Theory of Structures - I

Chapter 5. Influence Lines for Simple Structures [Part III of III]

Lecturer: Dr. Sanjeema Bajracharya

Contents

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

In lecture 8, we have learned how to determine the Influence lines and thus ILDs for support reactions, SF and BM at a given section for different types of beams as a unit load moves along its span.

In this lecture, we learn to use the ILDs to find the value of support reactions, shear force, and bending moment at a given section of a beam with given loading.

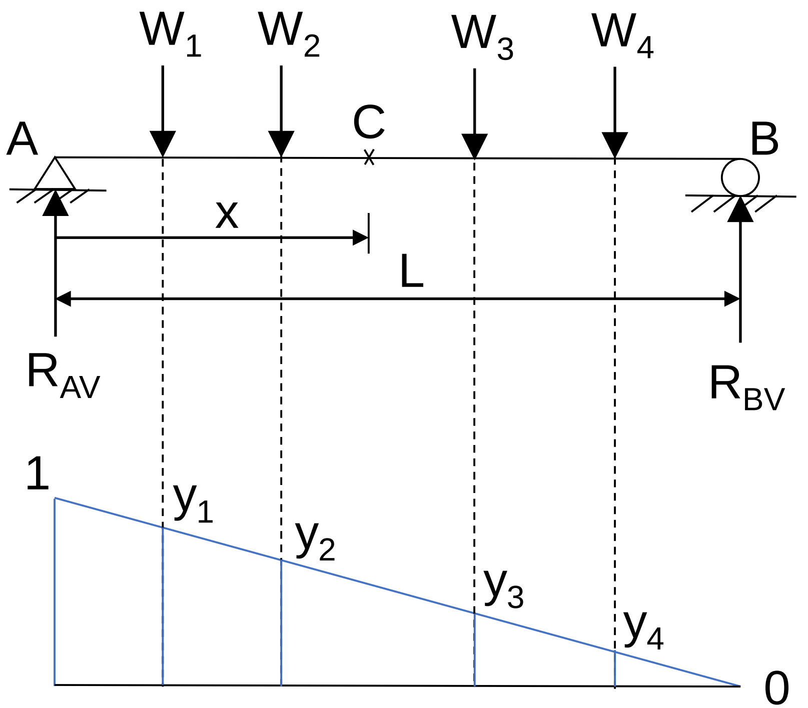
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Use of ILD (Point loads)

Let us consider a simply supported beam AB subjected to 4 concentrated loads W_1, W_2, W_3, and W_4 , for which we calculate the support reactions R_{AV}, R_{BV}, SF and BM at section C of the beam.

We have,
ILD for R_{AV} for simply supported beam as shown in the adjacent figure.
Let y_1, y_2, y_3, and y_4 be ordinates of ILD for R_{AV} at load points W_1, W_2, W_3, and W_4, respectively.



ILD for R_{AV}

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

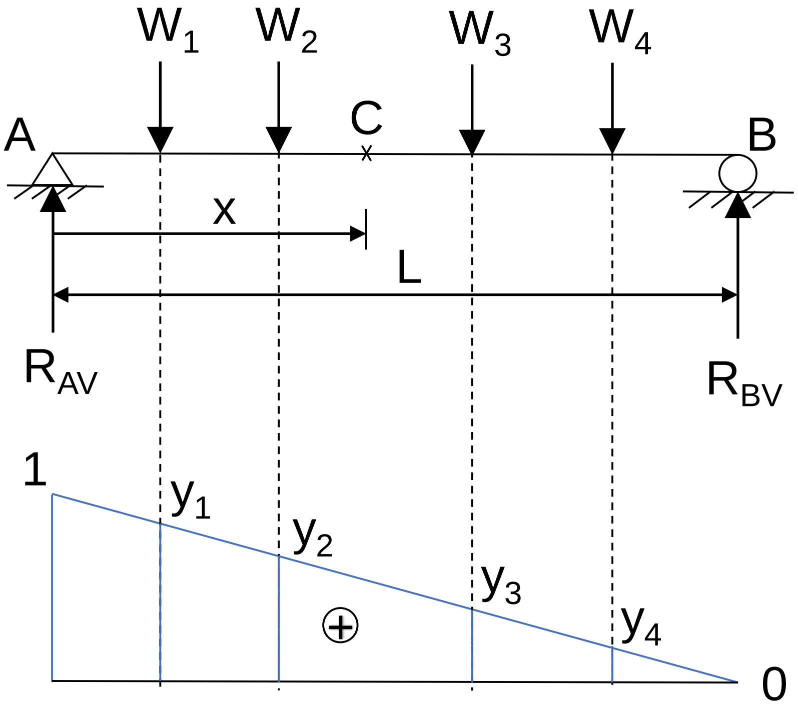
5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Use of ILD (Point loads)

Let y_1, y_2, y_3, and y_4 be ordinates of ILD for R_{AV} at load points W_1, W_2, W_3, and W_4, respectively.

Then,
R_{AV} due to the given loadings
$= W_1*y_1+W_2*y_2+W_3*y_3+W_4*y_4$

Note: Take into consideration the signs (+ve or -ve) of the ordinates as well.



ILD for R_{AV}

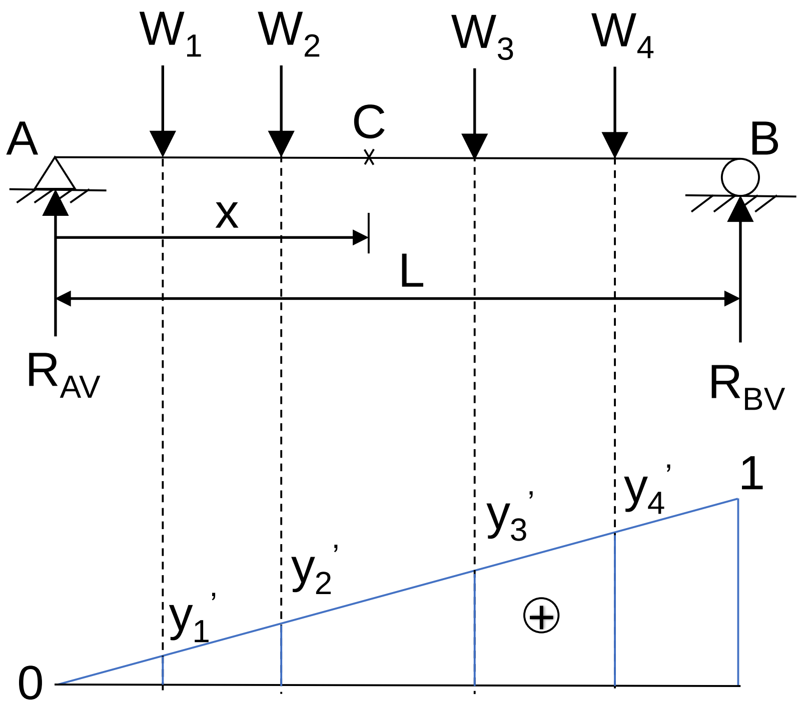
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Use of ILD (Point loads)

We have,
ILD for R_{BV} for simply supported beam as shown in the adjacent figure.
Let y_1', y_2', y_3', and y_4' be ordinates of ILD for R_{BV} at load points W_1, W_2, W_3, and W_4, respectively.

Then,
R_{BV} due to the given loadings
$= W_1*y_1'+W_2*y_2'+W_3*y_3'+W_4*y_4'$



ILD for R_{BV}

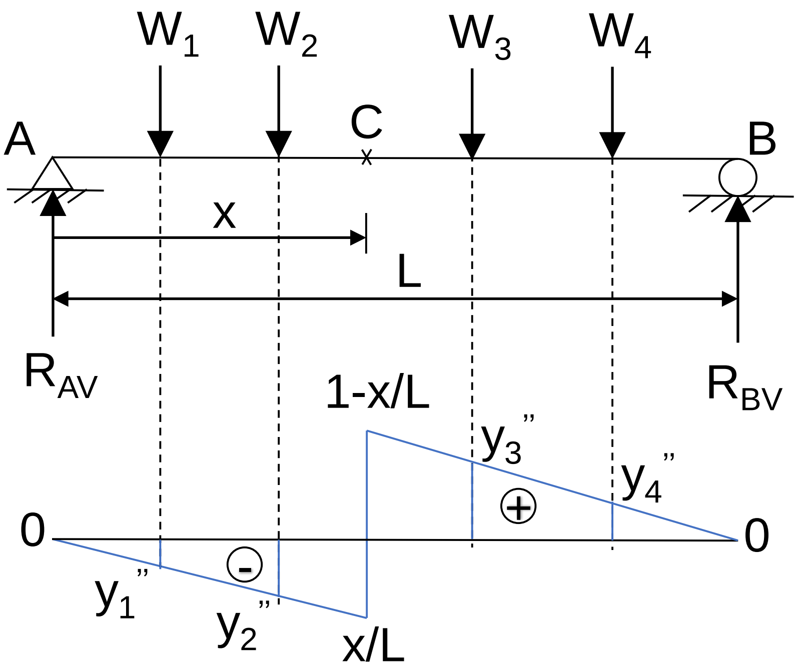
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Use of ILD (Point loads)

We have,
ILD for SF at C for simply supported beam as shown in the adjacent figure.
Let y_1'', y_2'', y_3'', and y_4'' be ordinates of ILD for SF at C at load points W_1, W_2, W_3, and W_4, respectively.

Then,
SF at C due to the given loadings
$= W_1 * -y_1'' + W_2 * -y_2'' + W_3 * y_3'' + W_4 * y_4''$



ILD for SF at C

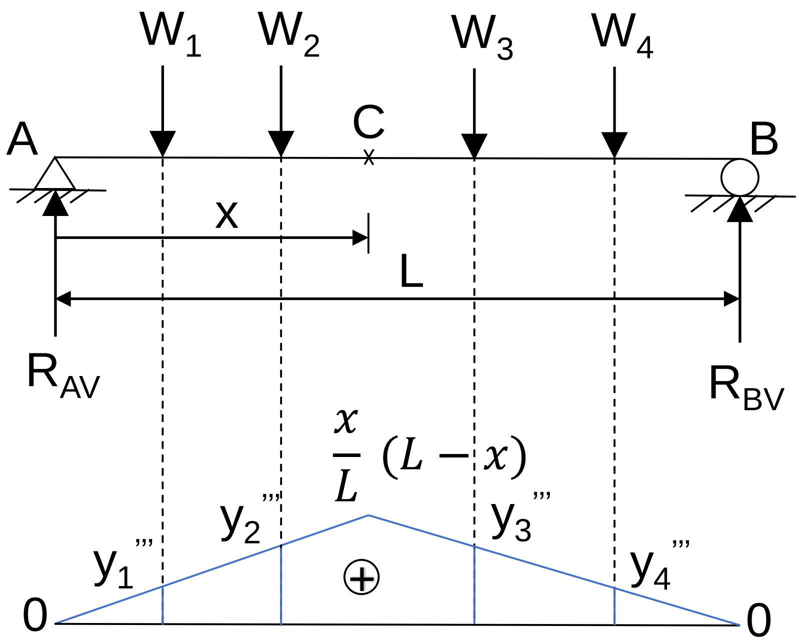
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Use of ILD (Point loads)

We have,
ILD for BM at C for simply supported beam as shown in the adjacent figure.
Let y_1''', y_2''', y_3''', and y_4''' be ordinates of ILD for BM at C at load points W_1, W_2, W_3, and W_4, respectively.

