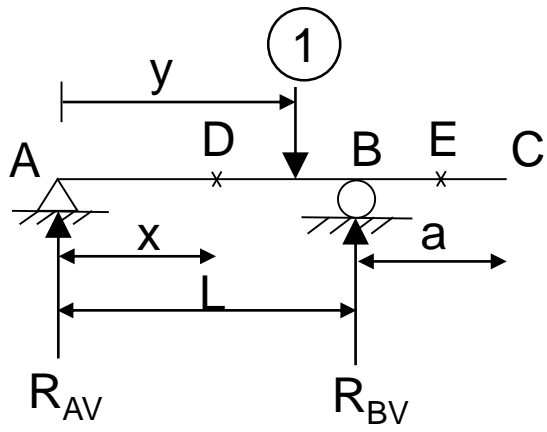
We have,

$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$



(i) $0 \leq y \leq x$

$$\begin{aligned} BM \text{ at } D &= R_{AV} * x - 1 * (x - y) \\ &= x - \frac{xy}{L} - x + y \\ &= \frac{y(L - x)}{L} \end{aligned}$$

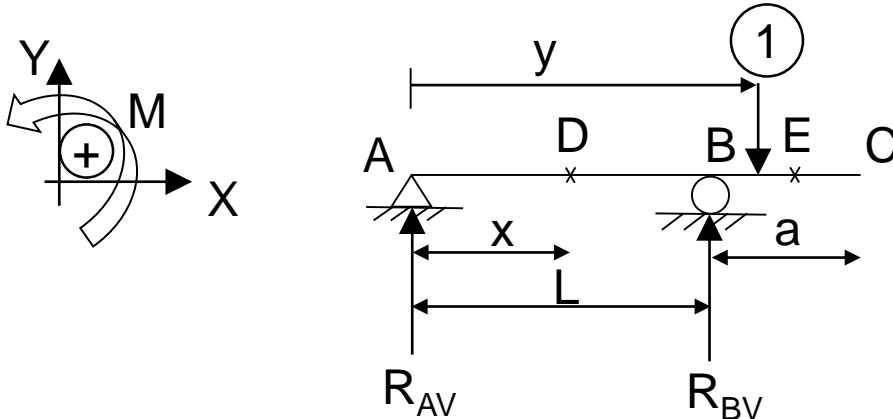


(ii) $x \leq y \leq L$

$$\begin{aligned} BM \text{ at } D &= R_{AV} * x \\ &= x * \frac{(L - y)}{L} \end{aligned}$$

5.8 Influence Lines for Bending Moment

[B] Single Overhang Beam (BM at D):



We have,

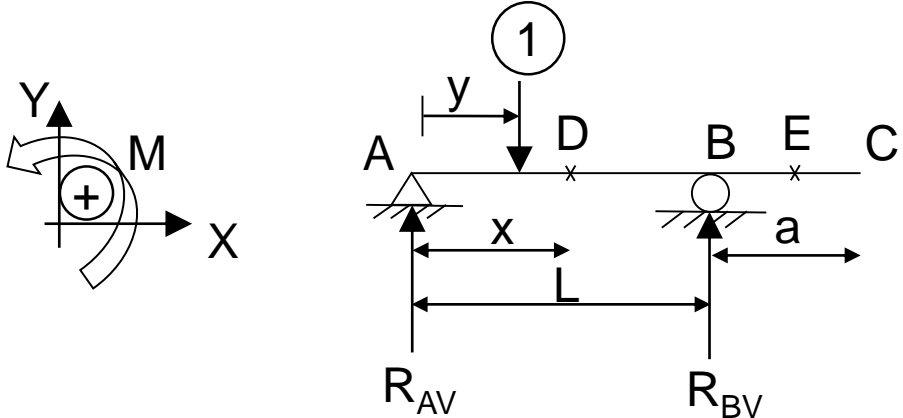
$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$

(iii) $L \leq y \leq L+a$

$$\begin{aligned}
 BM \text{ at } D &= R_{AV} * x \\
 &= x * \frac{(L-y)}{L}
 \end{aligned}$$

5.8 Influence Lines for Bending Moment

[B] Single Overhang Beam (BM at D):



We have,

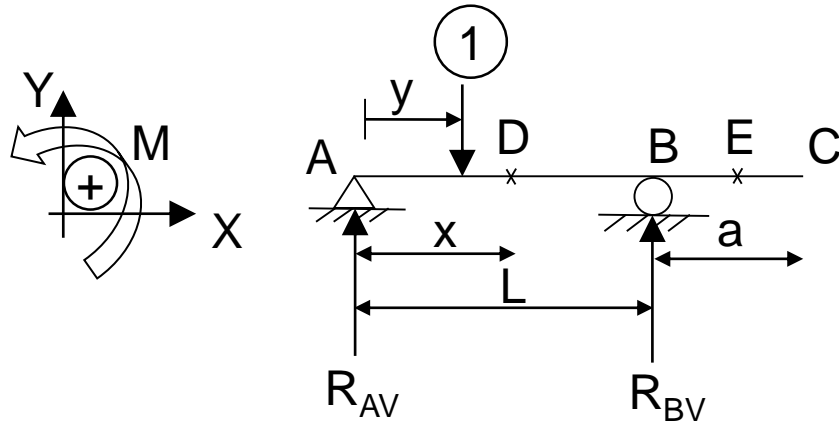
$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$

Unit load	BM at D	y=0	y=x	y=L	y=L+a
$0 \leq y \leq x$	$\frac{y(L-x)}{L}$	0	$\frac{x(L-x)}{L}$	-	-
$x \leq y \leq L$	$x * \frac{(L-y)}{L}$	-	$\frac{x(L-x)}{L}$	0	-
$L \leq y \leq L+a$	$x * \frac{(L-y)}{L}$	-	-	0	$x * \frac{(L-L-a)}{L}$ $= -\frac{ax}{L}$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.). New Delhi: Vikas Publishing House.

5.8 Influence Lines for Bending Moment

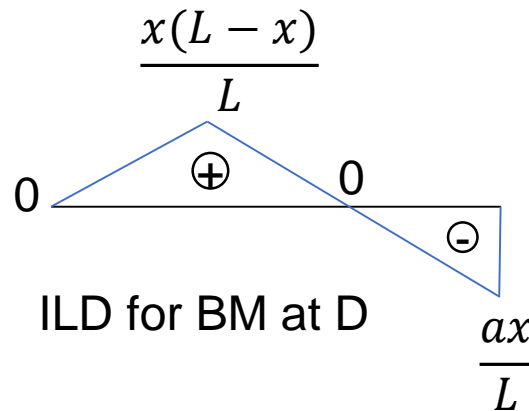
[B] Single Overhang Beam (BM at D):



We have,

$$R_{AV} = 1 - \frac{y}{L} \quad ; \quad R_{BV} = \frac{y}{L}$$

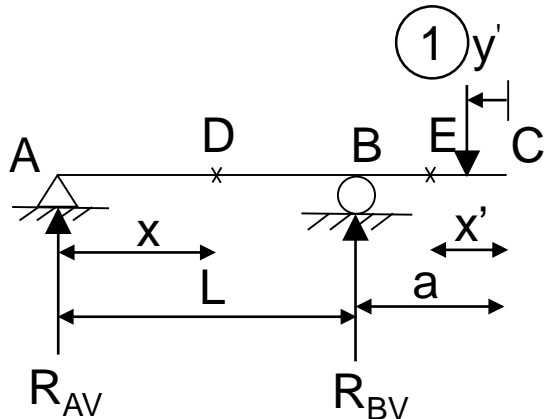
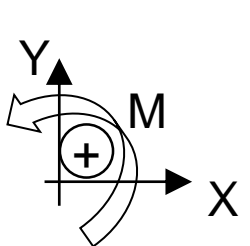
Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.



y=0	0		
y=x	$\frac{x(L-x)}{L}$	$\frac{x(L-x)}{L}$	
y=L		0	0
y=L+a			$-\frac{ax}{L}$

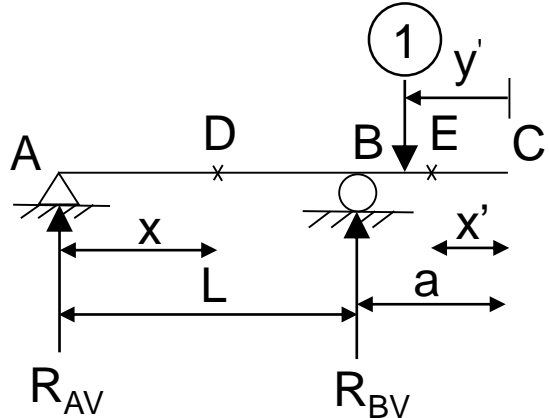
5.8 Influence Lines for Bending Moment

[B] Single Overhang Beam (BM at E):