Then,
BM at C due to the given loadings
$= W_1 * y_1''' + W_2 * y_2''' + W_3 * y_3''' + W_4 * y_4'''$



ILD for BM at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

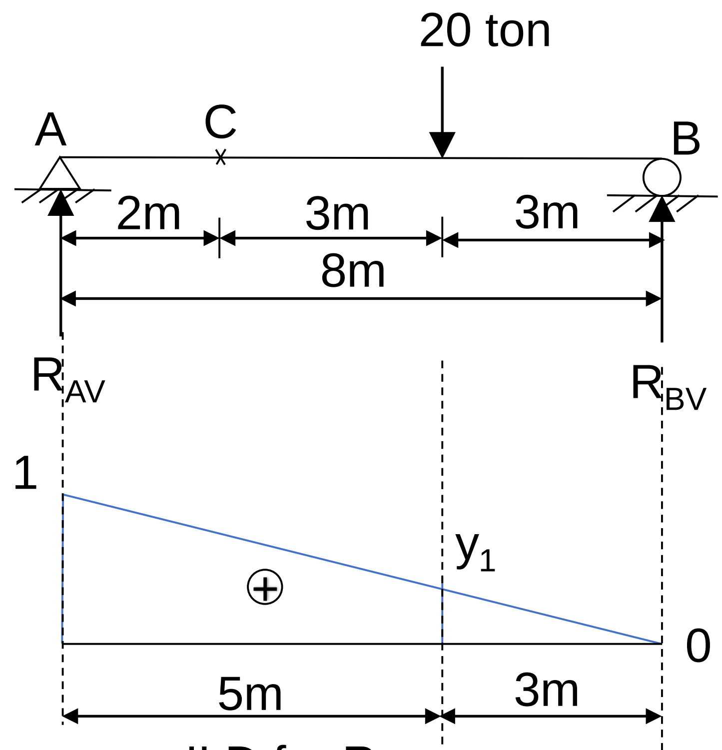
Numerical #1. Find the reaction at the supports, SF and BM at C of the given beam, using ILD.

We have,
ILD for $\mathbf{R_{AV}}$ for simply supported beam AB as shown in the adjacent figure.

Now,
Using similar triangles,

$$\frac{1}{5+3} = \frac{y_1}{3}$$

$$Or, y_1 = \frac{3}{8}$$

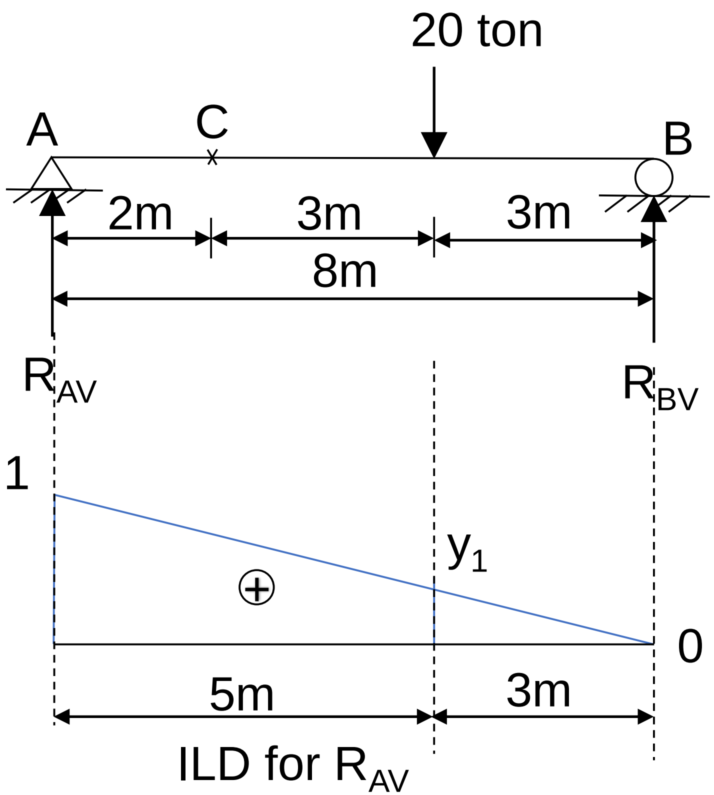


ILD for R_{AV}

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #1. Find the reaction at the supports, SF and BM at C of the given beam, using ILD.



We have,

$$y_1 = \frac{3}{8}$$

Then,

$$\boldsymbol{R_{AV}} = 20 * y_1 = 20 * \frac{3}{8} = \frac{15}{2} = \boldsymbol{7.5\ ton}$$

Ans.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

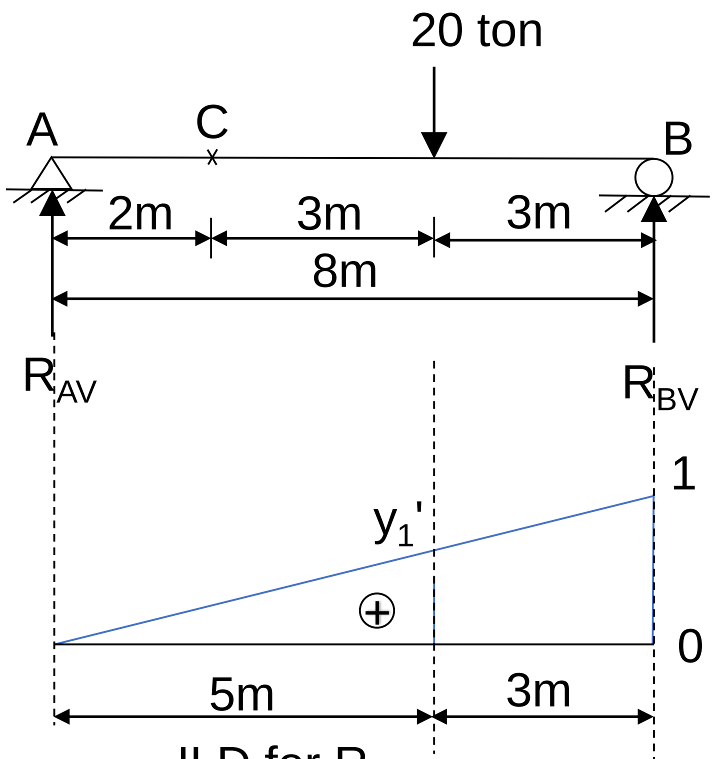
Numerical #1. Find the reaction at the supports, SF and BM at C of the given beam, using ILD.

We have,
ILD for $\mathbf{R_{BV}}$ for simply supported beam AB as shown in the adjacent figure.

Now,
Using similar triangles,

$$\frac{1}{5+3} = \frac{y_1'}{5}$$

$$Or, y_1' = \frac{5}{8}$$

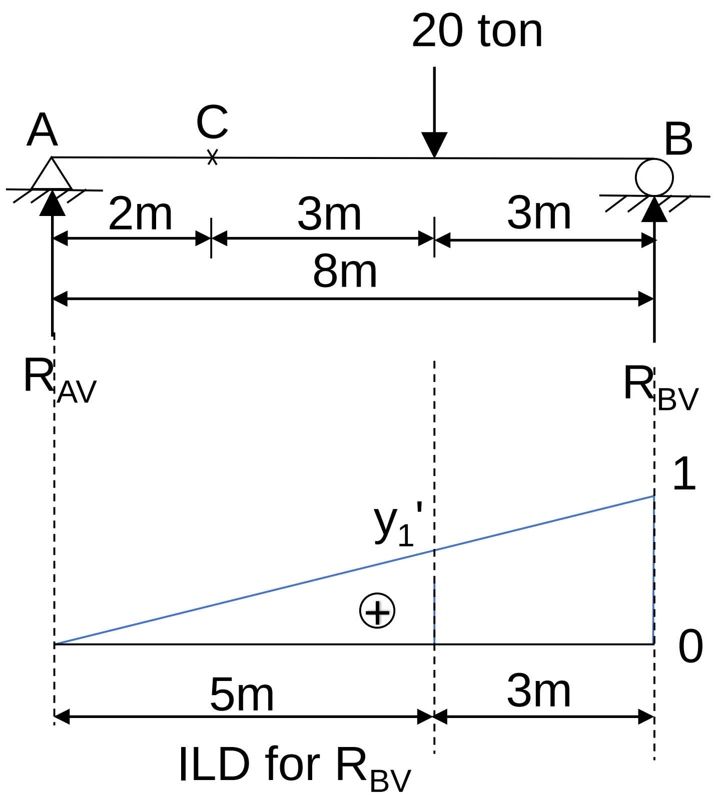


ILD for R_{BV}

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #1. Find the reaction at the supports, SF and BM at C of the given beam, using ILD.



We have,

$$y_1' = \frac{5}{8}$$

Then,

$$\boldsymbol{R_{BV}} = 20 * y_1' = 20 * \frac{5}{8} = \frac{25}{2} = \mathbf{12.5\ ton}$$

Ans.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

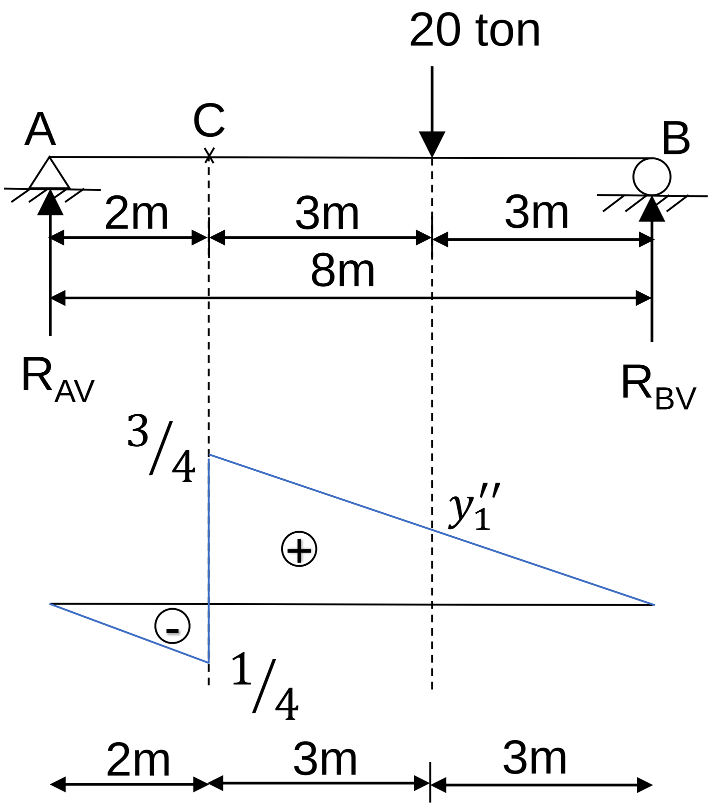
5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #1. Find the reaction at the supports, SF and BM at C of the given beam, using ILD.

We have,
ILD for **SF at C** for simply supported beam AB as shown in the adjacent figure, where
$\frac{x}{L} = \frac{2}{8} = \frac{1}{4}$ and $1 - \frac{x}{L} = 1 - \frac{1}{4} = \frac{3}{4}$

Now,
Using similar triangles,

$$\frac{3/4}{3+3} = \frac{y_1''}{3} \qquad Or, y_1'' = \frac{3}{8}$$



ILD for SF at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #1. Find the reaction at the supports, SF and BM at C of the given beam, using ILD.

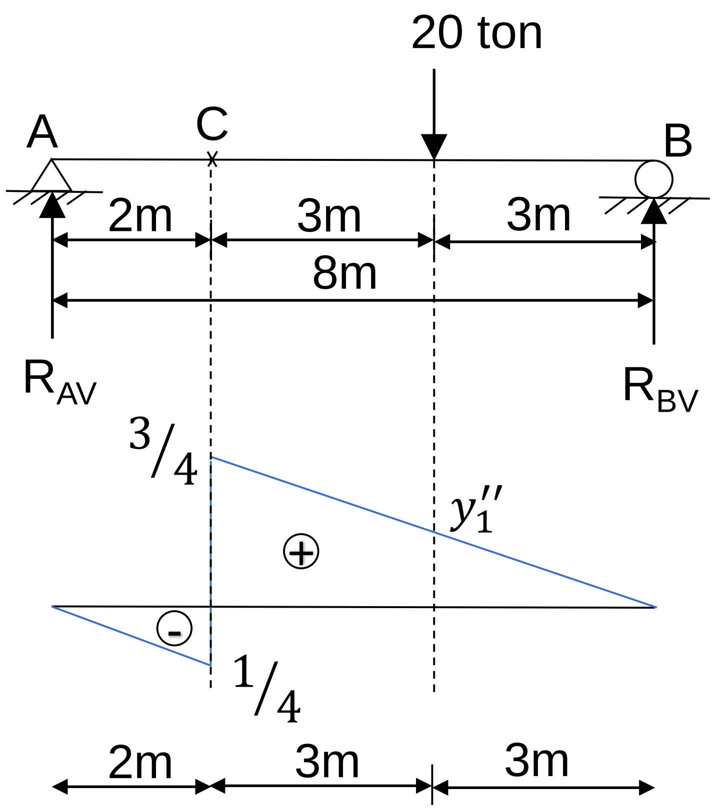
We have,

$$y_1'' = \frac{3}{8}$$

Then,

$$\boldsymbol{SF\ at\ C} = 20 * y_1'' = 20 * \frac{3}{8} = \frac{15}{2} = \mathbf{7.5}\ \boldsymbol{ton}$$

Ans.



ILD for SF at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

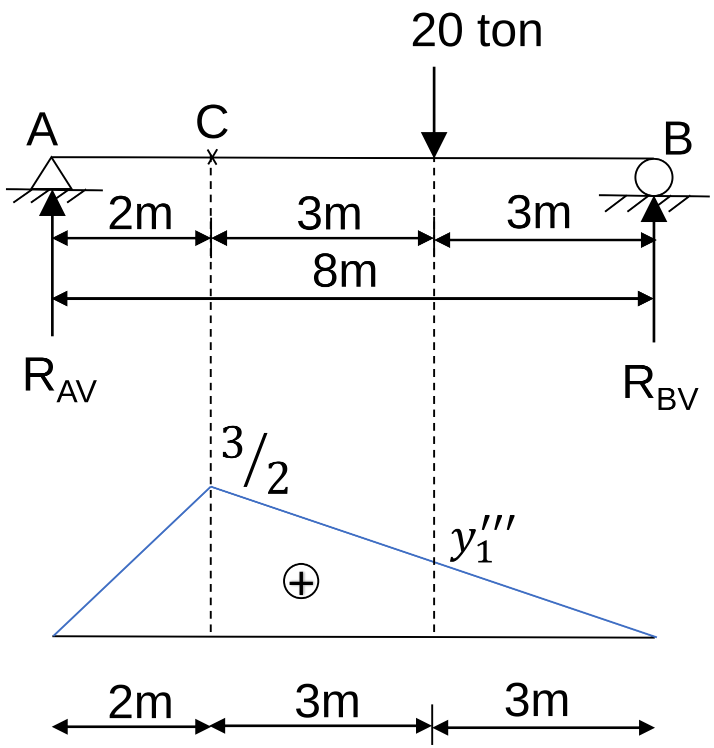
Numerical #1. Find the reaction at the supports, SF and BM at C of the given beam, using ILD.

We have,
ILD for **BM at C** for simply supported beam AB as shown in the adjacent figure, where

$$\frac{x}{L}(L-x) = \frac{2}{8}(8-2) = \frac{1}{4} * 6 = \frac{3}{2}$$

Now,
Using similar triangles,

$$\frac{3/2}{3+3} = \frac{y_1'''}{3} \qquad Or, y_1''' = \frac{3}{4}$$

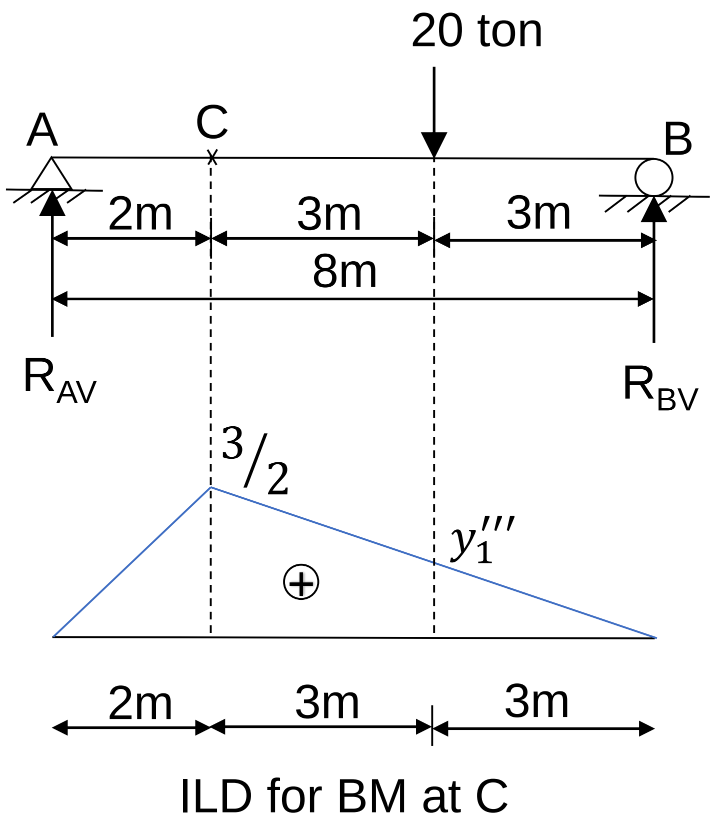


ILD for BM at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #1. Find the reaction at the supports, SF and BM at C of the given beam, using ILD.



We have,

$$y_1''' = \frac{3}{4}$$

Then,

$$\boldsymbol{BM\ at\ C} = 20 * y_1''' = 20 * \frac{3}{4} = \mathbf{15}\ \boldsymbol{ton}$$

Ans.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

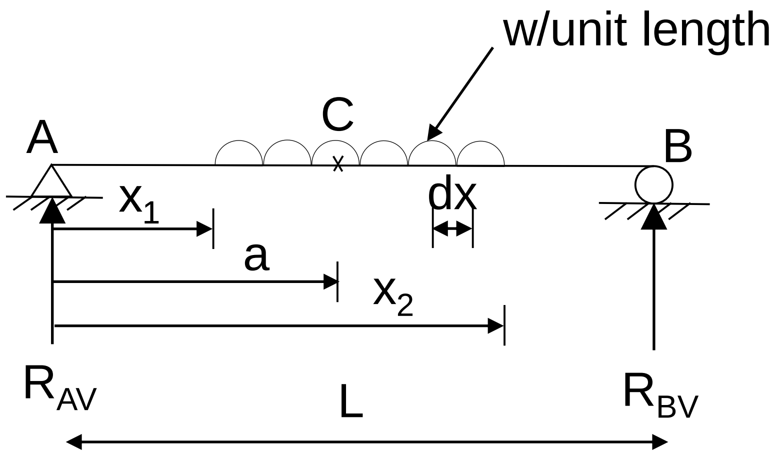
5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

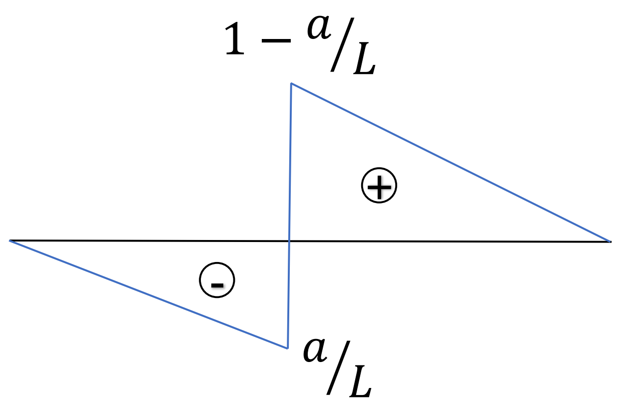
Use of ILD (Uniformly Distributed Load)

Let us consider a UDL w/unit length acting at a distance from x_1 to x_2 from support A on the simply supported beam AB of length L.

We need to compute the SF and BM at section C at a distance a from support A.

Then, we have,
ILD for SF at C for a simply supported beam is:

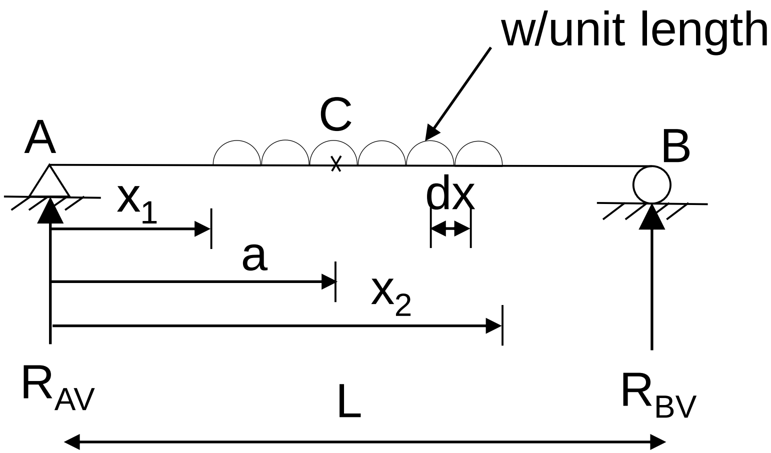




ILD for SF at C

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

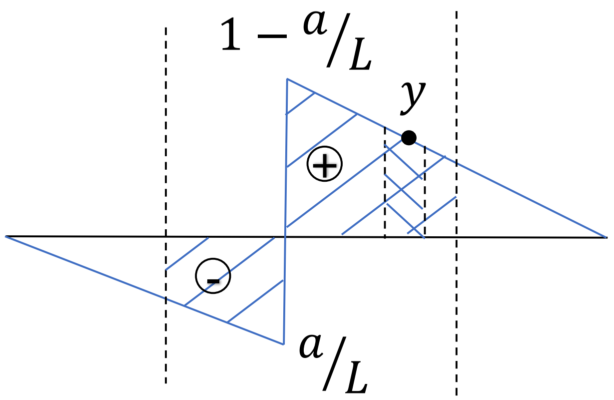
Use of ILD (Uniformly Distributed Load)



Here, for SF at C due to elemental load, w.dx = (w.dx).y,
Where, y is the ordinate of ILD for SF at C.
Then,
SF at C due to UDL from x_1 to x_2

$$= \int_{x_1}^{x_2} w.y\,dx = w\int_{x_1}^{x_2} y\,dx$$

= w*area of shaded portion
under the load of ILD for SF at C

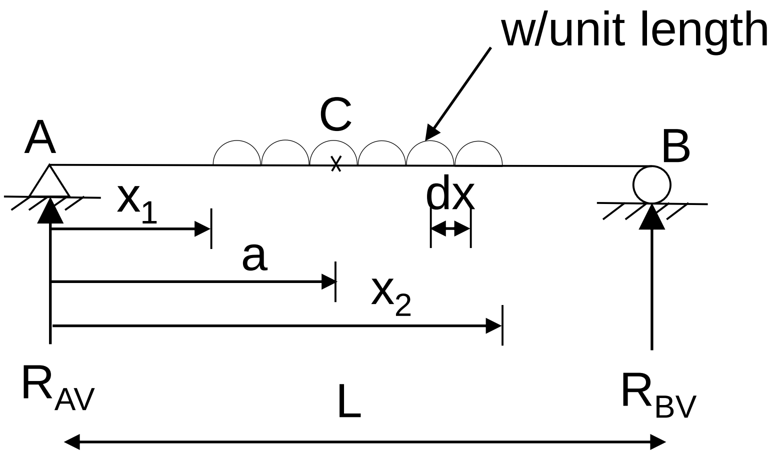


ILD for SF at C

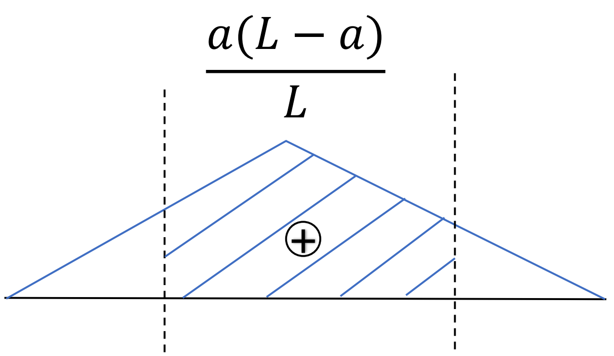
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Use of ILD (Uniformly Distributed Load)