(i) $0 \leq y' \leq x'$

BM at E = $y' - x'$

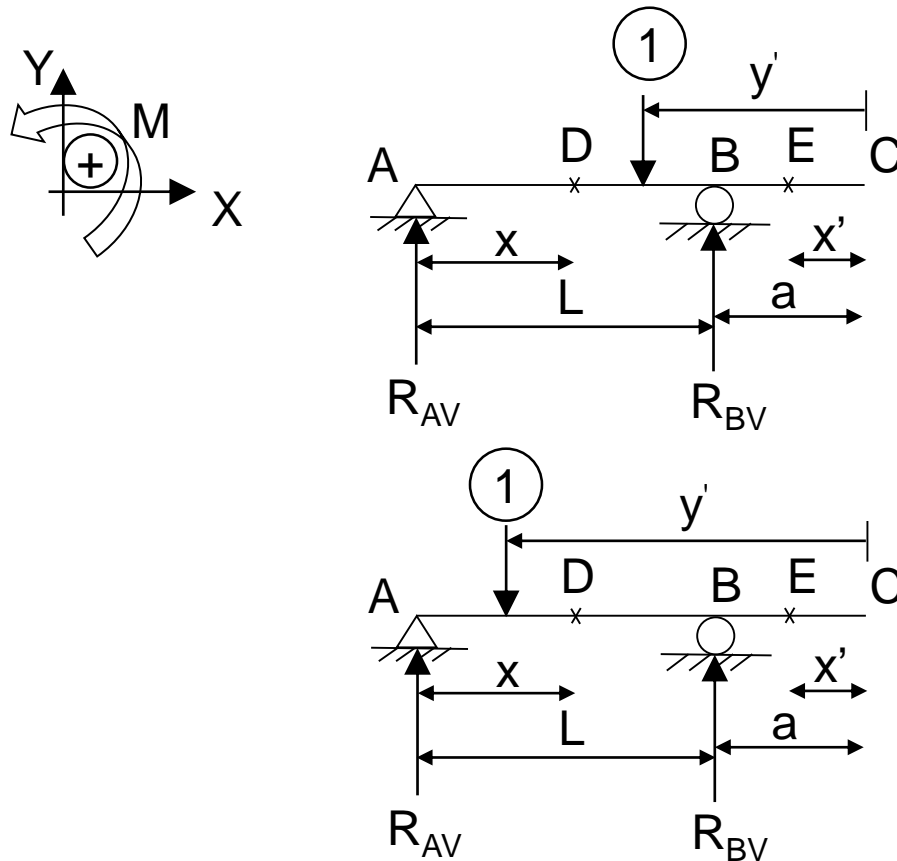


(ii) $x' \leq y' \leq a$

BM at E = 0

5.8 Influence Lines for Bending Moment

[B] Single Overhang Beam (BM at E):

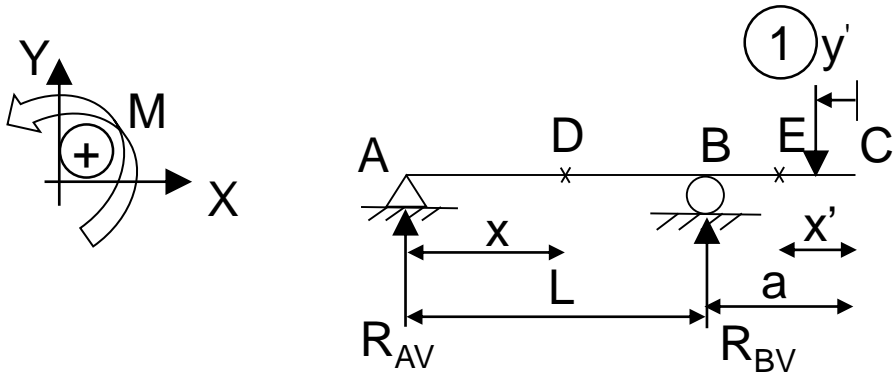


(iii) $a \leq y' \leq L+a$

BM at E = 0

5.8 Influence Lines for Bending Moment

[B] Single Overhang Beam (BM at E):