We have,
ILD for BM at C for a simply supported beam is:



ILD for BM at C

Similarly,
BM at C due to UDL
= w*area of shaded portion under the load of ILD for BM at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

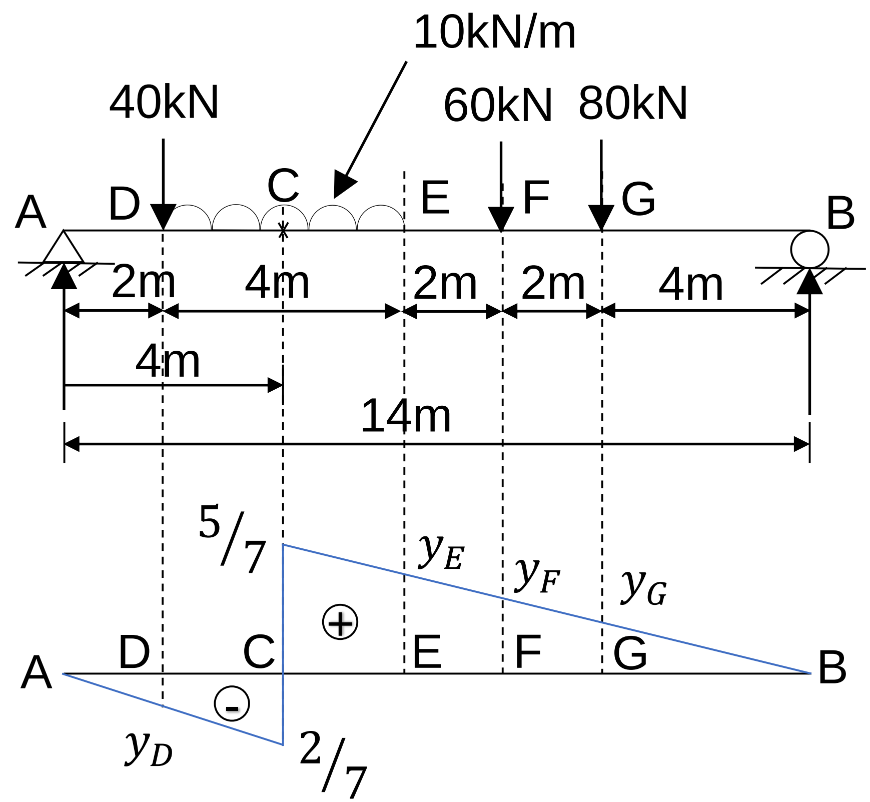
5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #2. Using ILD, determine the SF and BM at section C in the given simply supported beam.

We have,
ILD for SF at C for a simply supported beam, where
$\frac{x}{L} = \frac{4}{14} = \frac{2}{7}$ and $1 - \frac{2}{7} = \frac{5}{7}$

Now, using similar triangles, the ordinate at load point D is:

$$\frac{2/7}{4} = \frac{y_D}{2} \qquad Or, y_D = \frac{1}{7}$$

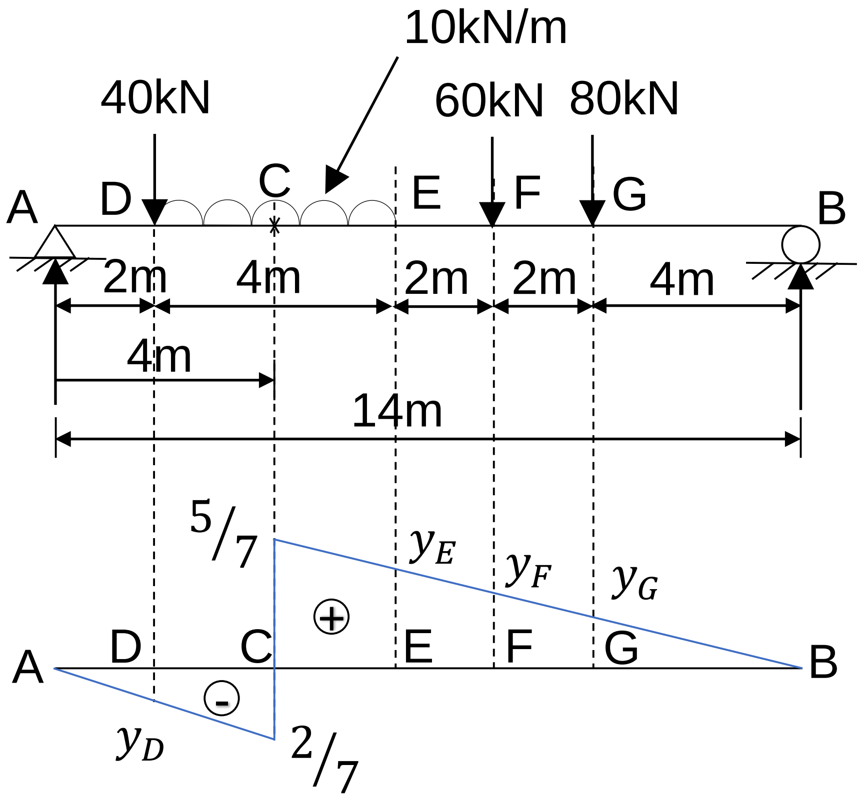


ILD for SF at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #2. Using ILD, determine the SF and BM at section C in the given simply supported beam.



ILD for SF at C

The ordinate at load point E is:

$$\frac{5/7}{10} = \frac{y_E}{8} \qquad Or, y_E = \frac{4}{7}$$

The ordinate at load point F is:

$$\frac{5/7}{10} = \frac{y_F}{6} \qquad Or, y_F = \frac{3}{7}$$

The ordinate at load point G is:

$$\frac{5/7}{10} = \frac{y_G}{4} \qquad Or, y_G = \frac{2}{7}$$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

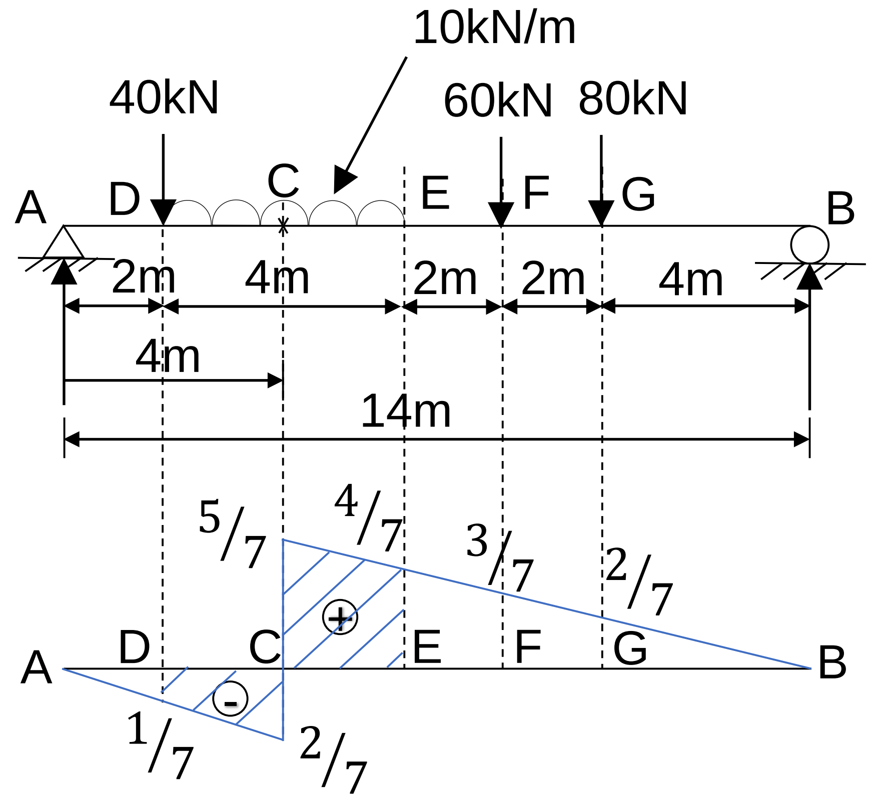
Numerical #2. Using ILD, determine the SF and BM at section C in the given simply supported beam.

We get,

the ordinates at load points D, E, F, and G in ILD for SF at C as $1/7$, $4/7$, $3/7$, and $2/7$, respectively.

Then,

SF at C = -40* y_D – 10*(trapezoidal) shaded area under UDL between D and C+ 10*(trapezoidal) shaded area under UDL between C and E + 60*$y_F + 80 * y_G$



ILD for SF at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #2. Using ILD, determine the SF and BM at section C in the given simply supported beam.

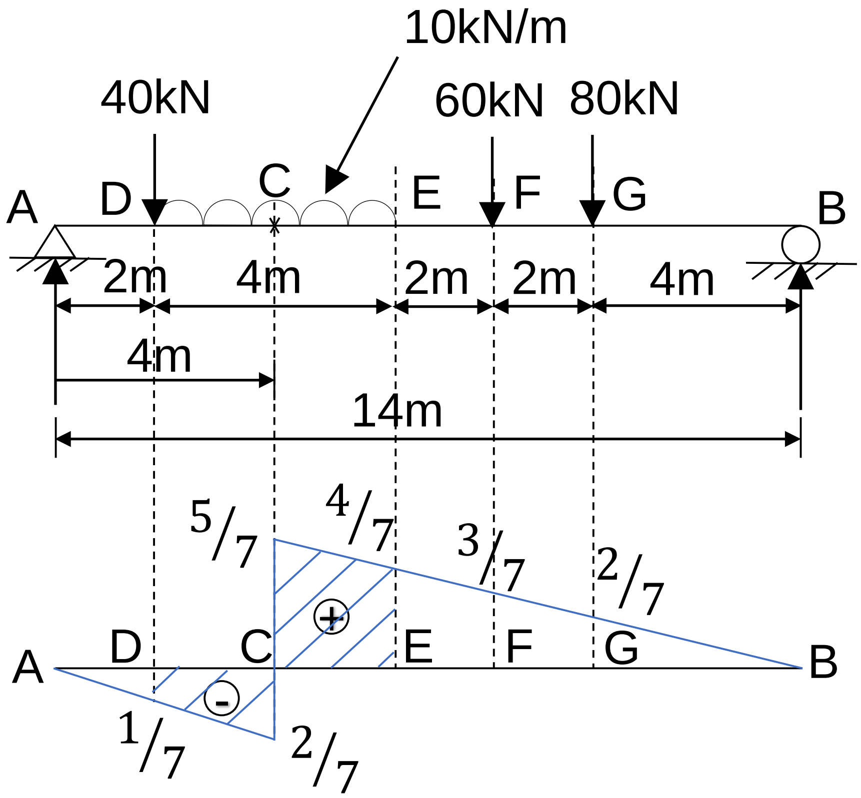
Then,

SF at C

$= 40^* \; -1/7 - \; 10^* \; 1/2 * (1/7 + 2/7)$ *2+ 10* $1/2 * (5/7 + 4/7)$*2+60*$3/7 + 80 * 2/7$

= 51.43 kN

Ans.