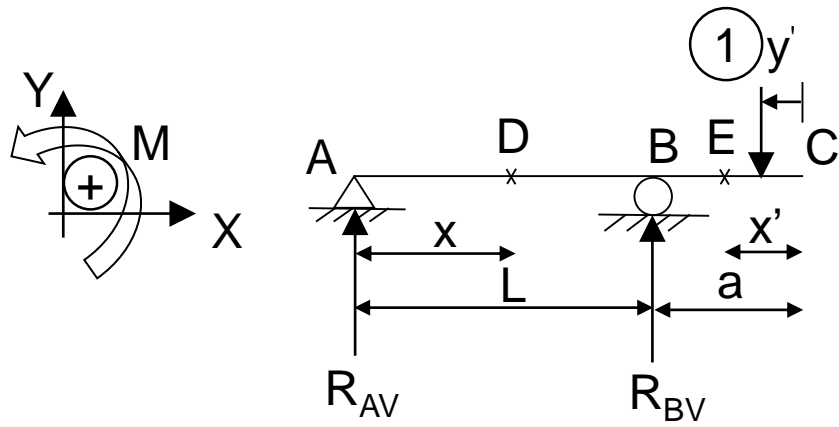


Unit load	SF at E	y'=0	y'=x'	y'=a	y'=L+a
$0 \leq y' \leq x'$	$y'-x'$	$-x'$	0	-	-
$x' \leq y' \leq a$	0	-	0	0	-
$a \leq y' \leq L+a$	0	-	-	0	0

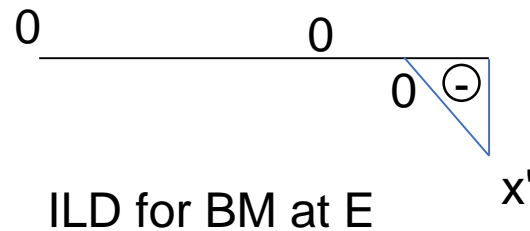
Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

5.8 Influence Lines for Bending Moment

[B] Single Overhang Beam (BM at E):



$y'=0$	$-x'$			
$y'=x'$	0	0		
$y'=a$		0	0	
$y'=L+a$			0	0



5.8 Influence Lines for Bending Moment

[C] Double Overhang Beam (BM at E):

We have,

$$\therefore R_{AV} = \frac{L + a - y}{L}$$

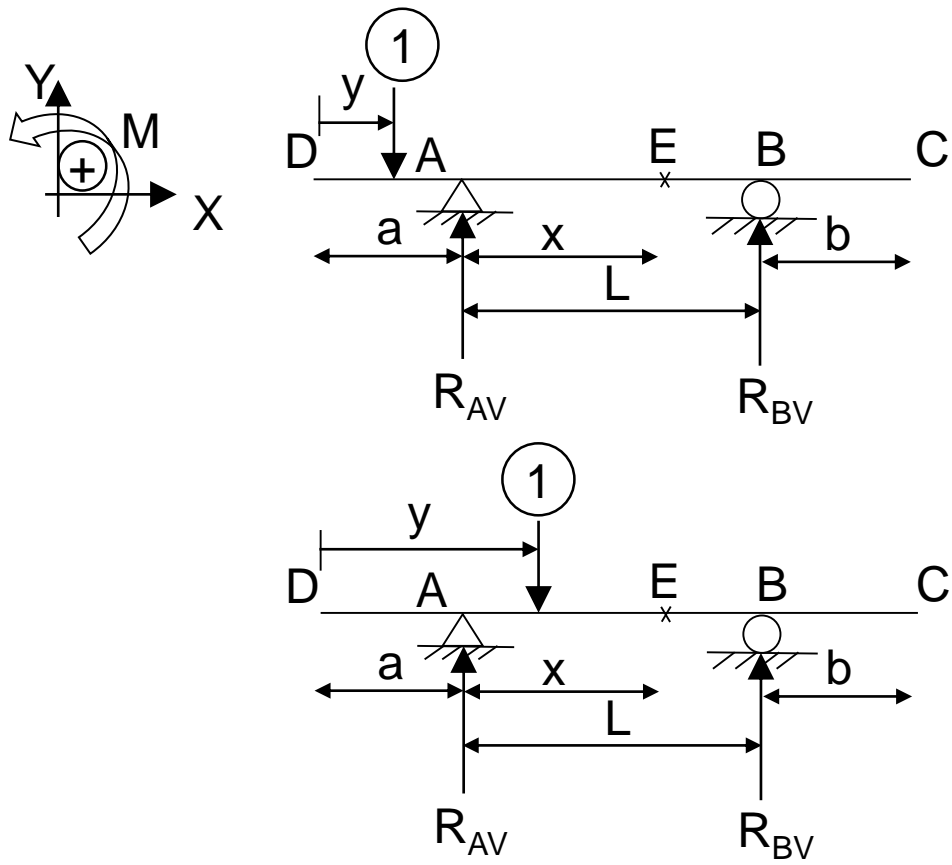
$$\therefore R_{BV} = \frac{y - a}{L}$$

(i) $0 \leq y \leq a$

$$\begin{aligned} BM \text{ at } E &= R_{AV} * x - 1 * (x + a - y) \\ &= x + \frac{x(a-y)}{L} - x - a + y \\ &= \frac{(a - y)(x - L)}{L} \end{aligned}$$

(ii) $a \leq y \leq a+x$

$$\begin{aligned} BM \text{ at } E &= R_{AV} * x - 1 * (x + a - y) \\ &= x + \frac{x(a-y)}{L} - x - a + y \\ &= \frac{(a - y)(x - L)}{L} \end{aligned}$$



Source: Bhavikatti, S. S. (2011). *Structural Analysis – I* (4th ed.). New Delhi: Vikas Publishing House.

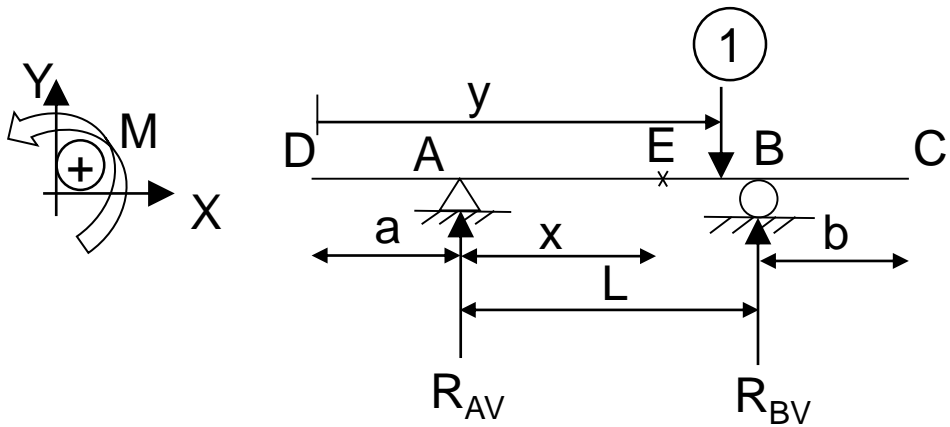
5.8 Influence Lines for Bending Moment

[C] Double Overhang Beam (BM at E):

We have,

$$\therefore R_{AV} = \frac{L + a - y}{L}$$

$$\therefore R_{BV} = \frac{y - a}{L}$$



(iii) $a + x \leq y \leq L + a$

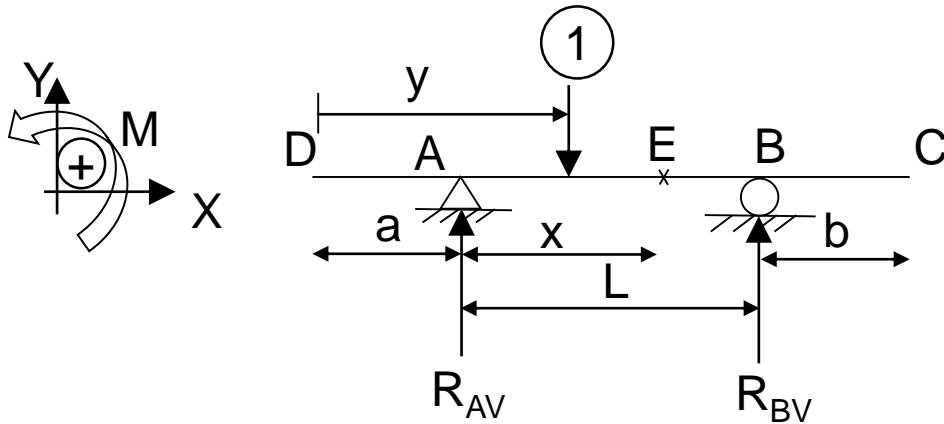
$$BM \text{ at } E = R_{AV} * x = \frac{x(L + a - y)}{L}$$