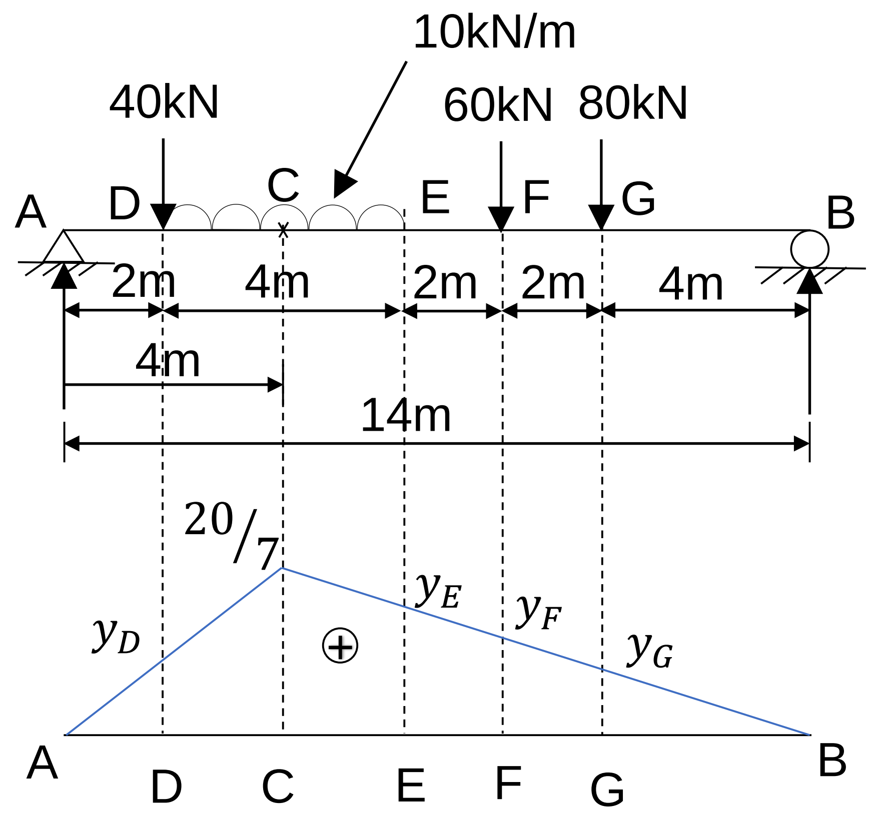


ILD for SF at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #2. Using ILD, determine the SF and BM at section C in the given simply supported beam.



ILD for BM at C

We have,
ILD for BM at C for a simply supported beam, where

$$\frac{x(L-x)}{L} = \frac{4(14-4)}{14} = \frac{20}{7}$$

Now, using similar triangles, the ordinate at load point D is:

$$\frac{20/7}{4} = \frac{y_D}{2} \qquad Or, y_D = \frac{10}{7}$$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #2. Using ILD, determine the SF and BM at section C in the given simply supported beam.

The ordinate at load point E is:

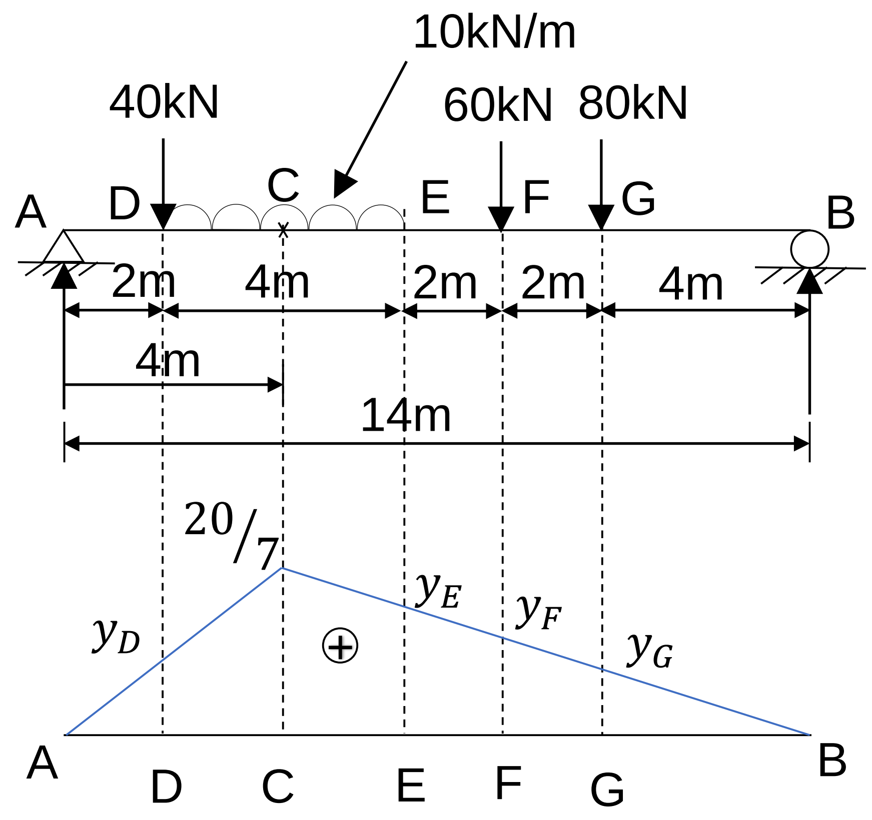
$$\frac{20/7}{10} = \frac{y_E}{8} \qquad Or, y_E = \frac{16}{7}$$

The ordinate at load point F is:

$$\frac{20/7}{10} = \frac{y_F}{6} \qquad Or, y_F = \frac{12}{7}$$

The ordinate at load point G is:

$$\frac{20/7}{10} = \frac{y_G}{4} \qquad Or, y_G = \frac{8}{7}$$



ILD for BM at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

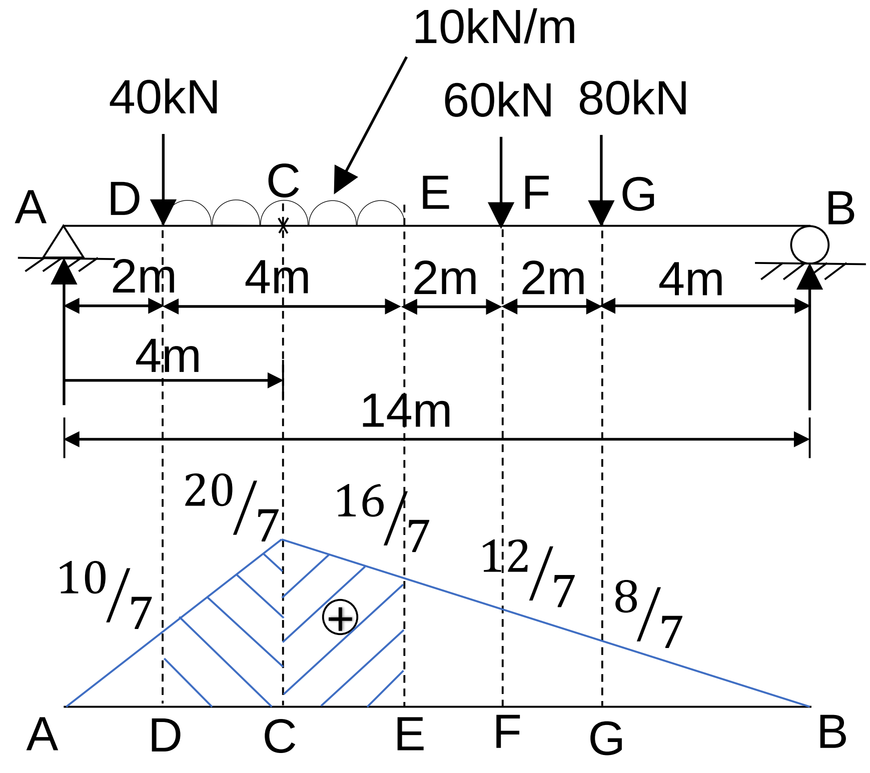
5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #2. Using ILD, determine the SF and BM at section C in the given simply supported beam.

We get,

the ordinates at load points D, E, F, and G in ILD for BM at C as $10/7$, $16/7$, $12/7$, and $8/7$, respectively.

Then,
BM at C
= 40*y_D+10*(trapezoidal) shaded area under UDL between D and C+ 10*(trapezoidal) shaded area under UDL between C and E + 60*$y_F + 80 * y_G$

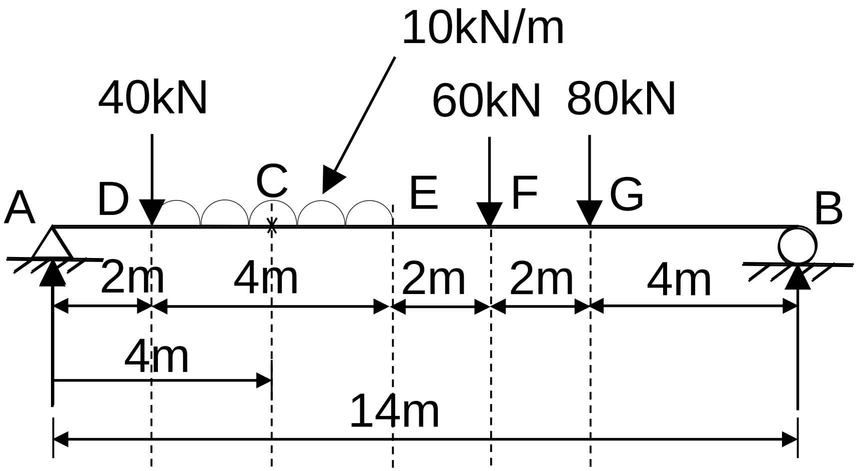


ILD for BM at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.9 Determination of Reactions, BM, and SF from ILDs for Different Loadings

Numerical #2. Using ILD, determine the SF and BM at section C in the given simply supported beam.



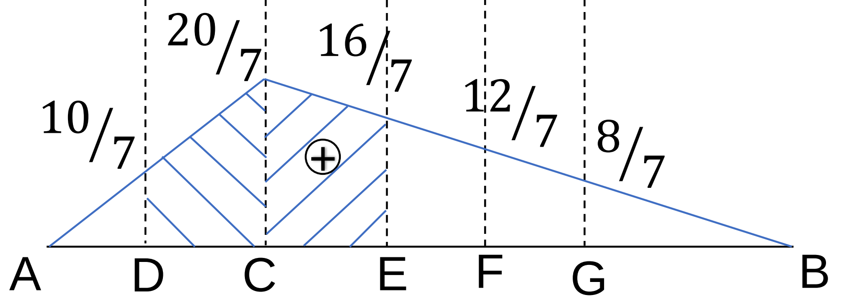
Then,

BM at C

= 40 * 10/7 + 10 * ½ *(10/7+20/7)*2 + 10 * ½ *(20/7+16/7)*2 + 60*12/7 + 80*8/7

= **345.71 kN-m**.