(iv) $L + a \leq y \leq L + a + b$

$$BM \text{ at } E = R_{AV} * x = \frac{x(L + a - y)}{L}$$

5.8 Influence Lines for Bending Moment

[C] Double Overhang Beam (BM at E):



We have,

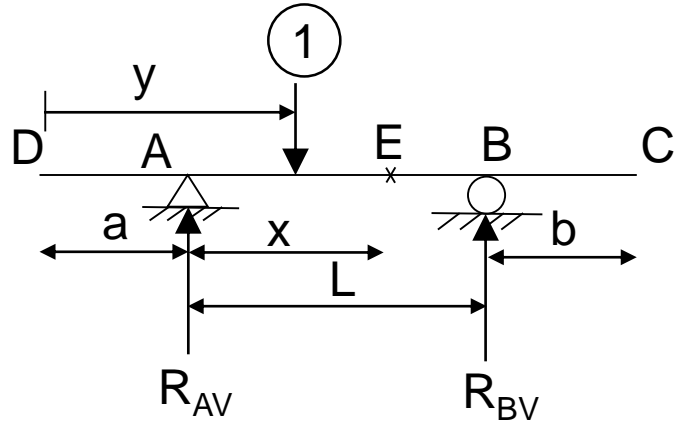
$$R_{AV} = \frac{L + a - y}{L}$$

$$R_{BV} = \frac{y - a}{L}$$

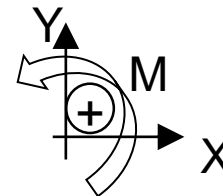
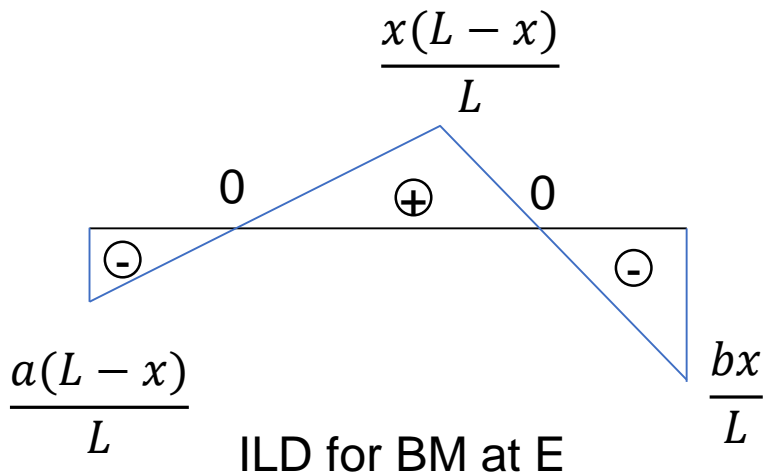
Unit load	BM at E	y=0	y=a	y=a+x	y=L+a	y=L+a+b
$0 \leq y \leq a$	$\frac{(a - y)(x - L)}{L}$	$-\frac{a(L - x)}{L}$	0	-	-	-
$a \leq y \leq x+a$	$\frac{(a - y)(x - L)}{L}$	-	0	$\frac{x(L - x)}{L}$	-	-
$x+a \leq y \leq L+a$	$\frac{x(L + a - y)}{L}$	-	-	$\frac{x(L - x)}{L}$	0	-
$L+a \leq y \leq L+a+b$	$\frac{x(L + a - y)}{L}$	-	-	-	0	$-\frac{bx}{L}$

5.8 Influence Lines for Bending Moment

[C] Double Overhang Beam (BM at E):



$y=0$	$-\frac{a(L-x)}{L}$			
$y=a$	0	0		
$y=a+x$		$\frac{x(L-x)}{L}$	$\frac{x(L-x)}{L}$	
$y=L+a$			0	0
$y=L+a+b$				$-\frac{bx}{L}$



-----End of Lecture#8-----

-----End of Part II of III for Chapter5-----

References

[1] Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.). New Delhi: Vikas Publishing House.