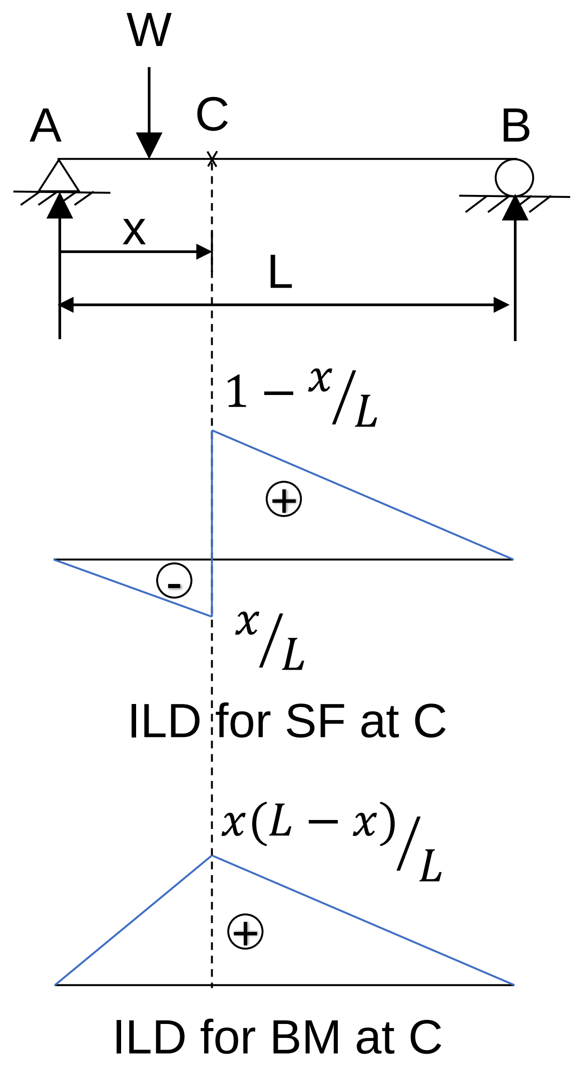
Ans.



ILD for BM at C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span



[A] Single point load
(i) Maximum SF and BM

Let W be the moving load and the values of maximum SF and BM is required at section C at a distance x from support A of the simply supported beam AB of span L.

		Location
Max –ve SF at C	W.x/L	When W is just to the left of C
Max +ve SF at C	W.$(1 - x/L)$	When W is just to the right of C
Max BM at C	W.x$((L-x)/L)$	When W is on C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

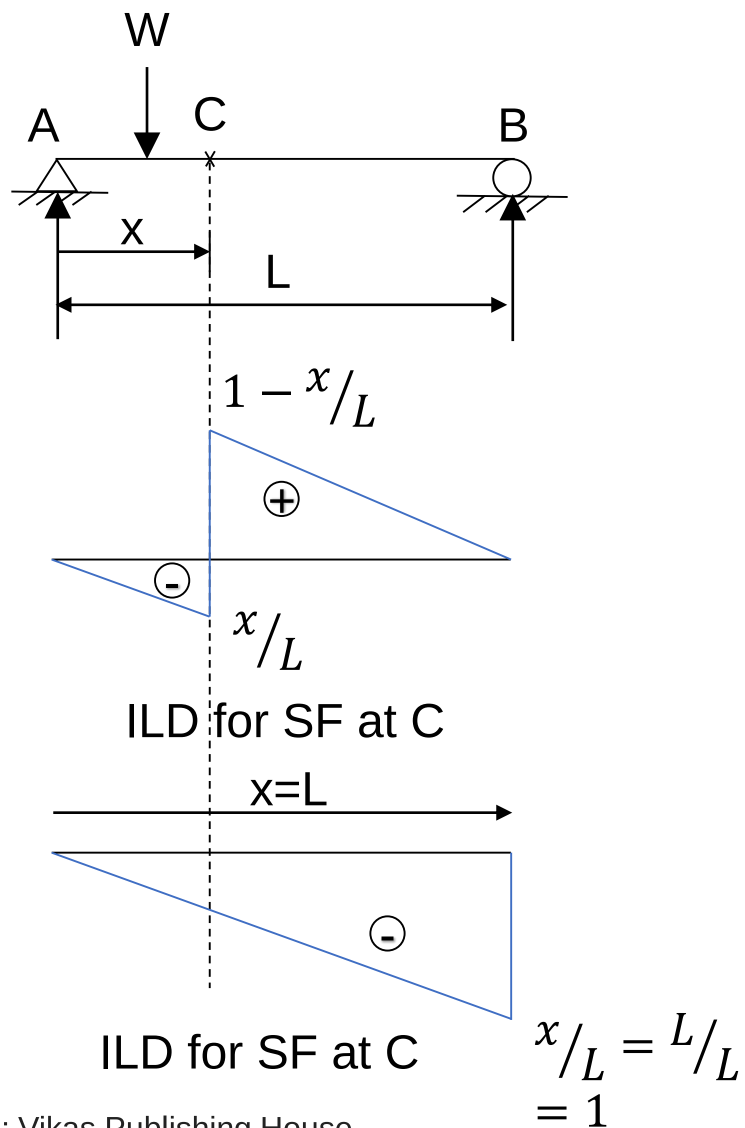
5.10 Most critical position of a load on a beam span

[A] Single point load
(ii) Absolute Maximum values anywhere in the beam

For this, the ordinate of ILD should be maximum.

ILD for SF at C
Negative SF ordinate is maximum when x=L and the value is equal to 1.

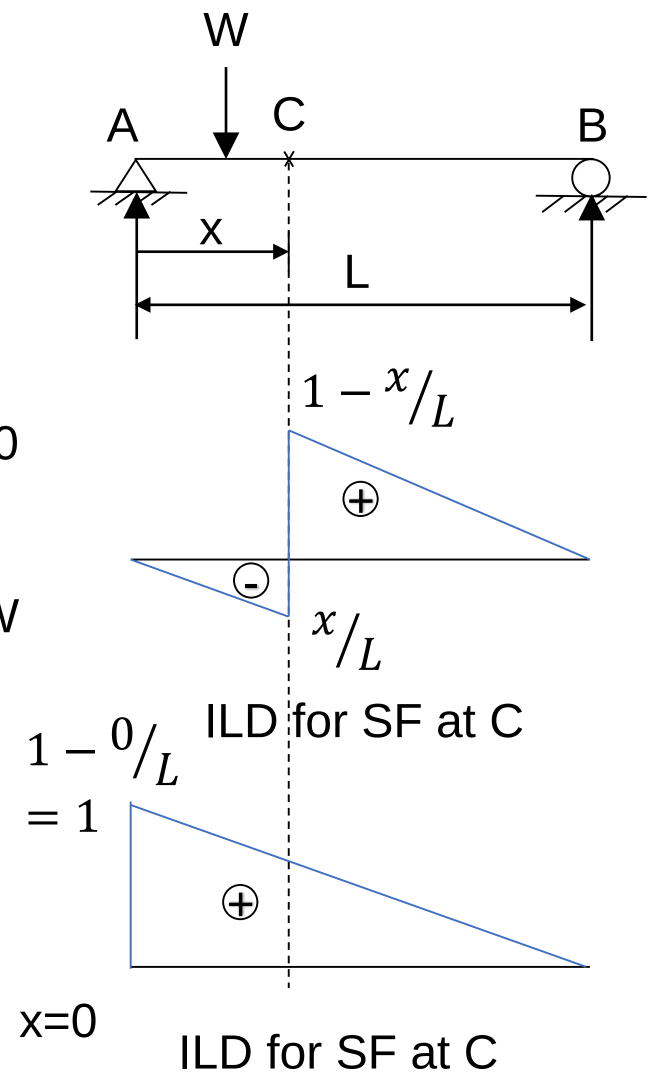
∴ Absolute maximum negative SF = W*1 = W and it occurs when W is at x=L, i.e. at B.



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span

[A] Single point load
(ii) Absolute Maximum values anywhere in the beam



ILD for SF at C

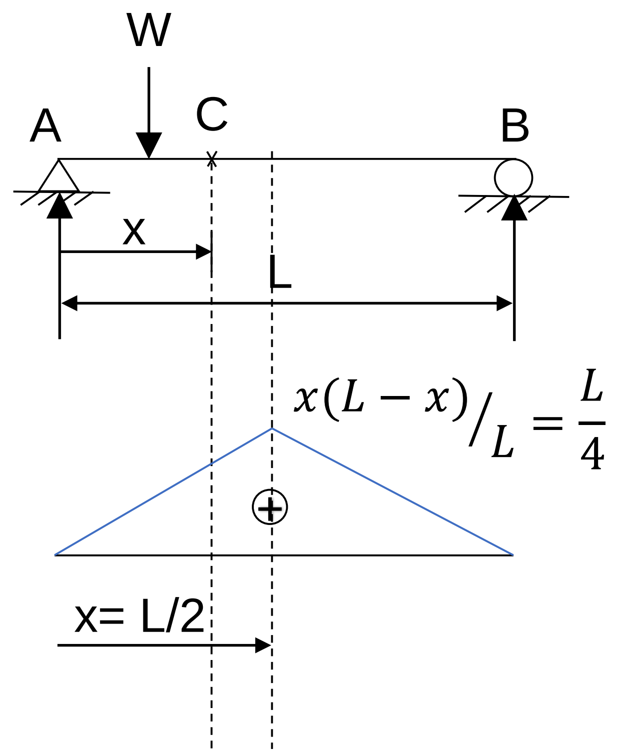
Positive SF ordinate is maximum when x=0 and the value is equal to 1.

∴ Absolute maximum positive SF = W*1 = W and it occurs when W is at x=0, i.e. at A.

∴ Absolute maximum SF = W

5.10 Most critical position of a load on a beam span

[A] Single point load
(ii) Absolute Maximum values anywhere in the beam



ILD for BM at C

ILD for BM at C

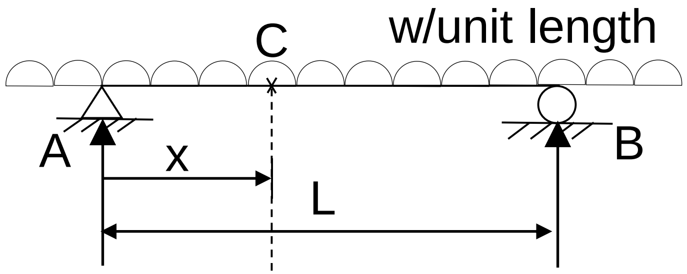
BM ordinate is maximum when x=L/2, i.e. when the load W is at mid-span, which is equal to:

$$\frac{L}{2} * (L - \frac{L}{2}) / _{L} = \frac{L}{4}$$

∴ Absolute maximum BM =W $* \frac{L}{4} = \frac{WL}{4}$ and it occurs at mid-span.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

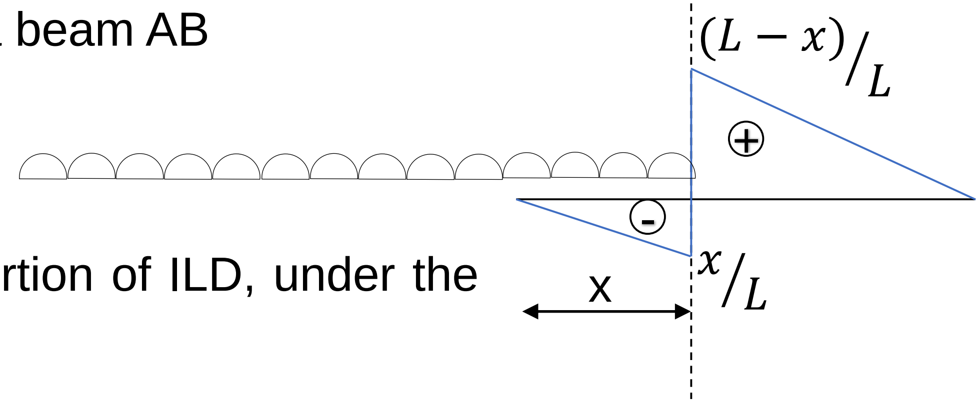
5.10 Most critical position of a load on a beam span



[B] UDL longer than the span (d>L)
(i) Maximum SF and BM at given section

Let a UDL of intensity **w** move from left to right across a beam AB

ILD for SF at C
Maximum –ve SF
= w * area of shaded portion of ILD, under the load



And, it occurs when UDL covers the portion AC only, as shown in the adjacent figure.

ILD for SF at C

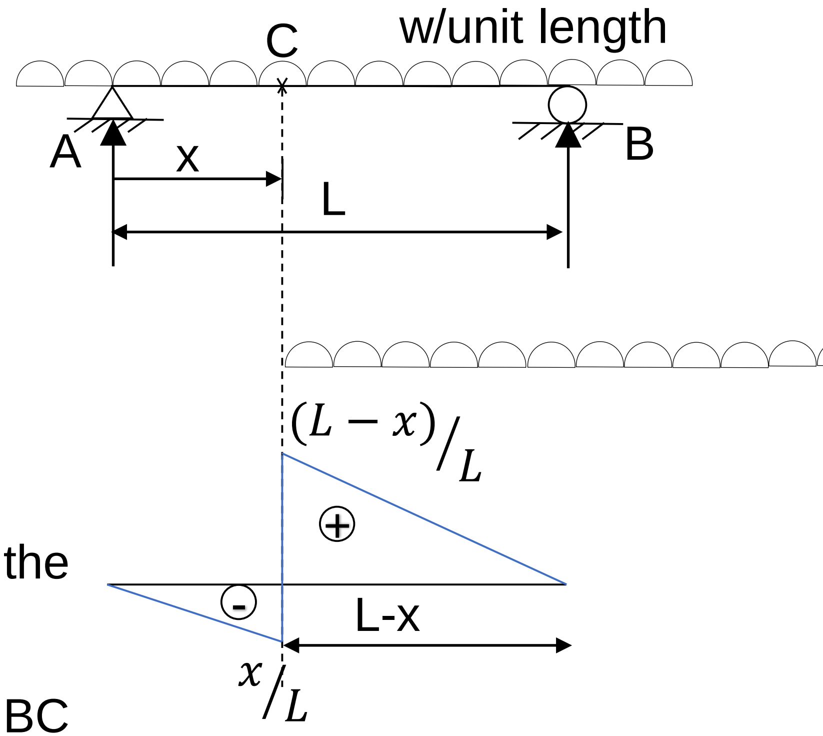
Then,
Maximum –ve SF at C = w* (½ *x* x/L) = $wx^2/2L$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span

[B] UDL longer than the span (d>L)
(i) Maximum SF and BM at given section

Let a UDL of intensity **w** move from left to right across a beam AB



ILD for SF at C

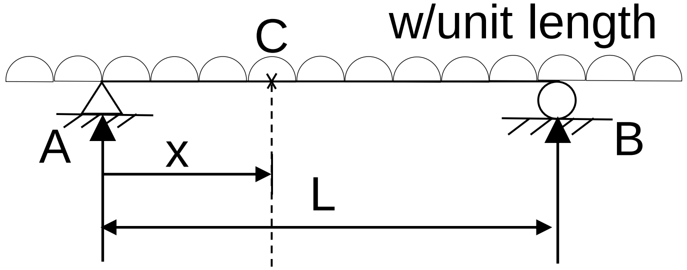
ILD for SF at C
Maximum +ve SF
= w * area of shaded portion of ILD, under the load

And, it occurs when UDL covers the portion BC only, as shown in the adjacent figure.

Then,
Maximum +ve SF at C = w* (½ *(L-x)* (L-x)/L) = $w(L-x)^2/2L$

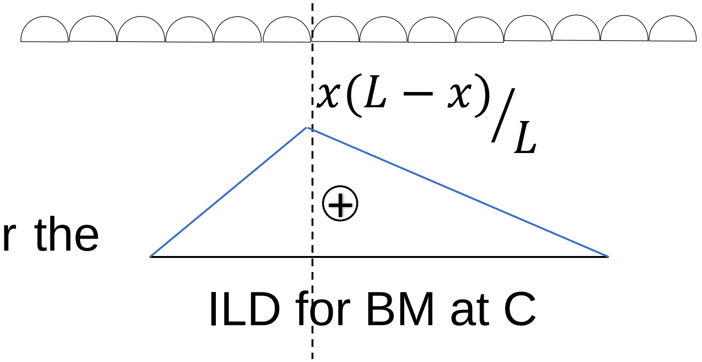
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span



[B] UDL longer than the span (d>L)
(i) Maximum SF and BM at given section

Let a UDL of intensity **w** move from left to right across a beam AB



ILD for BM at C
Maximum BM at C
= w * area of shaded portion of ILD, under the load

And, it occurs when UDL covers the entire span of the beam, as shown in the adjacent figure.

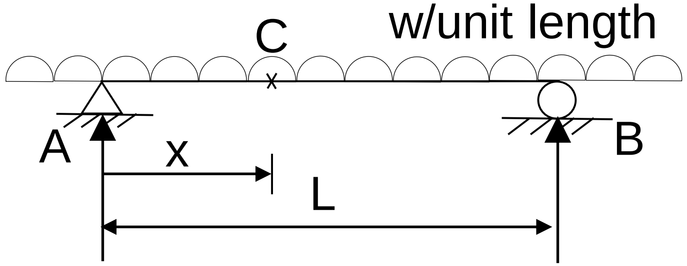
Then,

Maximum BM at C = w* (½ *L* x(L-x)/L) = wx(L-x)/2

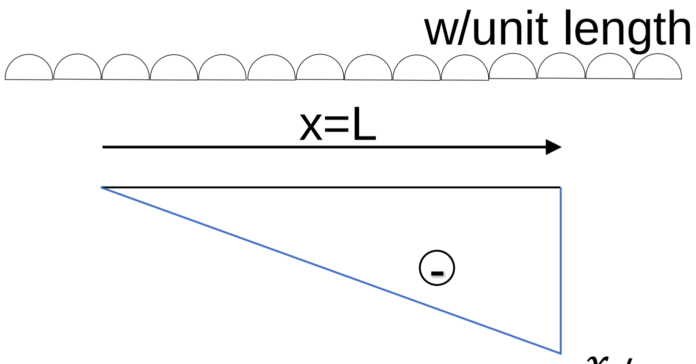
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span

[B] UDL longer than the span (d>L)
(ii) Absolute Maximum values anywhere in the beam



ILD for SF at C



$$x/L = L/L = 1$$

Negative SF ordinate is maximum when x=L and the value is equal to 1.

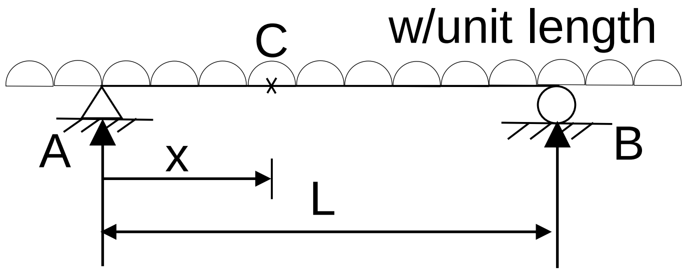
∴ Absolute maximum negative SF
=w*1/2*L*1
= wL/2

And, it occurs when the UDL occupies the entire span of the beam.

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span

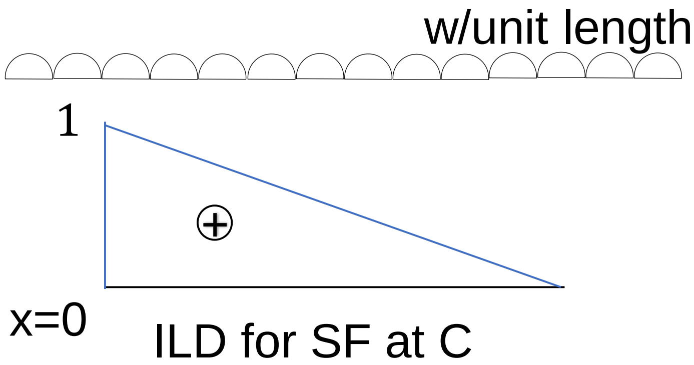
[B] UDL longer than the span (d>L)
(ii) Absolute Maximum values anywhere in the beam



ILD for SF at C

Positive SF ordinate is maximum when x=0 and the value is equal to 1.

∴ Absolute maximum positive SF
= w*1/2*L*1
= wL/2

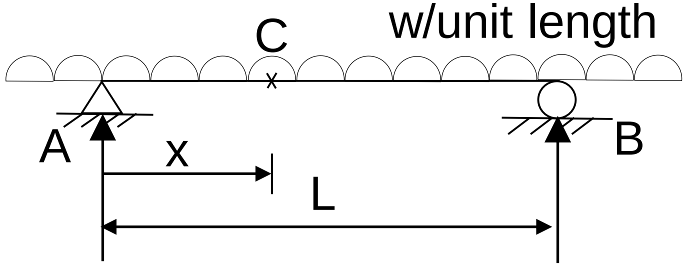


And, it occurs when the UDL occupies the entire span of the beam.

∴ Absolute maximum SF = wL/2

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span



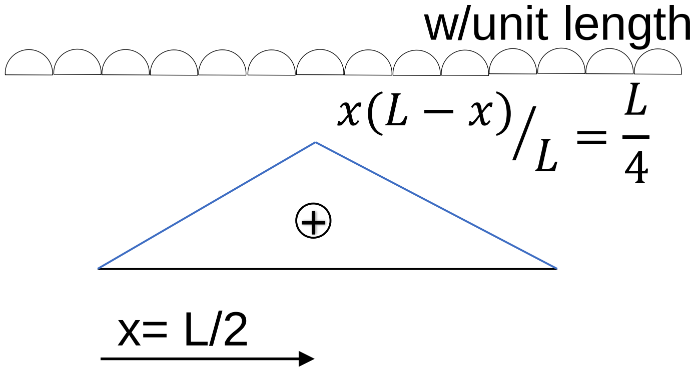
[B] UDL longer than the span (d>L)
(ii) Absolute Maximum values anywhere in the beam

ILD for BM at C

We have, Max BM at any section

$= {w * x(L-x)}/{2}$

The value of which will be maximum when x=L/2, i.e. at mid-span.



ILD for BM at C

∴ Absolute maximum BM

$= {w * \frac{L}{2}(L-\frac{L}{2})}/{2} = \frac{wL^2}{8}$

And, it occurs when the UDL occupies the entire span of the beam.

∴ Absolute maximum BM = $\frac{wL^2}{8}$

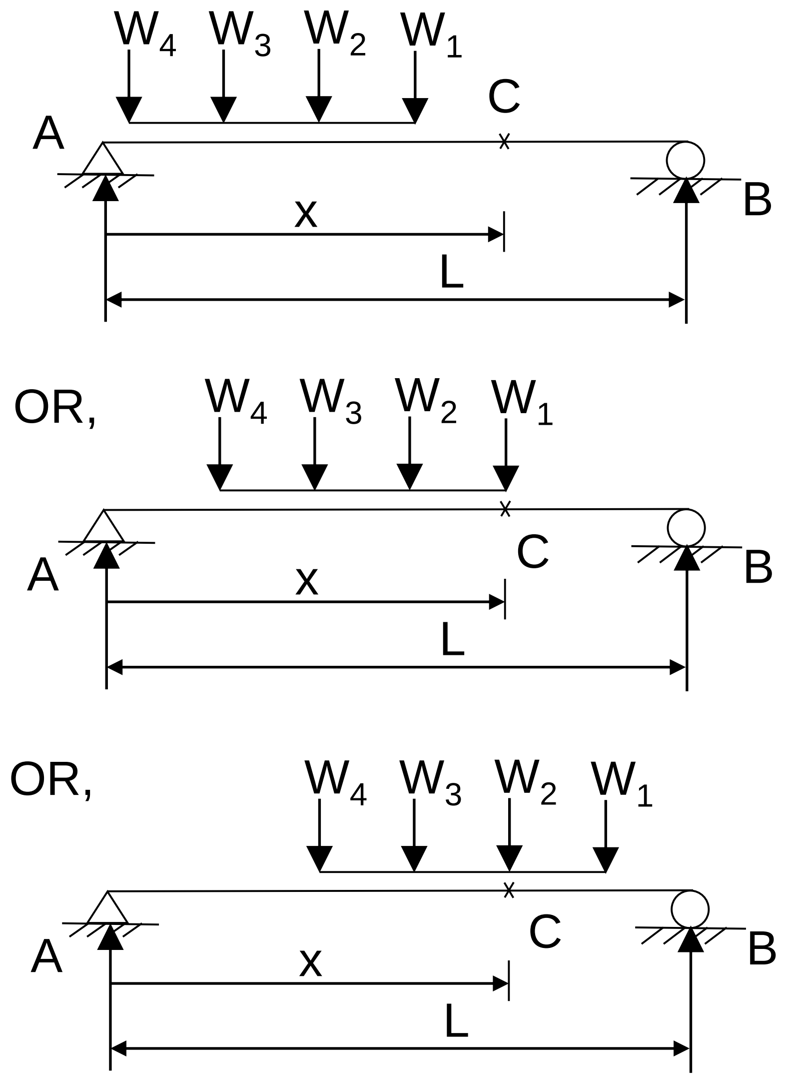
Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span

[C] A train of point loads
(i) Maximum SF at a section

The maximum negative or positive SF at a given section occurs when one of the loads is at the section itself, as shown in the adjacent figures.

From *trial and error*, we can determine which load should be placed over the section to get the maximum value.



Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

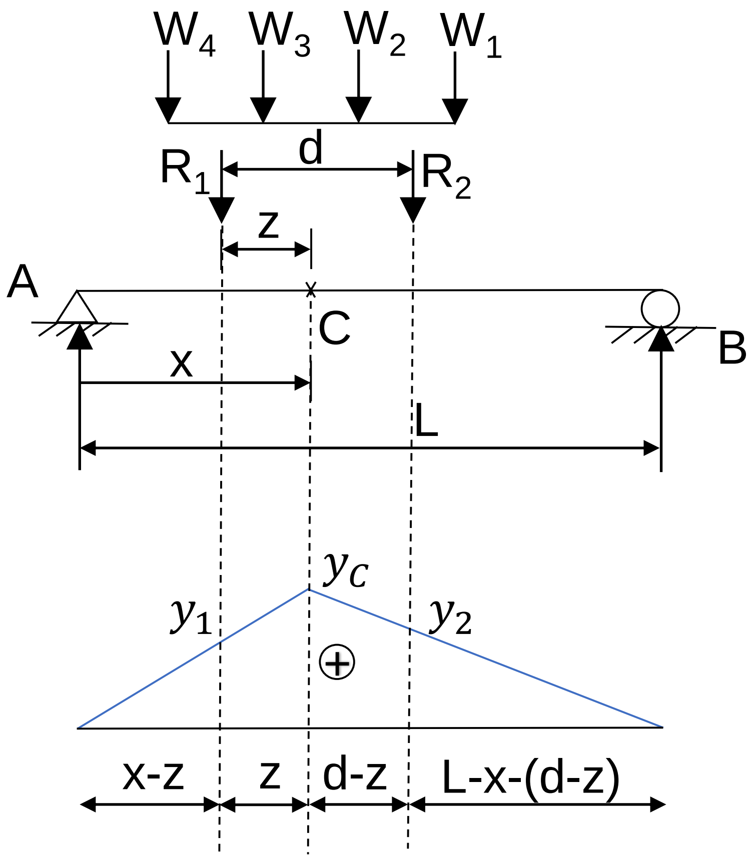
5.10 Most critical position of a load on a beam span

[C] A train of point loads
(ii) Maximum BM at a section

Let R_1 and R_2 be the resultant of loads on the left and the right of the section C, respectively.

Let the distance between R_1 and R_2 be d, and the distance of R_1 from section C be z.

Let the ordinate of ILD for moment at C be y_1 under R_1 and y_2 under R_2, and the maximum ordinate at C be y_c.

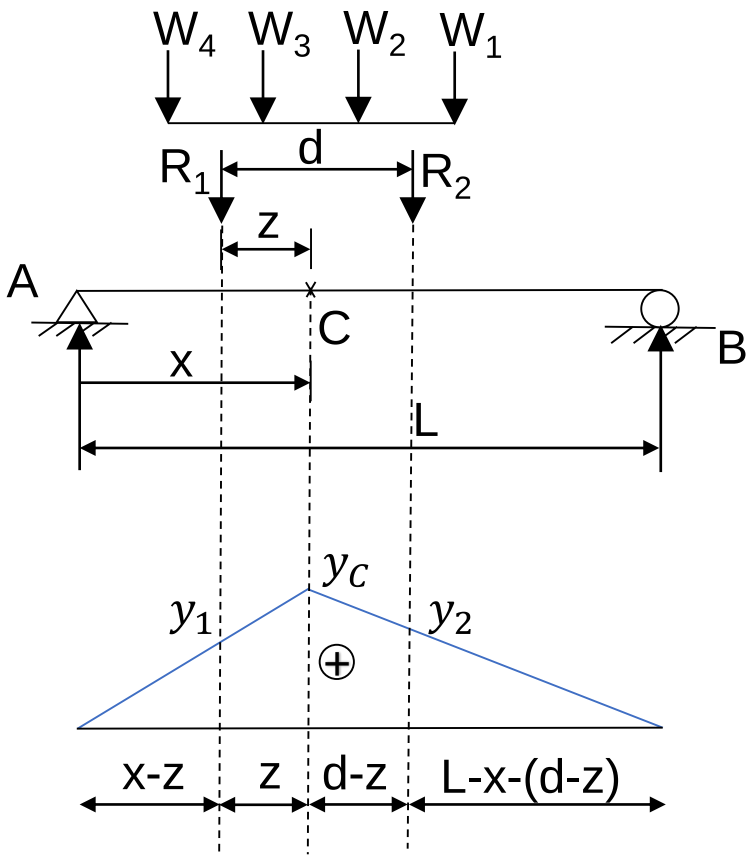


ILD for BM at C, M_C

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span

[C] A train of point loads
(ii) Maximum BM at a section



ILD for BM at C, M_C

Then, using similar triangles,

$$\frac{y_C}{x} = \frac{y_1}{x-z} \qquad Or,\ y_1 = \frac{x-z}{x} * y_C$$

And,

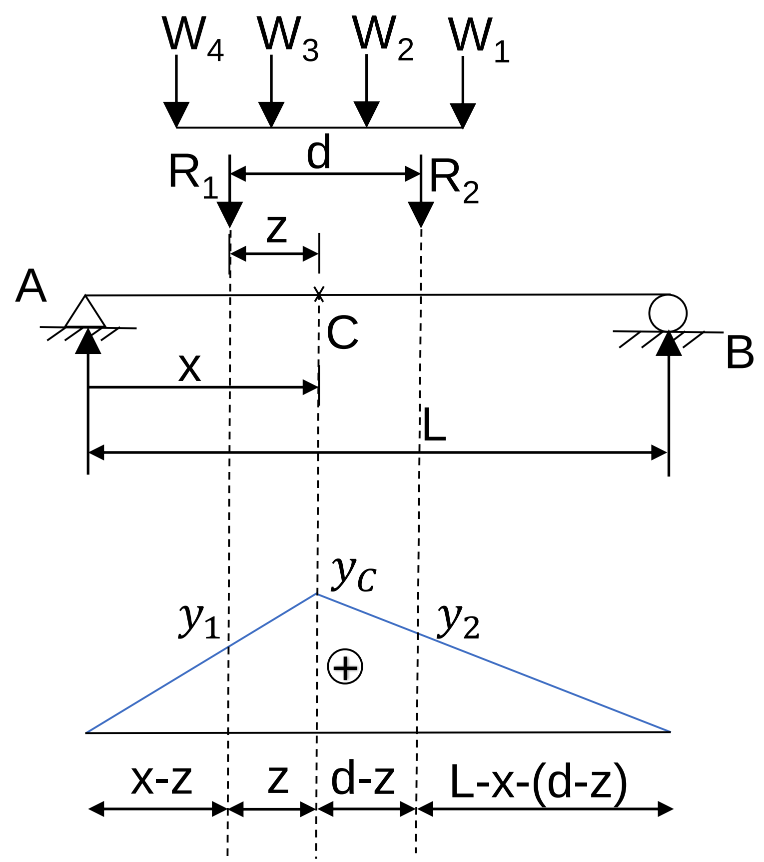
$$\frac{y_C}{L-x} = \frac{y_2}{L-x-d+z}$$

$$Or,\ y_2 = \frac{L-x-d+z}{L-x} * y_C$$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

5.10 Most critical position of a load on a beam span

[C] A train of point loads
(ii) Maximum BM at a section



ILD for BM at C, M_C

$$\therefore M_C = R_1 y_1 + R_2 y_2$$

$$= R_1 * \frac{x-z}{x} * y_C + R_2 * \frac{L-x-d+z}{L-x} * y_C$$

For M_C to be maximum,

$$\frac{dM_c}{dz} = 0$$

$$Or, R_1 y_C \left(-\frac{1}{x}\right) + R_2 y_C \left(\frac{1}{L-x}\right) = 0$$

$$Or, \left(\frac{R_1 y_C}{x}\right) = \left(\frac{R_2 y_C}{L-x}\right)$$

Source: Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.

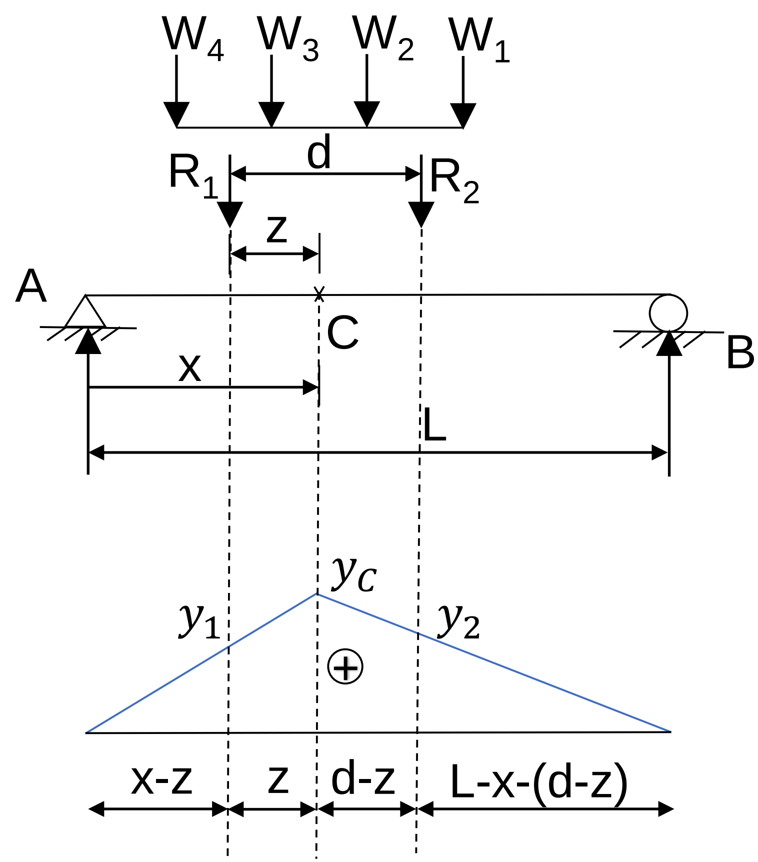
5.10 Most critical position of a load on a beam span

[C] A train of point loads
(ii) Maximum BM at a section

$$Or, \left(\frac{R_1 y_C}{x}\right) = \left(\frac{R_2 y_C}{L - x}\right)$$

$$Or, \left(\frac{R_1}{x}\right) = \left(\frac{R_2}{L - x}\right)$$

Hence, the average load on the left-side portion of the beam is the same as the average load on the right-side portion of the beam.



ILD for BM at C, M_C

------------End of Lecture#9----------------

------------End of Part III of III for Chapter5-------------

References

[1] Bhavikatti, S. S. (2011). *Structural Analysis –I* (4th ed.).New Delhi: Vikas Publishing House.